Problem Set 3: Sharing Quantum States

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1 Theory Exercises

1. In the section about superdense coding, we mentioned that Bob can do a Bell state measurement on the entangled pair of qubits to identify the bit string that Alice encoded. However if Bob has access to a quantum computer that can only do measurements in the computational basis, he still decode the bits Alice was trying to send. Draw a circuit that Bob must use to do this.

2. Alice is sending a total of 10 qubits to Bob using the BB84 protocol. For each qubit Alice randomly chooses a bit 0 or 1 with equal probability, Alice randomly chooses an encoding basis R or D with equal probability and Bob randomly chooses a measurement basis R or D with equal probability.

   (a) What is the expected number of bits that Alice and Bob will add to their keys? Assume Eve is not eavesdropping.

   (b) Eve is eavesdropping in the quantum channel, and measures all of the qubits in the R basis before they reach Bob. Now what is the expected number of bits that Alice and Bob add to their keys? What is the probability that the first bit in Alice's key is the same as the first bit in Bob's key?

   (c) If Eve measures in the $Y = \left\{ \frac{|0\rangle + i|1\rangle}{\sqrt{2}}, \frac{|0\rangle - i|1\rangle}{\sqrt{2}} \right\}$ basis instead, how do your answers for part (b) change?

2 Qiskit Excercise

The Qiskit problems can be found this jupyter notebook.