Homework 4

The Impulse Response and Convolution

The questions for Homework 4 are based on the examples given in [Section 6.8 (https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=207)](https://ebookcentral.proquest.com/lib/swansea-ebooks/reader.action?docID=3384197&ppg=207) of {% cite karris %}.

1. Confirm the result of [Example 5 (https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-5)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-5) from the notes using the convolution integral

$$h(t) * u_0(t) = \int_{-\infty}^{\infty} u_0(\tau)h(t-\tau)\,d\tau$$

1. Compute the impulse response $h(t) = i_s(t)$ (where $i_s(t) = \delta(t)$) in terms of R and L for the circuit shown in Fig. Q2 below. Use this result to compute the voltage $v_L(t)$ across the inductor.

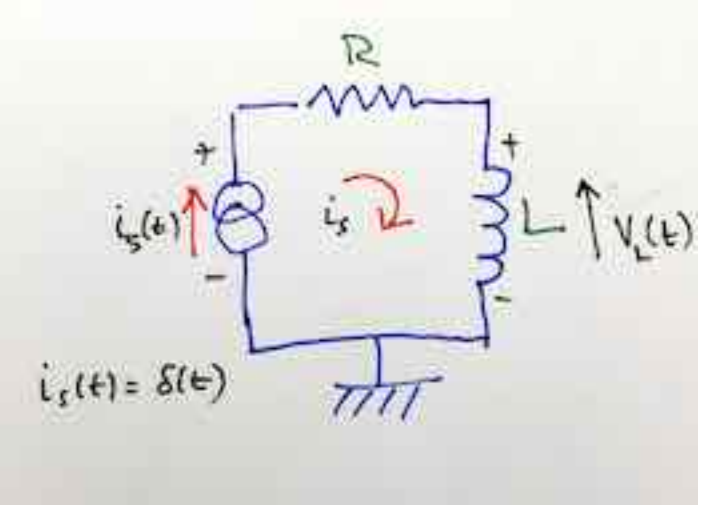

Fig. Q2: An RL Circuit

1. Redo the graphical convolution [Example 2 (https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-2)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-2) from the notes by forming $h(t-\tau)$ instead of $u(t-\tau)$. That is, use the convolution integral

$$\int_{-\infty}^{\infty} u(\tau)h(t-\tau)\,d\tau.$$

Confirm the result in MATLAB.

2. Redo the graphical convolution [Example 3 (https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-3)](https://cpjobling.github.io/eg-247-textbook/laplace_transform/5/convolution.html#Example-3) from the notes by forming $h(t-\tau)$ instead of $u(t-\tau)$. Confirm the result in MATLAB.
3. Derive the transfer function

$$H(s) = \frac{V_L(s)}{I_s(s)}$$

for the circuit of Fig. Q2.

Use this result to

a. Confirm the impulse response of this circuit $V_L(t)$.

b. Compute the step response

$$V_L(t) = \mathcal{L}^{-1}\left\{H(s)U_0(s)\right\}.$$

c. Validate this result in MATLAB.

4. For the network show in Fig. Q6 compute:

a. The transfer function

$$H(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)};$$

b. The response of this circuit to the input $v_{\mathrm{in}}(t) = u_0(t) - u_0(t-1)$.

c. Validate this result in MATLAB.

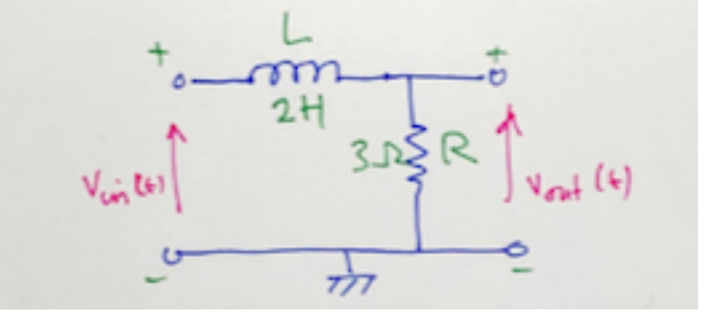

Fig. Q6: An LR Network

5. For the network shown in Fig. Q7 compute:

a. The transfer function

$$H(s) = \frac{V_{\mathrm{out}}(s)}{V_{\mathrm{in}}(s)};$$

b. Determine the step response of the network.

c. State the time constant of the network.

d. Validate this result in MATLAB.

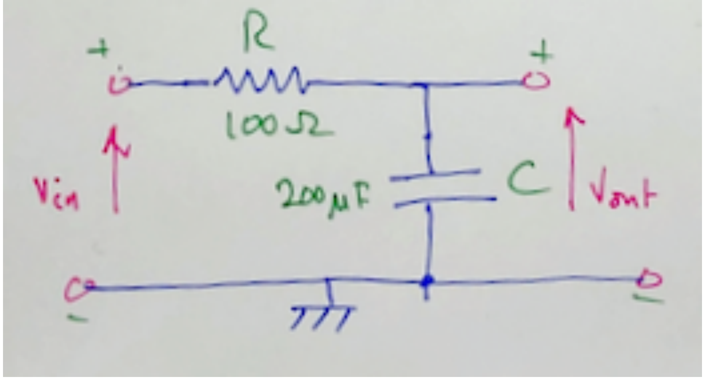

Fig. Q7: An RC Network

Answers to selected problems

- For question 2:

$$h(t) = i_s(t) = \left(\frac{1}{L}\right) e^{-\left(\frac{R}{L}\right)t}\, u_0(t);$$

$$v_L(t) = -\left(\frac{R}{L}\right) e^{-\left(\frac{R}{L}\right)t} u_0(t) + \delta(t).$$

- For question 5 the transfer function is

$$H(s) = \frac{s}{s + R/L}$$

  and the step response is:

$$v_L(t) = Le^{-\left(\frac{R}{L}\right)t} u_0(t).$$

- For question 6 the transfer function is

$$H(s) = \frac{3/2}{s + 3/2}$$

  and the pulse response is:

$$v_L(t) = \left(1 - e^{-1.5t}\right) u_0(t) - \left(1 - e^{-1.5(t-1)}\right) u_0(t-1).$$

- For question 7 the transfer function is

$$H(s) = \frac{50}{s + 50};$$

  the step response is

$$\left(1 - e^{-50t}\right) u_0(t)$$

  and the time constant is: $T = RC = 1/50$ s.

Reference

{% bibliography --cited %}