AI5002: Assignment 6

Pradyumn Sharma
AI21MTECH02001

latex codes from

https://github.com/96143/Assignment−6/blob/main/Assignment%206.tex

Python codes from

https://github.com/96143/Assignment−6/blob/main/Assignment_6.ipynb

1 Problem

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

1) all will bear 'X' mark.
2) not more than 2 will bear 'Y' mark.
3) at least one ball will bear 'Y' mark.
4) the number of balls with 'X' mark and 'Y' mark will be equal.

2 Solution

Let X be the number of balls with mark 'X'
Drawing a ball is a Bernoulli trial
So X has a Binomial distribution

$$P(X = x) = {}^{n}C_{x}p^{x}q^{n-x} \tag{2.0.1}$$

Here,
number of balls drawn = n = 6
probability of getting ball with 'X' mark = p = $\frac{10}{25}$ = $\frac{2}{5}$
q = 1 - p = $\frac{3}{5}$
Hence,

$$P(X = x) = {}^{6}C_{x}\left(\frac{2}{5}\right)^{x}\left(\frac{3}{5}\right)^{6-x} \tag{2.0.2}$$

1) probability that all will bear 'X' mark probability that all will bear 'X' mark = P(X=6) Putting x = 6 in (2.0.2)

$$\begin{aligned} P(X = 6) &= {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{6-6} \\ &= {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{0} \\ &= \left(\frac{2}{5}\right)^{6} \end{aligned}$$

2) probability that not more than 2 will bear 'Y' mark

$$\begin{aligned} &P(\textit{not more than } 2\ 'Y') \\ &= P(6X, 0Y) + P(5X, 1Y) + P(4X, 2Y) \\ &= {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{6-6} + {}^{6}C_{5}\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right)^{6-5} + {}^{6}C_{4}\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{6-4} \\ &= {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{0} + {}^{6}C_{5}\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right)^{1} + {}^{6}C_{4}\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{2} \\ &= 7\left(\frac{2}{5}\right)^{4} \end{aligned}$$

3) probability that at least one ball will bear 'Y' mark

$$\begin{aligned} P(\textit{at least one ball bear } 'Y') &= 1 - P(\textit{no ball bear } 'Y') \\ &= 1 - P(\textit{all balls bear } 'X') \\ &= 1 - P(X = 6) \\ &= 1 - {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{6-6} \\ &= 1 - {}^{6}C_{6}\left(\frac{2}{5}\right)^{6}\left(\frac{3}{5}\right)^{0} \\ &= 1 - \left(\frac{2}{5}\right)^{6} \end{aligned}$$

4) probability that the number of balls with 'X'

mark and 'Y' mark will be equal

$$
\begin{aligned}
P(X \ \& \ Y \ balls \ are \ equal) &= P(X = 3) \\
&= {}^6C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^{6-3} \\
&= {}^6C_6 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \\
&= 20 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^3 \\
&= \frac{864}{3125}
\end{aligned}
$$