AI5002: Assignment 6

Pradyumn Sharma
AI21MTECH02001

latex codes from

https://github.com/96143/Assignment−3/tree/main

1 Problem

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear 'X' mark.
2) not more than 2 will bear 'Y' mark.
3) at least one ball will bear 'Y' mark.
4) the number of balls with 'X' mark and 'Y' mark will be equal.

2 Solution

Let X be the number of balls with mark 'X'
Drawing a ball is a Bernoulli trial
So X has a Binomial distribution

$$\Pr(X = x) = {}^{n}C_{x} p^{x} q^{n-x} \tag{2.0.1}$$

Here,
number of balls drawn = n = 6
probability of getting ball with 'X' mark = p = $\frac{10}{25}$ = $\frac{2}{5}$
q = 1 - p = $\frac{3}{5}$
Hence,

$$\Pr(X = x) = {}^{6}C_{x} \left(\frac{2}{5}\right)^{x} \left(\frac{3}{5}\right)^{6-x} \tag{2.0.2}$$

1) probability that all will bear 'X' mark probability that all will bear 'X' mark = $\Pr(X = 6)$
Putting x = 6 in (2.0.2)

$$\Pr(X = 6) = {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{6-6}$$
$$= {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{0}$$
$$= \left(\frac{2}{5}\right)^{6}$$

2) probability that not more than 2 will bear 'Y' mark

$$\Pr(\textit{not more than } 2\ 'Y')$$
$$= \Pr(6X, 0Y) + \Pr(5X, 1Y) + \Pr(4X, 2Y)$$
$$= {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{6-6} + {}^{6}C_{5} \left(\frac{2}{5}\right)^{5} \left(\frac{3}{5}\right)^{6-5} + {}^{6}C_{4} \left(\frac{2}{5}\right)^{4} \left(\frac{3}{5}\right)^{6-4}$$
$$= {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{0} + {}^{6}C_{5} \left(\frac{2}{5}\right)^{5} \left(\frac{3}{5}\right)^{1} + {}^{6}C_{4} \left(\frac{2}{5}\right)^{4} \left(\frac{3}{5}\right)^{2}$$
$$= 7 \left(\frac{2}{5}\right)^{4}$$

3) probability that at least one ball will bear 'Y' mark

$$\Pr(\textit{at least one ball bear } 'Y') = 1 - \Pr(\textit{no ball bear } 'Y')$$
$$= 1 - \Pr(\textit{all balls bear } 'X')$$
$$= 1 - \Pr(X = 6)$$
$$= 1 - {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{6-6}$$
$$= 1 - {}^{6}C_{6} \left(\frac{2}{5}\right)^{6} \left(\frac{3}{5}\right)^{0}$$
$$= 1 - \left(\frac{2}{5}\right)^{6}$$

4) probability that the number of balls with 'X'

mark and 'Y' mark will be equal

$$\begin{aligned}\Pr(X \ \&\ Y \textit{ balls are equal}) &= \Pr(X = 3)\\ &= {}^{6}C_{3}\left(\frac{2}{5}\right)^{3}\left(\frac{3}{5}\right)^{6-3}\\ &= {}^{6}C_{6}\left(\frac{2}{5}\right)^{3}\left(\frac{3}{5}\right)^{3}\\ &= 20\left(\frac{2}{5}\right)^{3}\left(\frac{3}{5}\right)^{3}\\ &= \frac{864}{3125}\end{aligned}$$