AI5002: Assignment 5

Pradyumn Sharma
AI21MTECH02001

latex codes from

https://github.com/96143/Assignment−5/blob/main/Assignment%205.tex

1 Problem

Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X

2 Solution

When two fair dice are rolled, 6×6=36 observations are obtained.
Let X denote the sum of the numbers obtained when two fair dice are rolled.
So, X may have values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12

X	**Outcomes**	**No of Outcomes**	**Probability**
2	(1,1)	1	1/36
3	(1,2),(2,1)	2	2/36
4	(1,3),(2,2),(3,1)	3	3/36
5	(1,4),(2,3),(3,2),(4,1)	4	4/36
6	(1,5),(2,4),(3,3),(4,2),(5,1)	5	5/36
7	(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)	6	6/36
8	(2,6),(3,5),(4,4),(5,3),(6,2)	5	5/36
9	(3,6),(4,5),(5,4),(6,3)	4	4/36
10	(4,6),(5,5),(6,4)	3	3/36
11	(5,6),(6,5)	2	2/36
12	(6,6)	1	1/36

Thus,
The probability distribution table is

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Now we have to find the variance.

$$Var(X) = E[X^2] - (E[X])^2 \tag{2.0.1}$$

Finding E[X]

$$E[X] = \sum_{i=1}^{n} x_i p_i \tag{2.0.2}$$

$$\begin{aligned} E[X] = 2\times\frac{1}{36} + 3\times\frac{1}{18} + 4\times\frac{1}{12} + 5\times\frac{1}{9} \\ +6\times\frac{5}{36} + 7\times\frac{1}{6} + 8\times\frac{5}{36} + 9\times\frac{1}{9} \\ +10\times\frac{1}{12} + 11\times\frac{1}{18} + 12\times\frac{1}{36} \\ \implies E[X] = 7 \end{aligned}$$

Finding $E[X^2]$

$$E[X^2] = \sum_{i=1}^{n} (x_i)^2 p_i \qquad (2.0.3)$$

$$\begin{aligned} E[X^2] = {} & 2^2 \times \frac{1}{36} + 3^2 \times \frac{1}{18} + 4^2 \times \frac{1}{12} + 5^2 \times \frac{1}{9} \\ & +6^2 \times \frac{5}{36} + 7^2 \times \frac{1}{6} + 8^2 \times \frac{5}{36} + 9^2 \times \frac{1}{9} \\ & +10^2 \times \frac{1}{12} + 11^2 \times \frac{1}{18} + 12^2 \times \frac{1}{36} \end{aligned}$$

$$\implies E[X^2] = \frac{329}{6}$$

Using (2.0.1) Variance is given by

$$Var(X) = \frac{329}{6} - (7)^2$$

$$Var(X) = \frac{35}{6} = 5.83$$

Standard Deviation is given by

$$\sigma_x = \sqrt{Var(X)} \qquad (2.0.4)$$

$$\sigma_x = \sqrt{5.83} = 2.415$$