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Depth Profiles of Electrical Conductivity from Linear Combinations of Electromagnetic Induction Measurements

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ABSTRACT

Within the past 10 yr, frequency-domain electromagnetic (FEM) induction techniques have become more and more widely used in soil science and hydrology. Each measurement of apparent electrical conductivity represents a depth-weighted average of soil electrical conductivity. However, the depth weighting corresponding to each measurement may be very different from that required by the user. The simultaneous use of multiple measurements (with different depth weightings) should allow some aspects of the depth distribution of electrical conductivity to be inferred. We illustrate a method for obtaining linear combinations of FEM measurements to estimate the soil electrical conductivity within the depth interval of interest. The method relies on the simple fact that the measurement-system response is linear, so that a linear combination of apparent-conductivity readings corresponds to a linear combination of response functions. One can seek a linear combination of response functions that has desirable characteristics. This sets up an optimization problem that can be solved by standard methods, which avoids difficulties encountered with layered inversions to resolve nonlayered systems. The approach was applied to GEONICS EM31, EM34, and EM38 instruments in three examples: (i) single frequency measurements at vertical and horizontal dipole configurations, (ii) frequency measurements using the EM34 at all six possible configurations, and (iii) frequency measurements using the EM38 held at varying heights above the ground. The derived linear combinations were applied to field data from southern Australia. Soil conductivity profiles predicted using linear combinations showed good agreement with profiles measured with a conductivity probe.

THE USE of aboveground electromagnetic induction techniques to infer subsurface properties has become very popular in soil science within the past 10 yr. This has arisen due to the need to interpolate between point estimates, which are often costly to obtain, and to overcome the problems of spatial variability. In particular, electromagnetic methods are now routinely used in soil salinity and groundwater pollution studies (e.g., de Jong et al., 1979; Sweeney, 1984).

The electrical conductivity of a soil is a function of its water content and salt content, as well as the soil structure, texture, and mineralogy (e.g., Nadler and Frenkel, 1980). Electromagnetic methods measure a depth-weighted average of the soil electrical conductivity. This weighted average is termed the *apparent electrical conductivity* (EC_a). Often, the user requires more precise depth information on conductivity than can be obtained with any single measurement. In principle, the simultaneous use of multiple measurements, with different depth weightings, would allow better information on the depth distribution of electrical conductivity to be inferred.

Most often, the interpretation of electromagnetic data is done by mathematical inversions using layered-earth models (e.g., Raiche et al., 1985). These methods were originally designed for interpreting geological systems with distinct layering, however, and are not necessarily the most appropriate for soil science applications. Where subsurface layers are not sharply defined, this type of inversion is subject to considerable error.

Several authors (Rhoades and Corwin, 1981; Corwin and Rhoades, 1984, 1990; Slavich, 1990) have used multiple regressions of field data on instrument readings to obtain linear combinations of measurements that are sensitive to the depth intervals of interest. In principle, the resulting linear combinations are empirical and site specific. This means that much work is needed to transfer the results to other sites. Corwin and Rhoades (1982), Slavich (1987), and McNeill (1985) developed more general linear combinations, based on a knowledge of the individual instruments' depth-weighting functions, although in the former cases the approach was based on assumptions of constant electrical conductivities within certain depths. The site specificity of some previous studies is indicated by the fact that different linear combinations have been obtained for profiles that have increasing conductivity with depth, compared with profiles with decreasing conductivity with depth (Slavich, 1990). We provide an approach that is mathematically rigorous, to obtain linear combinations that are not site specific and do not assume a layered earth. The only requirement is that the depth-response functions for the instruments used (which define the depth of current penetration) are known and are independent of

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subsurface conductivity. This requirement is satisfied by a range of GEONICS frequency-domain electromagnetic induction instruments when used at low induction numbers (Wait, 1962; McNeill, 1980).

THEORY

The theory of FEM induction has been described by McNeill (1980). Briefly, an electromagnetic transmitter coil is placed on the ground surface and energized with an alternating current at an audio frequency. A primary electromagnetic wave is transmitted at the soil surface, which induces a secondary field in subsurface materials. The strength of the secondary field diminishes with depth. A receiving coil, placed a fixed distance from the transmitter, measures both the primary and secondary fields. (Coils may be oriented with either horizontal or vertical dipoles.) The ratio of the primary and secondary electromagnetic fields provides a measure of the EC_a of the soil.

The FEM instruments most commonly used in soil science are those manufactured by GEONICS (McNeill, 1986). The methods we describe relate specifically to these instruments, but they may be generalized to other manufacturers' models. The GEONICS FEM instruments are designed to be operated at a limited range of frequencies and intercoil spacings. They can be used with both dipoles oriented horizontally (HD) or both vertically (VD). The EM38 conductivity meter operates at a frequency of 13 200 Hz, and a fixed intercoil spacing of 1 m. Similarly, the EM31 operates at a frequency of 9800 Hz and a coil spacing of 3.7 m. The EM34 operates at either 6400, 1600, or 400 Hz, with intercoil spacings of 10, 20, or 40 m respectively.

The EC_a measured with each of these instruments is a depth-weighted conductivity, which can be expressed

$$EC_a = \int_0^\infty \phi(z)\, EC(z)\, dz \qquad [1]$$

where EC(z) is the electrical conductivity of the soil as a function of depth, and $\phi(z)$ is a weighting function, termed the *depth-response function* for the instrument. Under certain conditions (operation at low induction numbers: Wait, 1962; McNeill, 1980) the function $\phi(z)$ will be independent of the earth conductivity. For the GEONICS instruments, the depth-response function is dependent only on the coil spacing and dipole configuration (Fig. 1). The low-induction-number condition is usually met at low earth conductivities (McNeill, 1980), i.e., in nonsaline areas.

As can be seen from Fig. 1, the depth interval relating to a single measurement may be very broad, and is not sharply defined. However, the user may be interested in estimating the conductivity of the earth for a narrower, and better defined, depth interval. The use of multiple measurements (with different depth weightings) should allow more precise information to be obtained.

Under the low-induction-number approximation, the system is linear. This means that any linear combination of apparent conductivities measured by different instruments will have a response function that is the same linear combination of the response functions of the individual instruments. Thus it may be possible to choose linear combinations of aboveground measurements that have depth-response functions closer to those required by the investigator. For example, the user is often interested in the conductivity of the earth close to the soil surface. The GEONICS depth-reponse functions for HD and VD configurations have asymptotic forms:

$$\phi_H(z) \sim z^{-2}/4 \quad \text{as } z \to \infty, \qquad [2]$$

$$\phi_V(z) \sim z^{-2}/2 \quad \text{as } z \to \infty. \qquad [3]$$

Hence the linear combination $2\phi_H(z) - \phi_V(z)$ asymptotes to zero, and so is a much smaller weighting function than either individual function (Fig. 1). Similarly, McNeill (1985) suggests the linear combination $2\ \phi^{40}_H - \phi^{20}_H$ to reduce sensitivity to near-surface material, where ϕ^{40}_H and ϕ^{20}_H are HD measurements with the EM34 at 40- and 20-m coil spacings, respectively. We can see, therefore, the benefits of sensible linear combinations for obtaining information on the depth distribution of electrical conductivity.

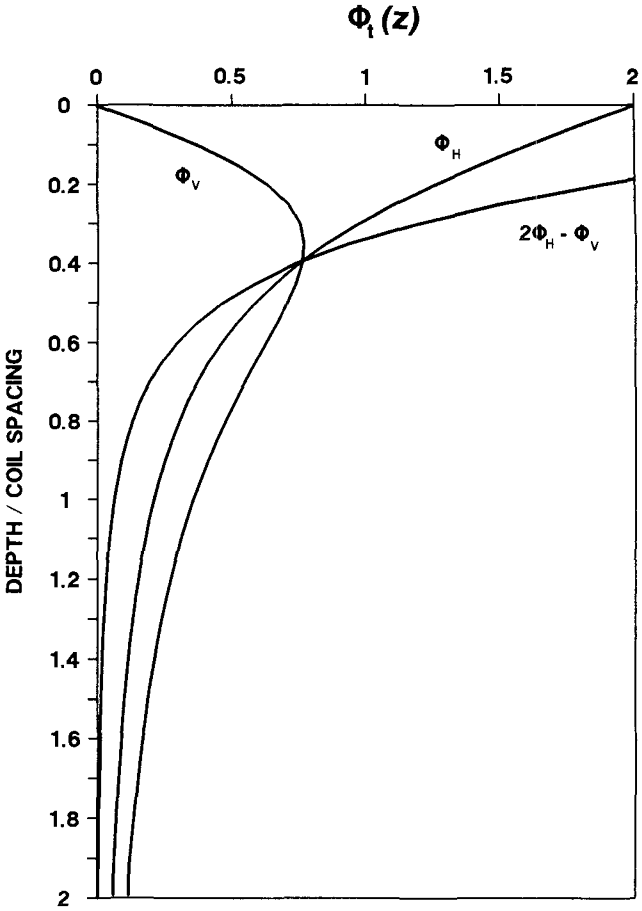

Fig. 1. Comparison of horizontal (ϕ_H) and vertical dipole (ϕ_V) response functions with the linear combination to minimize the deep response ($2\phi_H - \phi_V$).

In order to minimize the errors involved with this method, the coefficients for the linear combinations must remain small. GEONICS quotes an instrument accuracy of 10% at 0.2 dS m^{-1}. Each time an instrument reading is multiplied by a linear coefficient, this error will be multiplied by the same factor. In total, this means that the absolute error of any linear combination will be equal to the error for each individual measurement magnified by the sum of the absolute values of the coefficients.

A special type of linear combination method, which is widely used, involves mathematical inversions to solve for layered-earth models (e.g., McNeill, 1980). Software is commercially available for doing this (e.g., Interpex, 1988). The inversion method leads to each layer conductivity being a linear combination of apparent electrical conductivities. The depth-response function for this linear combination will often have a large component outside the relevant layer. Hence, in order to accurately estimate the conductivity of this layer, the method relies on cancellations of the response attributable to other layers. Where subsurface layers are not sharply defined, however, results of fitting layered models may be very misleading. Also, these methods have no intrinsic constraint to maintain small coefficients. Hence there is a possibility for very large errors to result, particularly if the response functions are highly correlated for the depth interval of interest, or if the response of all instruments to that interval is small.

As mentioned above, several authors have empirically determined linear combinations that appeared to measure the conductivity of the earth between certain depths at their field sites. While, in principle, the empirical fits will be site specific, they may in certain cases be reproducing optimal coefficients, which are not site specific. In saline areas, where the low-induction-number approximation does not hold, empirical methods may be the most practical means of determing these coefficients. In many cases (e.g., Rhoades and Corwin, 1981), however, these empirically determined coefficients were so large that small errors in measuring EC_a with individual instruments would have been greatly magnified. Also, there seems little reason to go to the considerable effort of empirically determining the coefficients for each site to be studied, if there exists a general method for determining them.

In the following, we develop a methodology that retains the advantages of a linear combination, without the site specificity of the empirical approaches, or the instabilities and approximations of inversion methods. Consider the linear combination

$$EC_t = \sum_{i=1}^{n} a_i \, EC_i, \qquad [4]$$

where EC_i is the apparent electrical conductivity measured with instrument configuration i, and a_i is the weighting coefficient for that configuration. Hence EC_t is some weighted average of apparent conductivity measurements (We drop the subscript a to denote apparent electrical conductivity, for simplicity.) This linear combination has the depth-response function

$$\phi_t(z) = \sum_{i=1}^{n} a_i \phi_i(z). \qquad [5]$$

where z is the depth below the soil surface and $\phi_i(z)$ is the depth response function of instrument i (which is assumed independent of conductivity). We normalize the values a_i such that

$$\int_0^\infty \phi_t(z) \, dz = 1. \qquad [6]$$

This constraint determines that, for readings made for an area in which the conductivity is constant with depth, the quantity EC_t will be equal to this conductivity. If the individual depth-response functions ϕ_i are defined such that

$$\int_0^\infty \phi_i(z) \, dz = 1, \qquad [7]$$

(this is satisfied for the GEONICS instruments provided they are placed on the ground surface), then the constraint in Eq. [6] becomes

$$\sum_{i=1}^{n} a_i = 1. \qquad [8]$$

If Eq. [7] is not satisfied, then a more general constraint should be used (see Appendix).

Suppose we are interested in a linear combination of instrument readings that will minimize the response from deep in the soil profile. The relative response from below depth L is equal to

$$\frac{\int_L^\infty \phi_t(z) \, EC(z) \, dz}{\int_0^\infty \phi_t(z) \, EC(z) \, dz}. \qquad [9]$$

Generally, however, we have little knowledge of the conductivity profile, $EC(z)$. Instead, we aim to minimize the magnitude of the function $\phi_t(z)$ below depth L. There are several ways of doing this. Here, we select coefficients a_i to minimize the function

$$S^2 = \int_L^\infty \left[\phi_t(z) \right]^2 dz = \int_L^\infty \left[\sum_{i=1}^{n} a_i \phi_i(z) \right]^2 dz \qquad [10]$$

Similarly, it is possible to minimize the response above a given depth. More generally, we may be interested in estimating the conductivity of some interval between depths U and L ($U < L$). In this case, we want to minimize the function

$$S^2 = \int_0^U \left[\phi_t(z) \right]^2 dz + \int_L^\infty \left[\phi_t(z) \right]^2 dz. \qquad [11]$$

The mathematics for this case is identical to the one-sided case, and so this will not be considered further.

One problem with this approach is that the minimum values of S^2 will commonly occur at very large values of coefficients a_i. The implication of this is that small errors in measured values EC_i will result in very large errors in the estimated conductivity EC_t. This is demonstrated more clearly below. The problem can be avoided, however, by including a damping parameter (λ) in Eq. [10], and minimizing the function

$$S'^2 = \int_L^\infty \left[\sum_{i=1}^{n} a_i \phi_i(z) \right]^2 dz + \lambda \sum_{i=1}^{n} a_i^2 \qquad [12]$$

where $\lambda > 0$. This is similar to the damping parameter used in regularization schemes. It is analogous to minimizing S^2 subject to the constraint $\Sigma a_i^2 < c$, where c is some constant determined by the value of λ. It is a matter of balancing our requirements for small S^2 with the requirement for small coefficients. In particular, if we can specify an acceptable accuracy for EC_t, we first minimize S^2 for $\lambda = 0$, then, if necessary, increase λ until the error term is within our specified limits.

To solve Eq. [12], we substitute for a_n using Eq. [8], and differentiate with respect to a_i. These calculations are shown in the Appendix. Setting the derivatives equal to zero, we have $n = 1$ equations to solve for $n - 1$ unknowns ($a_1,\ldots,a_n - 1$), so there exists a unique solution. The integrals may be calculated by trapezoidal integration. The coefficients are then determined by matrix inversion to solve the set of simultaneous equations.

The percentage response from within the required interval can be approximated by

$$F = \frac{\int_0^L \left| \phi_t(z) \right| dz}{\int_0^\infty \left| \phi_t(z) \right| dz}, \qquad [13]$$

and represents the relative response from above depth L in a profile of uniform electrical conductivity.

RESULTS AND DISCUSSION

Software to calculate linear combinations for chosen depth intervals can be obtained from the authors. We will present a few simple examples to illustrate the application of the method.

With all available Geonics FEM instruments (EM38, EM31, and EM34), a total of 10 separate measurements of EC_a may be made. In practice, however, the investigator is usually limited to only one available instrument. As a result, several investigators have used the EM38 held at different heights above the ground surface to provide depth information (e.g., Rhoades and Corwin, 1981). In the following, the theory developed above is applied to cases where: (i) only one coil spacing is used, but readings are made in both HD and VD configurations ($n = 2$), (ii) the EM34 is used in all possible configurations ($n = 6$), and (iii) the EM38 is held at varying heights above the ground ($n = 5$). (In this case, the constraint in Eq. [7] does not hold, and a more general constraint is used. See Appendix.)

The derived linear combinations were applied to data collected at a field site near Borrika, in southern Australia (Cook et al., 1992). Data are presented here for two bore sites (BEM05 and BEM08), where the actual depth profiles of EC_a had been measured to depths of 30 and 10 m, respectively (Fig. 2). They represent two very different shapes of conductivity profiles. Site BEM08 displays monotonically increasing EC_a with depth, whereas BEM05 shows more complicated variations in conductivity. At each of these sites, surface measurements of EC_a were made with all six configurations of the EM34, and with the EM31 in both HD and VD positions (Table 1).

Single Frequency, Coil Spacing

Several authors have used combinations of VD and HD measurements at a single frequency to minimize the deep response (e.g. Slavich, 1987, 1990; Corwin and Rhoades, 1982). In particular, a depth of 0.9 m has usually been chosen. Using the method described here, the linear combination for the EM38 that minimizes the response from below 0.9 m is given by

$$EC_t = 2.26\ EC_H - 1.26\ EC_V \qquad (F = 97\%).$$

where EC_H and EC_V are the HD and VD readings, respectively. (For the trapezoidal integration, we have used a subdivision width of 0.02 m, and have summed to a limit of 10 m.) The coefficients 2.19 and -1.19 will minimize the response below 1.0 m. As L becomes large, the coefficients for EC_H and EC_V that will minimize the response below L will approach the asymptotic forms discussed above.

The optimal coefficients to minimize the response below 0.9 m are in good agreement with the coefficients derived by Slavich (1987) (2.054 EC_H $-$ 1.054 EC_V). However, they differ from the coefficients derived empirically by Corwin and Rhoades (1982) (3.06 EC_H $-$ 1.133 EC_V $-$ 0.704 dS m^{-1}, $F = 92\%$) and from those derived by Slavich (1990) (0.828 EC_H + 0.394 EC_V $-$ 0.06 dS m^{-1}, $F = 69\%$). However these coefficients were derived to estimate the mean conductivity above 0.9 m, rather than to minimize the response below 0.9 m. The two approaches are quite different. Our coefficients will produce a weighted response function, concentrated within the chosen interval, but not necessarily constant within that interval.

Information concerning the structure of the EC_a at the two bores sites can be obtained from the surface measurements by using the linear combination 2 EC_H $-$ EC_V for each coil separation. The resulting linear combinations for 3.7-, 10-, 20-, and 40-m coil spacings are 0.69, 0.78, 0.84, and 0.78 dS m^{-1} for BEM05, and 0.14, 0.18, 0.52, and 0.84 dS m^{-1} for BEM08. These conductivities represent averages to increasingly greater depths. They show that the conductivity profile at BEM05 is relatively constant (i.e., shows no major trends) to approximately 20-m depth, while the profile at BEM08 increases with depth. The same can be seen from a comparison of individual HD measurements at different coil spacings, although it is more apparent through the use of the linear combinations. This is because the response functions for the linear combinations have a narrower depth range.

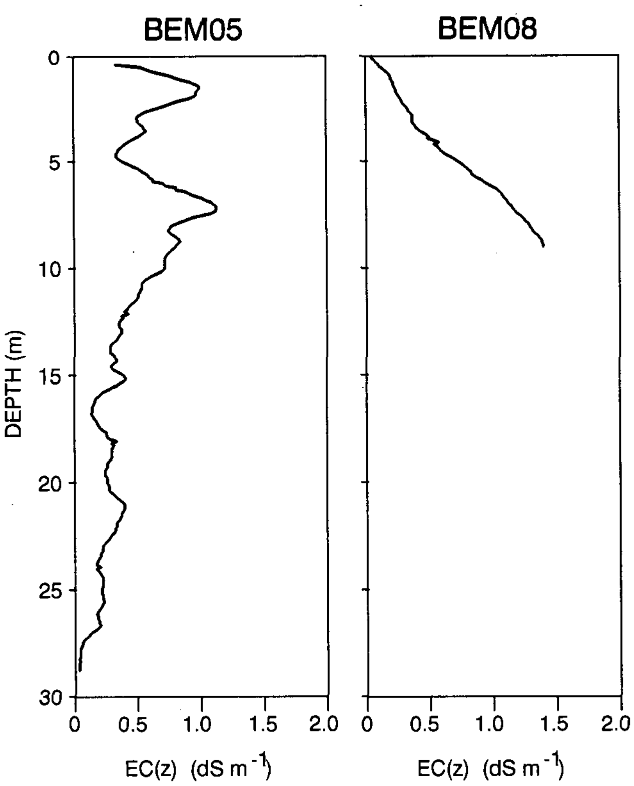

Fig. 2. Measured profiles of apparent electrical conductivity at the Borrika field site, southern Australia (Cook et al., 1992).

Table 1. Measured values of apparent electrical conductivity at boreholes BEM05 and BEM08 at the Borrika field site.

Configuration†	Apparent electrical conductivity	
	BEM05	BEM08
	dS m^{-1}	
EM31 VD	0.81	0.44
EM31 HD	0.75	0.29
10 m HD	0.62	0.34
10 m VD	0.46	0.50
20 m HD	0.59	0.54
20 m VD	0.34	0.56
40 m HD	0.54	0.63
40 m VD	0.30	0.42

† The EM31 and EM34 were used in horizontal (HD) and vertical (VD) dipole configurations, and at all possible coil separations. The EM31 has a fixed coil spacing of 3.7 m, while the EM 34 was used at coil spacings of 10, 20, and 40 m.

Table 2. Coefficients to minimize response for chosen intervals for the GEONICS EM34 frequency-domain electromagnetic induction instrument

Configuration†	Minimize response below 20 m				Minimize response above 5 m and below 20 m			Minimize response below 5 m	Minimize response above 20 and below 40 m
	$\lambda = 0$‡	$\lambda = 10^{-6}$	$\lambda = 10^{-5}$	$\lambda = 10^{-4}$	$\lambda = 0$	$\lambda = 10^{-5}$	$\lambda = 10^{-4}$	$\lambda = 10^{-4}$	$\lambda = 2 \times 10^{-5}$
10 m HD	20.1	0.04	−0.018	−0.107	61.7	−0.29	−0.21	0.505	−0.197
10 m VD	11.4	2.78	2.449	1.614	15.7	−1.08	−0.60	0.928	0.336
20 m HD	−38.4	0.05	0.146	0.256	−41.0	1.43	0.40	−0.347	0.674
20 m VD	11.7	−1.74	−1.243	−0.176	95.6	2.64	1.29	−1.616	−1.356
40 m HD	−4.4	−0.61	−0.534	−0.165	−165.1	−1.35	0.24	−0.291	−0.627
40 m VD	0.6	0.50	0.200	−0.422	34.1	−0.35	−0.11	1.821	2.170
Σa_i^2	2163	11.4	7.9	2.9	43298	12	2.3	7.2	7.5
$\Sigma\lvert a_i\rvert$	87	5.7	4.6	2.7	413	7	2.9	5.5	5.4
F	100%	99%	98%	93%	80%	57%	47%	32%	30%

† The EM34 was used in horizontal (HD) and vertical (VD) dipole configurations, and at 10-, 20-, and 40-m coil separations.
‡ Lambda is a damping parameter (Eq. [12]), which is used to limit the magnitude of the coefficients.

Use of EM34 in All Six Configurations

Suppose we take measurements with all six configurations of the EM34, and we wish to estimate the electrical conductivity of the soil above 20-m depth. Minimizing Eq. [12], we obtain coefficients shown in Table 2 (integrating to 200 m, with a subdivision width of 0.2 m). The depth-response functions for this linear combination are shown in Fig. 3. As can be seen, the response below 20 m is, in all cases, very small, with peak response at approximately 3-m depth. For $\lambda = 0$, the coefficients are very large ($\Sigma|a_i| = 87$), and so errors in measurement will be greatly magnified. For example, if the accuracy of measurement in each configuration is ± 0.02 dS m^{-1}, then the absolute error of the $\lambda = 0$ linear combination is ± 1.74 dS m^{-1}. Increasing λ reduces the magnitude of the coefficients. For values of $\lambda = 10^{-6}, 10^{-5}$, and 10^{-4}, $\Sigma|a_i| = 5.7, 4.6$, and 2.7, respectively (Table 2). The depth-response functions in Fig. 3 indicate that, at $\lambda = 10^{-6}$ and 10^{-5}, response below 20 m is very small ($F > 95\%$). For $\lambda = 10^{-4}$, response below 20 m is still relatively small ($F = 93\%$) but the coefficients are even smaller. This was considered the best combination.

Using the EC_i values measured at Site BEM05, and the coefficients to minimize the response below 20 m for $\lambda = 10^{-4}$, the linear combination yields $EC_t = \Sigma a_i EC_i = 0.55$ dS m^{-1}. The associated error is ± 0.06 dS m^{-1} (assuming an error of ± 0.02 dS m^{-1} for each measurement). This compares with the value of $\int \phi_t(z)EC(z)\,dz = 0.61$ dS m^{-1} using the measured conductivity profile (Fig. 4d). The two are in good agreement. The agreement between these two values decreases as λ decreases because of the magnification of errors discussed above. The average conductivity to 20-m depth is equal to 0.51 dS m^{-1}. However, EC_t need not necessarily provide an estimate of the average conductivity to 20 m since, for these coefficients, most of the response is attributable to the top 10 m of the profile. The relative contribution of different depths to the measured EC_a is depicted in Fig. 4c and 4d.

Suppose that we wish to estimate the conductivity between 5 and 20 m. This corresponds to the portion of the soil profile where the EC_w is 60 dS m^{-1} (Fig. 4a). Coefficients for this case ($\lambda = 0, 10^{-5}, 10^{-4}$) are given in Table 2, with depth-response functions in Fig. 5. In this case, it is more difficult to minimize

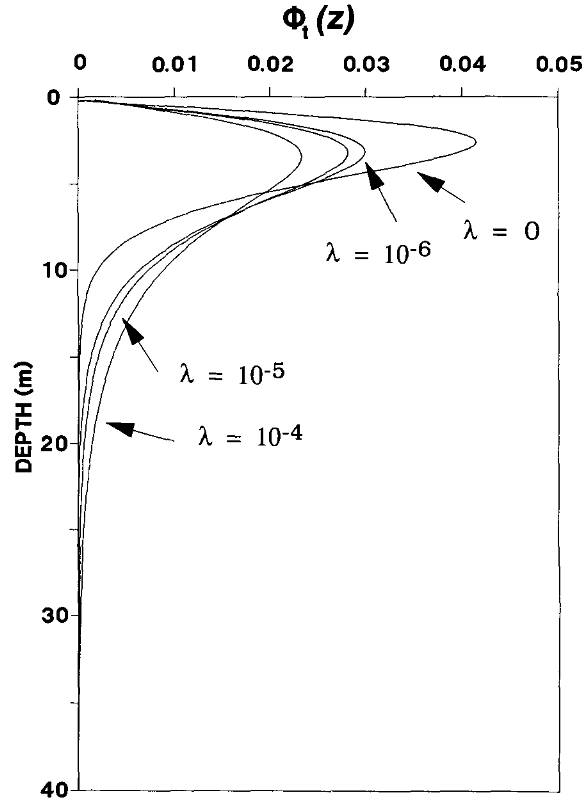

Fig. 3. Depth-response functions for linear combinations that minimize the response below 20 m. As λ increases, the magnitude of the coefficients decreases.

the response from outside the required interval and, at the same time, achieve small weighting coefficients. The best compromise for this case is considered to be $\lambda = 10^{-4}$, which gives $\Sigma|a_i| = 2.9$, and $F = 47\%$. Applying this to the same field data, using the measured values and the calculated coefficients for $\lambda = 10^{-4}$, yields $EC_t = \Sigma a_i EC_i = 0.36$ dS m^{-1}, with an error of ± 0.06 dS m^{-1}. This compares with the value of $\int \phi_t(z)EC_a(z) = 0.30$ dS m^{-1} calculated using the measured conductivity profile, and summing to 40-m depth.

In both of these examples, some of the discrepancy between EC_t and $\int \phi_t(z)EC(z)\,dz$ would be due to conductivity from below the 40-m depth, which is not

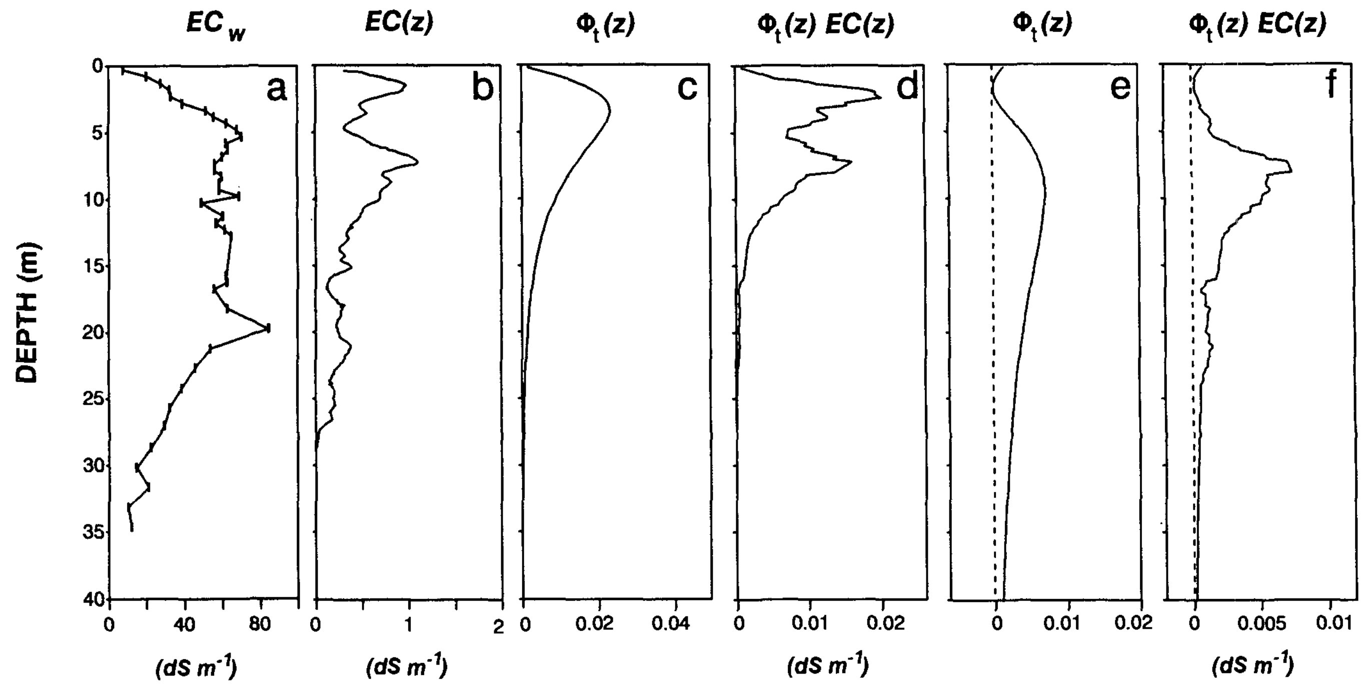

Fig. 4. Comparison of EM34 linear combinations and the measured soil electrical conductivity profile at site BEM05: (a) measured electrical conductivity of soil water, (b) measured soil electrical conductivity, (c) depth-response function to minimize response below 20 m, (d) product of curves b and c, (e) depth-response function to minimize response above 5 m and below 20 m, and (f) product of curves b and e. Data shown in curves a and b is from Cook et al. (1992).

included in the summation. In the former case, ϕ_t is negative below 40 m, and hence by summing to only

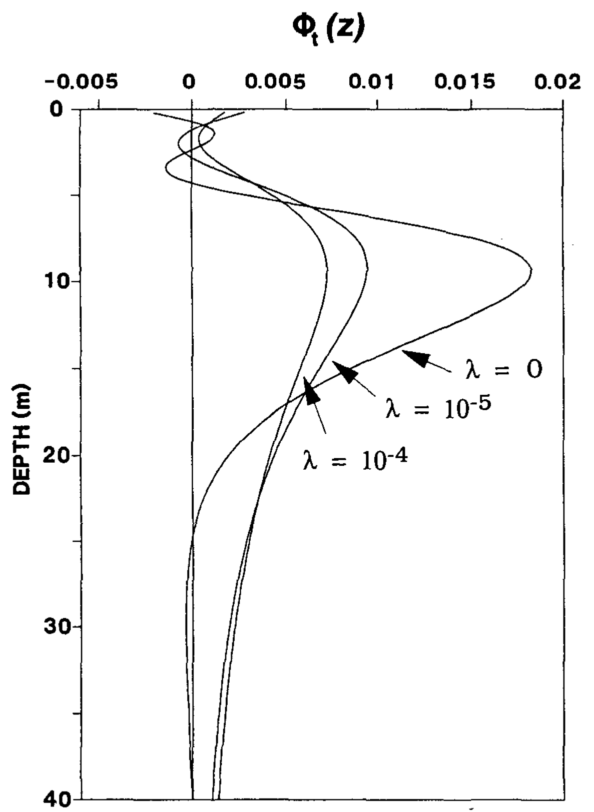

Fig. 5. Depth-response functions for linear combinations that minimize the response below 20 m and above 5 m. As λ increases, the magnitude of the coefficients decreases.

40 m, we overestimate $\int \phi_t(z)\mathrm{EC}(z)\, dz$. In the latter case, ϕ_t is positive below 40 m, and so we underestimate the value of the integral.

Linear combinations to maximize the response from above 5 m and between 20 and 40 m are also presented in Table 2. Comparing linear combinations that maximize the response above 5 m, between 5 and 20 m, and between 20 and 40 m should provide information on the structure of the $\mathrm{EC_a}$ profile. Applying these to the field data for BEM05 gives values of 0.37, 0.36, and 0.21 dS $\mathrm{m^{-1}}$, respectively. The linear combination that maximizes response between 20 and 40 m has clearly identified the lower conductivities present in the profile at depth (Fig. 2). This information was not obvious from the individual EM34 measurements. Applying the same linear combinations to the surface data for BEM08 gives values of 0.14, 0.72, and 0.21 dS $\mathrm{m^{-1}}$. Comparison of the two shallower linear combinations clearly shows that the $\mathrm{EC_a}$ profile is increasing with depth in the top 20 m. However, the deeper linear combination suggests that the conductivities below this decrease dramatically. This is not obvious from the individual EM34 measurements, or from the simple linear combinations given with a single frequency and coil spacing, and can only be determined by using a linear combination to reduce the response from the upper, high-conductivity layers. Although bore BEM08 did not penetrate below the 10-m depth, correlations with adjacent, deeper bores confirmed the presence of a low-conductivity layer between the 20- and 40-m depths.

Use of EM38 at Varying Heights Above the Ground

A third example involves using measurements with a single instrument (EM38) held at varying heights above the ground to obtain depth profiles. For our illustration, we chose heights above the ground (0, 0.3, 0.6, 0.9, and 1.2 m) to match those used by previous workers (Rhoades and Corwin, 1981). We also restricted our analysis to HD measurements.

To calculate the optimal coefficients using our minimization scheme, we varied the limits of integration in Eq. [12] to minimize both the response below 0.3 m and the response from above the soil surface. This is necessary because, when instruments are held above the ground, minimizing the response below any depth does not guarantee maximal response within the desired interval. The optimal coefficients (integrating to 10 m, with a subdivision width of 0.02 m) are a_1 = 1.58, a_2 = −0.06, a_3 = −0.04, a_4 = −2.21, a_5 = 0.31 (λ = 0, F = 54%).

In contrast to our result, Rhoades and Corwin (1981) determined the linear combination

$$- 0.1285\ EC_0 + 0.1446\ EC_{.03} + 5.3878\ EC_{0.6} - 17.4476\ EC_{0.9} + 15.0549\ EC_{1.2} - 0.1309\ \mathrm{dS\ m^{-1}}$$

to give the best estimate of conductivity in the 0- to 0.3-m interval (where $EC_{0.3}$ is the measurement made with the EM38 held at 0.3 m above the surface, etc.). The depth-response function for this linear combination is shown in Fig. 6. The fraction of response from within the required interval is only F = 24%. From conductivity profile data given by Rhoades and Corwin (1981), the conductivity below 0.3 m is much greater than above 0.3 m, and so the relative response from the desired interval is likely to be even less than this. The relative response from below 0.3 m may approach 90%. The linear combination only provides a useful measure of the conductivity above 0.3 m at this site because the conductivity in this zone is a relatively constant fraction of the conductivity below 0.3 m. These coefficients are, therefore, very site specific, and could not generally be transferred to other sites. Also, the coefficients for the empirical linear combination are very large, so any measurement errors will be greatly magnified ($\Sigma|a_i|$ = 38).

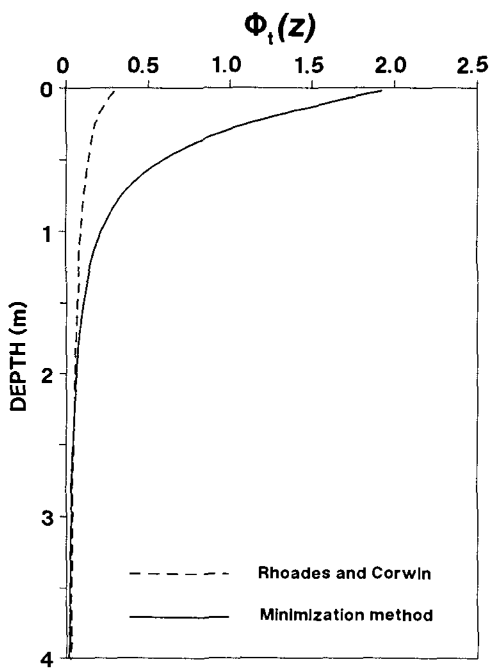

Fig. 6. Depth-response functions to estimate the conductivity to the 0.3-m depth using the coefficients of Rhoades and Corwin (1981) or coefficients to minimize the response below 0.3 m and above the soil surface.

CONCLUSIONS

The mathematical scheme described here provides a relatively simple method for determining linear combinations of FEM measurements that minimize the response within chosen intervals. Where the response is minimized by very large coefficients, however, associated measurement errors will be greatly magnified, and the linear combination will be of little use in practice. To avoid this problem, a damping parameter (λ) was added, and the scheme is designed to minimize the sum of the response outside the desired interval and the sum of the coefficients. If the choice of interval is inappropriate for the instruments used, then no linear combination will provide a suitable response function. Furthermore, it should not be assumed that minimizing the response outside certain bounds provides the mean conductivity within the same bounds. The response function within the bounds is unlikely to be constant, and will contain contributions from outside. However, as with all linear combinations methods, the depth-response function is known.

The mathematical scheme outlined here for determining linear combinations differs from previous attempts that have relied on multiple regressions of field data, in that it is not site specific and does not require calibration. In some cases, our method has been shown to provide good matches with previously published coefficients. In other cases, differences between our coefficients and empirically derived coefficients may be due to site specificity of the empirical coefficients. Also, our method provides a weighted conductivity, concentrated within the interval of interest, and not necessarily the mean for that interval. However, if no other information about the conductivity profile is available, then the linear combination produced by the minimization scheme is likely to give the best estimate of conductivity in the layer of interest. Nevertheless, in some cases, if the form of the conductivity profile was known, then it may be possible to use this information to derive better linear combinations. In particular, if it was known that the conductivity profile was increasing or decreasing with depth, better linear combinations could be achieved by concentrating on minimizing the contribution from areas with the highest conductivity.

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APPENDIX

We wish to minimize the function given in Eq. [12]:

$$S'^2 = \int_L^\infty \left[\sum_{i=1}^{n} a_i\phi_i(z) \right]^2 \mathrm{d}z + \lambda \sum_{i=1}^{n} a_i^2$$

subject to the constraint given in Eq. [6]. Under the assumption of Eq. [7], this constraint reduces to Eq. [8]. More generally, however, the constraint is

$$a_n = \frac{1 - \sum_{i=1}^{n-1} a_i \int_0^\infty \phi_i(z)\,\mathrm{d}z}{\int_0^\infty \phi_n(z)\,\mathrm{d}z} \quad \text{[A1]}$$

As discussed above, the constraint determines that, for readings made in an area in which the conductivity is constant with depth, EC_t will be equal to this conductivity.

We may rewrite Eq. [12] as

$$S'^2 = \int_L^\infty \left[\sum_{i=1}^{n-1} a_i\phi_i \right]^2 + 2a_n \int_L^\infty \left[\sum_{i=1}^{n-1} a_i\phi_i \right] \phi_n + \int_L^\infty a_n^2\phi_n^2 + \lambda \sum_{i=1}^{n-1} a_i^2 + \lambda a_n^2,$$

where all integrals are with respect to z. Substituting for a_n using Eq. [A1] and expanding, we get

$$\begin{aligned} S'^2 = {} & \int_L^\infty \left(\sum a_i\phi_i\right)^2 + 2\left(\sum a_i \int_L^\infty \phi_i\phi_n\right)\left(\int_0^\infty \phi_n\right)^{-1} \\ & - 2\left(\sum a_i \int_0^\infty \phi_i\right)\left(\sum a_i \int_L^\infty \phi_i\phi_n\right)\left(\int_0^\infty \phi_n\right)^{-1} \\ & + \left(\int_L^\infty \phi_n^2 + \lambda\right)\left(\int_0^\infty \phi_n\right)^{-2} \\ & - 2\left(\sum a_i \int_0^\infty \phi_i\right)\left(\int_L^\infty \phi_n^2 + \lambda\right)\left(\int_0^\infty \phi_n\right)^{-2} \\ & + \left(\sum a_i \int_0^\infty \phi_i\right)^2 \left(\int_L^\infty \phi_n^2 + \lambda\right)\left(\int_0^\infty \phi_n\right)^{-2} + \lambda \sum a_i^2, \end{aligned}$$

where all the summations are $i = 1, ..., n - 1$. Differentiating with respect to a_i gives

$$\begin{aligned} \partial S'^2/\partial a_i = {} & 2\sum_{j=1}^{n-1} a_j \int_L^\infty \phi_i\phi_j \\ & + 2\left(\int_L^\infty \phi_i\phi_n\right)\left(\int_0^\infty \phi_n\right)^{-1} - 2\sum_{j=1}^{n-1} a_j \\ & \left(\int_L^\infty \phi_n\phi_i \int_0^\infty \phi_j + \int_L^\infty \phi_n\phi_j \int_0^\infty \phi_i\right)\left(\int_0^\infty \phi_n\right)^{-1} \\ & - 2\int_0^\infty \phi_i \left(\int_L^\infty \phi_n^2 + \lambda\right)\left(\int_0^\infty \phi_n\right)^{-2} \\ & + 2\sum_{j=1}^{n-1} a_j \left(\int_0^\infty \phi_i\right)\left(\int_0^\infty \phi_j\right) \\ & \left(\int_L^\infty \phi_n^2 + \lambda\right)\left(\int_0^\infty \phi_n\right)^{-2} + 2\lambda a_i \end{aligned}$$

for $i = 1, ..., n - 1$. Setting each of these equal to zero, we have $n - 1$ equations to solve for $n - 1$ unknowns.

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