

# Worksheet 7

## To accompany Chapter 3.4 Transfer Functions

We will step through this worksheet in class.

You are expected to have at least watched the video presentation of [Chapter 3.4](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/transfer_functions) ([https://cpjobling.github.io/eg-247-textbook/laplace\\_transform/3/transfer\\_functions](https://cpjobling.github.io/eg-247-textbook/laplace_transform/3/transfer_functions)) of the [notes](https://cpjobling.github.io/eg-247-textbook) (<https://cpjobling.github.io/eg-247-textbook>) before coming to class. If you haven't watch it afterwards!

## Second Hour's Agenda

- Transfer Functions
- A Couple of Examples
- Circuit Analysis Using MATLAB LTI Transfer Function Block
- Circuit Simulation Using Simulink Transfer Function Block

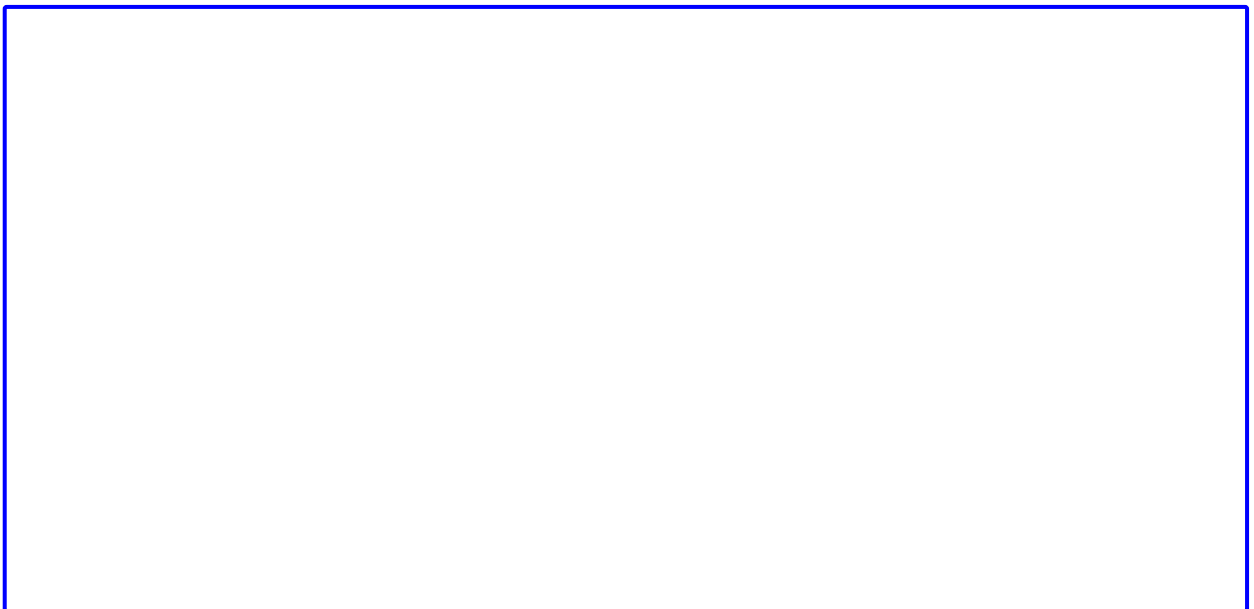
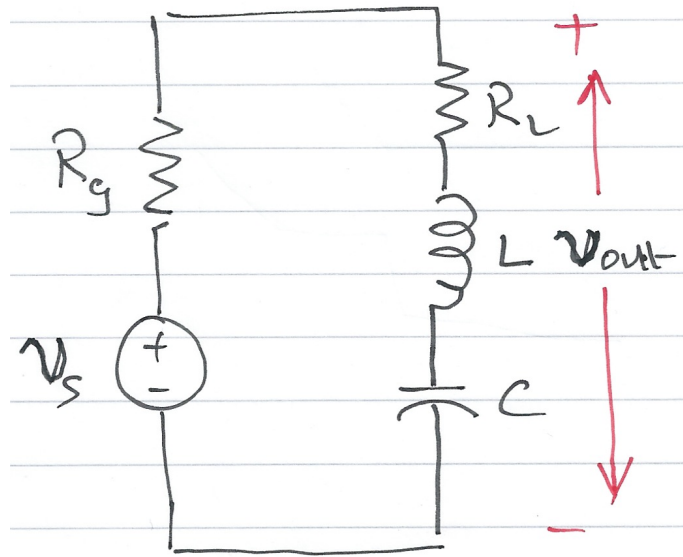
In [ ]:

```
% Matlab setup
clear all
format compact
```

## Transfer Functions for Circuits

### Example 6

Derive an expression for the transfer function  $G(s)$  for the circuit below. In this circuit  $R_g$  represents the internal resistance of the applied (voltage) source  $v_s$ , and  $R_L$  represents the resistance of the load that consists of  $R_L$ ,  $L$  and  $C$ .



## Sketch of Solution

- Replace  $v_s(t)$ ,  $R_g$ ,  $R_L$ ,  $L$  and  $C$  by their transformed (*complex frequency*) equivalents:  $V_s(s)$ ,  $R_g$ ,  $R_L$ ,  $sL$  and  $1/(sC)$
- Use the *Voltage Divider Rule* to determine  $V_{\text{out}}(s)$  as a function of  $V_s(s)$
- Form  $G(s)$  by writing down the ratio  $V_{\text{out}}(s)/V_s(s)$

## Worked solution.

Pencast: [ex6.pdf \(./worked\\_examples/ex6.pdf\)](#) - open in Adobe Acrobat Reader.

## Answer

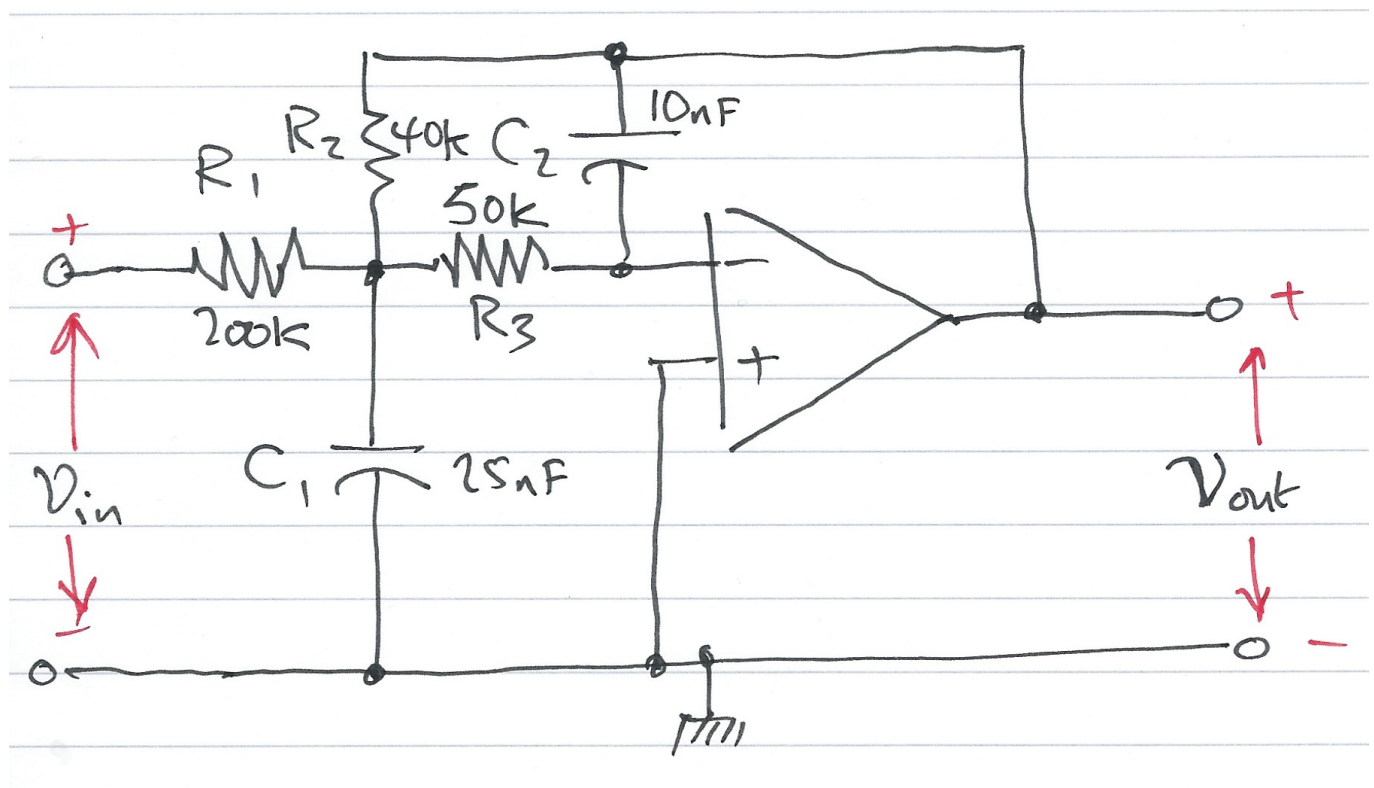
$$G(s) = \frac{V_{\text{out}}(s)}{V_s(s)} = \frac{R_L + sL + 1/sC}{R_g + R_L + sL + 1/sC}.$$

## Example 7

Compute the transfer function for the op-amp circuit shown below in terms of the circuit constants  $R_1$ ,  $R_2$ ,  $R_3$ ,  $C_1$  and  $C_2$ . Then replace the complex variable  $s$  with  $j\omega$ , and the circuit constants with their numerical values and plot the magnitude

$$|G(j\omega)| = \frac{|V_{\text{out}}(j\omega)|}{|V_{\text{in}}(j\omega)|}$$

versus radian frequency  $\omega$  rad/s.





## Sketch of Solution

- Replace the components and voltages in the circuit diagram with their complex frequency equivalents
- Use nodal analysis to determine the voltages at the nodes either side of the 50K resistor  $R_3$
- Note that the voltage at the input to the op-amp is a virtual ground
- Solve for  $V_{\text{out}}(s)$  as a function of  $V_{\text{in}}(s)$
- Form the reciprocal  $G(s) = V_{\text{out}}(s)/V_{\text{in}}(s)$
- Use MATLAB to calculate the component values, then replace  $s$  by  $j\omega$ .
- Plot  $|G(j\omega)|$  on log-linear "paper".

## Worked solution.

Pencast: [ex7.pdf](#) (../worked\_examples/ex7.pdf) - open in Adobe Acrobat Reader.

## Answer

$$G(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{-1}{R_1 ((1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sC_2R_3) + 1/R_2)}.$$

## The Matlab Bit

See attached script: [solution7.m](#) (matlab/solution7.m).

## Week 3: Solution 7

In [ ]:

```
syms s;
```

In [ ]:

```
R1 = 200*10^3;
R2 = 40*10^3;
R3 = 50*10^3;

C1 = 25*10^(-9);
C2 = 10*10^(-9);
```

In [ ]:

```
den = R1*((1/R1+ 1/R2 + 1/R3 + s*C1)*(s*R3*C2) + 1/R2);
simplify(den)
```

Result is:  $100*s*((7555786372591433*s)/302231454903657293676544 + 1/20000) + 5$

Simplify coefficients of  $s$  in denominator

In [ ]:

```
format long
denG = sym2poly(ans)
```

In [ ]:

```
numG = -1;
```

Plot

For convenience, define coefficients  $a$  and  $b$ :

In [ ]:

```
a = denG(1);
b = denG(2);
```

In [ ]:

```
w = 1:10:10000;
```

$$G(j\omega) = \frac{-1}{a\omega^2 - jb\omega + 5}$$

In [ ]:

```
Gs = -1./(a*w.^2 - j.*b.*w + denG(3));
```

In [ ]:

```
semilogx(w, abs(Gs))
xlabel('Radian frequency w (rad/s)')
ylabel('|Vout/Vin|')
title('Magnitude Vout/Vin vs. Radian Frequency')
grid
```

## Using Transfer Functions in Matlab for System Analysis

Please use the file [matlab/tf\\_matlab.m](#) ([tf\\_matlab.m](#)) to explore the Transfer Function features provide by Matlab. Use the *publish* option to generate a nicely formatted document.

## Using Transfer Functions in Simulink for System Simulation



The Simulink transfer function (**Transfer Fcn**) block shown above implements a transfer function representing a general input output function

$$G(s) = \frac{N(s)}{D(s)}$$

that it is not specific nor restricted to circuit analysis. It can, however be used in modelling and simulation studies.

### Example

Recast Example 7 as a MATLAB problem using the LTI Transfer Function block.

For simplicity use parameters  $R_1 = R_2 = R_3 = 1 \Omega$ , and  $C_1 = C_2 = 1 \text{ F}$ .

Calculate the step response using the LTI functions.

Verify the result with Simulink.

The Matlab solution: [example8.m](#) ([matlab/example8.m](#))

## MATLAB Solution

From a previous analysis the transfer function is:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{R_1 [(1/R_1 + 1/R_2 + 1/R_3 + sC_1)(sR_3C_2) + 1/R_2]}$$

so substituting the component values we get:

$$G(s) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{-1}{s^2 + 3s + 1}$$

We can find the step response by letting  $v_{\text{in}}(t) = u_0(t)$  so that  $V_{\text{in}}(s) = 1/s$  then

$$V_{\text{out}}(s) = \frac{-1}{s^2 + 3s + 1} \cdot \frac{1}{s}$$

We can solve this by partial fraction expansion and inverse Laplace transform as is done in the text book with the help of Matlab's `residue` function.

Here, however we'll use the LTI block that was introduced in the lecture.

Define the circuit as a transfer function

In [ ]:

```
G = tf([-1],[1 3 1])
```

step response is then:

In [ ]:

```
step(G)
```

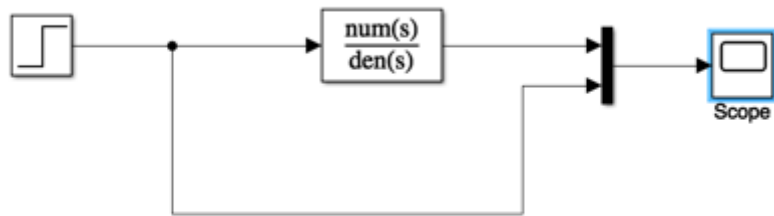
Simples!

## Simulink model

See [example\\_8.slx](#) ([matlab/example\\_8.slx](#))

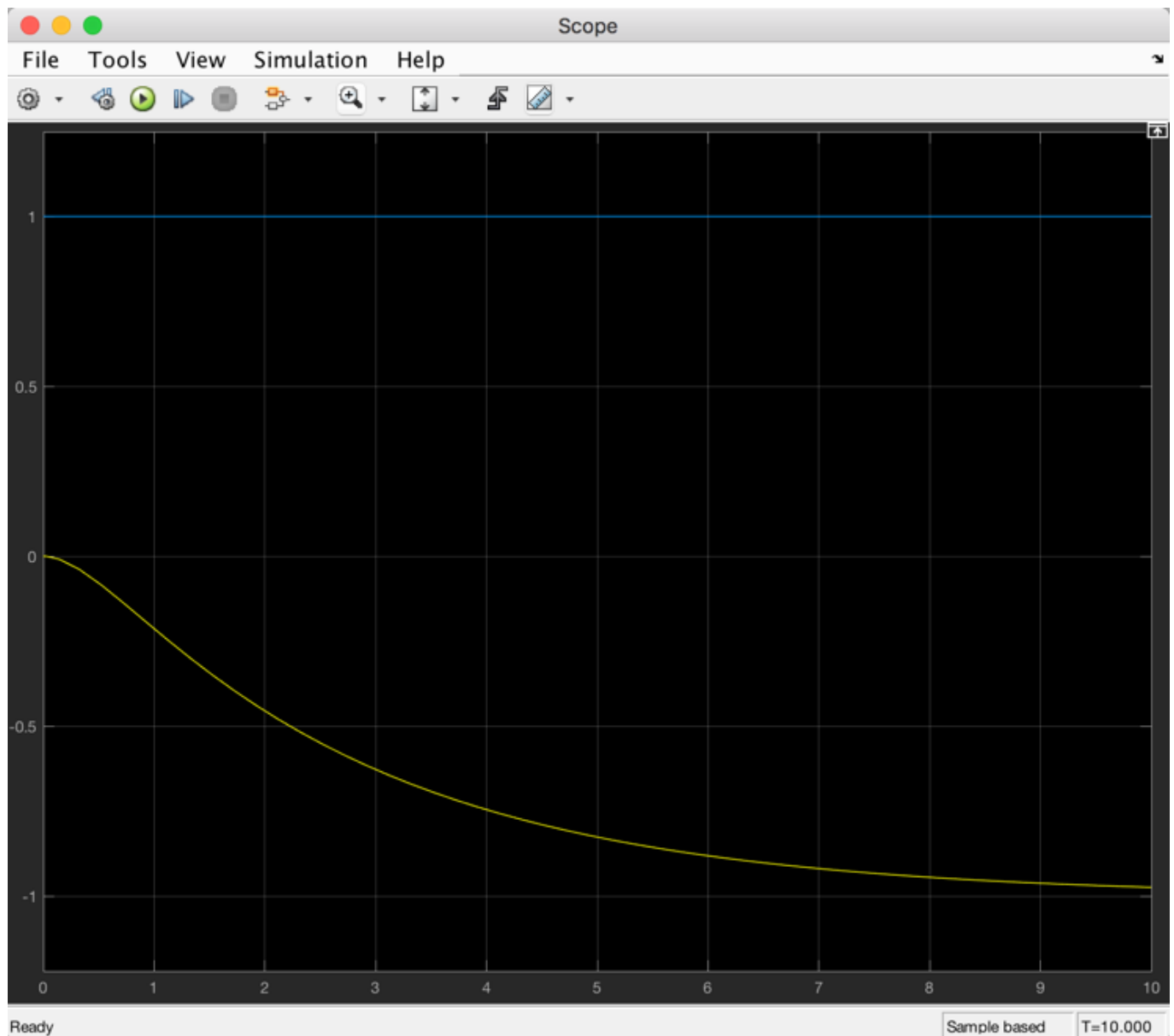
In [ ]:

```
open example_8
```





## Result



Let's go a bit further by finding the frequency response:

In [ ]:

```
bode(G)
```