Worksheet 4 Laplace Transform

Lecturer

Set up MATLAB

```
clear all
format compact
```

Worksheet 4

To accompany Chapter 3.1 Laplace Transform

We will step through this worksheet in class.

You are expected to have at least watched the video presentation of Chapter 3.1 of the notes before coming to class. If you haven't watch it afterwards!

Pingo

We will be using a web-based audience response system called **Pingo** for in-class quizzes and informal surveys.

Setup

Browse to: pingo.coactum.de.

When prompted: enter the **session ID**

The Session ID for this Course

194851

Icebreaker Questions

-> Launch Poll

End of setup

Plan B

If TurningPoint doesn't work, use this Google form instead

https://goo.gl/forms/EuyH6G7za2knqt862

First hour quiz

The Laplace and inverse Laplace transforms

Without conferring or looking it up, which of these integrals represents the Laplace transform?

A.	$\frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)\,e^{st}\,ds$	B.	$\int_{0}^{\infty} f(t)\,e^{-st}\,dt$
C.	$\int_{-\infty}^{t} f(\tau)\,g(t-\tau)\,d\tau$	D.	$\int_{-j\omega}^{+j\omega} f(t)\,e^{-j\omega t}\,dt$

-> Launch Poll

The Laplace and inverse Laplace transforms

Now discuss with your neigbours

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-> Launch Poll

The Laplace and inverse Laplace transforms

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-> Launch Poll

The Laplace and inverse Laplace transforms

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-> Launch Poll

Laplace transforms

Match the transform to the time-domain operator

Please confer

1.	$\int_{-\infty}^{t} f(\tau)\,d\tau$	A.	$\frac{F(s)}{s}+\frac{f(0^-)}{s}$
2.	$\lim_{t\to 0} f(t)$	B.	$sF(s)-f(0^-)$
3.	$\int_{0}^{t} f_1(\tau)f_2(t-\tau)\,d\tau$	C.	$\frac{\int_0^T f(t)\,e^{-sT}}{1-e^{-sT}}$
4.	$\frac{d}{dt} f(t)$	D.	$F_1(s)\,F_2(s)$

5.	$f(t+nT)$	E.	$\lim_{s\to\infty} sF(s)$

Properties of Laplace transforms

Match each of these mathematical properties to the associated Laplace transform property.

You should confer

1.	Linearity	A.	$f(t-a)\,u_0(t-a) \Leftrightarrow e^{-as}F(s)$
2.	Time Scaling	B.	$c_1 f_1(t) + c_2 f_2(t) + \cdots + c_n f_n(t) \Leftrightarrow c_1 F_1(s) + c_2 F_2(s) + \cdots + c_n F_n(s)$
3.	Time-shift	C.	$e^{-at}\,f(t) \Leftrightarrow F(s+a)$
4.	Frequency Shift	D.	$f(at) \Leftrightarrow (1/a)\,F(s/a)$

Name that property

What property is this?

$$\lim_{t\to\infty} f(t) \Leftrightarrow \lim_{s\to 0} sF(s)$$

A. Convolution in the time domain

B. Initial value theorem

C. Final value theorem

D. Differentiation in the time domain

E. Integration in the time domain

-> Launch Poll

Elementary signals

Match the elementary signal to it's Laplace transform

You may confer

1.	Dirac delta (unit impulse)	$\delta(t)$	A.	e^{-as}
2.	Unit step	$u_0(t)$	B.	$\frac{1-e^{-as}}{s}$
3.	Unit ramp	$u_1(t) = tu_0(t)$	C.	$\frac{1}{s}$
4.	Exponential decay	$e^{-at}u_0(t)$	D.	1
5.	Damped sinusoid	$e^{-at}\ \sin(\omega t)u_0(t)$	E.	$\frac{1}{s^2}$
				$\frac{1}{s+a}$

6.	Sampling function	$\delta(t-a)$	F.	
7.	Gating function	$u_0(t)-u_0(t-a)$	G.	$\dfrac{\omega}{(s+a)^2+\omega^2}$

End of first hour quiz

Is there anything in this quiz that you think we should go over in more detail in class?

-> Launch Poll

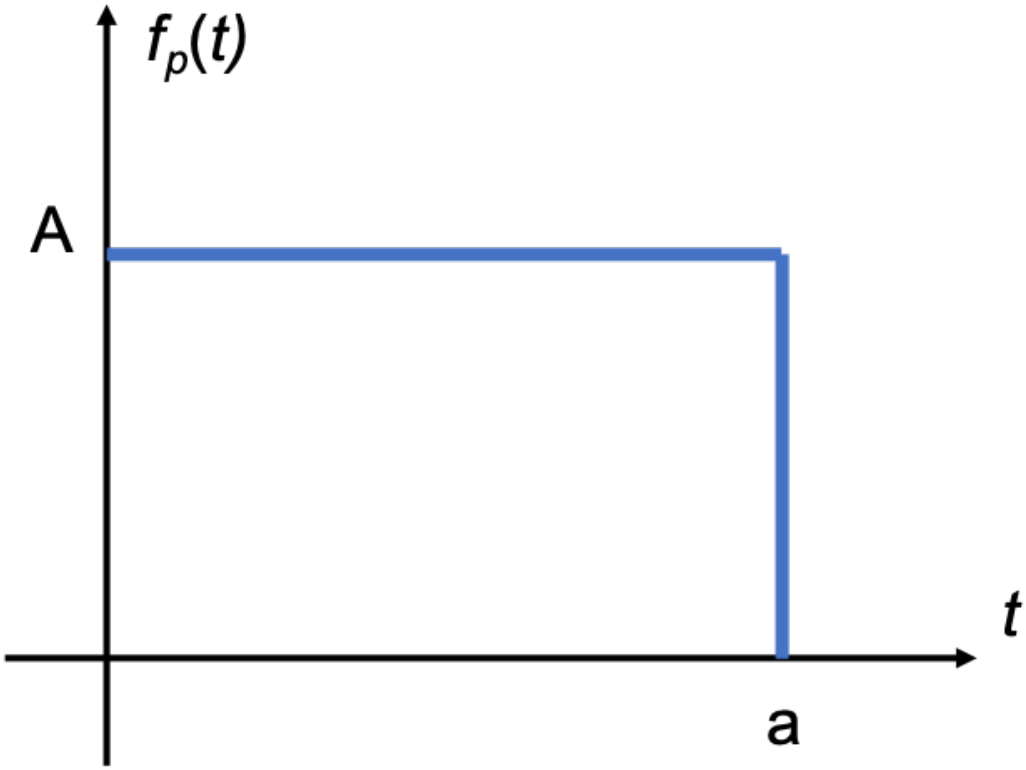
Laplace transforms of common waveforms

We will work through a few of the following on the board in class

- Pulse
- Linear segment
- Triangular waveform
- Rectangular periodic waveform (square wave)
- Half rectified sine wave

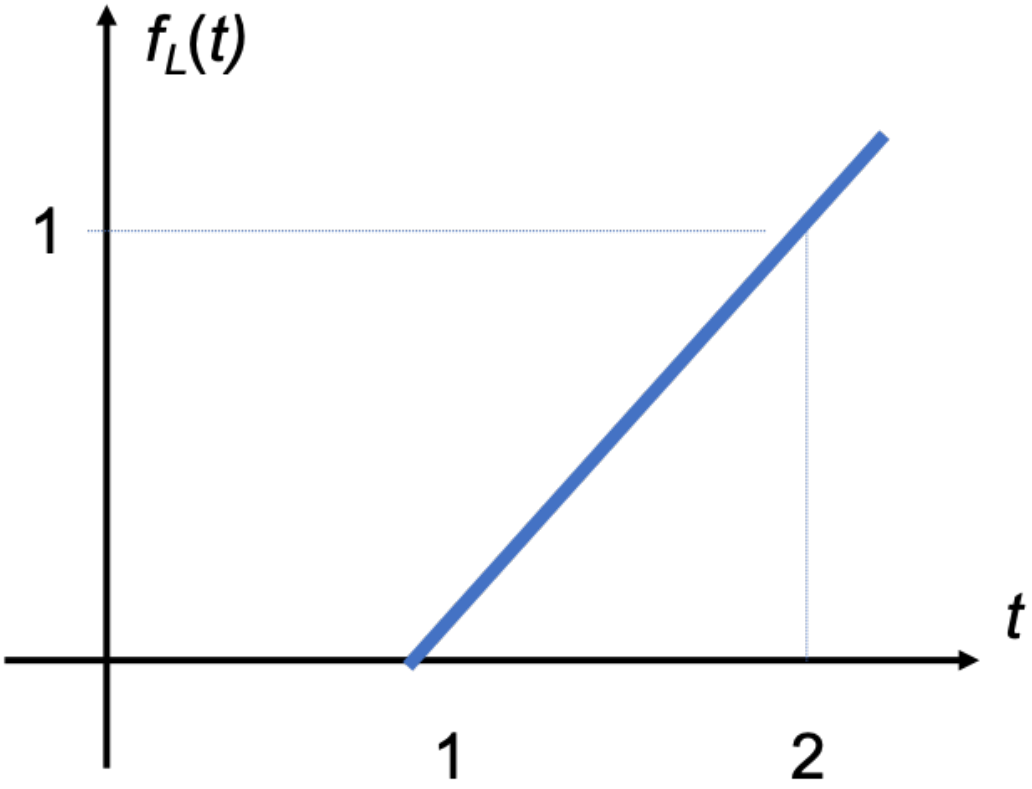
Pulse

Compute the Laplace transform of the pulse shown in the figure.


$f_p(t)$
A
t
a

Line segment

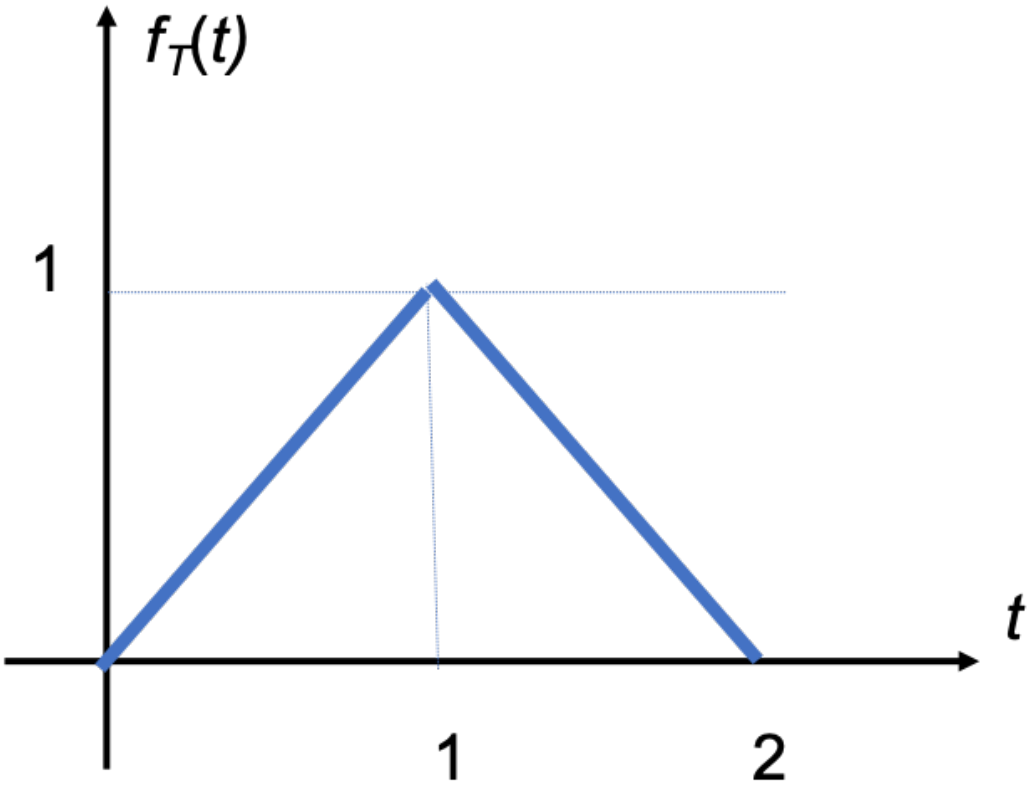
Compute the Laplace transform of the line segment shown below.



</pre>

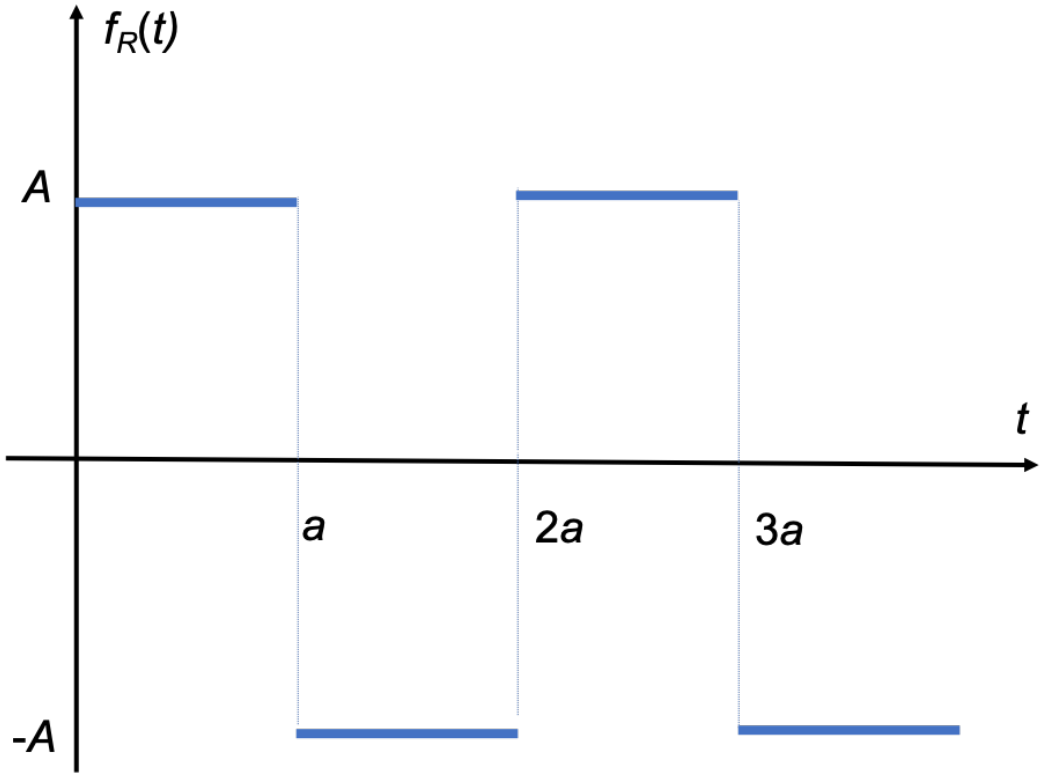
Triangular Pulse

Compute the Laplace transform of the triangular pulse shown below


$f_T(t)$
1
t
1
2

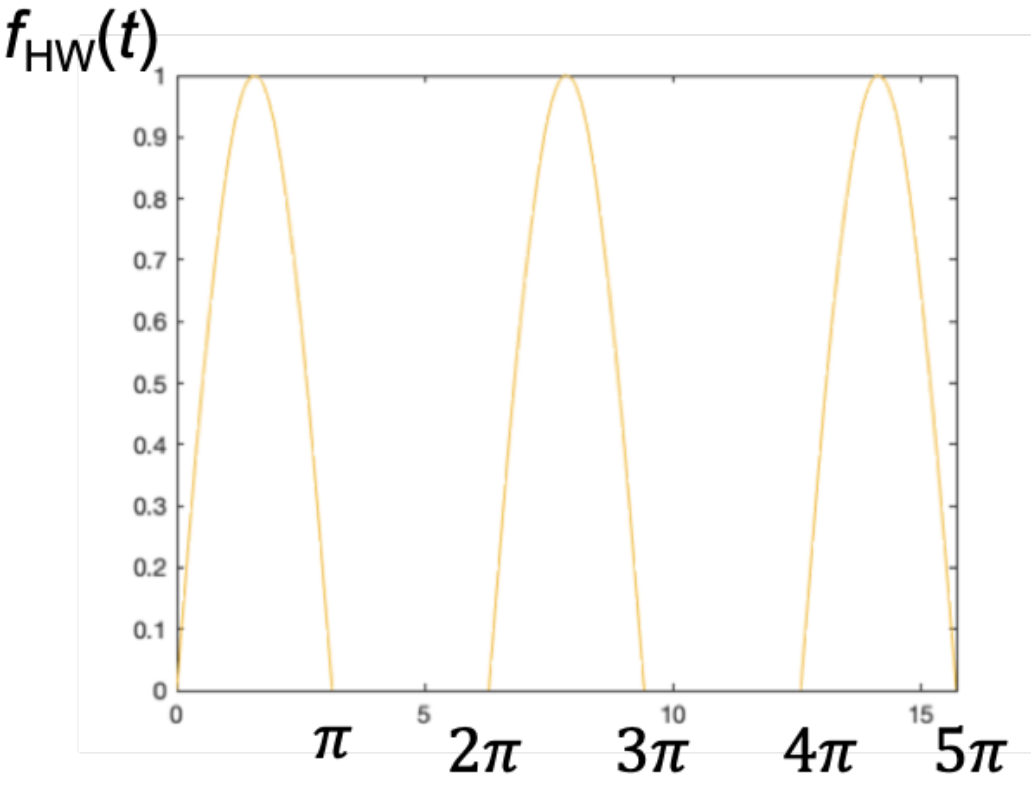
Square Wave

Compute the Laplace transform of the periodic function shown below.



Half-rectified Sinewave

Compute the Laplace Transform of the half-rectified sine wave shown below.



Homework

Attempt at least one of the end-of-chapter exercises from each question 1-7 of Section 2.7 of (Karris, 2012). Don't look at the answers until you have attempted the problems.

If we have time, I will work through one or two of these in class.

References

1. Karris, S. T. (2012). *Signals and systems with MATLAB computing and Simulink modeling*. Fremont, CA.: Orchard Publishing. Retrieved from https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197

Answers to in-class problems

1. $$Au_0(t) - Au_0(t-a) \Leftrightarrow \frac{A\left(1-e^{-as}\right)}{s}.$$

2. $$(t-1)u_0(t-1) \Leftrightarrow \frac{e^{-s}}{s}.$$

3. $$f_T(t) \Leftrightarrow \frac{\left(1-e^{-s}\right)^2}{s^2}.$$

4. $$f_R(t) \Leftrightarrow \frac{A\tanh\left(\frac{As}{2}\right)}{s}.$$

5. $$f_{HW}(t) \Leftrightarrow \frac{1}{\left(s^2+1\right)\left(1-e^{\pi s}\right)}.$$