AI5002: Assignment 3

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Download Python codes from

https://github.com/96143/Assignment−3/blob/main/assignment%203.ipnyb

and latex codes from

https://github.com/96143/Assignment−3/tree/main

1 Problem

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

2 Solution

Let,
A : Person has disease
B : Person does not have the disease
C : test result is positive

We need to find the Probability that the person has the disease given that his test result is positive

$$P(A|C) = \frac{P(A) \times P(C|A)}{P(B) \times P(C|B) + P(A) \times P(C|A)} \tag{2.0.1}$$

$P(A)$ = Probability that the person has disease

$$= 0.1\% = \frac{0.1}{100} = 0.001 \tag{2.0.2}$$

$P(B)$ = Probability that the person does not have the disease

$$= 99.9\% = \frac{99.9}{100} = 0.999 \tag{2.0.3}$$

$P(C|A)$ = Probability that the test result is positive, if the person has disease

$$= 99\% = \frac{99}{100} = 0.99 \tag{2.0.4}$$

$P(C|A^c)$ = Probability that the test result is positive, if the person does not have the disease

$$= 0.005 \tag{2.0.5}$$

Putting the values in the formula,

$$P(A|C) = \frac{0.001 \times 0.99}{0.999 \times 0.005 + 0.001 \times 0.99} \tag{2.0.6}$$

$$P(A|C) = \frac{99 \times 10^{-5}}{499.5 \times 10^{-5} + 0.001 \times 0.99} \tag{2.0.7}$$

$$P(A|C) = \frac{99 \times 10^{-5}}{10^{-5}[499.5 + 0.99]} \tag{2.0.8}$$

$$P(A|C) = \frac{99}{598.5} \tag{2.0.9}$$

$$P(A|C) = \frac{990}{5985} \tag{2.0.10}$$

$$P(A|C) = \frac{22}{133} \tag{2.0.11}$$

Therefore, the required probability is $\frac{22}{133}$