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Math H.W

1.1 Assume a simple model that predicts income
$\hat{income} = b_0$, with n datapoints
and Loss function $L(b_0) = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 + b_0^2$

In the above question, we consider y as income

$\therefore \quad L(b_0) = \sum_{i=1}^{n} (\hat{income}_i - income_i)^2 + b_0^2$

Replacing $\hat{income}$ with b_0, we get

$L(b_0) = \sum_{i=1}^{n} (b_0 - income_i)^2 + b_0^2$

For finding minimum of function : $\frac{df(x)}{dx} = 0$

$\therefore \quad \frac{dL(b_0)}{db_0} = 0$

$\frac{dL(b_0)}{db_0} = \frac{d}{db_0} \sum_{i=1}^{n} (b_0 - income_i)^2 + \frac{d(b_0^2)}{db_0} = 0$

$\therefore \sum_{i=1}^{n} \frac{d}{db_0} (b_0 - income_i)^2 + \frac{d(b_0^2)}{db_0} = 0$

Using formula $\frac{d(x^n)}{dx} = nx^{n-1}$

$\sum_{i=1}^{n} 2(b_0 - income_i) + 2b_0 = 0$

Expanding the summation function & removing constant 2

$\sum_{i=1}^{n} b_0 - \sum_{i=1}^{n} income_i + b_0 = 0$

$$\therefore \; n b_0 - \sum_{i=1}^{n} income_i + b_0 = 0$$

$$\therefore \; \sum_{i=1}^{n} income_i = n b_0 + b_0$$

$$\therefore \; \sum_{i=1}^{n} income_i = (n+1) b_0$$

$$\boxed{\therefore \; b_0 = \frac{1}{(n+1)} \sum_{i=1}^{n} income_i}$$

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1.2 Loss function: $L(b_0) = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 + b_0^2$

optimal value for $b_0 = \frac{1}{(n+1)} \sum_{i=1}^{n} income_i$ — ①

training data: income = $\{20000, 35000, 40000\}$ — ②
Here, n (number of datapoints) = 3 — ③

Substituting 2 & 3 in 1, we get

$b_0 = \frac{1}{(3+1)} (income_1 + income_2 + income_3)$

$= \frac{1}{4} (20000 + 35000 + 40000)$

$= \frac{1}{4} (95000)$

$\boxed{\therefore b_0 = 23750}$

Ans Based on the above training data, the optimal prediction of an unseen income given the model and the loss function = 23,750