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CSCE 3010
Homework #2
Due: 10/03/2016 11:59PM

1. Derive an expression for the step response of a Continuous Time LTI system. Hint: Start with the general Convolution Integral formula and substitute $x(t) = u(t)$ *and* $s(t) = y(t)$

$$x(t) = u(t) = \begin{cases} 1 \ if \ t > 0 \\ \frac{1}{2} \ if \ t = 0 \\ 0 \ if \ t < 0 \end{cases}$$

$$s(t) = y(t) = \int_{-\infty}^{\tau} h(\tau) d\tau$$

$$s(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau = \int_{0}^{\infty} h(t-\tau)d\tau = \int_{\infty}^{0} h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)d\tau$$

2. Given two Continuous Time functions defined as shown below.

$$\mathrm{x1\ (t)} = \begin{cases} -\mathrm{t} + 1 \text{ for } 1 > \mathrm{t} > 0 \\ 0, \text{otherwise} \end{cases}$$

$$\mathrm{x2\ (t)} = \mathrm{u(t)}$$

Compute the Convolution Integral $x1\,(t) * x2(t)$?

$$u(t) = \begin{cases} 1 \ if \ t > 0 \\ \frac{1}{2} \ if \ t = 0 \\ 0 \ if \ t < 0 \end{cases}$$

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$$

$$x_1(t) * x_2(t) = \int_{0}^{1} (-\tau+1)(1)d\tau = \left[-\frac{\tau^2}{2} + \tau\right]|_0^1 = \left[-\frac{1^2}{2} + 1 - 0\right] = \frac{1}{2} \ when \ t \geq 1$$

$$\left[\frac{-\tau^2}{2} + \tau\right] u(t) \ when \ 1 > t > 0$$

$$0 \ when \ 0 \geq t$$

$$\boxed{y(t) = \begin{cases} \frac{1}{2} when \ t \geq 1 \\ \frac{-t^2}{2} + t \ when \ 1 > t > 0 \\ 0 \ when \ 0 \geq t \end{cases}}$$

3. Given two Discrete Time functions defined as shown below.

$$\mathrm{x[n]} = (-2)^{\mathrm{n}} \mathrm{u[n]}$$

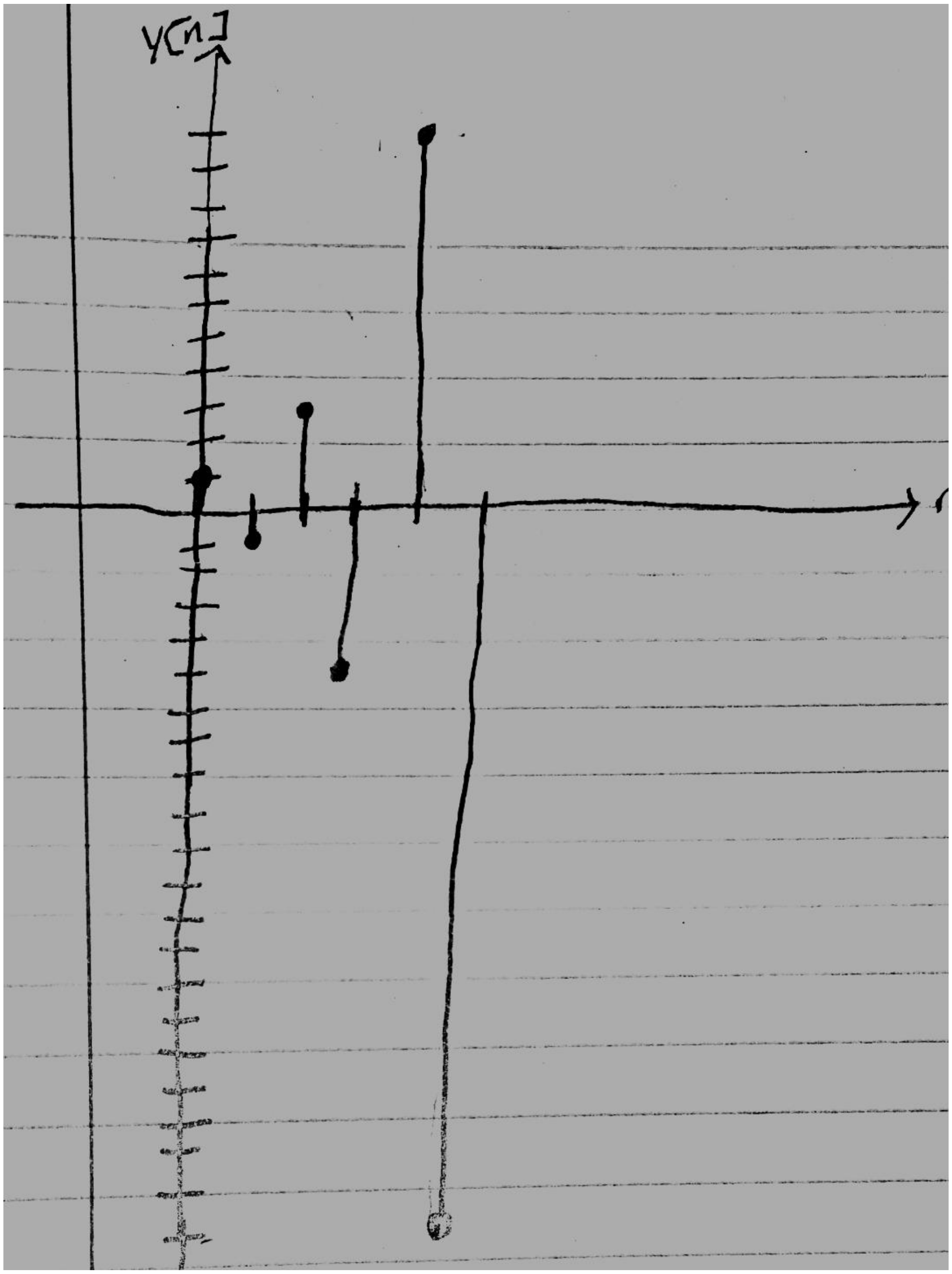
$$\mathrm{h[n]} = \mathrm{u[n]}$$

Compute the Convolution Sum $x[n] * h[n]$?

$$u[n] = \begin{cases} 1 \ if \ n \geq 0 \\ 0 \ if \ n < 0 \end{cases}$$

$$y[n] = \sum_{k=0}^{\infty} (-2)^k$$

$y[0] = 1; y[1] = -1; y[2] = 3; y[3] = -5; y[4] = 11; y[5] = -21;$



4. Change $h[n]$ = $u[n-3]$ in Problem 3 and re-compute the convolution sum.

$$u[n] = \begin{cases} 1 \; if \; n \geq 3 \\ 0 \; if \; n < 3 \end{cases}$$

$$y[n] = \sum_{k=0}^{n-3} (-2)^k$$

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$$y[3] = 1; y[4] = -1; y[5] = 3; y[6] = -5; y[7] = 11; y[8] = -21;$$

5. We have established that $x[n] * \delta[n]$ = $x[n]$. What will be the value of the convolution $x[n] * \delta[n - \beta]$? Explain your answer.

$$\delta[n] = \begin{cases} 1 \; if \; n = \beta \\ 0 \; if \; n \neq \beta \end{cases}$$

$$y[n] = x[n] * \delta[n-\beta] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-\beta-k]$$

$$\boxed{x[n] * \delta[n-\beta] = x[n-\beta]}$$

6. Is an LTI system with $h[n]$ = $u[n]$

$$u[n] = \begin{cases} 1 \; if \; n \geq 0 \\ 0 \; if \; n < 0 \end{cases}$$

   a. Causal? Yes, h[n] = 0 when n<0
   b. Stable? No, because $\sum_{k=-\infty}^{\infty} |u[n]| = \infty$. If it were less than ∞, it would be stable.
   c. Memoryless? No, because u[n] is 1 or 0 when n≠0. If h[n] were only 0 when n≠0, then it would be memoryless.

7. Consider two Discrete Time LTI systems—S1 and S2-- connected in cascade. The impulse response of the two systems are:

$$\mathrm{S1{:}\,h1} = \delta[\mathrm{n} + 1]$$
$$\mathrm{S2{:}\,h2} = \delta[\mathrm{n} - 3]$$

   What is the resultant impulse response of the cascade system?

$$\delta[n] = \begin{cases} 1 \; if \; n = 0 \\ 0 \; if \; n \neq 0 \end{cases}$$

$$h_1 * h_2 = \delta[n+1] * \delta[n-3]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k+1]\delta[n-3-k]$$

$$y[0] = 0; y[1] = 0; y[2] = 1; y[3] = 0;$$

$$\boxed{h[n] = \begin{cases} 1 \; if \; n = 2 \\ 0 \; if \; n \neq 2 \end{cases}}$$