AI5002: Assignment 5

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latex codes from

https://github.com/96143/Assignment−3/tree/main

1 Problem

Let Z denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of Z

2 Solution

Let X denotes the outcome of the first dice.Y is the outcome of the second dice.

$$\begin{aligned} X &\in 1,2,3,4,5,6 \\ Y &\in 1,2,3,4,5,6 \end{aligned} \tag{2.0.1}$$

Now,
Considering X and Y as independent.
PMF of X:

$$p_X(n) = \Pr(X = n) = \begin{cases} \frac{1}{6} & 1 \geq n \leq 6 \\ 0 & otherwise \end{cases} \tag{2.0.2}$$

PMF of Y:

$$p_Y(n) = \Pr(Y = n) = \begin{cases} \frac{1}{6} & 1 \geq n \leq 6 \\ 0 & otherwise \end{cases} \tag{2.0.3}$$

Let Z be the sum of the outcomes of two die. Since,

$$Z = X + Y \tag{2.0.4}$$

PMF of Z:

$$p_Z(n) = \Pr(Z = n) = \begin{cases} 0 & n < 1 \\ \frac{1}{6}\sum_{k=1}^{n-1} p_X(k) & 1 \geq n-1 \leq 6 \\ \frac{1}{6}\sum_{k=n-6}^{6} p_X(k) & 1 \geq n-6 \leq 6 \\ 0 & n > 12 \end{cases} \tag{2.0.5}$$

Using (2.0.2) and (2.0.3) in (2.0.5) we get

$$p_Z(n) = \Pr(Z = n) = \begin{cases} 0 & n < 1 \\ \frac{n-1}{36} & 2 \geq n \leq 7 \\ \frac{13-n}{36} & 7 \geq n \leq 12 \\ 0 & n > 12 \end{cases} \tag{2.0.6}$$

Now we have to find the variance.

$$Var(X) = E[X^2] - (E[X])^2 \tag{2.0.7}$$

Finding E[X]

$$E[X] = \sum_{k=1}^{n} k p_Z(k) \tag{2.0.8}$$

$$\begin{aligned} E[X] &= 0 + \sum_{k=2}^{7} k\left(\frac{k-1}{36}\right) + \sum_{k=7}^{12} k\left(\frac{13-k}{36}\right) + 0 \\ E[X] &= 7 \end{aligned} \tag{2.0.9}$$

Finding $E[X^2]$

$$E[X^2] = \sum_{k=1}^{n} k^2 p_Z(k) \tag{2.0.10}$$

$$\begin{aligned} E[X^2] &= 0 + \sum_{k=2}^{7} k^2\left(\frac{k-1}{36}\right) + \sum_{k=7}^{12} k^2\left(\frac{13-k}{36}\right) + 0 \\ E[X^2] &= \frac{329}{6} \end{aligned} \tag{2.0.11}$$

Using (2.0.7) Variance is given by

$$Var(X) = \frac{329}{6} - (7)^2$$

$$Var(X) = \frac{35}{6} = 5.83$$

Standard Deviation is given by

$$\sigma_x = \sqrt{Var(X)} \tag{2.0.12}$$

$$\sigma_x = \sqrt{5.83} = 2.415$$