Assignment 1

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Download python codes from

https://github.com/96143/Assignment−1/blob/main/assignment_1.ipynb

Download latex codes from

https://github.com/96143/Assignment−1/blob/main/Question%20(6.1.1%20and%206.1.2).tex

1 Problem 6.1.1

Let $X_1 \sim \mathcal{N}(0,1)$ and $X_2 \sim \mathcal{N}(0,1)$. Plot the CDF and PDF of

$$V = X_1^2 + X_2^2$$

2 Solution 6.1.1

$$F_v(V) = P(X_1^2 + X_2^2 \leq V)$$

$$F_v(V) = \iint\limits_{X_1^2+X_2^2\leq V} f_{X_1X_2}(x_1,x_2)dx_1dx_2 \tag{2.0.1}$$

$f_{X_1X_2}(x_1,x_2)$ is the joint probability of X_1 and X_2

Differenting (2.0.1) we get

$$f_V(v) = \int_{x_2=-\sqrt{V}}^{\sqrt{V}} \frac{1}{2\sqrt{V-x_2^2}} \Big(f_{X_1X_2}(\sqrt{V-x_2^2},x_2) + f_{X_1X_2}(-\sqrt{V-x_2^2},x_2)dx_2 \tag{2.0.2}$$

As we know that

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}}$$

$$e^{-}\frac{1}{(1-r^2)}\left[\frac{x^2}{\sigma_1^2} - \frac{2xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2}\right]$$

Put $\sigma_1 = 1$, $\sigma_2 = 1$, $r = 0$

$$f_{X_1X_2}(x_1,x_2) = \frac{1}{2\pi}e^{-(x_1^2+x_2^2)/2} \tag{2.0.3}$$

Substituting (2.0.3) in (2.0.2)

$$f_V(v) = \int_{x_2=-\sqrt{v}}^{\sqrt{v}} \frac{1}{2\sqrt{v-x_2^2}} \left[\frac{1}{2\pi}e^{-}\left(\frac{(\sqrt{v-x_2^2})^2+x_2^2}{2}\right) + \frac{1}{2\pi}e^{-}\left(\frac{-(\sqrt{v-x_2^2})^2+x_2^2}{2}\right)\right.$$

$$= \frac{1}{2\pi}\frac{1}{2}\int_0^{\sqrt{v}} \frac{2}{\sqrt{v-x_2^2}}\left(\frac{1}{2\pi}e^{-(v-x_2^2)+x_2^2/2} + \frac{1}{2\pi}e^{-(v-x_2^2)+x_2^2/2}\right)dx_2$$

$$= \frac{2}{2\pi}e^{-v/2}\int_0^{\sqrt{v}} \frac{1}{\sqrt{v-x_2^2}}dx_2$$

$$= \frac{2}{2\pi}e^{-v/2}\left[Sin^-1\left(\frac{x_2}{\sqrt{v}}\right)\right]_0^{\sqrt{v}}$$

$$= \frac{2}{2\pi}e^{-v/2}\pi/2$$

$$= \frac{1}{2}e^{-v/2}$$

Therefore

$$f_V(v) = \begin{cases} \frac{1}{2}e^{-v/2} & \text{for v} \geq 0 \\ 0 & \text{for v} < 0 \end{cases} \tag{2.0.4}$$

$$F_v(x) = \int_0^x f_v(v)dv$$

$$F_v(x) = \int_0^x \frac{1}{2}e^-v/2dv$$

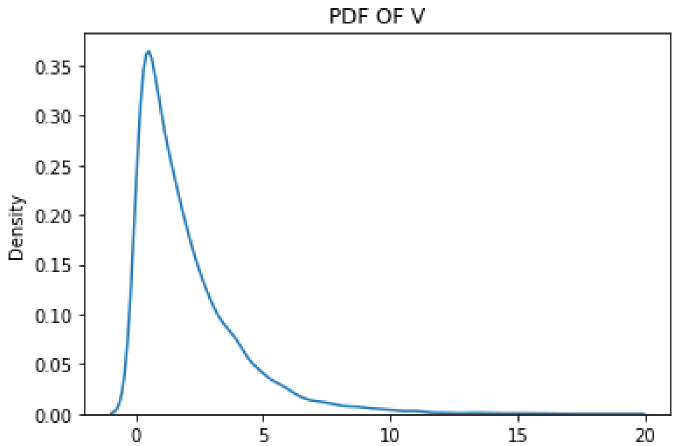
$$= \frac{1}{2}\left[\frac{e^{-v/2}}{-1/2}\right]_0^x$$

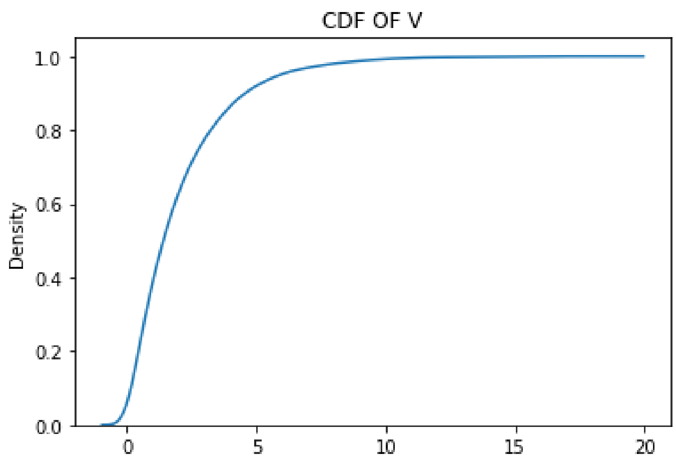
$$= -(e^{-x/2} - e^0$$

$$= -(e^{-x/2} - 1)$$

$$= 1 - e^{-x/2}$$

$$F_v(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2.0.5}$$





3 Problem 6.1.2

If

$$F_v(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find α

4 Solution 6.1.2

$$F_v(x) = \begin{cases} 1 - e^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{4.0.1}$$

Comparing (2.0.5) and (4.0.1) we get

$$\boxed{\alpha = \frac{1}{2}}$$