Using Laplace Transforms for Circuit Analysis

The preparatory reading for this section is Chapter 4 of Karris (https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=75#ppg=101) which

- presents examples of the applications of the Laplace transform for electrical solving circuit problems.

An annotatable copy of the full notes for this presentation will be distributed before the third class meeting as **Worksheet 6** in the handouts section for week 3 in the _*Content Library* of the **OneNote Class Notebook**. You can also view the notes for this presentation as a webpage (HTML (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.html)) and as a downloadable PDF file (https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/circuit_analysis.pdf).

Agenda

We look at applications of the Laplace Transform for

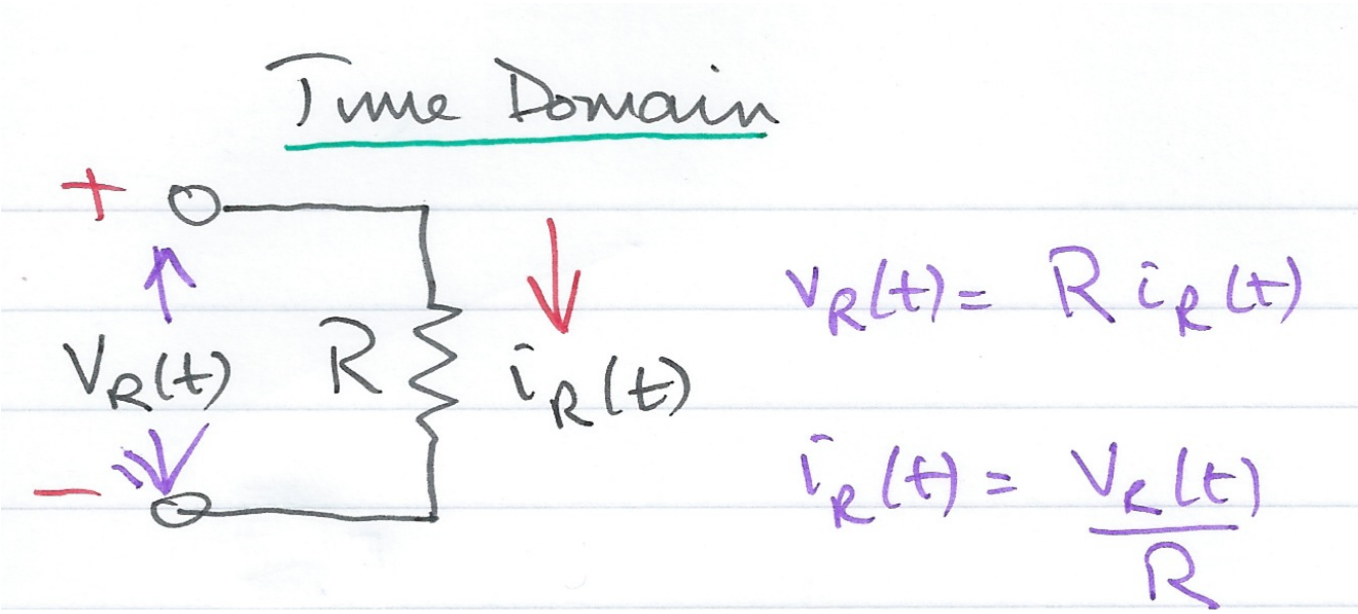
- Circuit transformation from Time to Complex Frequency

- Complex impedance

- Complex admittance

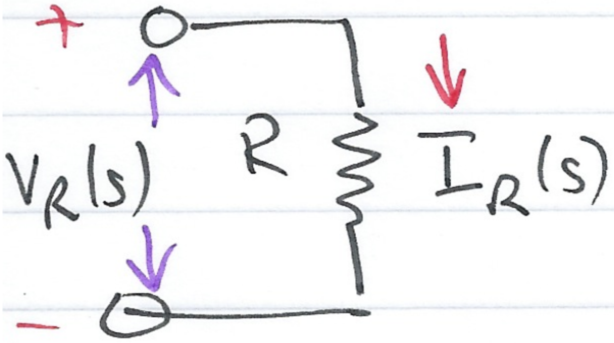
Circuit Transformation from Time to Complex Frequency

Resistive Network - Time Domain



Resistive Network - Complex Frequency Domain

Frequency Domain

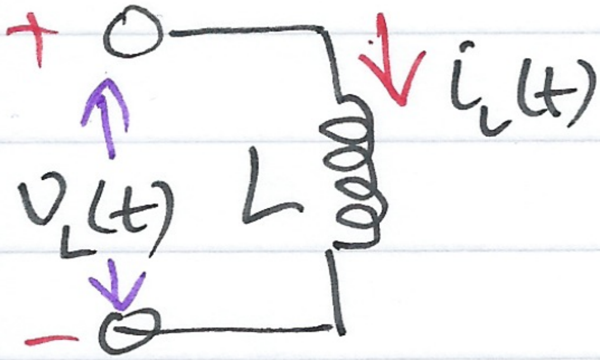


$$V_R(s) = R\, I_R(s)$$

$$I_R(s) = \frac{V_R(s)}{R}$$

Inductive Network - Time Domain

Time Domain

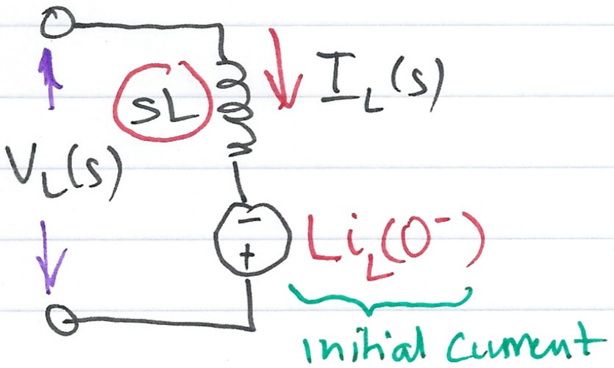


$$v_L(t) = L \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L} \int_{-\infty}^{t} v_L \, dt$$

Inductive Network - Complex Frequency Domain

Frequency Domain

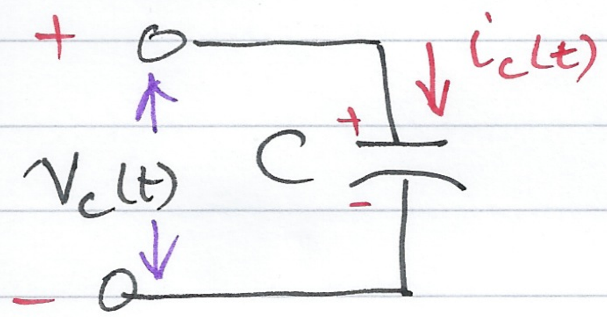


$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{Ls} + \frac{i_L(0^-)}{s}$$

Capacitive Network - Time Domain

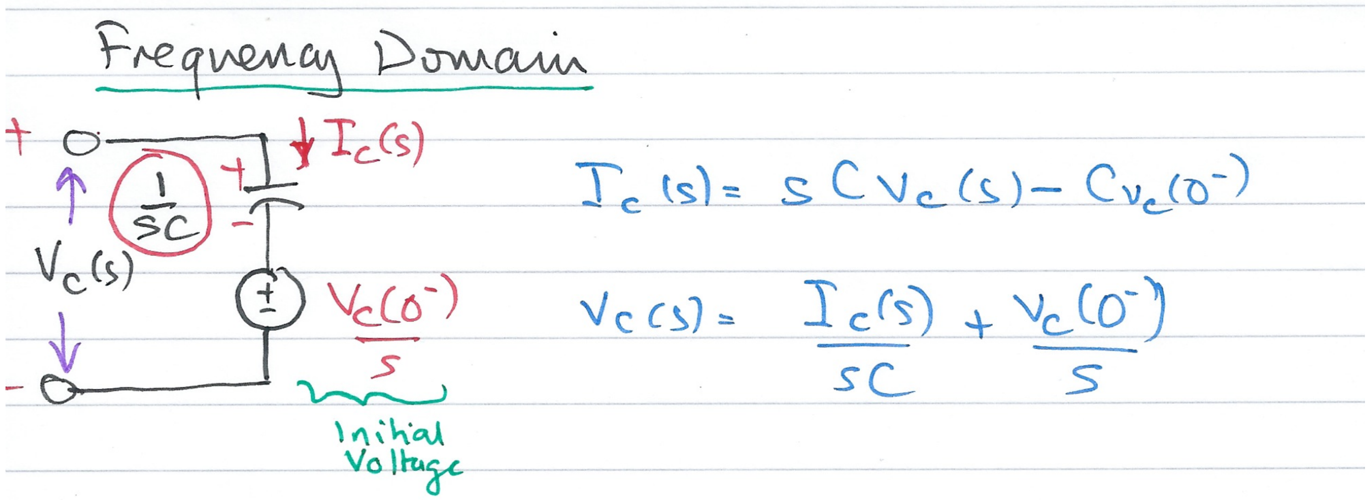
Time Domain



$$i_c(t) = C\frac{dv_c}{dt}$$

$$v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c\,dt$$

Capacitive Network - Complex Frequency Domain

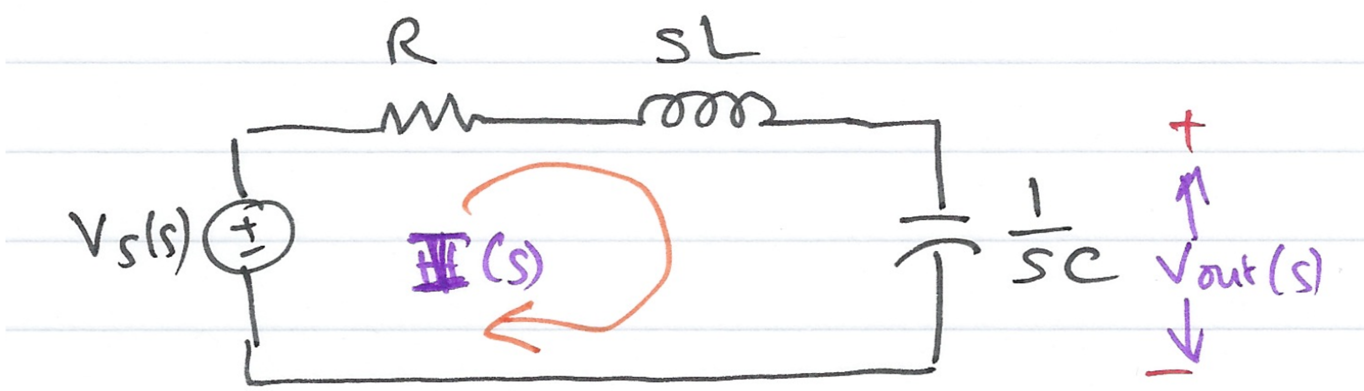


Examples

We will work through these in class. See worksheet6 (worksheet6).

Complex Impedance Z(s)

Consider the s-domain RLC series circuit, where the initial conditions are assumed to be zero.



For this circuit, the sum

$$R + sL + \frac{1}{sC}$$

represents that total opposition to current flow. Then,

$$I(s) = \frac{V_s(s)}{R + sL + 1/(sC)}$$

and defining the ratio $V_s(s)/I(s)$ as $Z(s)$, we obtain

$$Z(s) = \frac{V_s(s)}{I(s)} = R + sL + \frac{1}{sC}$$

The s-domain current $I(s)$ can be found from

$$I(s) = \frac{V_s(s)}{Z(s)}$$

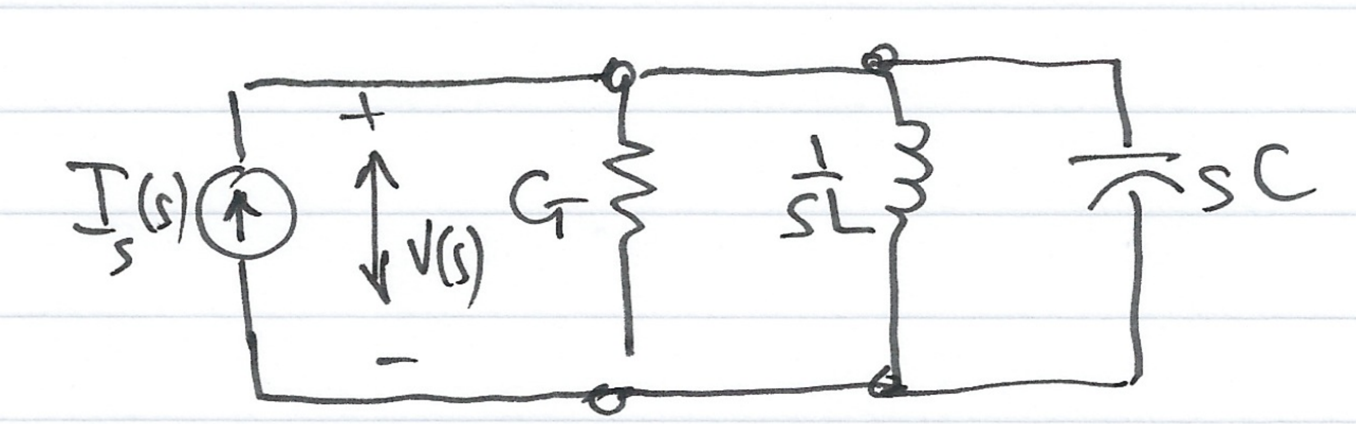
where

$$Z(s) = R + sL + \frac{1}{sC}.$$

Since $s = \sigma + j\omega$ is a complex number, $Z(s)$ is also complex and is known as the *complex input impedance* of this RLC series circuit.

Complex Admittance Y(s)

Consider the s-domain GLC parallel circuit shown below where the initial conditions are zero.



For this circuit

$$GV(s) + \frac{1}{sL}V(s) + sCV(s) = I_s(s)$$

$$\left(G + \frac{1}{sL} + sC\right) V(s) = I_s(s)$$

Defining the ratio $I_s(s)/V(s)$ as $Y(s)$ we obtain

$$Y(s) = \frac{I_s(s)}{V(s)} = G + \frac{1}{sL} + sC = \frac{1}{Z(s)}$$

The s-domain voltage $V(s)$ can be found from

$$V(s) = \frac{I_s(s)}{Y(s)}$$

where

$$Y(s) = G + \frac{1}{sL} + sC.$$

$Y(s)$ is complex and is known as the *complex input admittance* of this GLC parallel circuit.