1、证明：对于任意正整数d和任意常数a_d >0，有：$P(n) = \Sigma_{i=0}^{d} a_i n^i = \theta(n^d)$.

一方面，我们有：

$$\begin{aligned}
P(n) \quad &= \sum_{i=0}^{d} a_i \, n^i \\
&= (a_0 + a_1 n + \ldots + a_{d-1} n^{d-1}) + a_d n^d \\
&\geq (-|a_0| - |a_1| n - \ldots - |a_{d-1}| n^{d-1}) + a_d n^d \\
&\geq (-|a_0| n^{d-1} - |a_1| n^{d-1} - \ldots - |a_{d-1}| n^{d-1}) + a_d n^d \\
&\geq n^{d-1}(a_d n - \sum_{i}^{d-1} | \, a_i|)
\end{aligned} \qquad (1)$$

故取$c_1 = \frac{a_d}{2}$，当$n > n_0 = [2 \cdot \frac{\Sigma_i^{d-1} |a_i|}{a_d}] + 2$时，我们有

$$\begin{aligned}
P(n) - c_1 n^d \quad &\geq n^{d-1}(a_d n - \sum_{i}^{d-1} | \, a_i|) - \frac{a_d}{2} n^d \\
&= n^{d-1}(\frac{a_d}{2} n - \sum_{i}^{d-1} | \, a_i|) \\
&\geq n^{d-1}(\frac{a_d}{2}(2 \cdot \frac{\Sigma_i^{d-1} | \, a_i|}{a_d} - 1 + 2) - \sum_{i}^{d-1} | \, a_i|) \\
&\geq \frac{a_d}{2} n^{d-1} \\
&\geq 0
\end{aligned} \qquad (2)$$

因此$P(n) \geq c_1 n^d$

另一方面，我们有：

$$\begin{aligned}
P(n) \quad &= \sum_{i=0}^{d} a_i \, n^i \\
&= (a_0 + a_1 n + \cdots + a_{d-1} n^{d-1}) + a_d n^d \\
&\leq (|a_0| + |a_1| n + \cdots + |a_{d-1}| n^{d-1}) + |a_d| n^d \\
&\leq (|a_0| n^d + |a_1| n^d + \cdots + |a_{d-1}| n^d) + |a_d| n^d \\
&= (|a_0| + |a_1| + \cdots + |a_{d-1}| + |a_d|) n^d \\
&= \sum_{i=0}^{d} | \, a_i| n^d
\end{aligned} \qquad (3)$$

故取$c_2 = \sum_{i=0}^{d} | a_i|$，对任意的$n$，我们有$P(n) \le c_2 n^d$

综上，我们有 $n > n_0 = [2 \cdot \frac{\sum_i^{d-1} |a_i|}{|a_d|}] + 2$时，

$$c_1 n^d \le P(n) \le c_2 n^d \tag{4}$$

因此$P(n) = \Sigma_{i=0}^{d} a_i n^i = \theta(n^d)$.

2、证明：$f(n) = o\big(g(n)\big) \wedge g(n) = o\big(h(n)\big) \Longrightarrow f(n) = o\big(h(n)\big)$。

$\forall c > 0$,令$c = c_1 \cdot c_2$，$c_1, c_2 > 0$

由于$f(n) = o(g(n))$，对于$c_1 > 0$，存在n_1使得，$\forall n > n_1$时，$f(n) < c_1 g(n)$

同理$g(n) = o(h(n))$，对于$c_2 > 0$，存在n_2使得，$\forall n > n_2$时，$g(n) < c_2 h(n)$

存在$n_0 = max\{n_1, n_2\}$使得，$n > n_0 = max\{n_1, n_2\}$时，

$$f(n) < c_1 g(n) < c_1 \cdot c_2 h(n) < (c_1 c_2) h(n) = ch(n) \tag{5}$$

因此$f(n) = o(h(n))$