

# Worksheet 9

## To accompany Chapter 4.1 Trigonometric Fourier Series

We will step through this worksheet in class.

You are expected to have at least watched the video presentation of [Chapter 4.1](https://cpjobling.github.io/eg-247-textbook/fourier_series/1/trig_fseries) ([https://cpjobling.github.io/eg-247-textbook/fourier\\_series/1/trig\\_fseries](https://cpjobling.github.io/eg-247-textbook/fourier_series/1/trig_fseries)) of the [notes](https://cpjobling.github.io/eg-247-textbook) (<https://cpjobling.github.io/eg-247-textbook>) before coming to class. If you haven't watch it afterwards!

## Motivating Example

In the class I will demonstrate the Fourier Series demo (see [Notes \(trig\\_fs\)](#)).

## Symmetry in Trigonometric Fourier Series

There are simplifications we can make if the original periodic properties has certain properties:

- If  $f(t)$  is odd,  $a_0 = 0$  and there will be no cosine terms so  $a_n = 0 \forall n > 0$
- If  $f(t)$  is even, there will be no sine terms and  $b_n = 0 \forall n > 0$ . The DC may or may not be zero.
- If  $f(t)$  has *half-wave symmetry* only the odd harmonics will be present. That is  $a_n$  and  $b_n$  is zero for all even values of  $n$  (0, 2, 4, ...)

## Odd, Even and Half-wave Symmetry

### Recall

- An *odd* function is one for which  $f(t) = -f(-t)$ . The function  $\sin t$  is an *odd* function.
- An *even* function is one for which  $f(t) = f(-t)$ . The function  $\cos t$  is an *even* function.

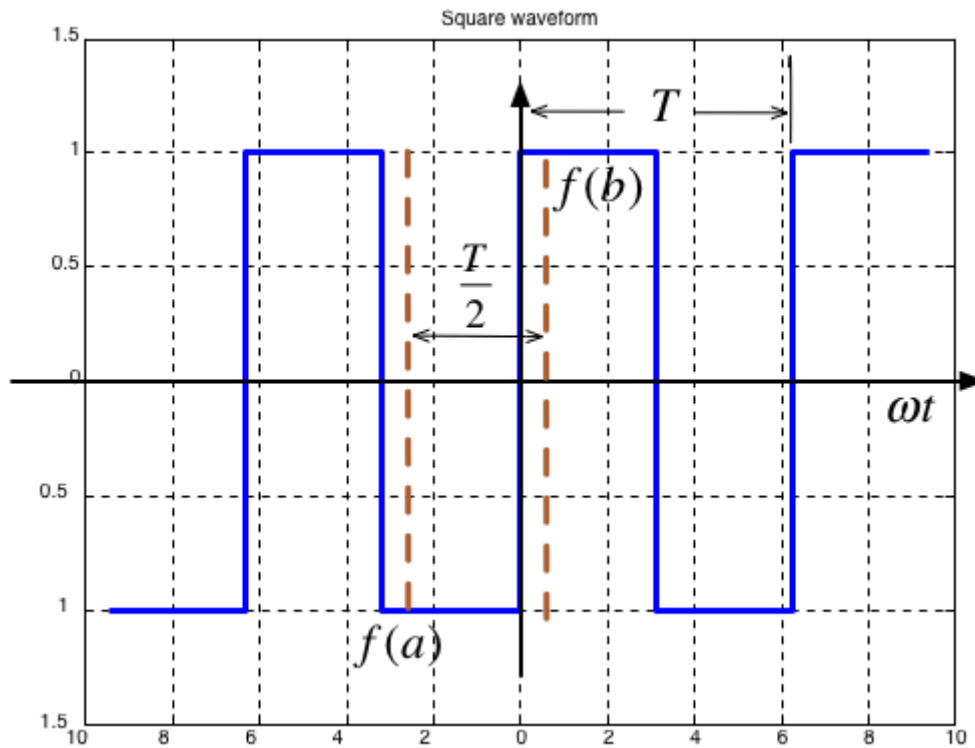
### Half-wave symmetry

- A periodic function with period  $T$  is a function for which  $f(t) = f(t + T)$
- A periodic function with period  $T$ , has *half-wave symmetry* if  $f(t) = -f(t + T/2)$

## Symmetry in Common Waveforms

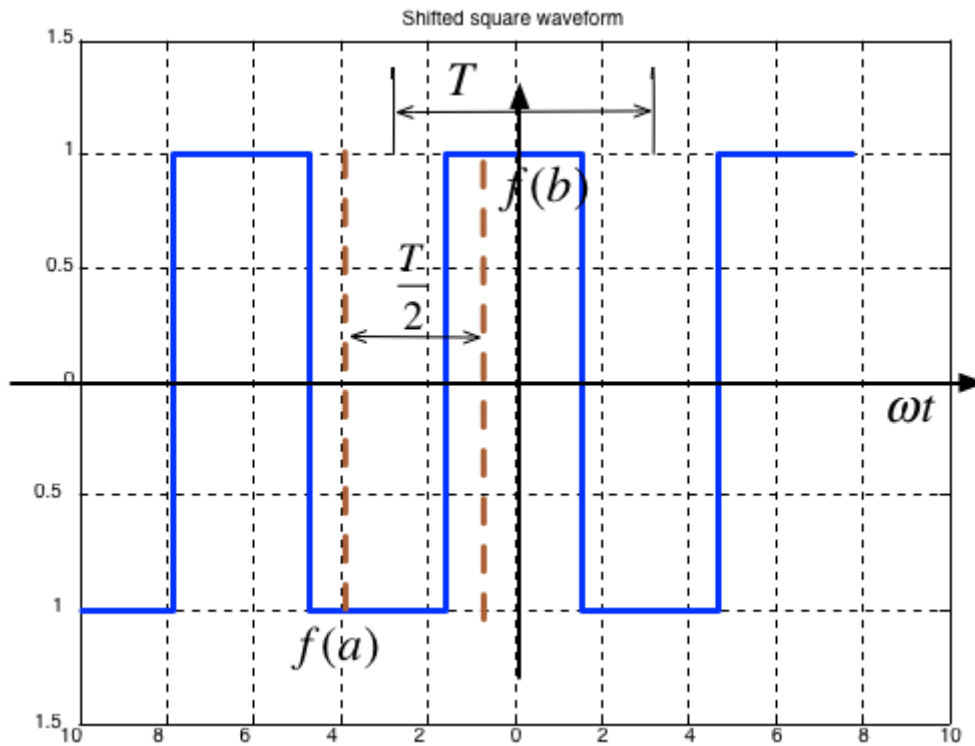
To reproduce the following waveforms (without annotation) publish the script [waves.m \(waves.m\)](#).

## Squarewave



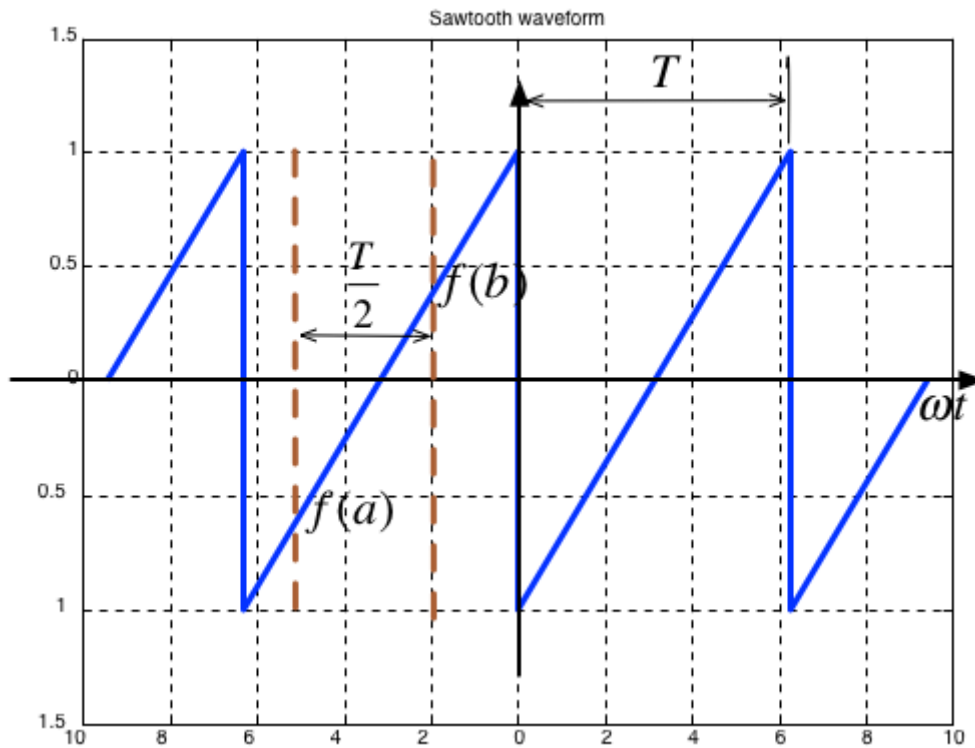
- Average value over period  $T$  is ...?
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

## Shifted Squarewave



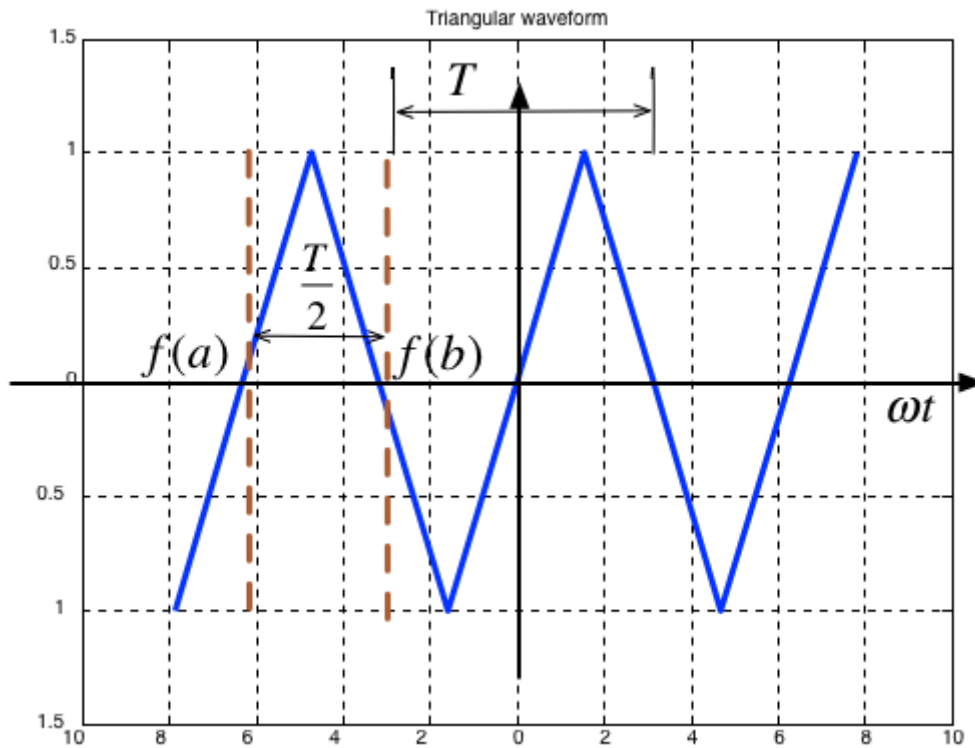
- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

## Sawtooth



- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

## Triangle

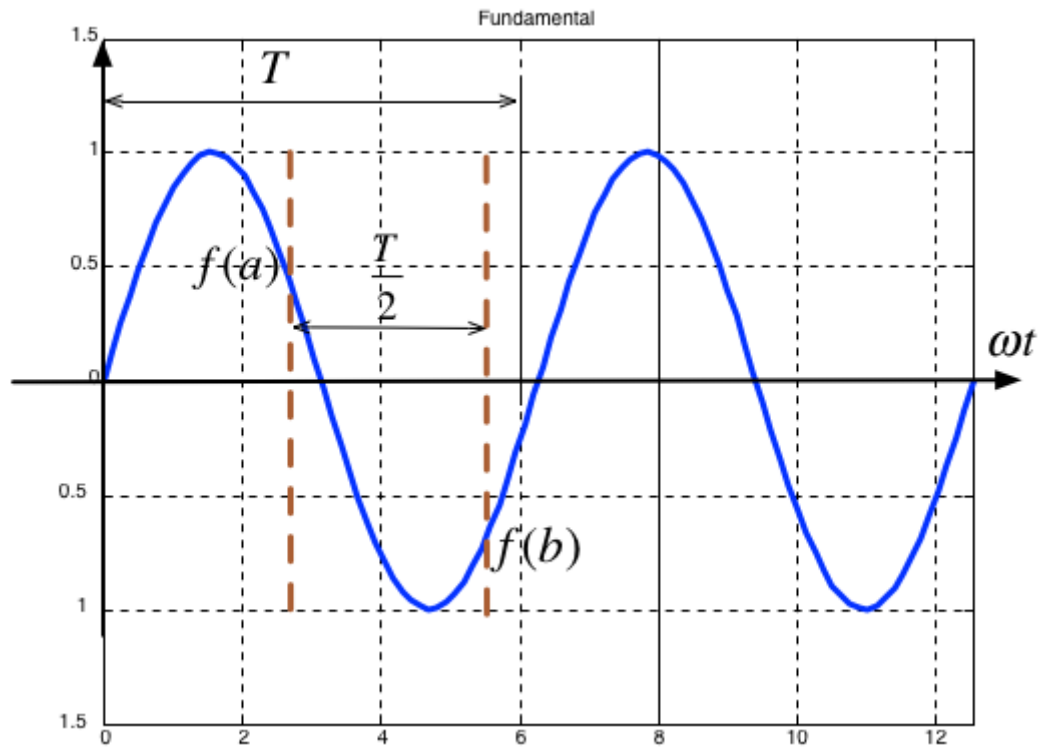


- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

## Symmetry in fundamental, Second and Third Harmonics

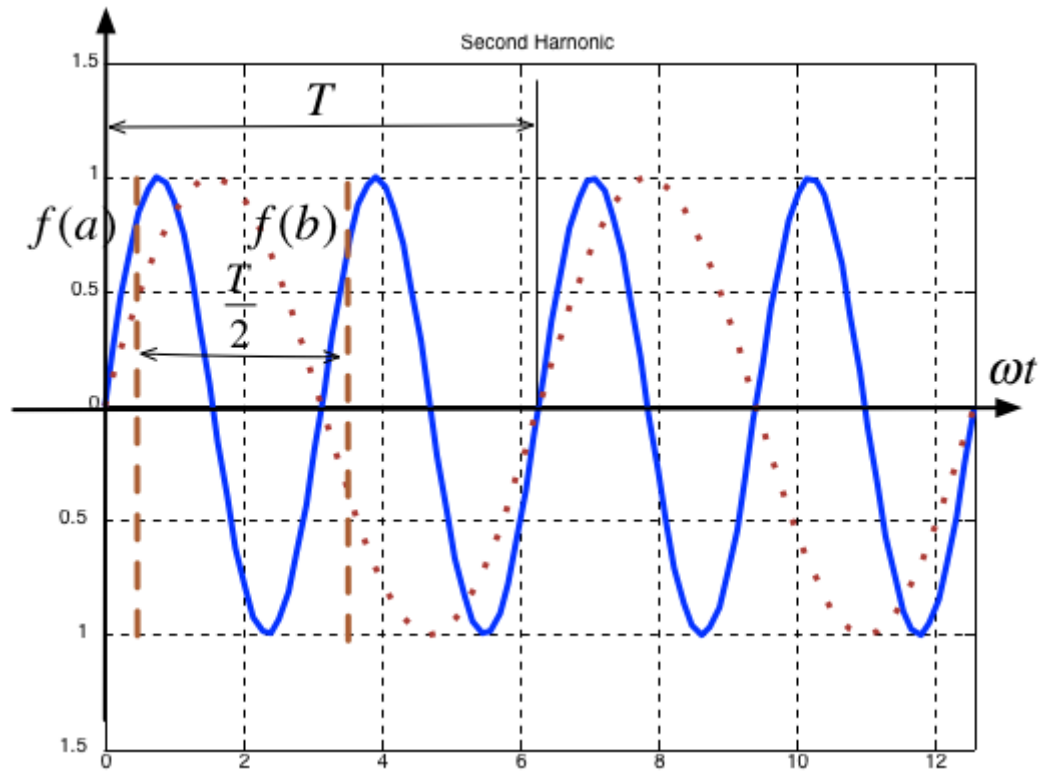
In the following,  $T/2$  is taken to be the half-period of the fundamental sinewave.

## Fundamental



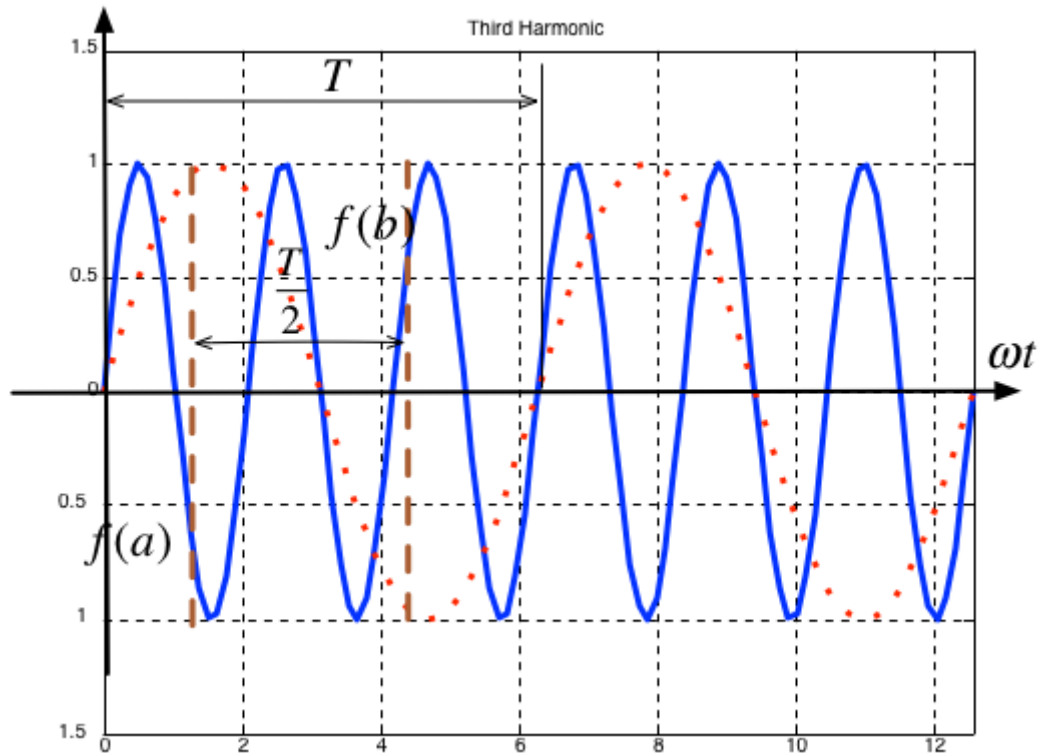
- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

## Second Harmonic



- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

### Third Harmonic



- Average value over period  $T$  is
- It is an **odd/even** function?
- It **has/has not** half-wave symmetry  $f(t) = -f(t + T/2)$ ?

### Some simplifications that result from symmetry

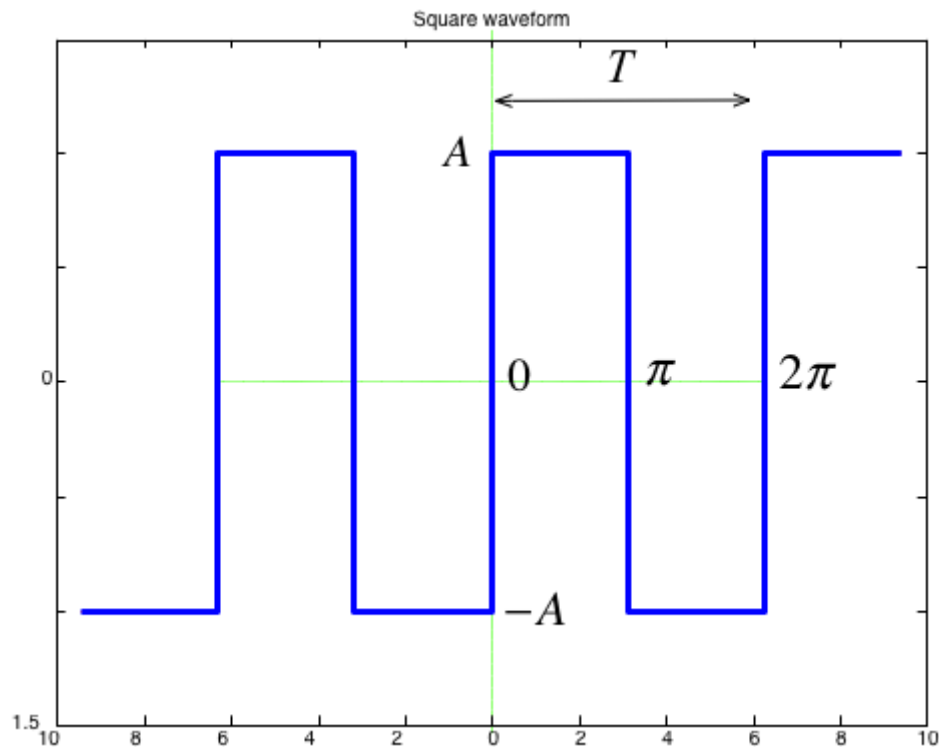
- The limits of the integrals used to compute the coefficients  $a_n$  and  $b_n$  of the Fourier series are given as  $0 \rightarrow 2\pi$  which is one period  $T$
- We could also choose to integrate from  $-\pi \rightarrow \pi$
- If the function is *odd*, or *even* or has *half-wave symmetry* we can compute  $a_n$  and  $b_n$  by integrating from  $0 \rightarrow \pi$  and multiplying by 2.
- If we have *half-wave symmetry* we can compute  $a_n$  and  $b_n$  by integrating from  $0 \rightarrow \pi/2$  and multiplying by 4.

(For more details see page 7-10 of the textbook)



# Computing coefficients of Trig. Fourier Series in Matlab

As an example let's take a square wave with amplitude  $\pm A$  and period  $T$ .



## Solution

Solution: See [square\\_ftrig.mlx](#) ([square\\_ftrig.mlx](#)). Script confirms that:

- $a_0 = 0$
- $a_i = 0$ : function is odd
- $b_i = 0$ : for  $i$  even - half-wave symmetry

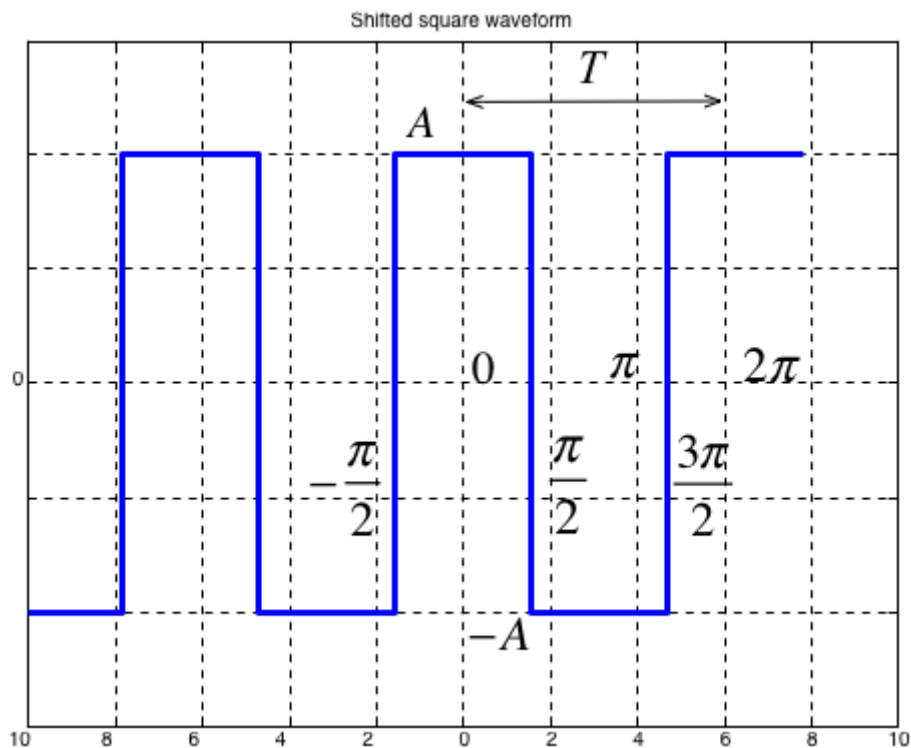
ft =

$$(4*A*\sin(t))/\pi + (4*A*\sin(3*t))/(3*\pi) + (4*A*\sin(5*t))/(5*\pi) + (4*A*\sin(7*t))/(7*\pi) + (4*A*\sin(9*t))/(9*\pi) + (4*A*\sin(11*t))/(11*\pi)$$

Note that the coefficients match those given in the textbook (Section 7.4.1).

$$f(t) = \frac{4A}{\pi} \left( \sin \Omega_0 t + \frac{1}{3} \sin 3\Omega_0 t + \frac{1}{5} \sin 5\Omega_0 t + \dots \right) = \frac{4A}{\pi} \sum_{n=\text{odd}} \frac{1}{n} \sin n\Omega_0 t$$

## Using symmetry - computing the Fourier series coefficients of the shifted square wave



- As before  $a_0 = 0$
- We observe that this function is even, so all  $b_k$  coefficients will be zero
- The waveform has half-wave symmetry, so only odd indexed coefficients will be present.
- Further more, because it has half-wave symmetry we can just integrate from  $0 \rightarrow \pi/2$  and multiply the result by 4.

See [shifted\\_sq\\_ftrig.mlx](#) ([shifted\\_sq\\_ftrig.mlx](#)).

`ft =`

```
(4*A*cos(t))/pi - (4*A*cos(3*t))/(3*pi) + (4*A*cos(5*t))/(5*pi) - (4*A*cos(7*t))/(7*pi) + (4*A*cos(9*t))/(9*pi) - (4*A*cos(11*t))/(11*pi)
```

Note that the coefficients match those given in the textbook (Section 7.4.2).

$$f(t) = \frac{4A}{\pi} \left( \cos \Omega_0 t - \frac{1}{3} \cos 3\Omega_0 t + \frac{1}{5} \cos 5\Omega_0 t - \dots \right) = \frac{4A}{\pi} \sum_{n=\text{odd}} (-1)^{\frac{n-1}{2}} \frac{1}{n} \cos n\Omega_0 t$$