

Developing FOB-CTMQC for use in simulating charge transport in organic semiconductors

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I, Matt Ellis, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

Abstract

Charge transfer in organic molecular systems are difficult to simulate due to (fast?) non-adiabatic transitions between Born-Oppenheimer energy surfaces. A range of techniques have been designed to overcome this, such as the Surface Hopping and Ehrenfest methods. However, they tend to suffer from unphysical overcoherence issues. In this report, I present an implementation of a fragment-orbital based coupled-trajectory mixed quantum-classical algorithm (FOB-CTMQC), which is designed for simulating charge transport in systems of tens to hundreds of organic molecules. This method uses an in-house analytical overlap method (AOM) within the framework of coupled-trajectory mixed quantum-classical (CTMQC) molecular dynamics. CTMQC enables the correct calculation of decoherence due to 2 new terms containing a quantity named the Quantum Momentum. While the AOM method allows the simulation of large systems, due to the method's analytic formulation of the off-diagonal elements of the Hamiltonian in terms of the overlap between singly occupied molecular orbitals. E.g. $H_{kl} = CS_{kl}$, where H_{kl} are the off-diagonal elements of the Hamiltonian, C is a constant of proportion and S_{kl} are the off-diagonal overlap elements. For many organic semiconductors we can assume pi-conjugation. Meaning only 1 optimized Slater p-orbital is needed for the calculation of the overlap.

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Chapter 1

Introduction

1.1 Charge Transport Regimes in Organic Semiconductors

1.1.1 Organic Semiconductors

Conductive polymers were first discovered in 1977 by Shirakawa et al [1, 2] for which they were awarded the Nobel prize in Chemistry. Recently these materials have become ubiquitous in many technologies, such as in organic solar cells[3], organic field-effect transistors (OFET) [4] and organic light-emitting diodes (OLED) [5]. While the other two technologies lag behind their inorganic counterparts, uptake of OLED screens is becoming increasingly popular especially in the smartphone and television market due to their flexibility, better colour representation and lower energy consumption than conventional backlit LCD displays. In fact IHS market's OLED market tracker predicts OLED to be the dominant technology in smartphone screens by 2020 [6] ¹. OLEDs have also found uses in lighting with their efficiency rivalling that of fluorescent tubes [7, 8]. Although, industry has made large strides in fabricating and using these materials the exact nature of the charge transport is still poorly understood. Conventional hopping and band theories break down in the regime of partial delocalisation of the charge carriers and atomistic simulations are required for a realistic picture.

Typically charge carrier mobilities in 'good' organic semiconductors (OSCs) fall

¹This is an online citation -Not sure whether I have cited correctly

between $1\text{--}10\text{ cm}^2\text{ V}^{-1}\text{s}^{-1}$ [9]. This is just beyond the range of hopping model validity ($\sim 1\text{ cm}^2\text{ V}^{-1}\text{s}^{-1}$) and below that of band theory ($> 50\text{ cm}^2\text{ V}^{-1}\text{s}^{-1}$). In this intermediate regime the charge carriers are typically not completely delocalised at the valence band edges (band regime) or localised to a single site/molecule (hopping regime) but delocalised over a few molecules. Without current analytic approaches being valid in this regime many computational approaches have been developed to investigate the underlying charge transport mechanisms [10].

1.1.2 Band-like Transport

Band theory paragraph...

1.1.3 Hopping-like Transport

Need way more citations here -see Gajdos AOM paper it has lots of good references

Hopping theories assume the charge carrier is localised on one site and can hop from site to site in a series of discrete hops [10]. There are various underlying mechanisms for this. For example, the presence of the charge carrier at a site can alter the nuclear geometry. The distorted nuclear geometry can make it harder for the charge carrier to move onto the next site, creating a metastable state and trapping the charge carrier. The deformation in the nuclear geometry is called a small polaron.

Polaronic hopping theories have been used to relative success and one of the key tools used in visualising this process (assuming harmonic response) are the Marcus Parabolas. These show how the free energy and reaction coordinates change after a charge transfer i.e. from initial to final diabatic states. The term ‘diabatic state’

isn't well defined and can refer to different things in different formulations. In this work a diabatic state can be imagined as the charge carrier localised on a single molecule. This is discussed later in more detail in 3.0.1

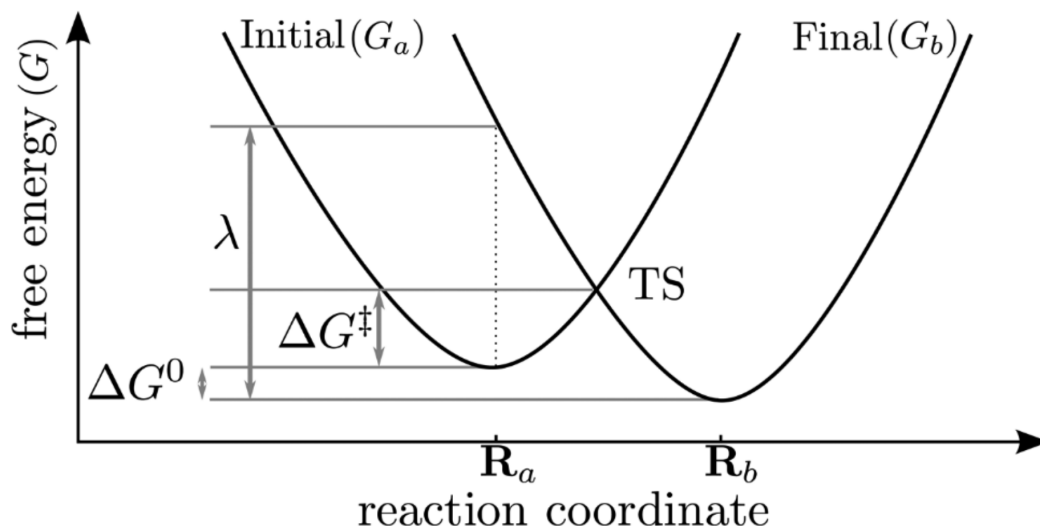


Figure 1.1: Marcus parabolas depicting the relationship between the free energy in the system and the reaction coordinate at 0 electronic coupling. This figure was taken from Oberhofer et al [10]

Figure 1.1 above defines various important quantities for calculating the mobility in materials displaying hopping-like transport. The initial parabola describes the change in free energy with respect to the reaction coordinate for the initial state, for example when the charge is located on site 1. The final parabola describes the change in free energy when the charge is located on site 2 e.g. when the charge has relocated to site 2. The transition state (TS) is a point where the energies of the initial and final states are the same. This degenerate point is the only point at which the charge can move from the initial to final state as other points would result in a non-zero jump in energy between the 2 parabolas. The diabatic activation energy ΔG^\ddagger defines the energy required to get to this transition state from the minima of the initial parabola. The driving force ΔG^0 is the difference in minima of the 2 parabolas, the reorganisation energy λ defines the energy required to change the

reaction coordinate from the final state minima to the initial state minima without changing electronic state.

Figure 1.1 can change when there is a non-zero electronic coupling (H_{ab}) between diabatic states. This parameter increases the chance of moving between the initial and final diabatic states by lowering the diabatic activation energy i.e. the energy required to transition from state 1 to 2. This is visualised in figure 1.2 In figure 1.2

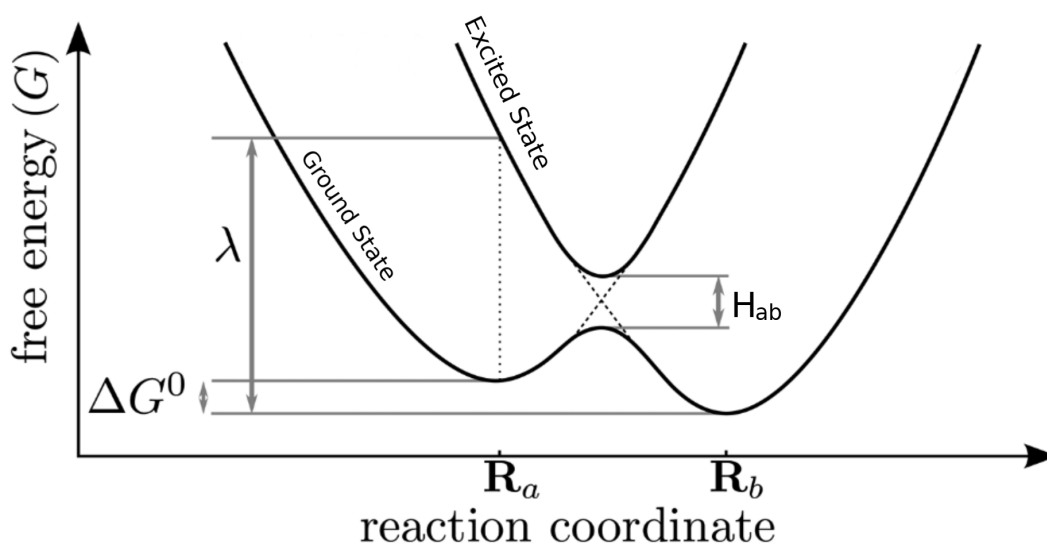


Figure 1.2: Graph depicting the change in free energy for a change in reaction coordinate for non-zero electronic coupling. Adapted from [10]

above the diabatic activation energy has been lowered to $\Delta G^\ddagger = \Delta G^\ddagger - H_{ab}$ making it easier for charge carriers to transition between diabatic states. In our formalism this means it is easier for charge carriers to move between sites i.e. delocalise. We see also that instead of being described totally by 2 parabolas there are 2 new adiabatic potential energy surfaces arising -the ground and excited state. The amount that these new potential energy surfaces diverge from the diabatic wells is dependent on an adiabaticity factor which is proportional to the ratio between the electronic coupling, H_{ab} , and the re-organisation energy, λ . This has been discussed in detail in multiple papers [10–13] .

In fact for systems with couplings larger than $H_{ab} > \frac{3}{8}\lambda$ the diabatic activation

energy vanishes completely [13], meaning that there is no energy cost in transitioning between diabatic states. Beyond this regime hopping theories cannot be accurately applied. Unfortunately, at room temperatures thermal fluctuations means the mean free path of the charge carriers is comparable to the intermolecular spacing. As such band theories too are inapplicable [10, 14, 15]. Much beyond this regime the energy cost to transition to higher adiabatic potential energy surfaces becomes prohibitively high and the system travels on a single state. In these situations the Born-Oppenheimer approximation is valid. However, in this work I will be looking into the regime in between the band and hopping-like transport where we currently don't have analytical theories to describe charge transport. For this I will be using non-adiabatic atomistic simulations, namely a technique called CTMQC.

1.2 Atomistic Simulations of Nonadiabatic Processes

In simulating processes involving electronic transfers a key approximation used in conventional molecular dynamics (MD) breaks down. That is the Born-Oppenheimer or adiabatic approximation [16]. This approximation relies on the fact that nuclei are more massive than electrons and are approximately stationary with respect to electron movement (need ref). This results in nuclear evolution that is governed by a single, adiabatic, potential energy surface. However, in many interesting processes, such as electron transfer, non-radiative decay and photochemical processes, electronic transitions between adiabatic potential energy surfaces occur (need ref). Simulating these processes requires non-adiabatic molecular dynamics (NAMD) techniques to be developed to correctly capture dynamical properties.

There have been many techniques proposed for use in NAMD such as the quantum classical Liouville equation (need ref), multiple spawning (need ref) or nonadiabatic Bohmian dynamics (need ref)². However, two of the most popular are trajectory

²See first Frederica paper

surface hopping (need ref) and mean-field approaches (need ref). This is probably due to their relative simplicity to implement (need ref), efficiency for large systems (need ref) and proven efficacy in a wide variety of situations (need ref). In all of these approaches the general aim is to treat as much of the system as possible with (computationally cheaper) classical mechanics. While handling all necessary parts with quantum mechanics [17]. In Surface Hopping, Ehrenfest and CTMQC one treats the nuclear subsystem classically and the electronic one quantum mechanically. The nuclei are propagated using a velocity verlet algorithm according to Newton's laws. The electrons are propagated using a fourth order Runge Kutta algorithm according to the time-dependent Schrödinger equation. This is normally expanded as a linear combination of adiabatic or diabatic states. The nuclei and electrons can also interact. Taking account of this interaction is where these different atomistic simulation techniques differ.

1.2.1 Surface Hopping and Ehrenfest Dynamics

How Surface Hopping and Ehrenfest alter the potential energy surface to affect the nuclear dynamics -Introduce the terms non-adiabatic coupling vectors (NACV) and NACEs also mention that what trajectory based methods are.

- Brief introduction to NAMD.
 - ~~Why isn't normal MD good enough?~~
 - ~~What sort of processes need NAMD to simulate them?~~
 - Characteristics of NAMD -Trivial Crossings, avoided crossings.
 - Give current standard methods and their advantages/disadvantages
- Briefly explain the AOM method.
 - How it works - The analytic off-diagonal elements of hamiltonian (Spencer -both)
 - What the alternatives are (Gajdos paper)
 - Why this is preferable (Gajdos)
 - What are the limitations
- Explain CTMQC
 - Intro -It was derived from exact factorisation (maybe put derivation in appendix)
 - Give the equations. -Ehrenfest + QM term
 - How does it compare to other techniques -decoherence, first principles
 - Why combine it with FOB-CTMQC? -Expensive DFT etc...
 - What sort of systems will I be looking at? Charge transfer in organic semiconductors -1D and 2D systems

Then Results

Chapter 2

CTMQC

2.1 Exact Factorisation

CTMQC comes from taking the semi-classical limit of an exact factorisation of the molecular wavefunction into its constituent electronic and nuclear components [18]. Where the electronic component is parametrically dependent on the nuclear coordinates, \mathbf{R} . This is shown below in eq (2.1) where χ is the nuclear wavefunction and Φ is the electronic one.

$$\Psi(\mathbf{R}, \mathbf{r}, t) = \Phi_{\mathbf{R}}(\mathbf{r}, t) \chi(\mathbf{R}, t) \quad (2.1)$$

In the above equation (and throughout this report) I will denote nuclear coordinates and electronic coordinates R and r respectively. The nuclear and electronic wavefunctions then obey separate, but coupled, time-dependent schrödinger equations for spatial and temporal evolution. This representation has proven to be useful in furthering understanding through exact solutions of small toy-model systems (need ref ¹). However, in this report I will be focussing on the semi-classical limit of these equations (CTMQC) and give some early results of a combination of this and the AOM method explained previously in section 3.0.1. The equations for the evolution of the electronic and nuclear wavefunctions in the exact factorisation

¹see deconstruction paper

[18] are given below:

$$\hbar \frac{\delta}{\delta t} \Phi_{\mathbf{R}}(\mathbf{r}, t) = (\hat{H}_{BO} + \hat{U}_{en}[\Phi_{\mathbf{R}}, \chi] - \varepsilon(\mathbf{R}, t)) \Phi_{\mathbf{R}}(\mathbf{r}, t) \quad (2.2)$$

$$\hbar \frac{\delta}{\delta t} \chi(\mathbf{R}, t) = \left(\sum_{v=1}^{N_n} \frac{[-\hbar \nabla_v + \mathbf{A}_v(\mathbf{R}, t)]^2}{2M_v} + \varepsilon(\mathbf{R}, t) \right) \chi(\mathbf{R}, t) \quad (2.3)$$

Where \hat{H}_{BO} is the Born-Oppenheimer Hamiltonian, that is $\hat{T}_e + \hat{W}_{ee} + \hat{W}_{nn} + \hat{V}_{en}$. Where \hat{T}_e is the electronic kinetic energy operator, $\hat{W}_{ee/nn}$ is the electron-electron/nuclei-nuclei interaction and V_{en} is the electronic-nuclear potential.

The \hat{U}_{en} is an electronic-nuclear coupling operator (ENCO). This is defined as

$$\hat{U}_{en}[\Phi_{\mathbf{R}}, \chi] = \sum_{v=1}^{N_{nuc}} \frac{1}{M_v} \left[\frac{[-\hbar \nabla_v - \mathbf{A}_v(\mathbf{R}, t)]^2}{2} + \left(\frac{-\hbar \nabla_v \chi}{\chi} + \mathbf{A}_v(\mathbf{R}, t) \right) \left(-\hbar \nabla_v - \mathbf{A}_v(\mathbf{R}, t) \right) \right] \quad (2.4)$$

Where the \mathbf{A}_v is a time-dependent vector potential (TDVP), given by $\langle \Phi_{\mathbf{R}}(t) | -\hbar \nabla_v \Phi_{\mathbf{R}} \rangle_{\mathbf{r}}$ and M_v is the mass of nuclei v . Finally $\varepsilon(\mathbf{R}, t)$ is a time-dependent scalar potential energy surface (TDPES), given by $\langle \Phi_{\mathbf{R}}(t) | \hat{H}_{BO} + \hat{U}_{en}^{coup} - \hbar \frac{\delta}{\delta t} | \Phi_{\mathbf{R}}(t) \rangle_{\mathbf{r}}$.

The effects of the TDPES, TDVP and the ENCO have been investigated in multiple works [19–23]. The TDPES and TDVP are both responsible for the evolution of the system [20]. The TDPES provides exact classical forces on the nuclei. In fact, an alternative independent-trajectory semi-classical scheme has been investigated using these exact forces [19]. This found the TDPES is responsible for the splitting of the nuclear wavepacket in regions of high non-adiabaticity by taking the shape

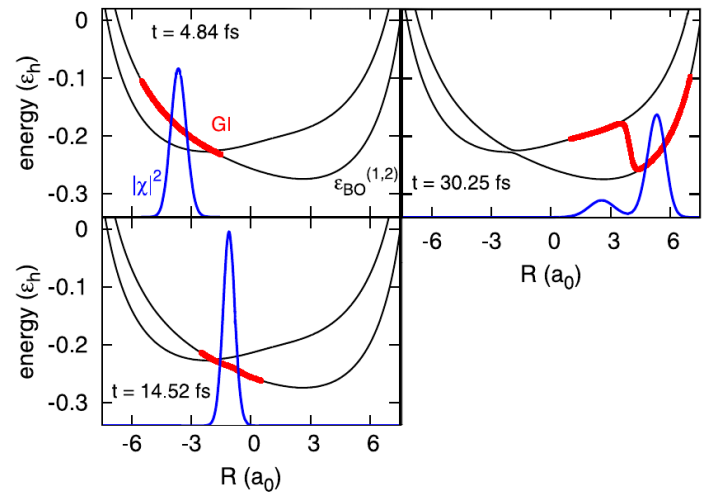


Figure 2.1: A demonstration of how the TDPES can cause the splitting of the nuclear wavepacket in non-adiabatic regions. The red line represents the TDPES and the blue is the nuclear density. Adapted from [19]

of a step function between the 2 adiabatic potentials. This is demonstrated in figure

2.1. Finally the electronic-nuclear coupling

operator (ENCO) is responsible for the non-adiabatic effects in the system such as electronic nonadiabtic transitions and decoherence [20].

2.2 Approximations in CTMQC

Six approximations have been made in the derivation of CTMQC, these are discussed in detail in Ref. [24]. In the interest of completeness I have summarised them below.

2.2.1 Classical Nuclei

Techniques that include nuclear quantum effects (NQE); such as multiple spawning [25], ring-polymer surface hopping [26] and nonadiabatic Bohmian dynamics [27, 28] although extremely accurate, cannot be applied to hundreds or thousands of molecules. This is due to their high computational cost. Further, in many systems of interest NQEs are negligible, especially at room temperature. For this reason the classical limit of the nuclear Schrödinger equation (2.3) is taken when deriving the CTMQC equations.

2.2.2 Neglect the ENCO in the TDPES

The electron-nuclei coupling operator is omitted in the expression for the time-dependent potential energy surface. This is justified as the first term $([-\hbar\nabla_v - \mathbf{A}_v(\mathbf{R},t)]^2)$ contains a second order derivative which is expensive to calculate and has a negligible effect compared to the second term in the ENCO [29]. However, the rest of the ENCO is equal to zero when averaged over $\Phi_{\mathbf{R}}(\mathbf{r},t)$ so it does not contribute to the TDPES.

2.2.3 Derivative of the Adiabatic Coefficients

The derivative of the adiabatic coefficients appears in the electronic evolution equations. However, we can re-write the derivative of the adiabatic coefficients

in terms of their modulus and phase:

$$\nabla_v C_l^{(I)}(t) = \left[\underbrace{\frac{\nabla_v |C_l^{(I)}(t)|}{|C_l^{(I)}(t)|}}_{\text{(Term 1)}} + \underbrace{\frac{i}{\hbar} \nabla_v \gamma_l^{(I)}(t)}_{\text{(Term 2)}} \right] C_l^{(I)}(t) \quad (2.5)$$

It has been found that the first term is negligible compared to the second [19, 21, 22] so it doesn't need to be calculated and we can remove it. It was also assumed that the NACVs are localised in space meaning that, after some algebra, the spatial derivative of the adiabatic coefficient can be written as:

$$\nabla_v C_l^{(I)}(t) = \frac{i}{\hbar} \nabla_v \gamma_l^{(I)}(t) C_l^{(I)}(t) = -\frac{i}{\hbar} \int^t dt' \nabla_v \epsilon_l^{(I)} C_l^{(I)}(t) = -\frac{i}{\hbar} \mathbf{f}_l^{(I)} C_l^{(I)}(t) \quad (2.6)$$

Where $\epsilon_l^{(I)}$ is the energy of the l^{th} adiabatic potential energy surface for trajectory I, $C_l^{(I)}$ is the adiabatic expansion coefficient for state l and trajectory I. The $\mathbf{f}_l^{(I)}$ is the time-integrated adiabatic force.

2.2.4 Gaussians as Nuclear Wavepackets

In order to calculate the quantum momentum -the new term in CTMQC. Knowledge of the nuclear distribution is needed. To this end the nuclear wavepacket is assumed to take the shape of a Gaussian. This is centred on the atomic coordinate with a width σ . In this work I have used a constant width throughout, with plans to implement dynamic Gaussian width calculations later. However, the nuclei are still propagated classically, the width parameter is only used in the calculation of the quantum momentum.

2.2.5 Separating the Effects of Decoherence and NACVs

So as to not introduce any population transfer (due to the quantum momentum) when the NACV is zero a fifth approximation has been introduced. Namely the quantum momentum depends on pairs of states -l,k. This enables the separation of the 'competing' effects of the NACV and the Quantum Momentum.

2.3 The CTMQC equations

Chapter 3

FOB Formalism

3.0.1 FOB Formalism

The effect of the nuclei on the electrons is normally handled via the Hamiltonian. This is dependent on nuclear positions and is in turn used in the Schrödinger equation to propagate the electron dynamics. Often the construction of the Hamiltonian is carried out using density function theory (DFT). However, for large, dynamic systems this becomes too computationally expensive and a different technique would have to be used. In this work I will rely on an Analytical Overlap Method (AOM) [14] to calculate the off-diagonal elements of the Hamiltonian and the diagonal elements will be calculated via a forcefield.

3.0.1.1 AOM

AOM assumes that the off-diagonal elements of the Hamiltonian are proportional to the off-diagonal elements of the overlap matrix between 2 singly occupied molecular orbitals (SOMO). For π -conjugated systems,

Chapter 4

General Conclusions

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Appendix A

Derivations

A.1 Preservation of the Norm

Appendix B

Another Appendix About Things

(things)

Appendix C

Colophon

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