1 A Standard RBC model with Multiple Shocks

Consider the following representative agent's problem

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \prod_{i=0}^{t} \beta_i \left[\log c_t + \psi_t \log(1-h_t)\right]$$

$$\begin{aligned}
k_{t+1} + c_t &= (1-\delta_t) k_t + A_t k_t^{\alpha} h_t^{1-\alpha}, \forall t \\
c_t &\geq 0, h_t \in [0,1], k_0 \text{ given} \\
\log A_t/\overline{A} &= \rho^A \log A_{t-1}/\overline{A} + \sigma_A \varepsilon_t^A \\
\log \beta_t/\overline{\beta} &= \rho^B \log \beta_{t-1}/\overline{\beta} + \sigma_B \varepsilon_t^B \\
\log \delta_t/\overline{\delta} &= \rho^D \log \delta_{t-1}/\overline{\delta} + \sigma_D \varepsilon_t^D \\
\log \psi_t/\overline{\psi} &= \rho^{\psi} \log \psi_{t-1}/\overline{\psi} + \sigma_{\psi} \varepsilon_t^{\psi}
\end{aligned}$$

where β_i is discounter factor at period i, $\beta_0 = 1$. ψ_t is the utility for labor; δ_t is the depreciation rate; A_t is technology. All these four variables are time-varying and follow a stochastic process. $\varepsilon_t^A, \varepsilon_t^B, \varepsilon_t^D, \varepsilon_t^{\psi}$ follows standard normal distribution and are all realized at the beginning of period t.

Altogether, we have eight "endogenous" variables $\{c_t, k_{t+1}, h_t, r_t, A_t, \beta_t, \delta_t, \psi_t\}$, and eight equations

$$\begin{aligned}
\frac{1}{c_t} &= \beta_{t+1} (1 + r_{t+1} - \delta_{t+1}) \\
k_{t+1} + c_t &= (1-\delta_t) k_t + A_t k_t^{\alpha} h_t^{1-\alpha} \\
\frac{\psi_t}{1-h_t} &= \frac{(1-\alpha) A_t (k_t/h_t)^{\alpha}}{c_t} \\
r_t &= \alpha A_t (k_t/h_t)^{\alpha-1} \\
\log A_t/\overline{A} &= \rho^A \log A_{t-1}/\overline{A} + \sigma_A \varepsilon_t^A \\
\log \beta_t/\overline{\beta} &= \rho^B \log \beta_{t-1}/\overline{\beta} + \sigma_B \varepsilon_t^B \\
\log \delta_t/\overline{\delta} &= \rho^D \log \delta_{t-1}/\overline{\delta} + \sigma_D \varepsilon_t^D \\
\log \psi_t/\overline{\psi} &= \rho^{\psi} \log \psi_{t-1}/\overline{\psi} + \sigma_{\psi} \varepsilon_t^{\psi}
\end{aligned}$$

where the first three equations are first-order conditions associated with the above problem, the fourth is the definition of r_t, and the remaining are the stochastic process.