

# AI5002: Assignment 6

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latex codes from

<https://github.com/96143/Assignment-6/blob/main/Assignment%206.tex>

## 1 PROBLEM

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

- 1) all will bear 'X' mark.
- 2) not more than 2 will bear 'Y' mark.
- 3) at least one ball will bear 'Y' mark.
- 4) the number of balls with 'X' mark and 'Y' mark will be equal.

## 2 SOLUTION

Let X be the number of balls with mark 'X'  
Drawing a ball is a Bernoulli trial  
So X has a Binomial distribution

$$P(X = x) = {}^nC_x p^x q^{n-x} \quad (2.0.1)$$

Here,

number of balls drawn = n = 6

probability of getting ball with 'X' mark = p =  $\frac{10}{25}$   
=  $\frac{2}{5}$

q = 1 - p =  $\frac{3}{5}$

Hence,

$$P(X = x) = {}^6C_x \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{6-x} \quad (2.0.2)$$

- 1) probability that all will bear 'X' mark probability that all will bear 'X' mark = P(X=6)

Putting x = 6 in (2.0.2)

$$\begin{aligned} P(X = 6) &= {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^{6-6} \\ &= {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \\ &= \left(\frac{2}{5}\right)^6 \end{aligned}$$

- 2) probability that not more than 2 will bear 'Y' mark

$$\begin{aligned} P(\text{not more than 2 'Y'}) &= P(6X, 0Y) + P(5X, 1Y) + P(4X, 2Y) \\ &= {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^{6-6} + {}^6C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^{6-5} + {}^6C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^{6-4} \\ &= {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 + {}^6C_5 \left(\frac{2}{5}\right)^5 \left(\frac{3}{5}\right)^1 + {}^6C_4 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2 \\ &= 7 \left(\frac{2}{5}\right)^4 \end{aligned}$$

- 3) probability that at least one ball will bear 'Y' mark

$$\begin{aligned} P(\text{at least one ball bear 'Y'}) &= 1 - P(\text{no ball bear 'Y'}) \\ &= 1 - P(\text{all balls bear 'X'}) \\ &= 1 - P(X = 6) \\ &= 1 - {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^{6-6} \\ &= 1 - {}^6C_6 \left(\frac{2}{5}\right)^6 \left(\frac{3}{5}\right)^0 \\ &= 1 - \left(\frac{2}{5}\right)^6 \end{aligned}$$

- 4) probability that the number of balls with 'X'

mark and 'Y' mark will be equal

$$\begin{aligned}
 P(X \text{ \& } Y \text{ balls are equal}) &= P(X = 3) \\
 &= {}^6C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^{6-3} \\
 &= {}^6C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \\
 &= 20 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \\
 &= \frac{864}{3125}
 \end{aligned}$$