Transfer Functions

The preparatory reading for this section is Chapter 4.4 of Karris (https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=75#ppg=113) which discusses transfer function models of electrical circuits.

An annotatable copy of the full notes for this presentation will be distributed before the third class meeting as **Worksheet 7** in the handouts section for week 3 in the _*Content Library* of the **OneNote Class Notebook.** You can also view the notes for this presentation as a webpage (HTML (https://cpjobling.github.io/eg-247-textbook/laplace_transform/4/transfer_functions.html)) and as a downloadable PDF file (https://cpjobling.github.io/eg-247-textbook/laplace_transform/4/transfer_functions.pdf).

Agenda

- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

In [21]:

```
% Matlab setup
cd ../matlab
pwd
clear all
format compact
```

```
ans =
    '/Users/eechris/dev/eg-247-textbook/content/laplace_transform/ma
tlab'
```

Transfer Functions for Circuits

When doing circuit analysis with components defined in the complex frequency domain, the ratio of the output voltage $V_{\text{out}}(s)$ to the input voltage $V_{\text{in}}(s)$ *under zero initial conditions* is of great interest.

This ratio is known as the *voltage transfer function* denoted $G_v(s)$:

$$G_v(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

Similarly, the ratio of the output current $I_{\text{out}}(s)$ to the input current $I_{\text{in}}(s)$ *under zero initial conditions*, is called the *cuurent transfer function* denoted $G_i(s)$:

$$G_i(s) = \frac{I_{\text{out}}(s)}{I_{\text{in}}(s)}$$

In practice, the current transfer function is rarely used, so we will use the voltage transfer function denoted:

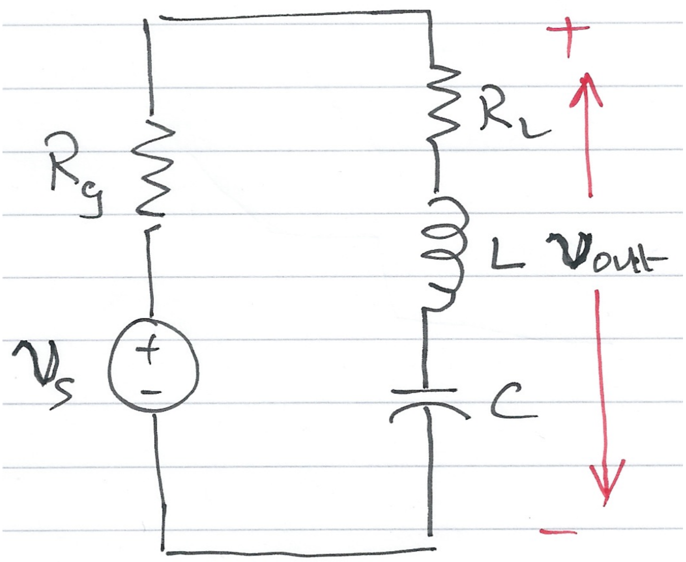
$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)}$$

Examples

See worksheet7 (worksheet7) for the worked solutions to the examples. We will work through these in class. Here' I'll demonstrate the MATLAB solutions.

Example 6

Derive an expression for the transfer function $G(s)$ for the circuit below. In this circuit R_g represents the internal resistance of the applied (voltage) source v_s, and R_L represents the resistance of the load that consists of R_L, L and C.



Sketch of Solution

- Replace $v_s(t)$, R_g, R_L, L and C by their transformed (*complex frequency*) equivalents: $V_s(s)$, R_g, R_L, sL and $1/(sC)$
- Use the *Voltage Divider Rule* to determine $V_{\text{out}}(s)$ as a function of $V_s(s)$
- Form $G(s)$ by writing down the ratio $V_{\text{out}}(s)/V_s(s)$

Worked solution.

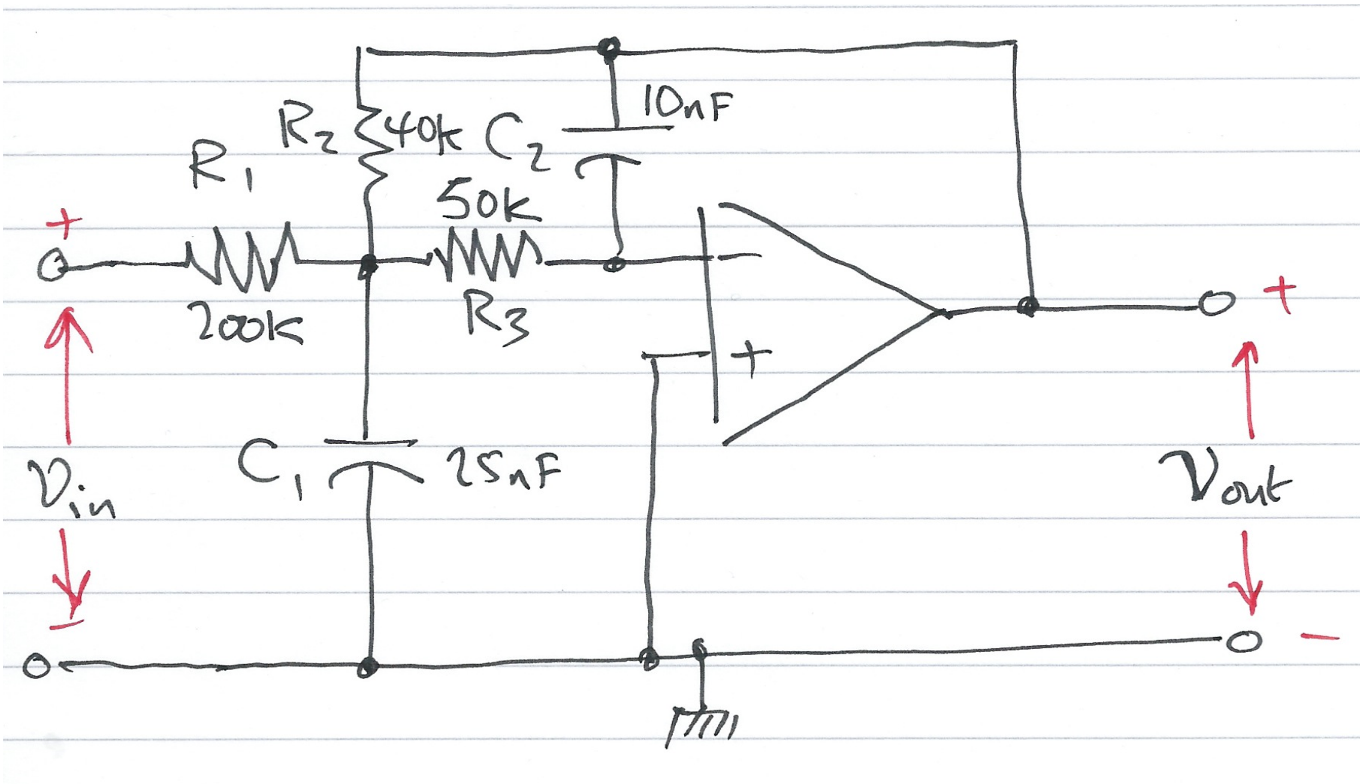
Pencast: ex6.pdf (../worked_examples/ex6.pdf) - open in Adobe Acrobat Reader.

Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants R_1, R_2, R_3, C_1 and C_2.



Then replace the complex variable s with $j\omega$, and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency ω rad/s.

Sketch of Solution

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents

- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor R_3

- Note that the voltage at the input to the op-amp is a virtual ground

- Solve for $V_{\text{out}}(s)$ as a function of $V_{\text{in}}(s)$

- Form the reciprocal $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$

- Use MATLAB to calculate the component values, then replace s by $j\omega$.

- Plot

$$|G(j\omega)|$$

  on log-linear "paper".

Worked solution.

Pencast: ex7.pdf (../worked_examples/ex7.pdf) - open in Adobe Acrobat Reader.

Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1\left(\left(1/R_1 + 1/R_2 + 1/R_3 + sC_1\right)\left(sC_2R_3\right) + 1/R_2\right)}.$$

The Matlab Bit

See attached script: solution7.m (../matlab/solution7.m).

Week 3: Solution 7

In [22]:

```
syms s;
```

In [23]:

```
R1 = 200*10^3;
R2 = 40*10^3;
R3 = 50*10^3;

C1 = 25*10^(-9);
C2 = 10*10^(-9);
```

In [24]:

```
den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2);
simplify(den)
```

```
ans =
100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5
```

Simplify coefficients of s in denominator

In [25]:

```
format long
denG = sym2poly(ans)
```

```
denG =
   0.000002500000000   0.005000000000000   5.000000000000000
```

In [26]:

```
numG = -1;
```

Plot

For convenience, define coefficients a and b:

In [27]:

```
a = denG(1);
b = denG(2);
```

$$G(j\omega) = \frac{-1}{a\omega^2 - jb\omega + 5}$$

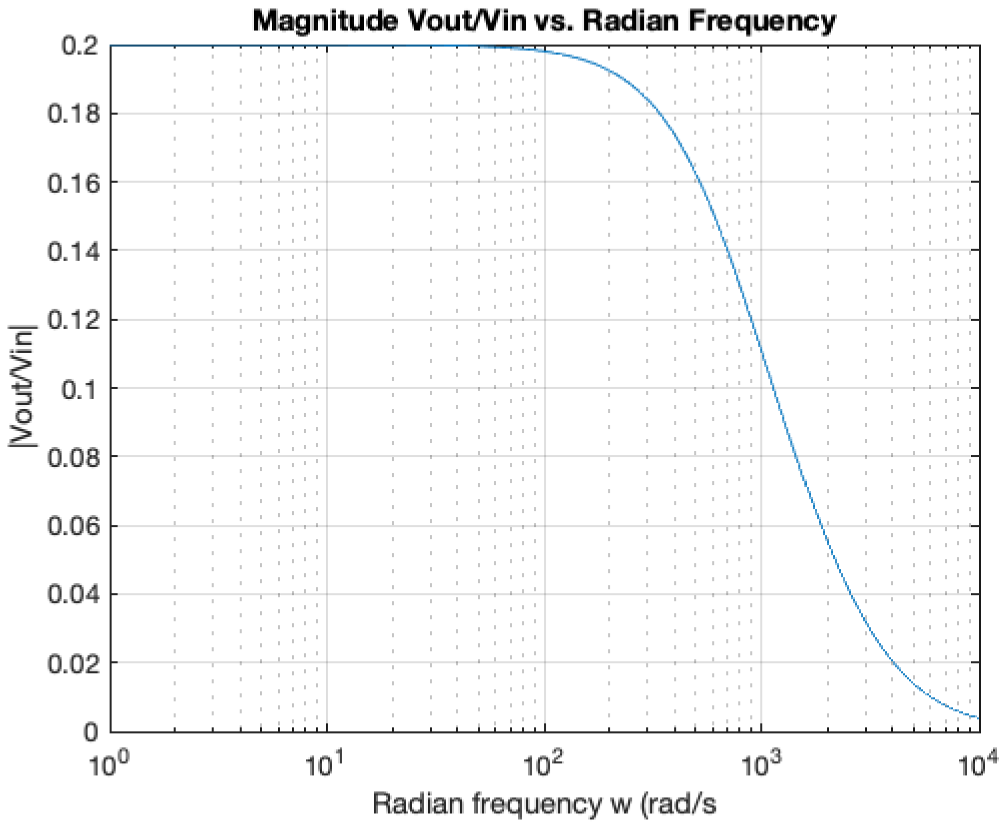
In [28]:

```
w = 1:10:10000;
Gs = -1./(a*w.^2 - j.*b.*w + denG(3));
```

Plot

In [29]:

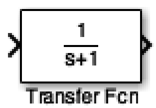
```
semilogx(w, abs(Gs))
xlabel('Radian frequency w (rad/s')
ylabel('|Vout/Vin|')
title('Magnitude Vout/Vin vs. Radian Frequency')
grid
```



Using Transfer Functions in MATLAB for System Analysis

Please use the file tf_matlab.m (../matlab/tf_matlab.m) to explore the Transfer Function features provide by MATLAB. Open the file as a Live Script to see a nicely formatted document.

Using Transfer Functions in Simulink for System Simulation



The Simulink transfer function (**`Transfer Fcn`**) block implements a transfer function

The transfer function block represents a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

and is not specific nor restricted to circuit analysis.

It can, however be used in modelling and simulation studies.

Example

Recast Example 7 as a MATLAB problem using the LTI Transfer Function block.

For simplicity use parameters $R_1 = R_2 = R_3 = 1\ \Omega$, and $C_1 = C_2 = 1$ F.

Calculate the step response using the LTI functions.

Verify the result with Simulink.

The Matlab solution: [example8.m (../matlab/example8.m)](../matlab/example8.m)

MATLAB Solution

From a previous analysis the transfer function is:

$$G(s) = \frac{V_{\mathrm{out}}}{V_{\mathrm{in}}} = \frac{-1}{R_1\left[(1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sR_3C_2) + 1/R_2\right]}$$

so substituting the component values we get:

$$G(s) = \frac{V_{\mathrm{out}}}{V_{\mathrm{in}}} = \frac{-1}{s^2 + 3s + 1}$$

We can find the step response by letting $v_{\mathrm{in}}(t) = u_0(t)$ so that $V_{\mathrm{in}}(s) = 1/s$ then

$$V_{\mathrm{out}}(s) = \frac{-1}{s^2 + 3s + 1}.\frac{1}{s}$$

We can solve this by partial fraction expansion and inverse Laplace transform as is done in the text book with the help of MATLAB's `residue` function.

Here, however we'll use the LTI block.

Define the circuit as a transfer function

In [30]:

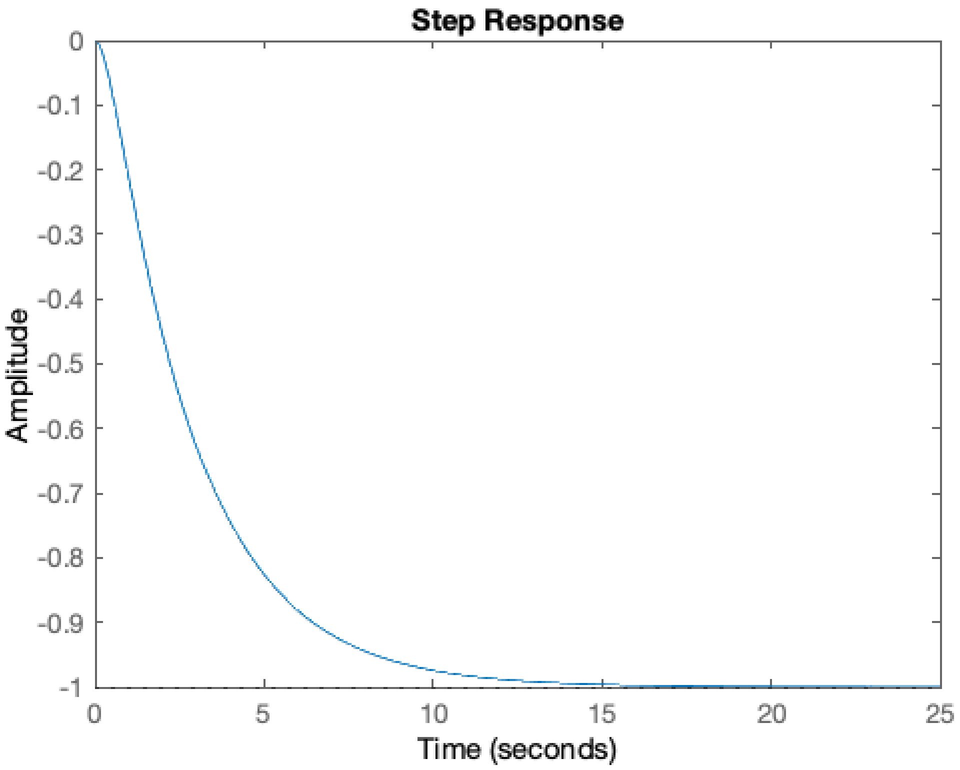
```
G = tf([-1],[1 3 1])
```

```
G =

       -1
  -------------
  s^2 + 3 s + 1

Continuous-time transfer function.
```

step response is then:

In [31]:

```
step(G)
```



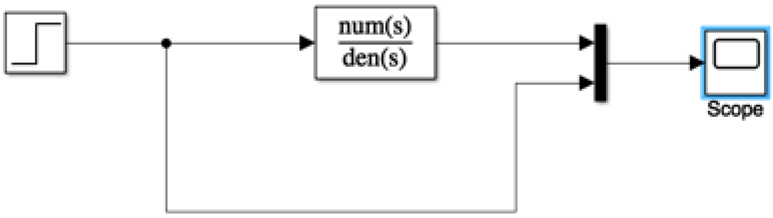
Simples!

Simulink model

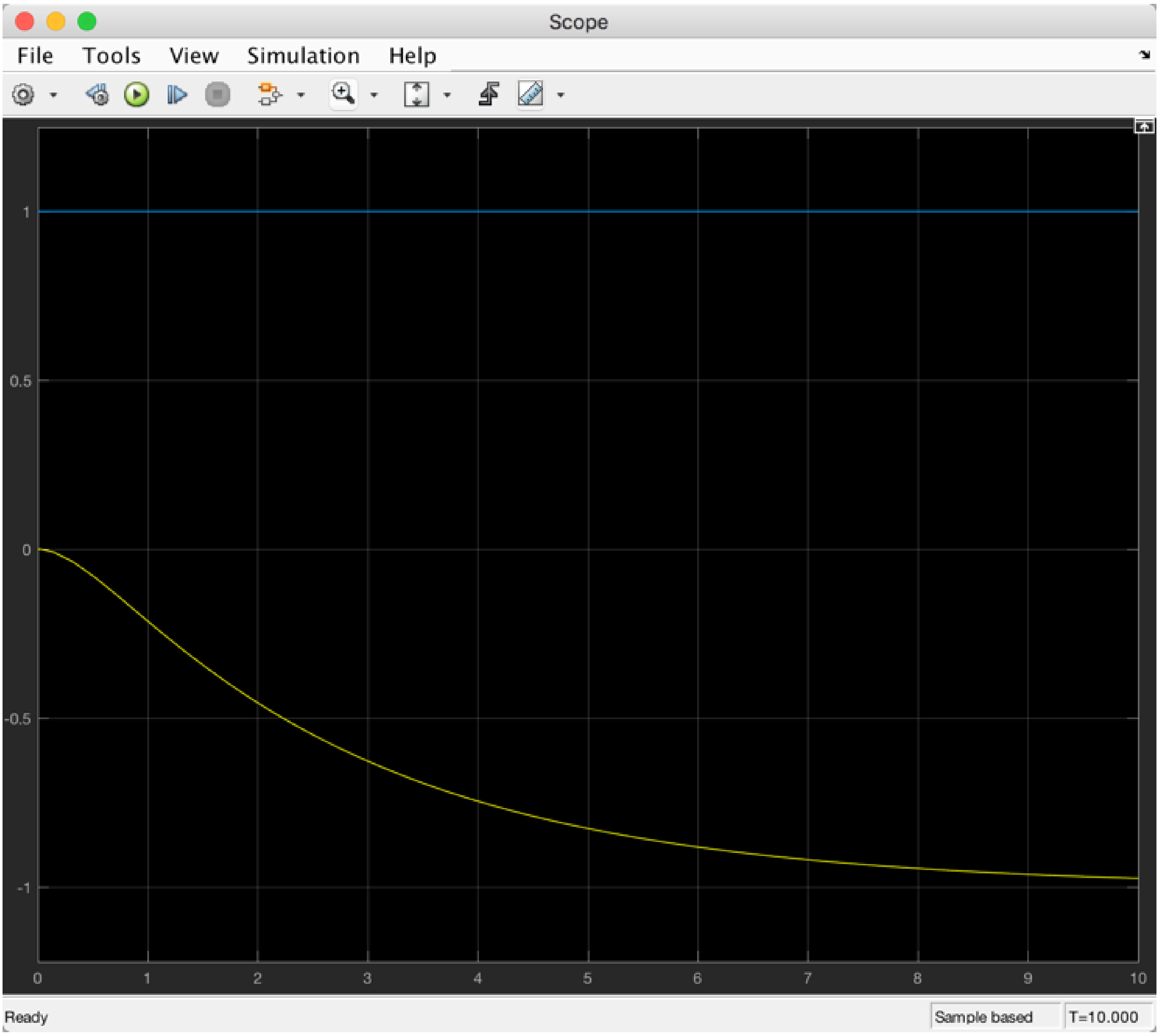
See [example_8.slx (../matlab/example_8.slx)](../matlab/example_8.slx)

In [32]:

```
open example_8
```



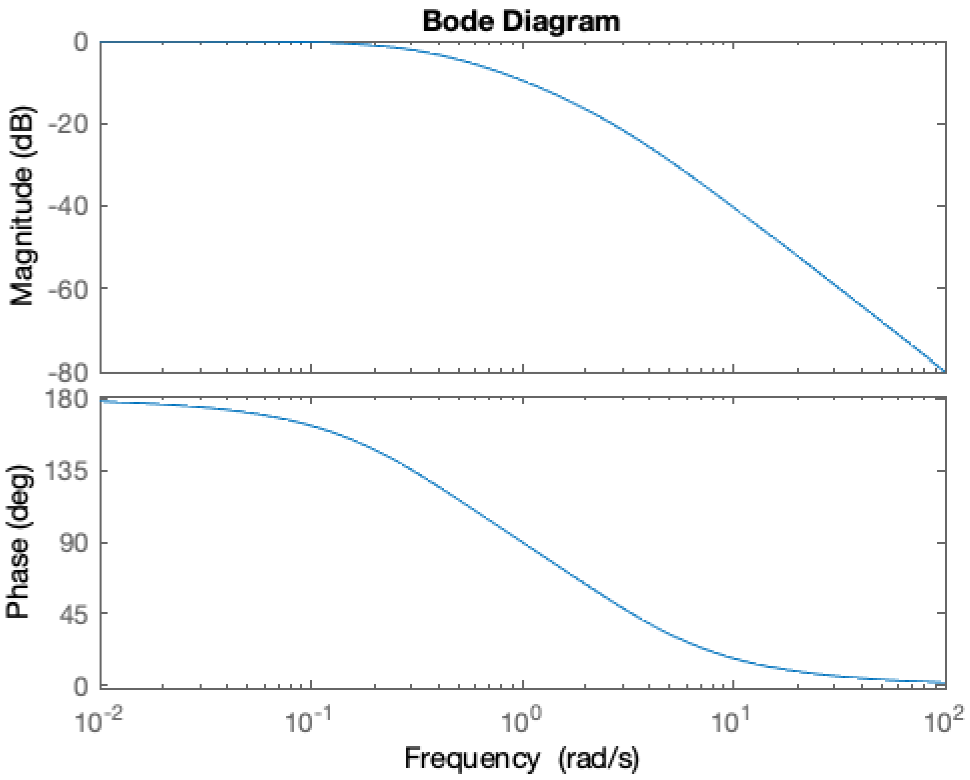
Result



Let's go a bit further by finding the frequency response:

In [33]:

```
bode(G)
```


Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
Phase (deg)
180
135
90
45
0
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)