POLITECNICO DI MILANO
DIPARTIMENTO DI ENERGIA

HANDOUTS 2024-25

POLITECNICO DI MILANO

DEPARTMENT OF ENERGY - NUCLEAR ENGINEERING DIVISION

Nuclear Design and Technology

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Version: 17/09/2024

The content of this document is intended as a collection of exercises, useful for the understanding of the subject ("design-by-analysis" approach) and for the preparation to the exam. Part of the exercises will be solved during the Course (exercise + flipped sessions), part will be left for autonomous self-evaluation.

Table 1: Design specifications and material properties.

Fuel assembly (FA) representative of average core conditions		
FA thermal power	MW	6.641
Number of fuel pins in the FA	-	284
Linear heat rate at fuel midplane, *q'(0)*	kW m^{-1}	26.03
Pin pitch	mm	13.6
Coolant inlet temperature	°C	400
Coolant outlet temperature	°C	480
Coolant properties – Liquid Pb		
Average heat capacity (uniform along the channel)	J kg^{-1} $°C^{-1}$	146
Average density (uniform along the channel)	kg m^{-3}	10520
Average dynamic viscosity coefficient (uniform along the channel)	Pa s	0.002
Average thermal conductivity (uniform along the channel)	W m^{-1} $°C^{-1}$	15.5
Fuel pins		
External diameter, *D*	mm	8.50
Cladding thickness, *t*	mm	0.600
Active length, H_a	mm	1100
Total length, H_{tot}	mm	2100
Fuel – solid pellets (MOX)		
Density	%TD	88
$T_{melting}$ (fresh fuel)	°C	2730
Diameter	mm	6.98
Linear coefficient of thermal expansion (reference 25°C)	$°C^{-1}$	$12 \cdot 10^{-6}$
Young's modulus	GPa	170
Poisson coefficient	-	0.31
Cladding – Ferritic-martensitic steel T91 (UNS No. K90901)		
$T_{melting}$	°C	1500
Average thermal conductivity	W m^{-1} $°C^{-1}$	28.7
Linear coefficient of thermal expansion (reference 25°C)	$°C^{-1}$	$12 \cdot 10^{-6}$
Young's modulus	GPa	170
Poisson coefficient	-	0.3
Yield strength	MPa	↲
$536.1 - 4.878 \cdot 10^{-1} T + 1.6 \cdot 10^{-3} T^2 - 3 \cdot 10^{-6} T^3 + 8 \cdot 10^{-10} T^4$		T (°C)

The design specifications of a Lead Fast Reactor are given in Table 1. The fuel assembly is constituted by a closed, square array of 17x17 pins, as illustrated in Figure 1.

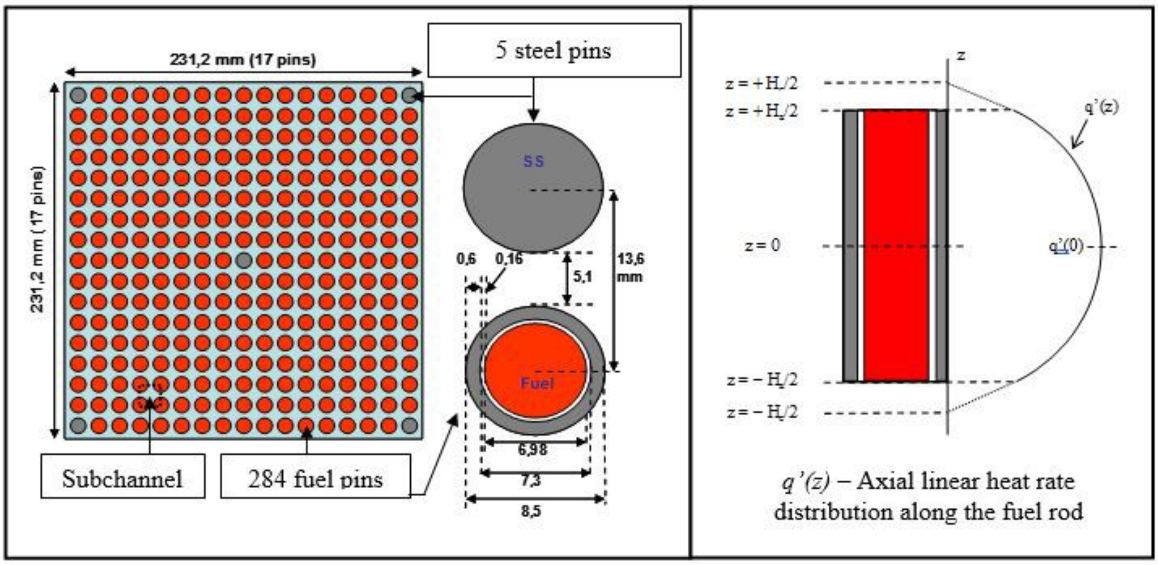

Figure 1: Fuel assembly sketch and geometry description of the fuel pin.

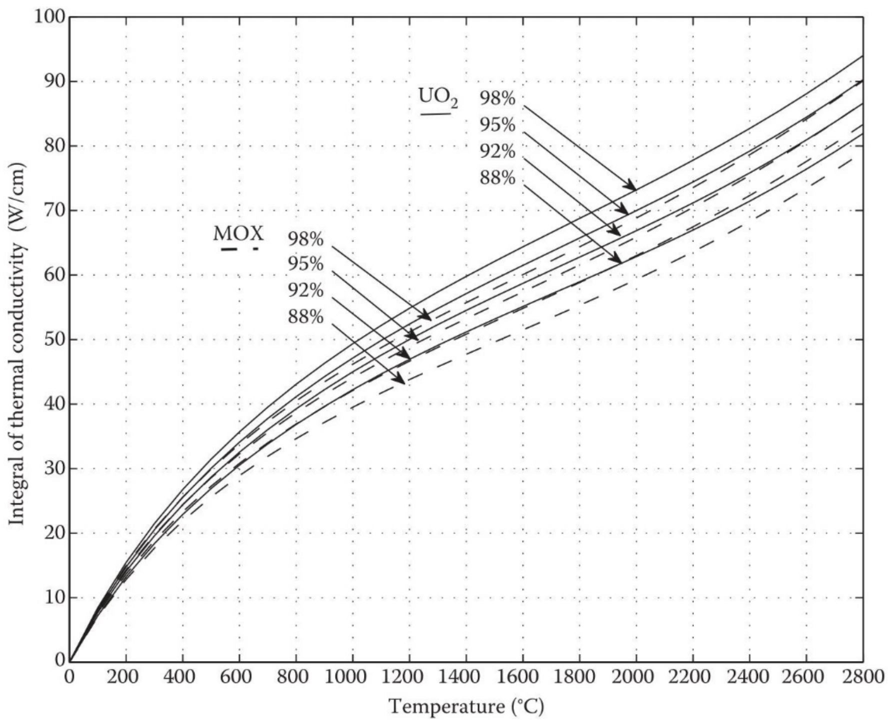

Figure 2: Integral of thermal conductivity (from 0°C to the selected temperature) of UO_2 and $(U,Pu)O_2$ fresh fuels at different densities (expressed as % of the Theoretical Density).

In the analysis, employ the integral of thermal conductivity reported in Figure 2. Please, note that the temperature of the fuel is expressed in °C, the integral (whose lower limit is 0°C) is in W cm^{-1}, and refers to different fuel density values with respect to the theoretical density. Moreover, to evaluate the fuel-cladding thermal conductance, neglect the contribution due to radiative heat transport and adopt the following correlation to estimate the filling gas thermal conductivity (initial filling gas at beginning of life: 100% He):

$$k_{He} = 15.8 \cdot 10^{-4} \cdot \Theta^{0.79} \tag{1}$$

where k_{He} (W m^{-1} K^{-1}) is He thermal conductivity and Θ is the He average temperature (K).

Cold geometry analyses

I) Referring to the fuel assembly representative of the ***average core conditions***:

a) Determine the coolant mass flow rate, average velocity, and pressure drop along the generic subchannel specified in Figure 1. In the calculation of the pressure drop, neglect the inlet/outlet losses and those due to the grid spacers. Employ the following correlation to calculate the Darcy friction factor:

$$f_{Darcy} = 0.184 \cdot Re^{-0.20} \tag{2}$$

applicable for Reynolds numbers spanning in $3 \cdot 10^4 < Re < 1 \cdot 10^6$**.**

b) Estimate the average heat transfer coefficient between the coolant and the fuel pin. In particular, employ one of the following correlations, discussing the choice, verifying the range of applicability, and commenting the impact on the results (by means of a sensitivity analysis):

$$Nu = 0.58 \cdot \left(\frac{D_h}{D}\right)^{0.55} \cdot Pe^{0.45} \qquad 80 \le Pe \le 4000;\ 1.1 \le x \le 1.5$$

$$Nu = 4.5 + 0.014 \cdot Pe^{0.8} \qquad \text{No Pe limitation} \tag{3}$$

In these correlations, Nu is the Nusselt number, Pe is the Peclet number, D_h (m) is the hydraulic diameter, D (m) is the cladding outer diameter, and *x* (-) is the pin–pitch to cladding outer diameter ratio.

II) Considering the fuel pin representative of the ***average conditions*** in the above considered fuel assembly, with an axial linear heat rate distribution in the active region obeying to the equation

$$q'(z) = q'(0) \cdot \cos\left(\frac{\pi z}{H_e}\right) \tag{4}$$

being z the axial coordinate and H_e the extrapolated length of the fuel stack (see Figure 1):

a) Evaluate the average linear heat rate (q'_{av}) and the extrapolated length of the fuel pin.

b) Draw the radial temperature profile at the midplane of the fuel pin active length (z = 0). In particular, evaluate the cladding outer and inner temperatures, and the fuel outer and maximum temperatures as well.

c) Draw the axial profile of the cladding temperature at the outer and inner radius.

d) Determine the axial coordinate $\bar{z}$ with respect to the fuel pin midplane where the maximum cladding outer temperature is reached, and the corresponding cladding outer temperature.

e) Estimate the minimum value of the linear heat rate, at the midplane of the active length of the fuel pin, leading to fuel pellet cracking due to thermal stresses. Indicate the radial positions where the fracture stress is reached and where the brittle-ductile transition occurs. In this analysis, consider the following correlation for the tensile rupture stress - σ_R (MPa) - of the MOX fuel:

$$\sigma_R = 9.81 \cdot \left(13.6 + \frac{T}{4500}\right) \qquad \text{if } T \leq 1733$$
$$\sigma_R = 9.81 \cdot (66 - 0.03 \cdot T) \qquad \text{if } T > 1733 \tag{5}$$

where T (°C) is the local temperature. In this analysis, consider an *effective* thermal conductivity of the fuel equal to 2.5 W m^{-1} K^{-1}.

III) Considering the subchannel representative of the most demanding operating condition of the core (***hot channel***), assuming the same extrapolated length of **II)**-a) and the same mass flow rate of **I)**-a), considering a total peak factor (i.e., the ratio between the maximum linear heat rate $q'(0)_{max}$ and the average value in the core q'_{av}) equal to 1.5:

a) Determine the hot channel outlet coolant temperature and the axial coordinate $\bar{z}$ with respect to the fuel pin midplane where the maximum cladding outer temperature is reached. Discuss whether this value is compatible or not with the corrosion of the employed steel, considering an asymptotic oxide layer thickness (whose thermal conductivity is equal to 1 W m^{-1} K^{-1}) given by:

$$\delta_c = 1 \qquad \text{if } T_c \leq 380$$

$$\delta_c = 0.26 \cdot (T_c - 380) + 1 \qquad \text{if } T_c > 380 \tag{6}$$

where δ_c (μm) is the oxide thickness and T_c (°C) is the temperature of the non-oxidized cladding outer surface.

b) Evaluate an "average core" throttle which guarantees:

- The *hot channel* outlet temperature to be equal to the *average channel* one (480°C).
- The maximum cladding outer temperature reached in the *hot channel* to be equal to that reached in the *average channel*.

c) Draw the radial temperature profile at midplane of the hottest fuel pin active length (z = 0). In particular, evaluate the cladding outer and inner temperatures, and the fuel outer and maximum temperatures as well.

d) Draw the axial profile of the cladding temperature at the outer and inner radius.

e) Estimate the value of the linear heat rate leading to fuel melting, q'_{melt}, and indicate the axial position with respect to the midplane where it first occurs. In the analysis, employ the following value for the integral of thermal conductivity towards the melting of the fuel:

$$\int_{0°C}^{T_{melt}} k \, dT = 78 \text{ W cm}^{-1} \tag{7}$$

where k is the fuel thermal conductivity and T_{melt} its melting temperature.

f) Considering a peak linear heat rate of 750 W cm^{-1}, and keeping T_{melt} as in Table 1, determine whether melting occurs at the fuel midplane. If melting occurs, estimate the ratio of the molten area with respect to the total fuel area.

g) Calculate the linear heat rate causing the yielding of cladding, considering only the effects due to the radial temperature gradient in the cladding. Estimate the axial and the radial position through the cladding wall, where the first yielding occurs.

h) Estimate the fuel pin internal pressure causing the yielding of the cladding, due only to mechanical loads (for simplicity, assume zero external pressure).

i) Analyse the impact of helium embrittlement in the considered T91 ferritic-martensitic steel, estimating the ^{4}He content in atoms ppm subsequent to an irradiation period of one year (around 20 dpa) and discussing whether it can constitute an issue for the safe operation of the fuel pins. In the analysis, compare the performance of T91 to that of a classic AISI 304 austenitic steel. The compositions of the two alloys are reported in Table 2 (estimate the average molar mass *x* of the other elements (impurities) present in each steel).

Consider the following values at the midplane of the fuel rod for the thermal and the fast component of the neutron flux: $\Phi_{th} = 1 \cdot 10^{13}$ neutrons cm^{-2} s^{-1}, $\Phi_v = 1 \cdot 10^{15}$ neutrons cm^{-2} s^{-1}. Assume that all the helium production is due to the following reactions, whose effective cross sections are expressed in millibarns:

Thermal flux	
$^{10}B(n,\alpha)$	$\sigma(^{10}B) = 3.837 \cdot 10^6$
$^{58}Ni + n \rightarrow {}^{59}Ni + \gamma$	$\sigma(^{58}Ni) = 4.4 \cdot 10^3$
$^{59}Ni + n \rightarrow {}^{56}Fe + \alpha$	$\sigma(^{59}Ni) = 1.3 \cdot 10^4$
Fast flux	
$^{10}B(n,\alpha)$	$\sigma(^{10}B) = 623$
$Fe(n, \alpha)$	$\sigma(Fe) = 0.23$
$Cr(n, \alpha)$	$\sigma(Cr) = 0.20$
$Ni(n, \alpha)$	$\sigma(Ni) = 4.20$

In the analysis of AISI 304, assume that only a *small* fraction of the initial nickel is consumed, in order to decouple the ordinary differential equations relative to the nickel balance in the thermal and fast neutron flux components. Discuss the applicability of this approximation to the T91 steel.

A posteriori, verify the validity of the assumption above and that the production of helium due to the thermal component of the neutron flux on nickel is negligible with respect to the fast flux component on nickel and to other elements' contributions.

Critically discuss if other kinds of material embrittlement must be considered in the conditions experienced by the T91 cladding in both the *average* and *hot channels*.

Table 2: T91 and AISI 304 compositions.

AISI 304 (density 8.32 g cm^{-3})			
Element	**wt. %**	**Molar mass (g mol^{-1})**	**Isotopic abundance (at.%)**
Fe	70	55.487	
Cr	19	51.996	
Ni	9	58.690	^{58}Ni 68.3, ^{59}Ni 0.0
Mn	0.8	54.938	
Si	0.5	28.086	
Mo	0.2	95.940	
C	0.06	12.011	
B	0.0005	10.811	^{10}B 19.8
Impurities	0.4395	x	
T91 (density 7.76 g cm^{-3})			
Element	**wt. %**	**Molar mass (g mol^{-1})**	**Isotopic abundance (at.%)**
Fe	Balance	55.487	
Cr	8.5	51.996	
Ni	0.13	58.690	^{58}Ni 68.3, ^{59}Ni 0.0
Mn	0.45	54.938	
Si	0.35	28.086	
Mo	0.925	95.940	
C	0.1	12.011	
V	0.22	50.94	
Impurities	0.17	x	

Hot geometry analyses

IV) Referring to the ***hot pin*** identified at **III)**, determine the radial temperature distribution at the midplane of the fuel pin, analysing *separately* the following "hot effects" at the beginning of life (neglect the effects of plutonium redistribution):

a) Employing a three-zones modelling approach and considering the undeformed *cold geometry*, consider a fuel restructuring process which leaves unaffected the outermost region of the pellet (ρ_0 = 0.88 ρ_{TD}) and leads (proceeding towards the centre) to the formation of a region with equiaxed grains (ρ_{eqax} = 0.95· ρ_{TD}, T_{eqax} = 1600°C), a region with columnar grains (ρ_{clmn} = 0.98 ρ_{TD}, T_{clmn} = 1800°C), and a central hole. Evaluate, varying ρ_0 and $q'(0)_{max}$, whether fuel restructuring leads always to a decrease of the maximum fuel temperature. In solving this task, employ the integral of thermal conductivity of Figure 2 in the analysis of the columnar and equiaxed grain regions, and adopt the following correlation for the thermal conductivity in the as-fabricated region (in particular, when varying ρ_0):

$$k(T,P) = \left[\frac{1}{A_0 + B_0 \cdot T} + \frac{C}{T^2} \cdot \exp\left(-\frac{D}{T}\right)\right] \cdot (1-P)^{2.5}$$

$A_0 = 3.08 \cdot 10^{-2}$ (8)
$B_0 = 2.516 \cdot 10^{-4}$
$C = 4.715 \cdot 10^{9}$
$D = 16361$

where T (K) is the temperature and P (-) is the porosity.

b) Considering an un-restructured fuel microstructure (i.e., with a homogeneous density of $\rho_0 = 0.88 \cdot \rho_{TD}$), estimate the reduction of the gap due to the differential thermal expansion of fuel and cladding. Draw the radial temperature profile at the fuel pin midplane, determined on a trial-and-error basis (iterative procedure), and verify that nowhere fuel-cladding contact occurs.

c) Verify that the temperature field experienced by the fuel in the cases **IV)**-a) and **IV)**-b) is lower than in case **III)**-c). Discuss if the superposition principle can be applied to evaluate the overall benefit in terms of maximum fuel temperature reduction when both the "hot effects" evaluated in **IV)**-a) and **IV)**-b) are considered acting *together* (as in reality).

Hot effects analyses

V) Discuss the "hot effects" influencing the fuel pin performance under irradiation that induce a higher temperature regime with respect to that obtained neglecting the "hot effects".

a) Considering only the effects of plutonium redistribution (i.e., neglecting fuel restructuring and gap reduction), estimate the percentage decrease of the linear heat rate necessary to guarantee the same fuel maximum temperature found in the *hot channel* (**III)**-c)). In the analysis, consider the same fuel outer temperature found in **III)**-c), and the following relationship to account for plutonium redistribution:

$$\frac{q'''(r)}{q_0} = \frac{c(r)}{c_0} = 1 + D\left\{\exp\left[-2\alpha\left(\frac{r-r^*}{R_{fo}}\right)\right] - 2\cdot\exp\left[-\alpha\left(\frac{r-r^*}{R_{fo}}\right)\right]\right\} \quad (9)$$

where D = 0.01, α = 10 and r* are empirical constants. In particular, r^*/R_{fo} is such that it must guarantee the conservation of plutonium in the redistribution. In Eq. 9, R_{fo} is the fuel outer radius, $q'''(r)$ and c(r) are the power density radial distribution ($W\,cm^{-3}$) and the radial plutonium concentration ($g\,cm^{-3}$), respectively. Moreover, q_0''' and c_0 are the initial values of the former and the latter quantity, assumed to be uniform along the radius.

Verification of cladding thermal creep

VI) Evaluate whether the cladding creep is compatible or not with a fuel pin expected life of 4 years, considering the most critical section. In the analysis, consider: as design values for the internal and external pressure 12 MPa and 0 MPa, respectively; the continuous line by Kawasaki et al. fitting the experimental data reported in Figure 3; and the following expression for the Larson-Miller parameter (P_{LM}):

$$P_{LM} = T \cdot (29.1146 + \log_{10}(t_{rupt})) \tag{10}$$

where T (K) is the average cladding temperature, t_{rupt} (h) the creep failure time, and σ (MPa) the average *effective* stress through the thickness. Discuss the impact of cladding oxidation, adopting the formula given in **III)**-a) for the asymptotic oxide layer thickness, having a thermal conductivity of 1 W m^{-1} K^{-1}.

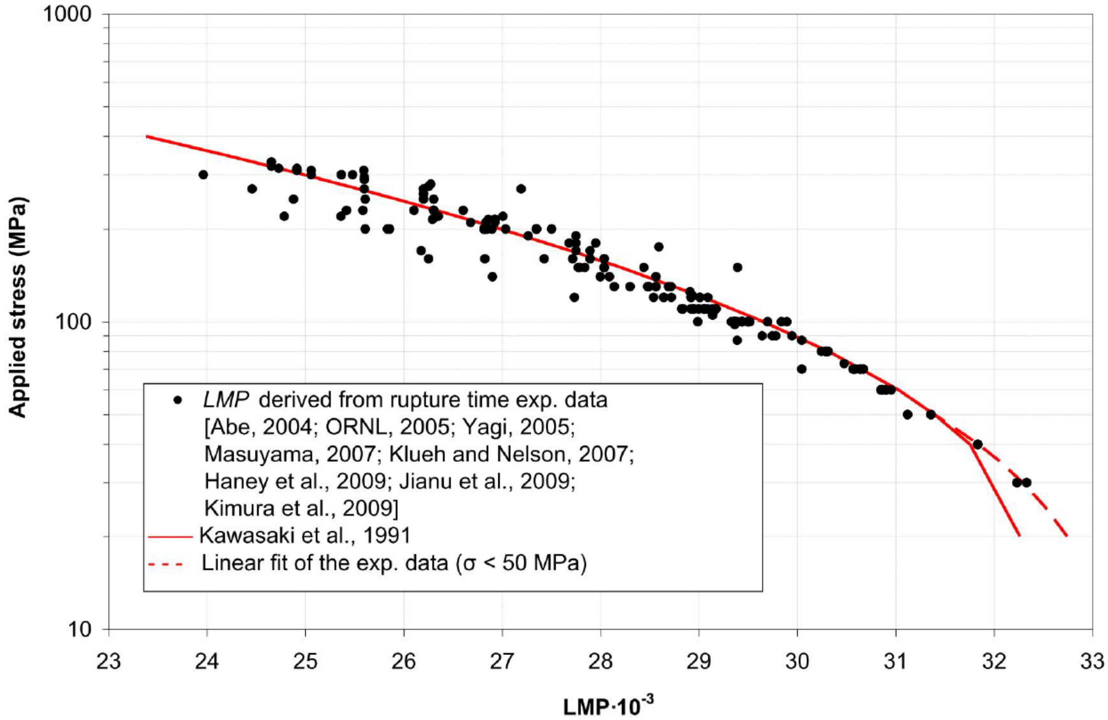

Figure 3: *Effective* stress leading to creep failure vs. Larson-Miller parameter for the considered cladding steel.