AI5002: Assignment 7

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Download all latex codes from

https://github.com/96143/Assignment−7/blob/main/assignment%207.tex

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https://github.com/96143/Assignment−7/blob/main/Assignment_7.ipynb

1 Problem

Find the probability distribution of number of doublets in three throws of a pair of dice.

2 Solution

Let X denotes the outcome of the first dice.Y is the outcome of the second dice.

Probability of getting a doublet = $\Pr(X = Y)$
Probability of not getting a doublet = $\Pr(X \neq Y)$
Now,
Considering X and Y as independent.
Probability of getting a doublet:

$$\Pr(X = Y) = \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} + \frac{1}{6}\times\frac{1}{6} = \frac{1}{6} \tag{2.0.1}$$

Probability of not getting a doublet:

$$\begin{aligned}\Pr(X \neq Y) &= 1 - \Pr(X = Y)\\ &= 1 - \frac{1}{6}\\ &= \frac{5}{6}\end{aligned} \tag{2.0.2}$$

Let p be the probability of getting a doublet and q be the probability of not getting a doublet.
It is a binomial distribution. So using probability distribution of Binomial. Let Z be the no of doublets in three throws of a dice.

$$\Pr(Z = z) = {}^{n}C_{z}\,p^{z}q^{n-z} \tag{2.0.3}$$

Probability of getting no doublets:

$$\begin{aligned}\Pr(Z = 0) &= {}^{3}C_{0}\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{3-0}\\ &= \left(\frac{5}{6}\right)^{3} = \frac{125}{216}\end{aligned} \tag{2.0.4}$$

Probability of getting 1 doublet:

$$\begin{aligned}\Pr(Z = 1) &= {}^{3}C_{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{3-1}\\ &= 3\times\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{2} = \frac{75}{216}\end{aligned} \tag{2.0.5}$$

Probability of getting 2 doublets:

$$\begin{aligned}\Pr(Z = 2) &= {}^{3}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{3-2}\\ &= 3\times\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right) = \frac{15}{216}\end{aligned} \tag{2.0.6}$$

Probability of getting 3 doublets:

$$\begin{aligned}\Pr(Z = 3) &= {}^{3}C_{3}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{3-3}\\ &= \left(\frac{1}{6}\right)^{3} = \frac{1}{216}\end{aligned} \tag{2.0.7}$$

$\therefore$ The required probabilities are shown in the figure:

X	0	1	2	3
P(X)	125/216	75/216	15/216	1/216