AI5002: Assignment 8

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1 Problem

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $\Pr(3V \geq 2U)$ is

2 Solution

$$U = \mathcal{N}\left(0, \frac{1}{4}\right)$$

$$V = \mathcal{N}\left(0, \frac{1}{9}\right)$$

To determine-

$$\Pr(3V \geq 2U) = \Pr(3V - 2U \geq 0)$$

Let $Y = 3V - 2U$

$$\therefore E[Y] = 3E[V] - 2E[U]$$

$$\implies E[Y] = 0$$

Also,

$$Var[Y] = 3^2 Var[V] + 2^2 Var[U]$$

$$\implies Var[Y] = 9 \times \frac{1}{9} + 4 \times \frac{1}{4}$$

$$\implies Var[Y] = 2$$

$$\therefore Y = \mathcal{N}(0, 2)$$

W is a Gaussian Random variable with mean zero and variance 2.

$$\Pr(3V - 2U \geq 0) = \Pr(Y \geq 0)$$

Since, Y is symmetric about the zero mean axis

$$\therefore \Pr(Y \geq 0) = \frac{1}{2}$$