3112. Count Valid Paths in a Tree

Difficulty : Hard

https://leetcode.com/problems/count-valid-paths-in-a-tree

There is an undirected tree with `n` nodes labeled from `1` to `n`. You are given the integer `n` and a 2D integer array `edges` of length `n - 1`, where `edges[i] = [u_i, v_i]` indicates that there is an edge between nodes `u_i` and `v_i` in the tree.

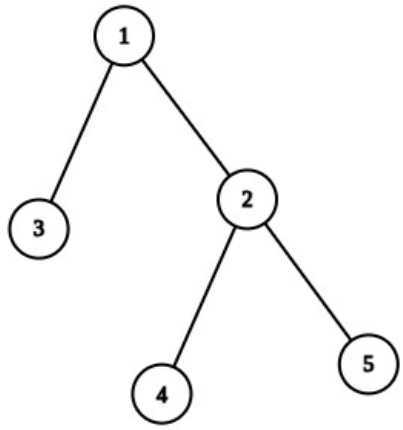
Return *the **number of valid paths** in the tree*.

A path `(a, b)` is **valid** if there exists **exactly one** prime number among the node labels in the path from `a` to `b`.

Note that:

- The path `(a, b)` is a sequence of **distinct** nodes starting with node `a` and ending with node `b` such that every two adjacent nodes in the sequence share an edge in the tree.
- Path `(a, b)` and path `(b, a)` are considered the **same** and counted only **once**.

Example 1:

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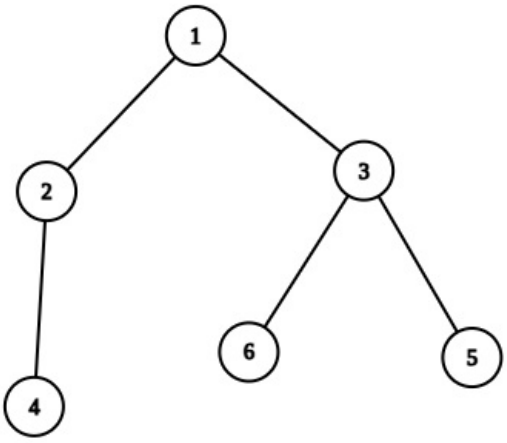
Input: n = 5, edges = [[1,2],[1,3],[2,4],[2,5]]
Output: 4
Explanation: The pairs with exactly one prime number on the path between them are:
- (1, 2) since the path from 1 to 2 contains prime number 2.
- (1, 3) since the path from 1 to 3 contains prime number 3.
- (1, 4) since the path from 1 to 4 contains prime number 2.
- (2, 4) since the path from 2 to 4 contains prime number 2.
It can be shown that there are only 4 valid paths.
```

Example 2:



Input: n = 6, edges = [[1,2],[1,3],[2,4],[3,5],[3,6]]
Output: 6
Explanation: The pairs with exactly one prime number on the path between them are:
- (1, 2) since the path from 1 to 2 contains prime number 2.
- (1, 3) since the path from 1 to 3 contains prime number 3.
- (1, 4) since the path from 1 to 4 contains prime number 2.
- (1, 6) since the path from 1 to 6 contains prime number 3.
- (2, 4) since the path from 2 to 4 contains prime number 2.
- (3, 6) since the path from 3 to 6 contains prime number 3.
It can be shown that there are only 6 valid paths.

Constraints:

- $1 <= n <= 10^5$
- edges.length == n - 1
- edges[i].length == 2
- $1 <= u_i, v_i <= n$
- The input is generated such that edges represent a valid tree.