# 1 Natural Deduction

$$\frac{A \quad B}{A \wedge B}\ \wedge I \qquad\qquad \frac{A \wedge B}{A}\ \wedge E_l \qquad\qquad \frac{A \wedge B}{B}\ \wedge E_r$$

$$\frac{\begin{array}{c}[A]^1\\ \vdots \\ B\end{array}}{A \Rightarrow B}\ \Rightarrow I^1 \qquad\qquad \frac{A \Rightarrow B \quad A}{B}\ \Rightarrow E$$

$$\frac{}{A \vee \neg A}\ \mathcal{TND}$$

$$\frac{A}{\forall X.A}\ \forall I^* \qquad\qquad \frac{\forall X.A}{[B/X](A)}\ \forall E$$

* means that A does not depend on any hypothesis in which X is free

$$\frac{[B/X](A)}{\exists X.A}\ \exists I \qquad\qquad \frac{\exists X.A \qquad \begin{array}{c}[[c/X](A)]^1\\ \vdots \\ C\end{array} \qquad c \in \sum_0^{sk} new}{C}\ \exists E^1$$

$$\frac{}{A = A}\ = I \qquad\qquad \frac{A = B \qquad C[A]_p}{[B/p]C}\ = E$$

$$\frac{}{A \Leftrightarrow A}\ \Leftrightarrow I \qquad\qquad \frac{A \Leftrightarrow B \qquad C[A]_p}{[B/p]C}\ \Leftrightarrow E$$

## 1.1 Derived Rules

$$\frac{A}{A \vee B}\ \vee I_l \qquad\qquad \frac{B}{A \vee B}\ \vee I_r$$

$$\frac{A \vee B \quad \begin{array}{c}[A]^1\\ \vdots \\ C\end{array} \quad \begin{array}{c}[B]^1\\ \vdots \\ C\end{array}}{C}\ \vee E^1$$

$$\frac{\begin{array}{c}[A]^1\\ \vdots \\ C\end{array} \quad \begin{array}{c}[A]^1\\ \vdots \\ \neg C\end{array}}{\neg A}\ \neg I^1 \qquad\qquad \frac{\neg\neg A}{A}\ \neg E$$

$$\frac{\neg A \quad A}{\bot}\ \bot I \qquad\qquad \frac{\bot}{A}\ \bot E$$

## 2 Tableau

$$\frac{(A \land B)^T}{\begin{array}{c}A^T \\ B^T\end{array}} \mathcal{T}_0\land \qquad \frac{(A \land B)^F}{A^F \mid B^F} \mathcal{T}_0\lor$$

$$\frac{(\neg A)^T}{A^F} \mathcal{T}_0\neg^T \qquad \frac{(\neg A)^F}{A^T} \mathcal{T}_0\neg^F$$

$$\frac{\begin{array}{l}A^\alpha \\ \vdots \\ A^\beta \quad (\alpha \neq \beta)\end{array}}{\perp} \mathcal{T}_0\perp$$

$$\frac{(\forall X.A)^T \quad Y new}{([Y/X](A))^T} \mathcal{T}_1^f\forall \qquad \frac{(\forall X.A)^F \quad free(\forall X.A) = X_1, \cdots, X_k \quad f \in \sum_k^{sk} new}{([f(X_1, \cdots, X_k)/X](A))^F} \mathcal{T}_1^f\exists$$

$$\frac{\begin{array}{l}A^\alpha \\ \vdots \\ B^\beta \quad (\alpha \neq \beta) \quad \sigma(A) = \sigma(B)\end{array}}{\perp : \sigma} \mathcal{T}_1^f\perp$$

### 2.1 Derived Rules

$$\frac{(A \to B)^T}{A^F \mid B^T} \qquad \frac{(A \to B)^F}{\begin{array}{c}A^T \\ B^F\end{array}} \qquad \frac{\begin{array}{c}A^T \\ (A \to B)^T\end{array}}{B^T}$$

$$\frac{(A \lor B)^T}{A^T \mid B^T} \qquad \frac{(A \lor B)^F}{\begin{array}{c}A^F \\ B^F\end{array}}$$

$$\frac{(A \iff B)^T}{\begin{array}{c}A^T \\ B^T\end{array} \mid \begin{array}{c}A^F \\ B^F\end{array}} \qquad \frac{(A \iff B)^F}{\begin{array}{c}A^T \\ B^F\end{array} \mid \begin{array}{c}A^F \\ B^T\end{array}}$$

$$\frac{(\exists X.A)^F \quad Y new}{([Y/X](A))^F} \qquad \frac{(\exists X.A)^T \quad free(\forall X.A) = \{X_1, \cdots, X_k\} \quad f \in \sum_k^{sk} new}{([f(X_1, \cdots, X_k)/X](A))^T}$$

# 3 CNF

$$\frac{C \vee (A \vee B)^T}{C \vee A^T \vee B^T} \vee^T \qquad \frac{C \vee (A \vee B)^F}{(C \vee A^F) \wedge (C \vee B^F)} \vee^F$$

$$\frac{C \vee \neg A^T}{C \vee A^F} \neg^T \qquad \frac{C \vee \neg A^F}{C \vee A^T} \neg^F$$

$$\frac{(\forall X.A)^T \vee C \quad Z \notin (free(A) \cup free(C))}{([Z/X](A))^T \vee C} \forall^T$$

$$\frac{(\forall X.A)^F \vee C \quad free(\forall X.A) = \{X_1, \cdots, X_k\} \quad f \in \sum_k^{sk} new}{([f(X_1, \cdots, X_k)/X](A))^F \vee C} \forall^F$$

## 3.1 Derived Rules

$$\frac{C \vee (A \Rightarrow B)^T}{C \vee A^F \vee B^T} \Rightarrow^T \qquad \frac{C \vee (A \Rightarrow B)^F}{(C \vee A^T) \wedge (C \vee B^F)} \Rightarrow^F$$

$$\frac{C \vee (A \wedge B)^T}{(C \vee A^T) \wedge (C \vee B^T)} \wedge^T \qquad \frac{C \vee (A \wedge B)^F}{C \vee A^F \vee B^F} \wedge^F$$

$$\frac{(\exists X.A)^T \vee C \quad free(\forall X.A) = \{X_1, \cdots, X_k\} \quad f \in \sum_k^{sk} new}{([f(X_1, \cdots, X_k)/X](A))^T \vee C} \exists^T$$

$$\frac{(\exists X.A)^F \vee C \quad Z \notin (free(A) \cup free(C))}{([Z/X](A))^F \vee C} \exists^F$$

## 4 Resolution

$$\frac{P^T \vee A \qquad\qquad P^F \vee B}{A \vee B}\ \mathcal{R}_0$$

$$\frac{A^T \vee C \quad B^F \vee D \quad \sigma = mgu(A, B)}{(\sigma(C)) \vee (\sigma(D))}\ \mathcal{R}_1$$

$$\frac{A^\alpha \vee B^\beta \vee C \quad \sigma = mgu(A, B)}{(\sigma(A)) \vee (\sigma(C))}\ \mathcal{R}_1$$

## 5 Unification

$$\frac{\epsilon \wedge f(A^1, \cdots, A^n) =^? f(B^1, \cdots, B^n)}{\epsilon \wedge A^1 =^? B^1 \wedge \cdots \wedge A^n =^? B^n}\ \mathcal{U}_{dec}$$

$$\frac{\epsilon \wedge A =^? A}{\epsilon}\ \mathcal{U}_{triv}$$

$$\frac{\epsilon \wedge X =^? A \quad X \notin free(A) \quad X \in free(\epsilon)}{[A/X](\epsilon \wedge X =^? A)}\ \mathcal{U}_{elim}$$

$$\frac{\epsilon \wedge f(X^1, \cdots, X^n) =^? g(Y^1, \cdots, Y^n)}{\bot}\ \mathcal{U}\bot$$

## 6 Tableau Calculus for $\mathcal{ALC}$ ($\mathcal{T}_{\mathcal{ALC}}$)

$$\frac{x : \varphi \sqcap \psi}{x : \varphi \quad x : \psi}\ \mathcal{T}_\sqcap$$

$$\frac{x : \varphi \sqcup \psi}{x : \varphi \mid x : \psi}\ \mathcal{T}_\sqcup$$

$$\frac{x : \forall R.\varphi \quad xRy}{y : \varphi}\ \mathcal{T}_\forall$$

$$\frac{x : \exists R.\varphi}{xRy \quad y : \varphi}\ \mathcal{T}_\exists$$

$$\frac{x : \varphi \quad x : \overline{\varphi}}{\bot}\ \mathcal{T}_\bot$$