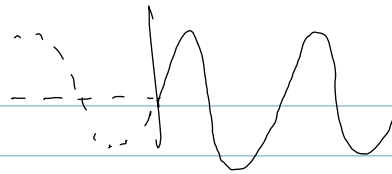Example 3

$$\sin \omega_0 t \; u_0(t) \iff \frac{\pi}{j2}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right]$$

$$+ \frac{\omega}{\omega_0^2 - \omega^2}$$

$$\sin \omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$= \frac{e^{j\omega_0 t} u_0(t) - e^{-j\omega_0 t} u_0(t)}{2j}$$

$$= \frac{\pi \delta(\omega-\omega_0) - \pi\delta(\omega+\omega_0)}{2j} +$$

$$\frac{1}{2j}\left[\frac{1}{j(\omega-\omega_0)} - \frac{1}{j(\omega+\omega_0)}\right]$$

$$= \frac{\pi}{j2}\left[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right] +$$

$$-\frac{1}{2}\left[\frac{1}{\omega+\omega_0} - \frac{1}{\omega-\omega_0}\right]$$

$$\frac{\omega-\omega_0-(\omega+\omega_0)}{(\omega+\omega_0)(\omega-\omega_0)} = \frac{-2\omega_0}{\omega^2 - \omega_0^2}$$

$$\implies \frac{\omega_0}{\omega^2 - \omega_0^2}$$