

Lecturer: Set up MATLAB

In [17]:

```
clear all
format compact
```

## Worksheet 3

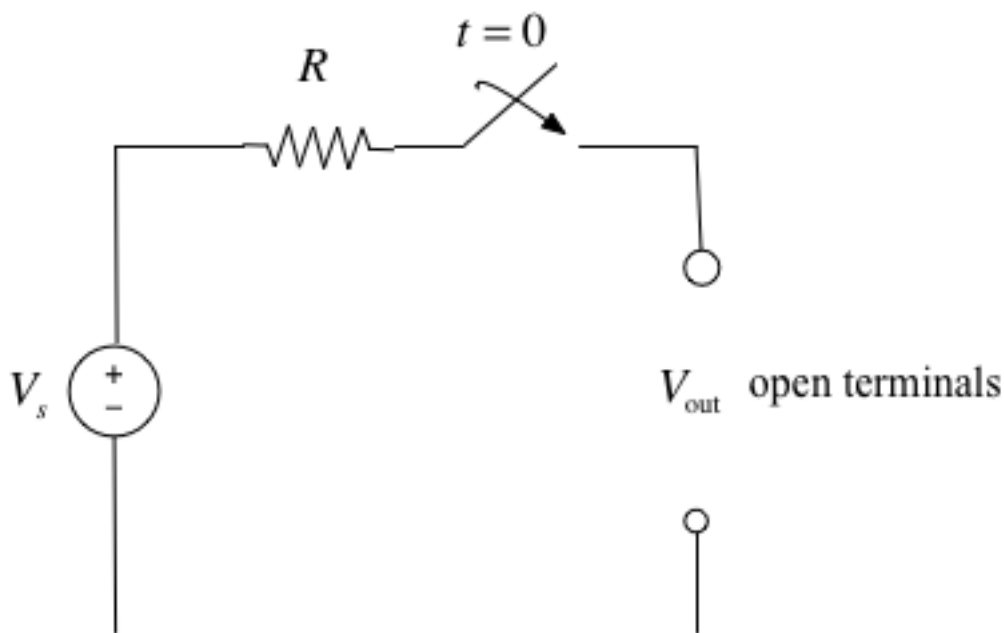
### To accompany Chapter 2 Elementary Signals

We will step through this worksheet in class.

You are expected to have at least watched the video presentation of [Chapter 2](https://cpjobling.github.io/eg-247-textbook/elementary_signals/index) ([https://cpjobling.github.io/eg-247-textbook/elementary\\_signals/index](https://cpjobling.github.io/eg-247-textbook/elementary_signals/index)) of the [notes](https://cpjobling.github.io/eg-247-textbook) (<https://cpjobling.github.io/eg-247-textbook>) before coming to the first class. If you haven't watch it afterwards!

## Elementary Signals

Consider this circuit, and assume that the switch is closed at  $t = 0$  and that all components are ideal:



Q1: What happens **before**  $t = 0$ ?

1.  $v_{\text{out}} = \text{undefined}$
2.  $v_{\text{out}} = 0$
3.  $v_{\text{out}} = V_s$
4.  $v_{\text{out}} = 1/2$
5.  $v_{\text{out}} = \infty$

-> **Open Poll**

Q2: What happens **after**  $t = 0$ ?

1.  $v_{\text{out}} = \text{undefined}$
2.  $v_{\text{out}} = 0$
3.  $v_{\text{out}} = V_s$
4.  $v_{\text{out}} = 1/2$
5.  $v_{\text{out}} = \infty$

-> **Open Poll**

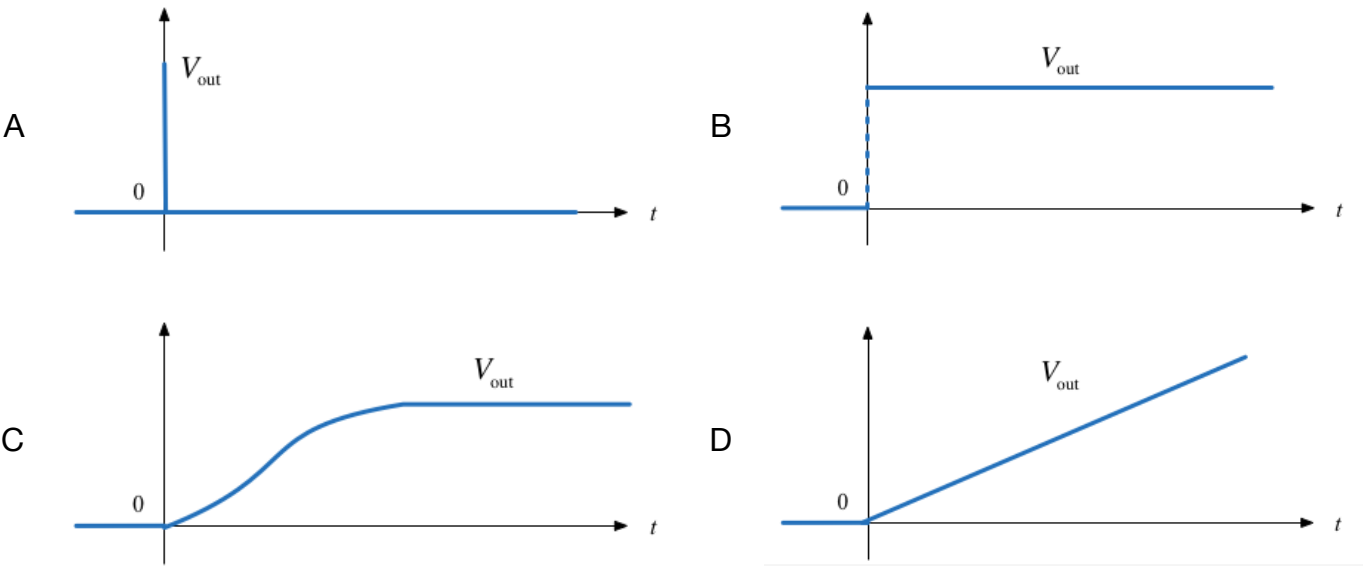
Q3: What happens **at**  $t = 0$ ?

1.  $v_{\text{out}} = \text{undefined}$
2.  $v_{\text{out}} = 0$
3.  $v_{\text{out}} = V_s$
4.  $v_{\text{out}} = 1/2$
5.  $v_{\text{out}} = \infty$

-> **Open Poll**

Q4: What does the response of  $V_{out}$  look like?

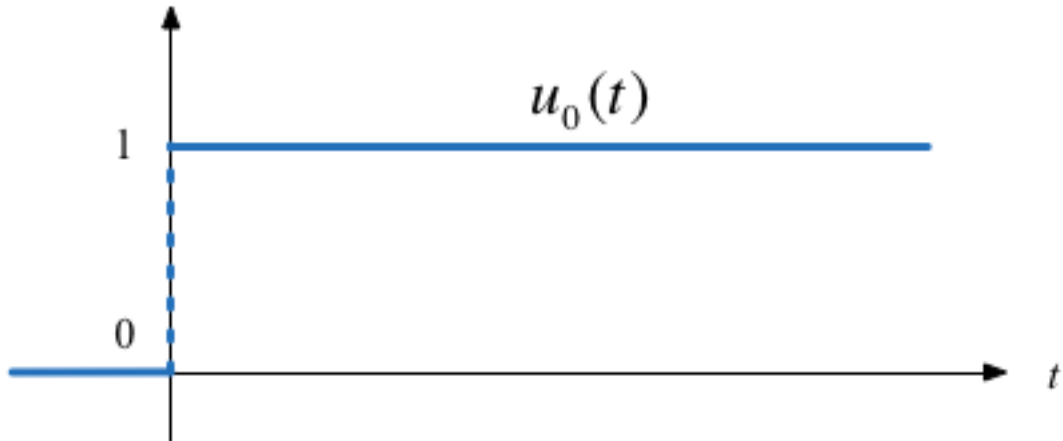
Circle the picture you think is correct on your handout.



-> Open Poll

# The Unit Step Function

$$u_0(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



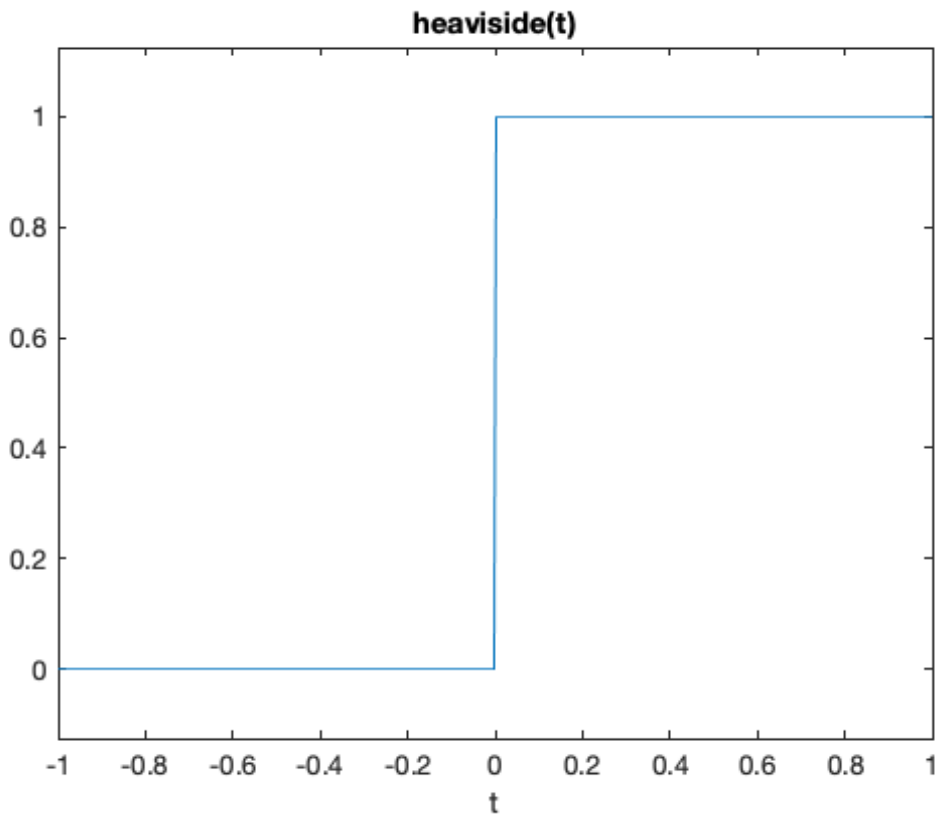
## In Matlab

In Matlab, we use the `heaviside` function (Named after [Oliver Heaviside](http://en.wikipedia.org/wiki/Oliver_Heaviside) ([http://en.wikipedia.org/wiki/Oliver\\_Heaviside](http://en.wikipedia.org/wiki/Oliver_Heaviside))).

In [16]:

```
syms t
ezplot heaviside(t), [-1, 1])
heaviside(0)
```

```
ans =
    0.5000
```



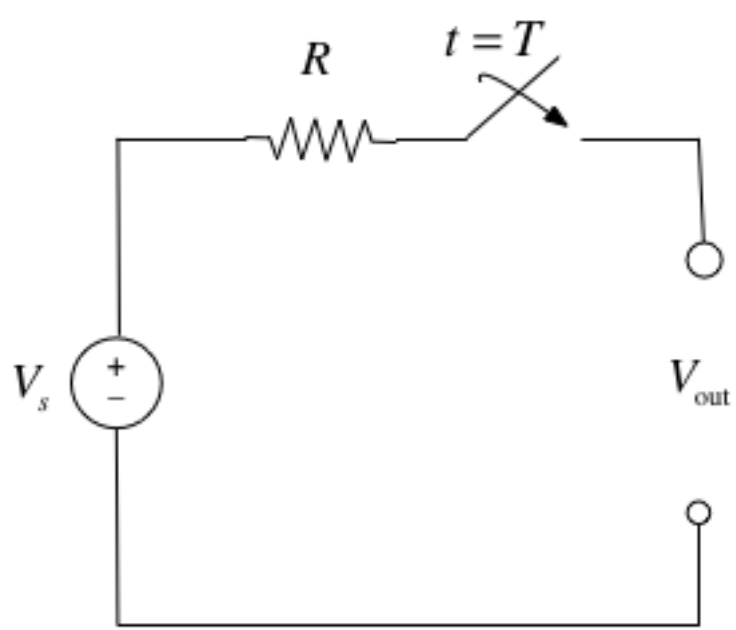
See: [heaviside\\_function.m \(matlab/heaviside\\_function.m\)](http://matlab/heaviside_function.m)

Note that, so it can be plotted, Matlab defines the *heaviside function* slightly differently from the mathematically ideal unit step:

$$\text{heaviside}(t) = \begin{cases} 0 & t < 0 \\ 1/2 & t = 0 \\ 1 & t > 0 \end{cases}$$

# Circuit Revisited

Consider the network shown below, where the switch is closed at time  $t = T$ .



Express the output voltage  $v_{out}$  as a function of the unit step function, and sketch the appropriate waveform.



## Simple Signal Operations

### Amplitude Scaling

Sketch  $Au_0(t)$  and  $-Au_0(t)$



**Time Reversal**

Sketch  $u_0(-t)$





# Time Delay and Advance

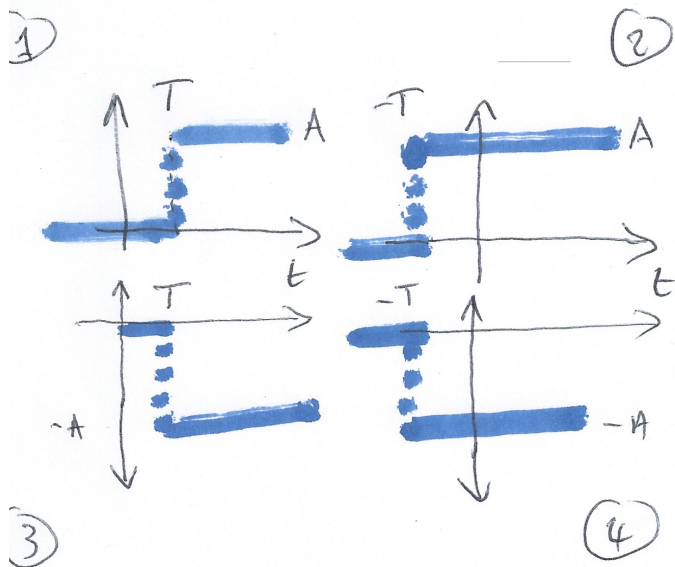
Sketch  $u_0(t - T)$  and  $u_0(t + T)$



## Examples

### Example 1

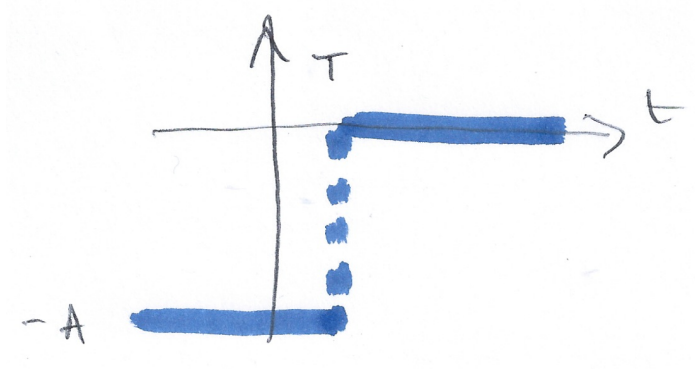
Which of these signals represents  $-Au_0(t + T)$ ?



-> Open Poll

### Example 2

What is represented by



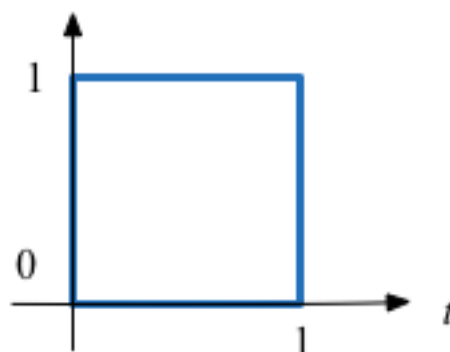
1.  $-Au_0(t + T)$
2.  $-Au_0(-t + T)$
3.  $-Au_0(-t - T)$
4.  $-Au_0(t - T)$

-> Open Poll

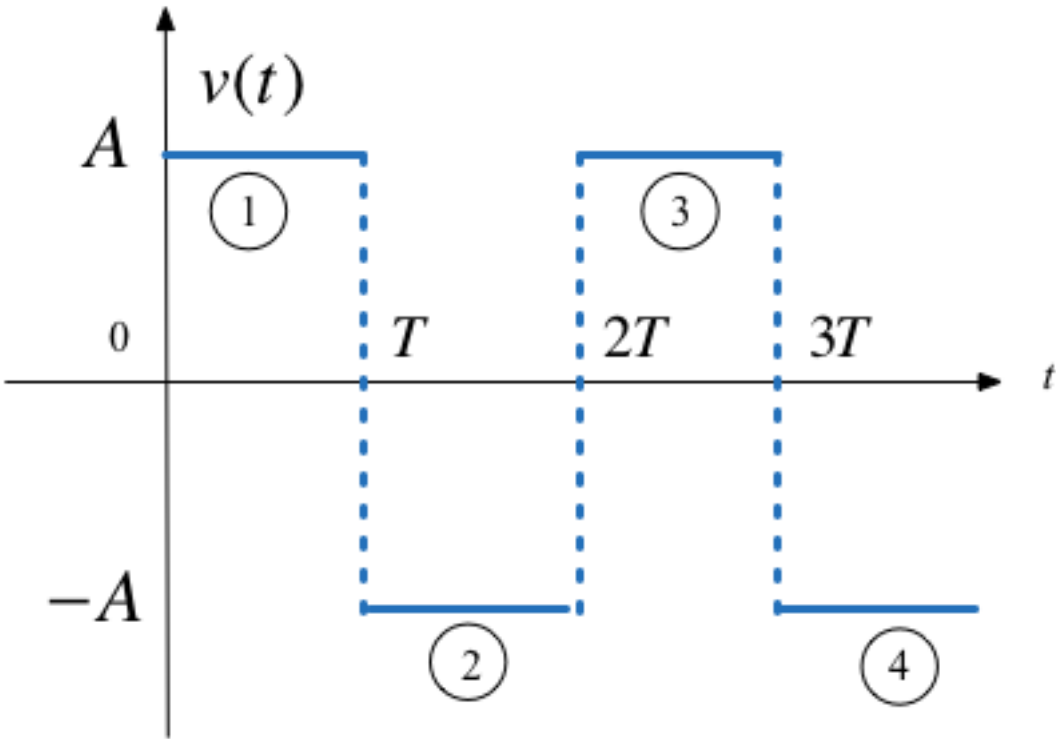
## Synthesis of Signals from Unit Step

Unit step functions can be used to represent other time-varying functions such as rectangular pulses, square waves and triangular pulses.

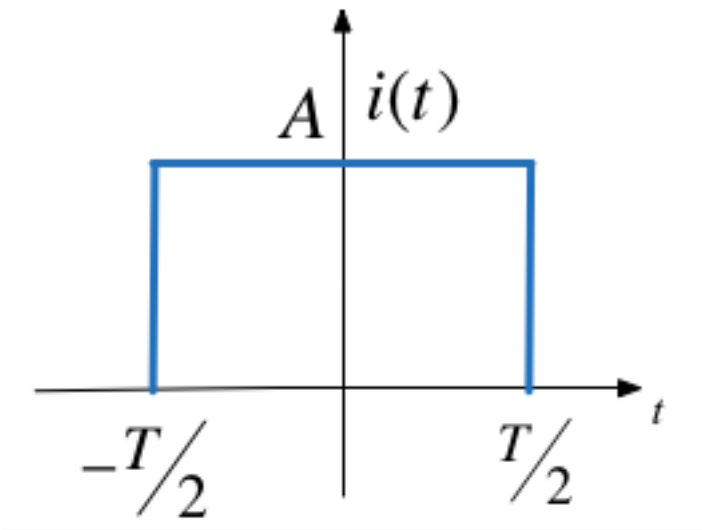
### Synthesize Rectangular Pulse



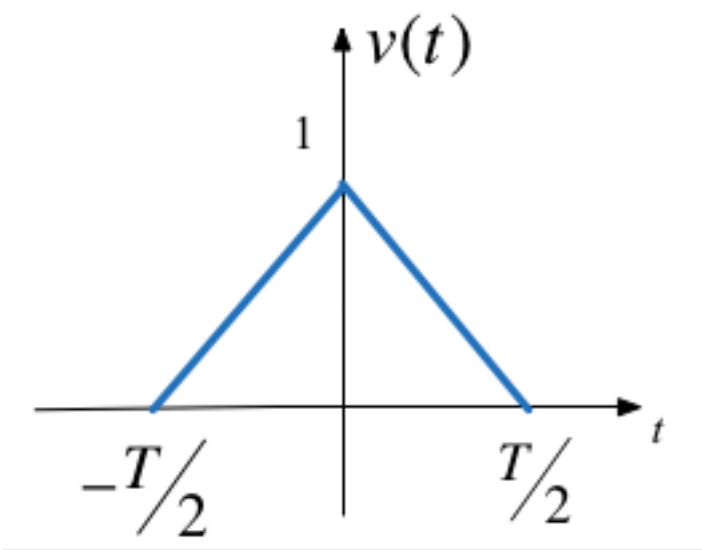
Synthesize Square Wave



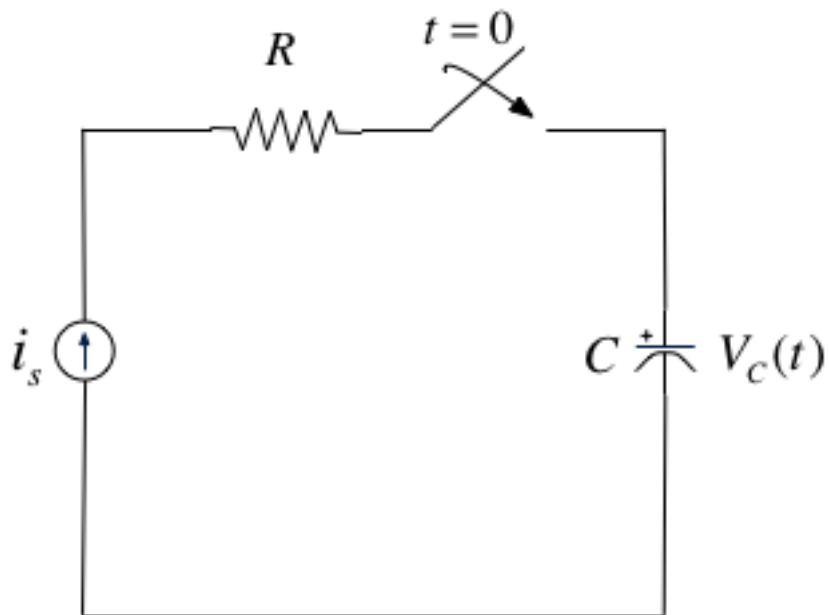
Synthesize Symmetric Rectangular Pulse



Synthesize Symmetric Triangular Pulse



# The Ramp Function

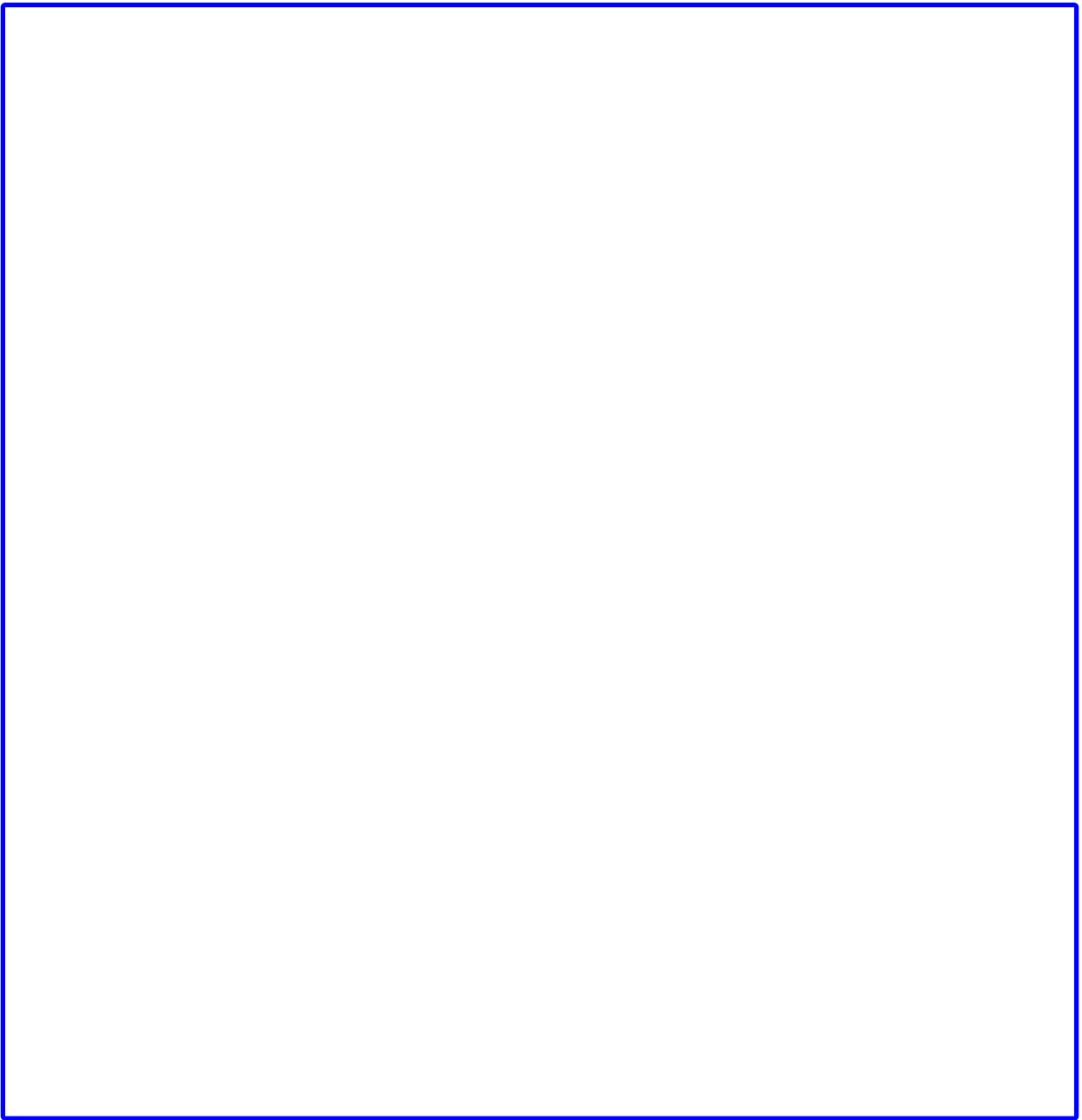


In the circuit shown above  $i_s$  is a constant current source and the switch is closed at time  $t = 0$ .

Show that the voltage across the capacitor can be represented as

$$v_C(t) = \frac{i_s}{C} t u_0(t)$$

and sketch the wave form.



The unit ramp function is defined as

$$u_1(t) = \int_{-\infty}^t u_0(\tau) d\tau$$

so

$$u_1(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$

and

$$u_0(t) = \frac{d}{dt} u_1(t)$$

## Note

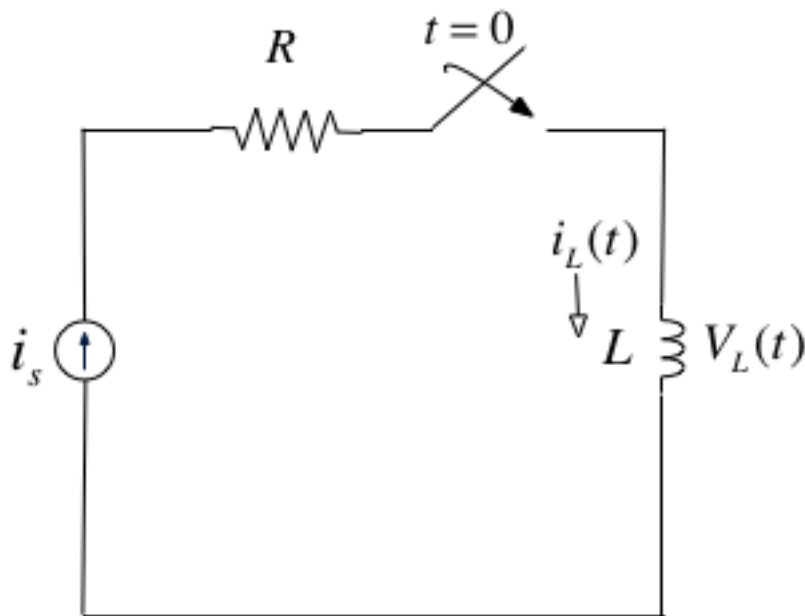
Higher order functions of  $t$  can be generated by the repeated integration of the unit step function.

For future reference, you should determine  $u_2(t)$ ,  $u_3(t)$  and  $u_n(t)$  for yourself and make a note of the general rule:

$$u_{n-1} = \frac{1}{n} \frac{d}{dt} u_n(t)$$

Details are given in equations 1.26—1.29 in the textbook.

## The Dirac Delta Function



In the circuit shown above, the switch is closed at time  $t = 0$  and  $i_L(t) = 0$  for  $t < 0$ . Express the inductor current  $i_L(t)$  in terms of the unit step function and hence derive an expression for  $v_L(t)$ .





## Notes

To solve this problem we need to invent a function that represents the derivative of the unit step function. This function is called  $\delta(t)$  or the *dirac delta* function (named after [Paul Dirac](http://en.wikipedia.org/wiki/Paul_Dirac) ([http://en.wikipedia.org/wiki/Paul\\_Dirac](http://en.wikipedia.org/wiki/Paul_Dirac))).

## The delta function

The unit impulse or the delta function, denoted as  $\delta(t)$ , is the derivative of the unit step.

This function is tricky because  $u_0(t)$  is discontinuous at  $t = 0$  but it must have the properties

$$\int_{-\infty}^t \delta(\tau) d\tau = u_0(t)$$

and

$\delta(t) = 0$  for all  $t \neq 0$ .

## Sketch of the delta function



## Important properties of the delta function

See the accompanying [notes \(index\)](#).

## Examples

### Example 3

Evaluate the following expressions

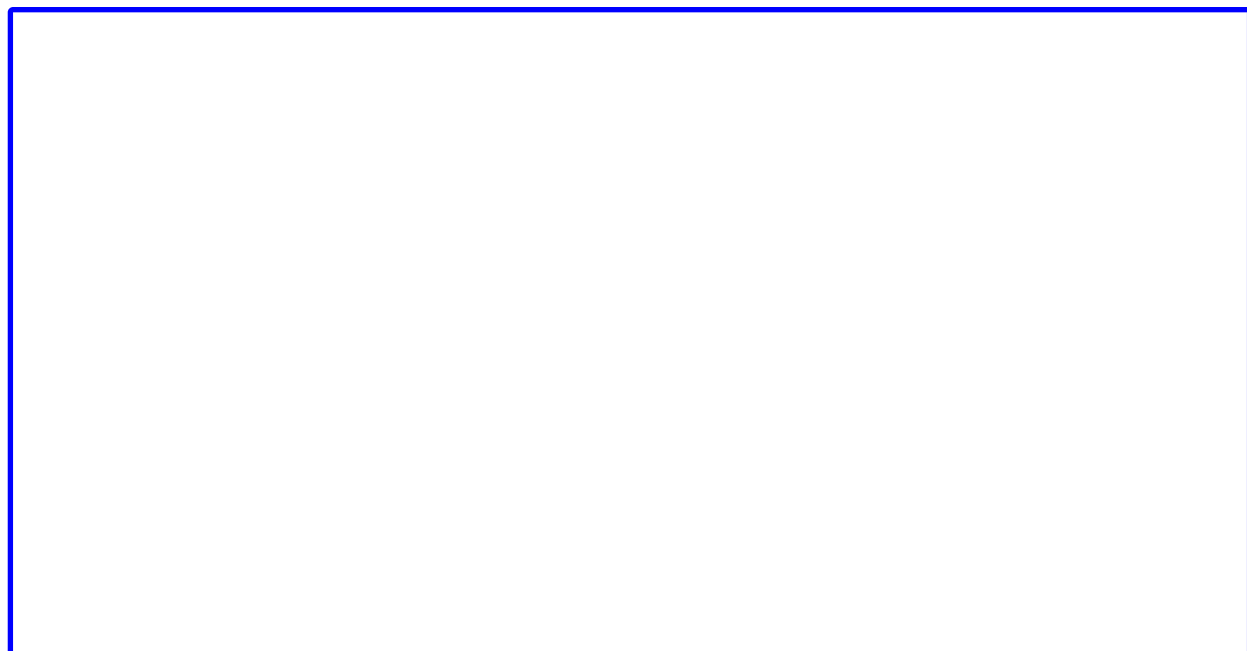
$$3t^4\delta(t - 1)$$



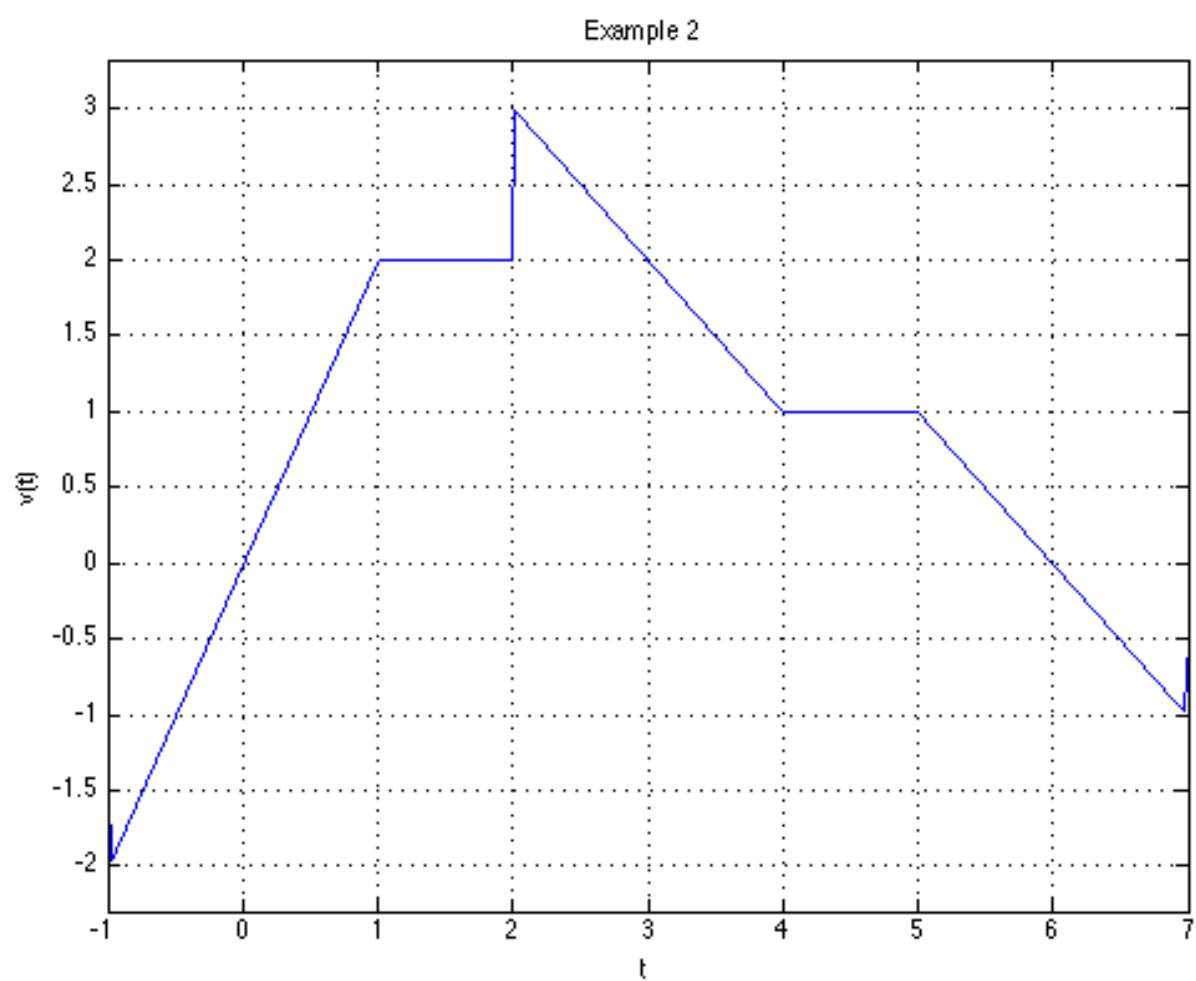
$$\int_{-\infty}^{\infty} t\delta(t-2)dt$$



$$t^2\delta'(t-3)$$



#### Example 4



(1) Express the voltage waveform  $v(t)$  shown above as a sum of unit step functions for the time interval  $-1 < t < 7$  s



Using the result of part (1), compute the derivative of  $v(t)$  and sketch its waveform.



# Lab Work

In the first lab, next Tuesday, we will solve further elementary signals problems using MATLAB and Simulink following the procedure given between pages 1-17 and 1-22 of the Karris. We will also explore the heaviside and dirac functions.

## Answers to in-class questions

Mathematically

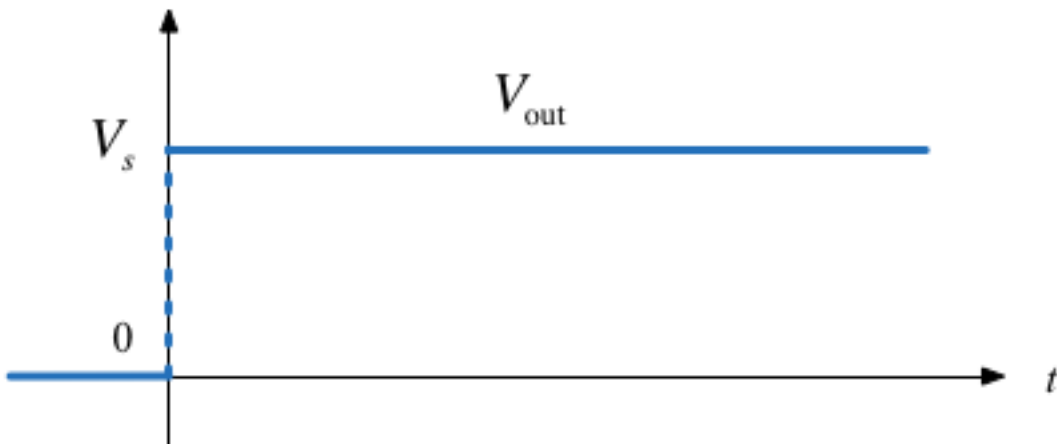
Q1.  $v_{\text{out}} = 0$  when  $-\infty < t < 0$  (answer 2)

Q2.  $v_{\text{out}} = V_s$  when  $0 < t < \infty$  (answer 3)

Q3.  $v_{\text{out}} = \text{undefined}$  when  $t = 0$  (answer 1)

$V_{\text{out}}$  jumps from 0 to  $V_s$  instantaneously when the switch is closed. We call this a discontinuous signal!

Q4: The correct image is:



Example 1: Answer 3.

Example 2: Answer 2.