

PROBLEM:

ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}(\triangle ADE) = \text{ar}(\triangle BCF)$

Theory: Parallelograms on the same base and in between the same parallels are equal in area.

Given: ABCD, DCFE and ABFE are parallelograms.

Solution Statement:

We can see that the sides of a triangle ADE and BCF are also the opposite sides of a given parallelogram. Now we can show both the triangles are congruent using congruency property. We know that congruent triangles are equal areas.

SOLUTION:

parallelogram ABCD lies between same parallel lines AD and BC

$$\therefore AD = BC \dots (1)$$

parallelogram DECF lies between same parallel lines DE and CF

$$\therefore DE = CF \dots (2)$$

parallelogram ABEF lies between same parallel lines AE and FB

$$\therefore EA = FB \dots (3)$$

In $\triangle ADE$, $\triangle BCF$

$$\therefore AD = BC$$

$$\therefore DE = CF$$

$$\therefore EA = FB$$

$$\therefore \triangle ADE = \triangle BCF$$

Hence, Proved

Termux commands :

The input parameters for this construction are

| Symbol | Value | Description |
|------------|--|-------------|
| a | 4.5 | EA |
| b | 4.5 | BC |
| c | 10 | CD |
| d | 2.5 | DE |
| θ_1 | $25\pi/180$ | $\angle BC$ |
| θ_2 | $120\pi/180$ | $\angle DE$ |
| θ_3 | $2\pi/3$ | $\angle AE$ |
| θ_4 | $35\pi/180$ | $\angle CD$ |
| E | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ | Point E |

To Prove:

$$\text{Ar}(\triangle ADE) = \text{Ar}(\triangle BCF) \quad (1)$$

$$\mathbf{v1} = \mathbf{A} - \mathbf{D} \quad (2)$$

$$\mathbf{v2} = \mathbf{D} - \mathbf{E} \quad (3)$$

Area of the triangle $\triangle ADE$ is given by

$$\text{Ar}(\triangle ADE) = 1/2 \times \|\mathbf{v1} \times \mathbf{v2}\|$$

$$\mathbf{v3} = \mathbf{B} - \mathbf{C} \quad (4)$$

$$\mathbf{v4} = \mathbf{C} - \mathbf{F} \quad (5)$$

Area of the triangle $\triangle BCF$ is given by

$$\text{Ar}(\triangle BCF) = 1/2 \times \|\mathbf{v3} \times \mathbf{v4}\|$$

$$\text{Ar}(\triangle ADE) = \text{Ar}(\triangle BCF) \quad (6)$$

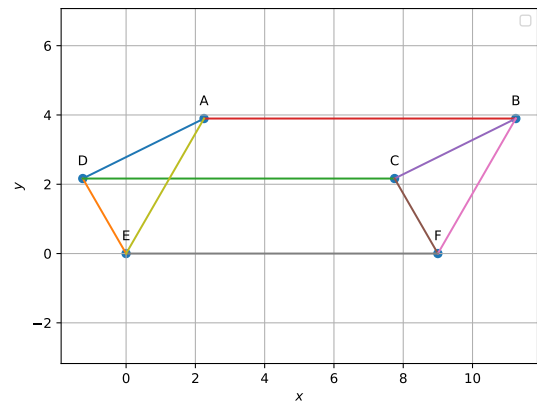
0.1 Construction

Figure of Construction

The below python code realizes the above construction:
<https://github.com/9705701645/FWC/blob/main/lines4.py>