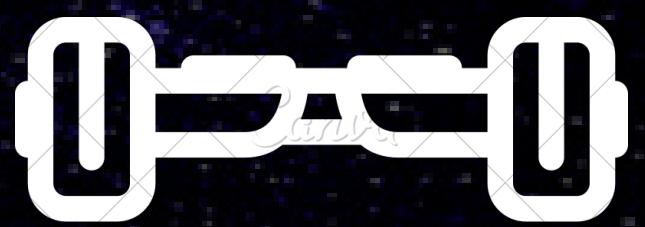


Robobot ROBOT MÓVIL: Diseño, análisis e Implementación de control LQR para un Robot tipo balancín



Integrantes:

Belén Lopez

Jheny Huanca

Weimar Condori

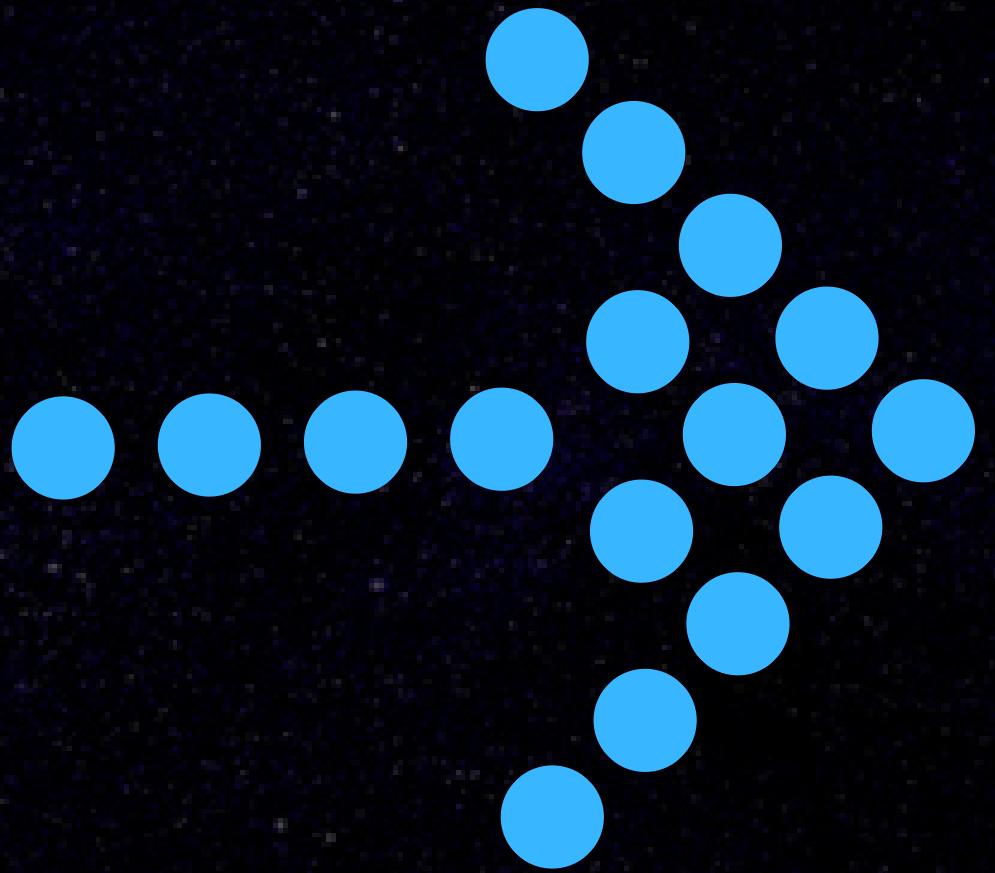
Brayan Durán

Marcelo Velasquez

Docente:

Ing. Gabriel Rojas

f(x)

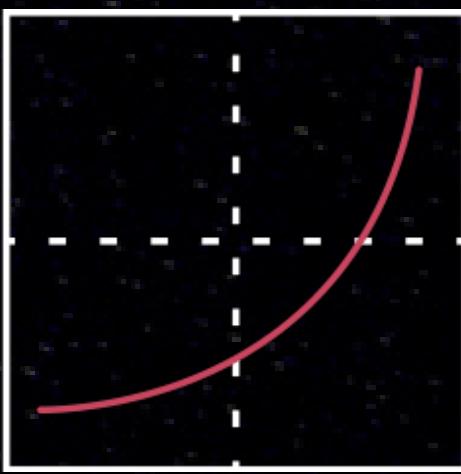
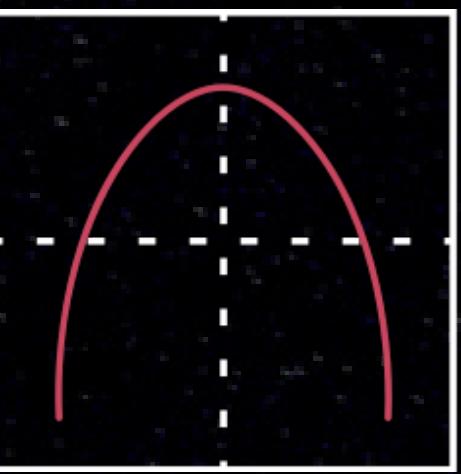
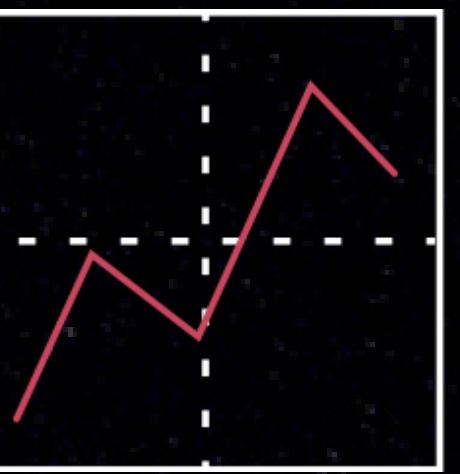


DISEÑO CONTROLADOR LOR

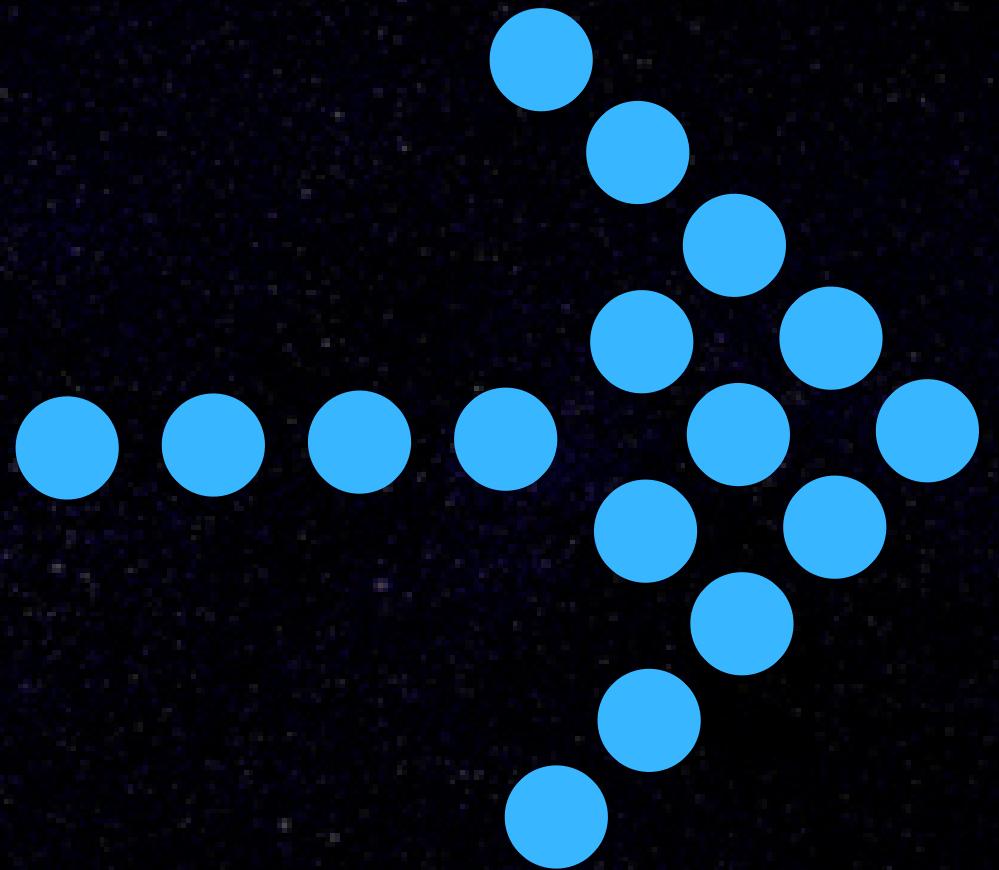
ANÁLISIS DE ENERGÍAS

A right-angled triangle is shown with its hypotenuse labeled 'hyp'. The vertical leg is labeled 'opp' (opposite) and the horizontal leg is labeled 'adj' (adjacent). The angle between the horizontal leg and the hypotenuse is labeled θ .

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$



f(x)

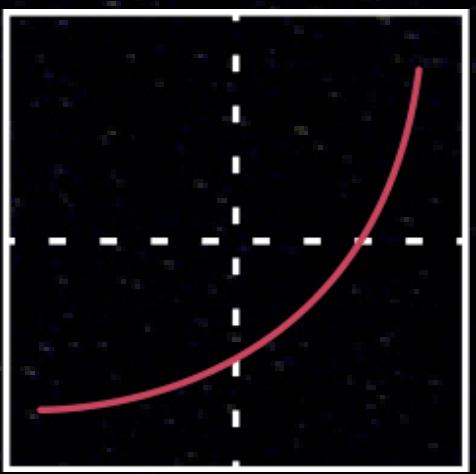
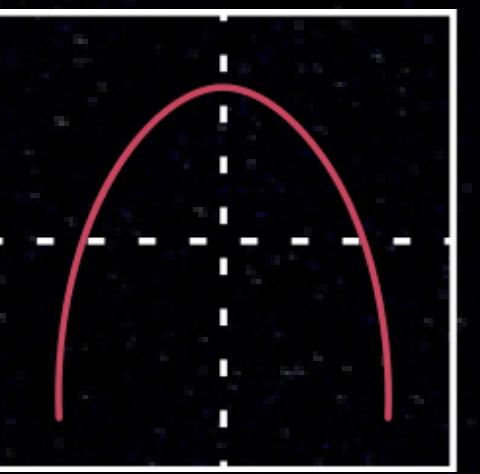
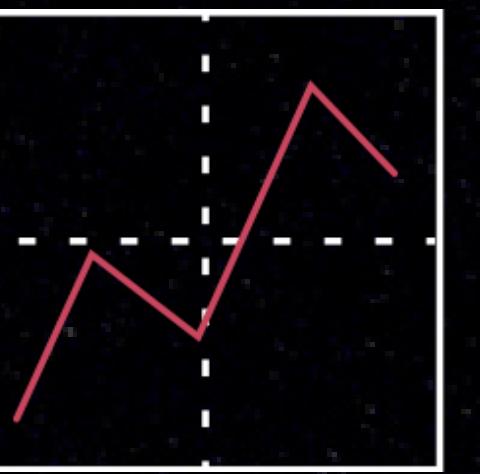


LAGRANGIANO

ANÁLISIS DE ENERGÍAS

A right-angled triangle diagram illustrating the sine function. The vertical leg is labeled "opp" (opposite), the horizontal leg is labeled "adj" (adjacent), and the hypotenuse is labeled "hyp". The angle between the horizontal leg and the hypotenuse is labeled θ . A small square at the vertex indicates a right angle.

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$



ANÁLISIS DE ENERGÍAS

Energía cinética translacional de la rueda:

$$E_{c1} = \frac{1}{2} * m_r * \dot{x}^2$$

Energía cinética rotacional de la rueda:

$$E_{c2} = \frac{1}{2} * J_r * \dot{\phi}^2 = \frac{1}{2} * J_r * \frac{\dot{x}^2}{r^2}$$

Energía cinética rotacional del péndulo:

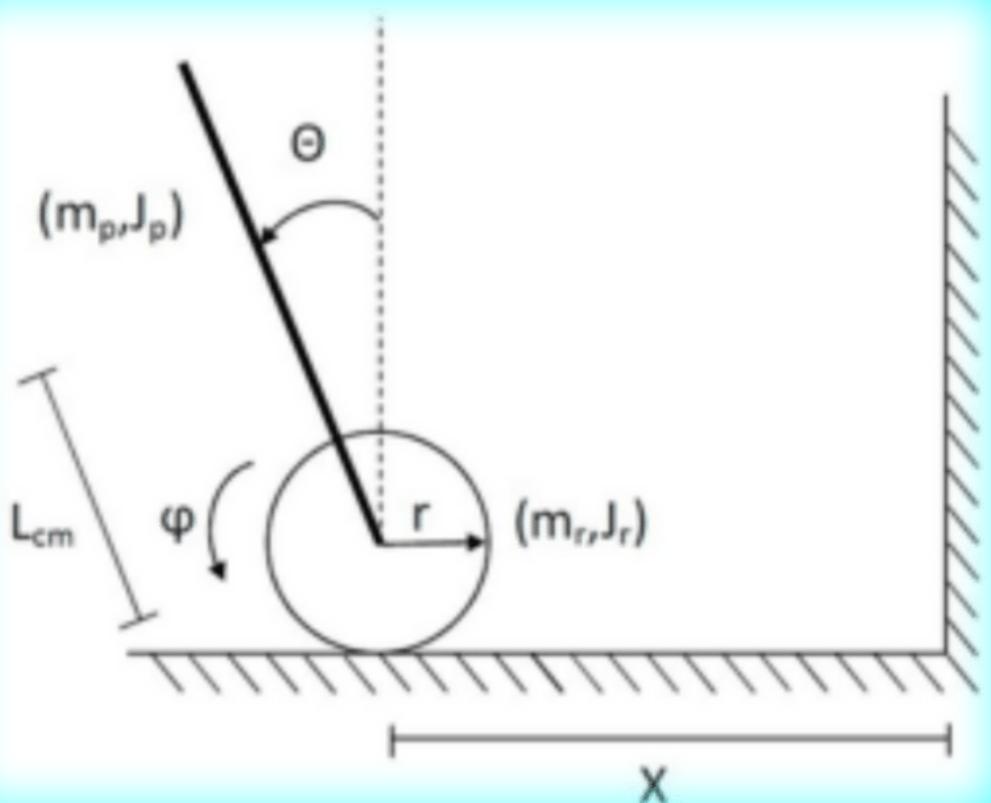
$$E_{c3} = \frac{1}{2} * J_p * \dot{\theta}^2$$

Energía cinética translacional del péndulo:

$$E_{c4} = \frac{1}{2} * m_p * (\dot{x}_{cm}^2 + \dot{y}_{cm}^2)$$

$$E_{c4} = \frac{1}{2} * m_p * ((x + L\sin\theta)')^2 + ((L\cos\theta)')^2$$

$$E_{c4} = \frac{1}{2} * m_p * (L^2\dot{\theta}^2 + \dot{x}^2 + 2L\dot{x}\dot{\theta}\cos\theta)$$



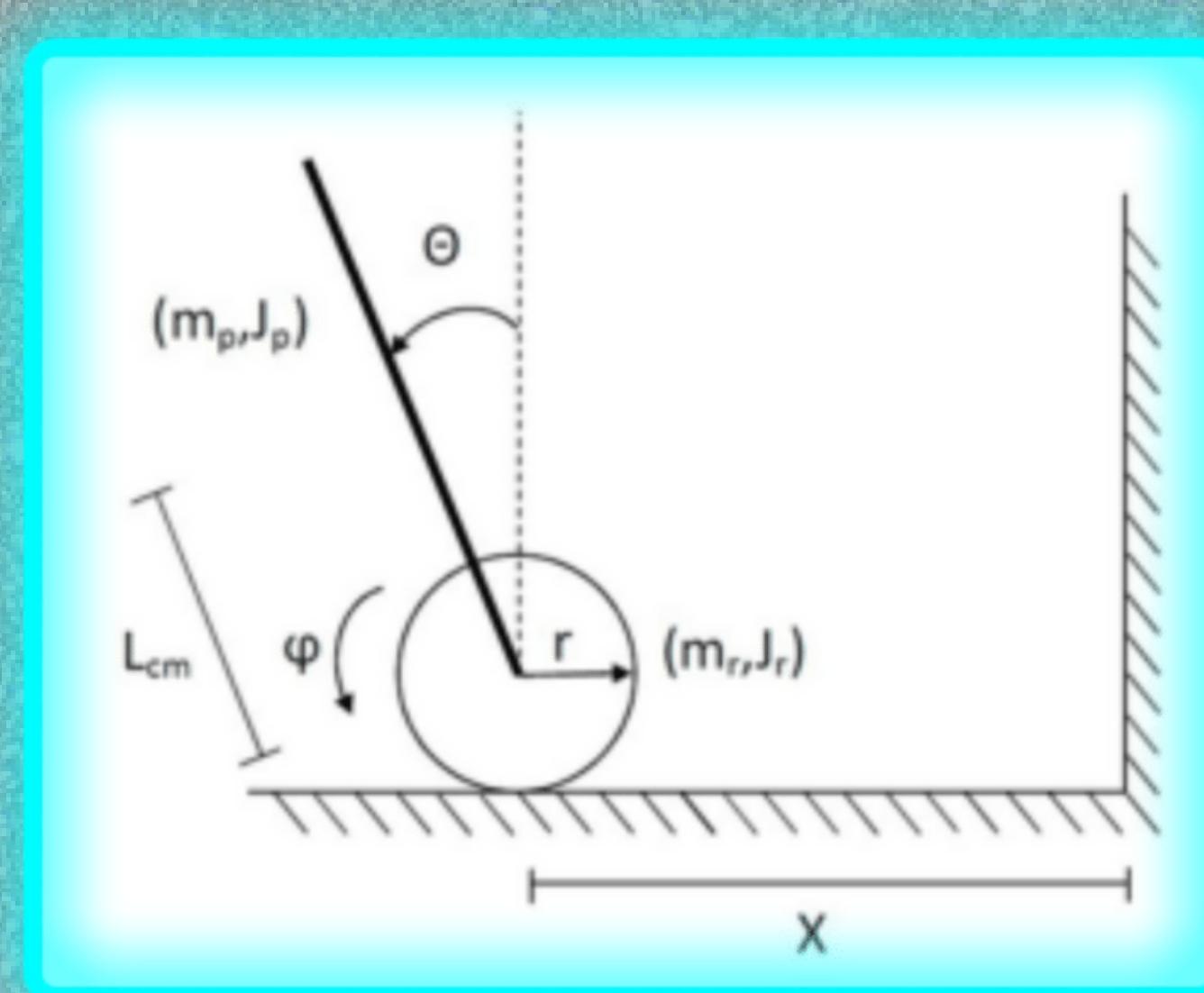
Energía potencial del péndulo:

$$E_p = m_p * g * L * \cos\theta$$

LAGRANGIANO DEL SISTEMA

CONSIDERACIONES:

- modelo en dos dimensiones
- masa en un plano paralelo a las ruedas
- no deslizamiento entre ruedas y piso



- Donde:
- m_r : masa de la rueda
- m_p : masa del péndulo
- J_r : Momento de inercia de la rueda
- J_p : Momento de inercia del péndulo
- r : radio de la rueda
- L : distancia del eje de las ruedas al centro de masa del péndulo
- g : gravedad
- x : posición del eje de la rueda
- \dot{x} : velocidad del eje de la rueda
- \ddot{x} : aceleración del eje de la rueda
- θ : ángulo del péndulo respecto a la vertical
- $\dot{\theta}$: velocidad angular del péndulo respecto a la vertical
- $\ddot{\theta}$: aceleración angular del péndulo respecto a la vertical

LAGRANGIANO DEL SISTEMA

$$L = \frac{1}{2} \left(m_r + \frac{J_r}{r^2} + m_p \right) \dot{x}^2 + \frac{1}{2} (J_p + m_p L^2) \dot{\theta}^2 + m_p L \dot{x} \dot{\theta} \cos \theta - m_p * g * L * \cos \theta$$

APLICACIÓN DE EULER-LAGRANGE

Coordenadas generalizadas:

$$q_1 = x$$

$$q_2 = \theta$$

$$\dot{q}_1 = \dot{x}$$

$$\dot{q}_2 = \dot{\theta}$$

Ecuación de Euler-Lagrange con primera coordenada generalizada

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = F$$

$$\left(m_r + \frac{J_r}{r^2} + m_p \right) \ddot{x} + m_p L \cos(\theta) \ddot{\theta} - m_p L \sin(\theta) \dot{\theta}^2 = F$$

Ecuación de Euler-Lagrange con segunda coordenada generalizada

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} = 0$$

$$(J_p + m_p L^2) \ddot{\theta} + m_p L \cos(\theta) \ddot{x} - m_p g L \sin \theta = 0$$

LINEALIZACIÓN

Los elementos no lineales de las ecuaciones de movimiento son 3:

$$f_1(\theta) = \cos(\theta)$$

$$f_2(\theta) = \sin(\theta)$$

$$f_3(\dot{\theta}) = \dot{\theta}^2$$

$$f_1(\theta) = \cos(\theta)$$

$$f_1'(\theta) = -\sin(\theta)$$

$$L_1(\theta) = \cos(180) - \sin(180) * (\theta - 0)$$

$$L_1(\theta) = -1 - 0(\theta - 0)$$

$$L_1(\theta) = -1$$

$$f_2(\theta) = \sin(\theta)$$

$$f_2'(\theta) = \cos(\theta)$$

$$L_2(\theta) = \sin(180) + \cos(180) * (\theta - 0)$$

$$L_2(\theta) = 0 - 1(\theta)$$

$$L_2(\theta) = -\theta$$

$$f_3(\dot{\theta}) = \dot{\theta}^2$$

$$f_3'(\dot{\theta}) = 2\dot{\theta}$$

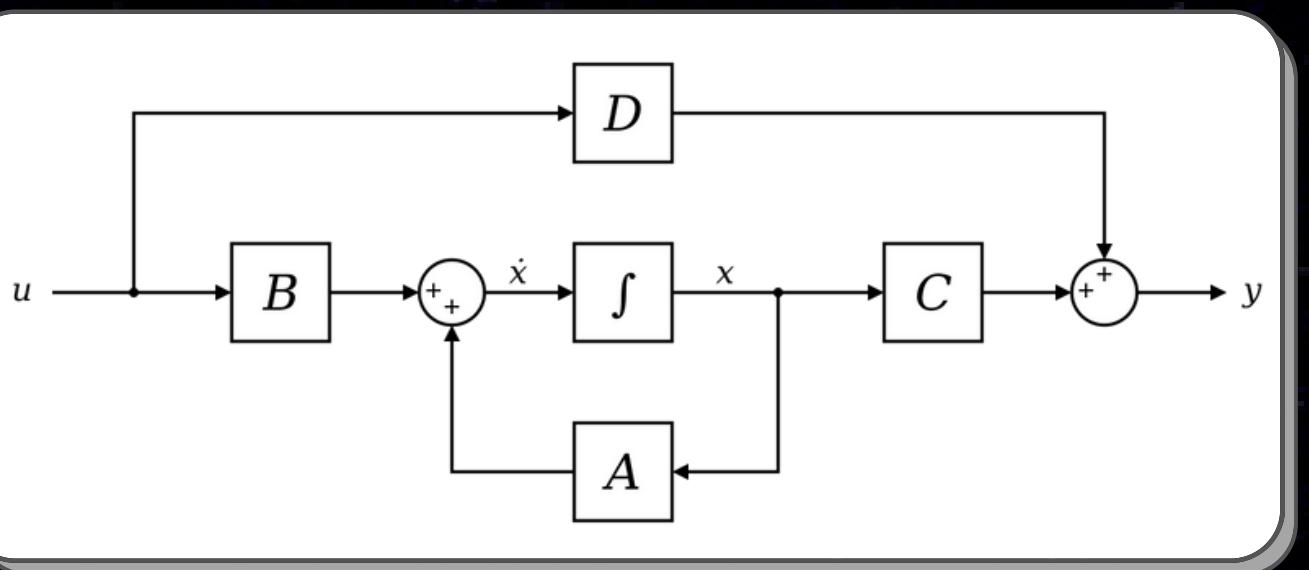
$$L_3(\dot{\theta}) = 0^2 + 2(0)(\dot{\theta} - 0)$$

$$L_3(\dot{\theta}) = 0$$

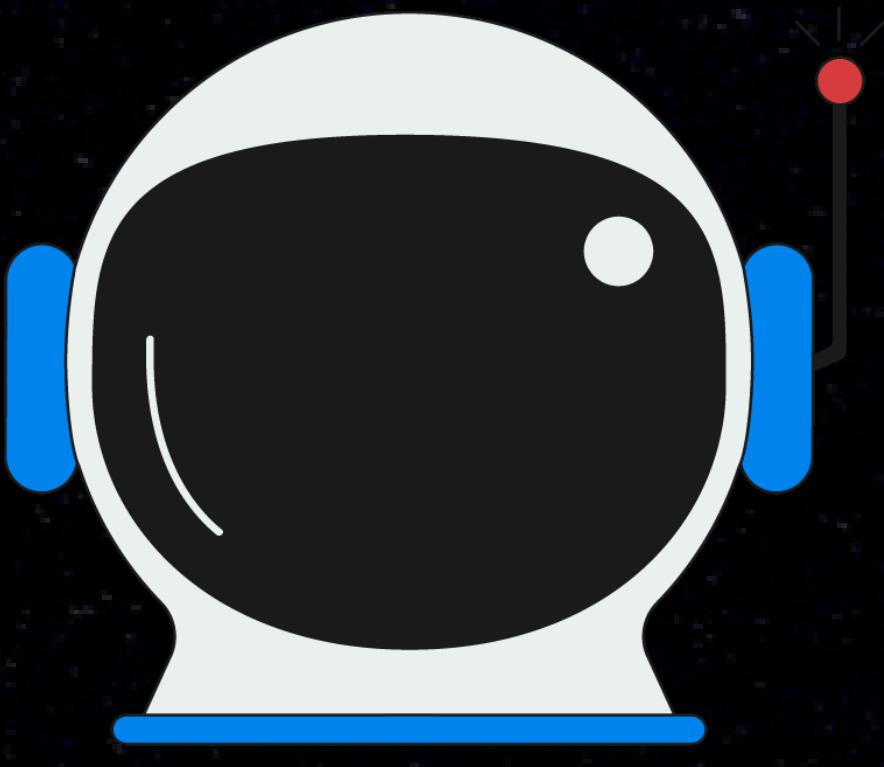
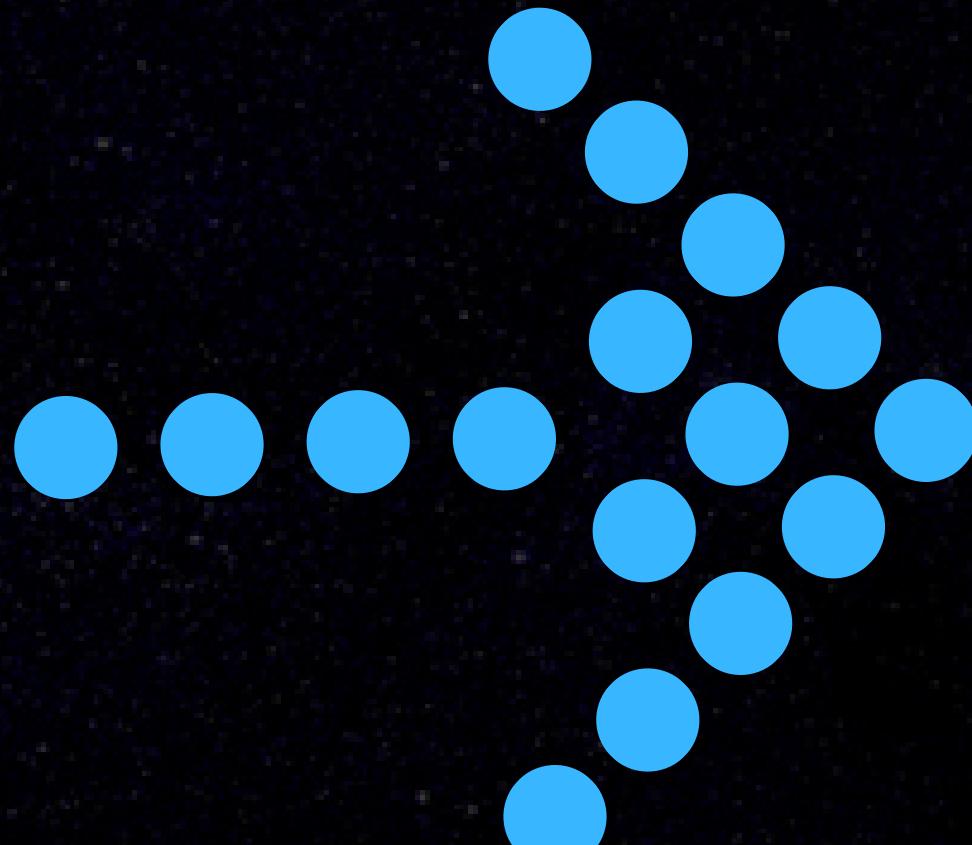
Reemplazando en las ecuaciones originales:

$$\left(m_r + \frac{J_r}{r^2} + m_p\right)\ddot{x} - m_p L \ddot{\theta} = F$$

$$(J_p + m_p L^2)\ddot{\theta} - m_p L \ddot{x} + m_p g L \theta = 0$$



ESPAZIO DE ESTADOS DEL SISTEMA



OBTENCIÓN DE VARIABLES DE ESTADO

$$x_1 = x \rightarrow x_1 = x_2$$

$$x_2 = \dot{x} \rightarrow x_2 = \dot{x}$$

$$x_3 = \theta \rightarrow x_3 = x_4$$

$$x_4 = \dot{\theta} \rightarrow x_4 = \dot{\theta}$$

ESPAZIO DE ESTADOS

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} - \frac{m_p^2 g L^2 * \left(m_r + \frac{J_r}{r^2} + m_p \right)}{\left(m_r + \frac{J_r}{r^2} + m_p \right) * \left(J_p m_r + \frac{J_p J_r}{r^2} + J_p m_p + m_p m_r L^2 + \frac{m_p L^2 J_r}{r^2} \right)} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{m_p^2 L^2 + J_p m_r + \frac{J_p J_r}{r^2} + J_p m_p + m_p m_r L^2 + \frac{m_p L^2 J_r}{r^2}}{\left(m_r + \frac{J_r}{r^2} + m_p \right) * \left(J_p m_r + \frac{J_p J_r}{r^2} + J_p m_p + m_p m_r L^2 + \frac{m_p L^2 J_r}{r^2} \right)} \\ 0 \\ \frac{m_p L}{J_p m_r + \frac{J_p J_r}{r^2} + J_p m_p + m_p m_r L^2 + \frac{m_p L^2 J_r}{r^2}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

ESPAZIO DE ESTADOS

Valores constantes:

- $m_r=0.036$
- $m_p=0.779$
- $J_r = 0.00005$
- $J_p = 0.002128$
- $r=0.05$
- $L=0.05226$
- $g = 9.81$

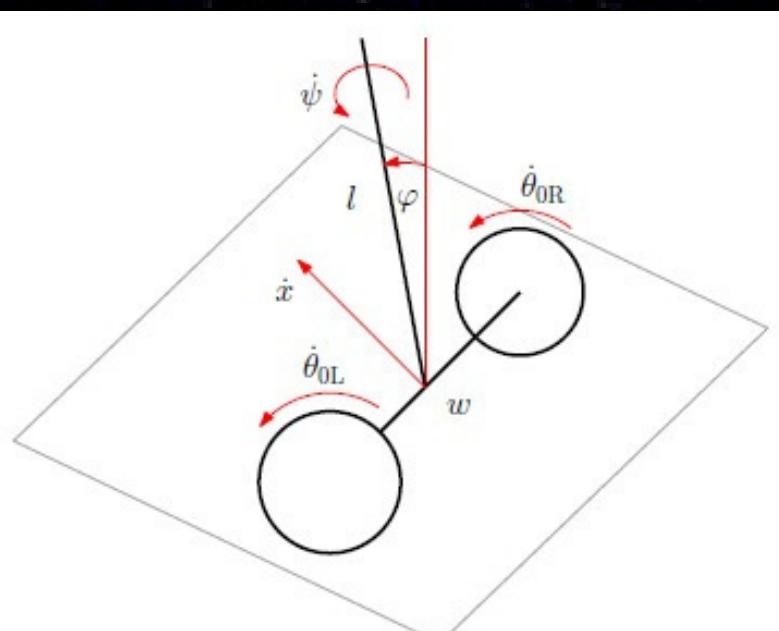
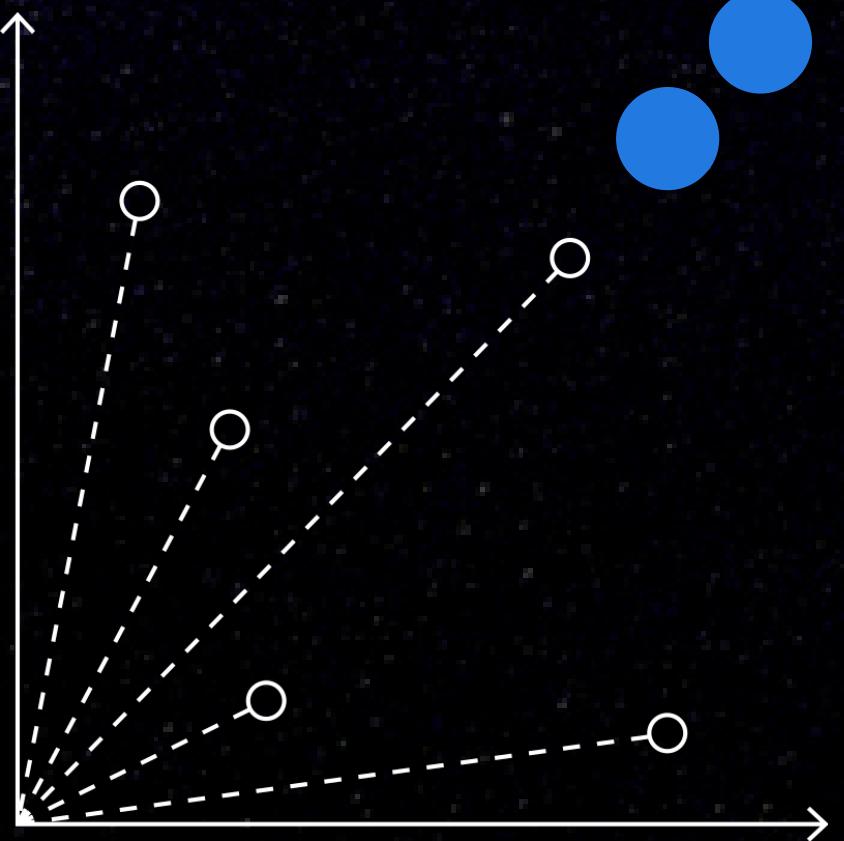
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -8.5751 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -175.8810 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 44.8891 \\ 0 \\ 429.4311 \end{bmatrix}$$

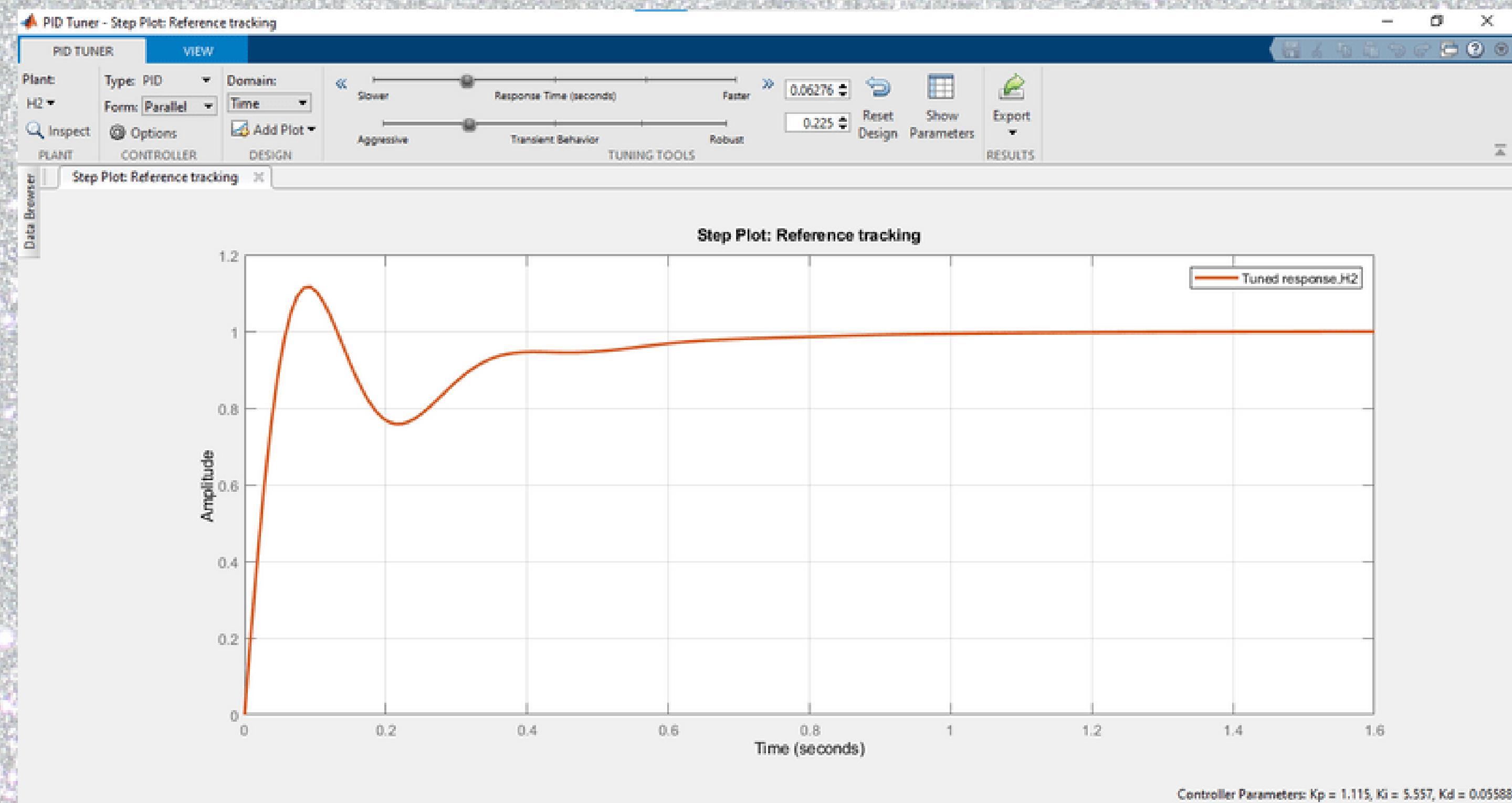
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

SIMULACIÓN DEL CONTROL PID



ESTADO DEL SISTEMA



ESTADO DEL SISTEMA

Parámetros de control

$$K_p = 1.115$$

$$K_i = 5.557$$

$$K_d = 0.05588$$

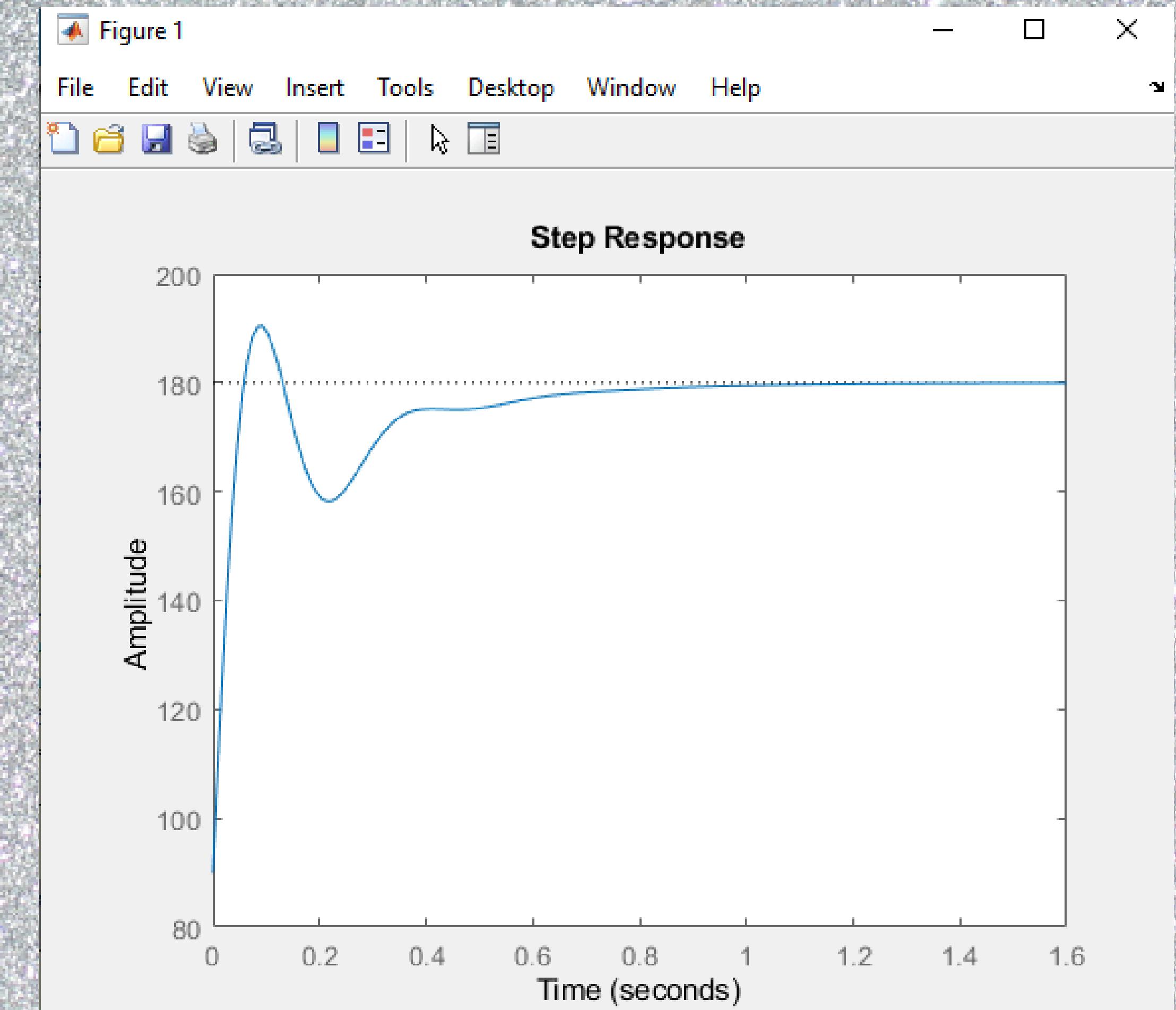
Desempeño

$$\text{SETTLING TIME} = 0.7 \text{ s}$$

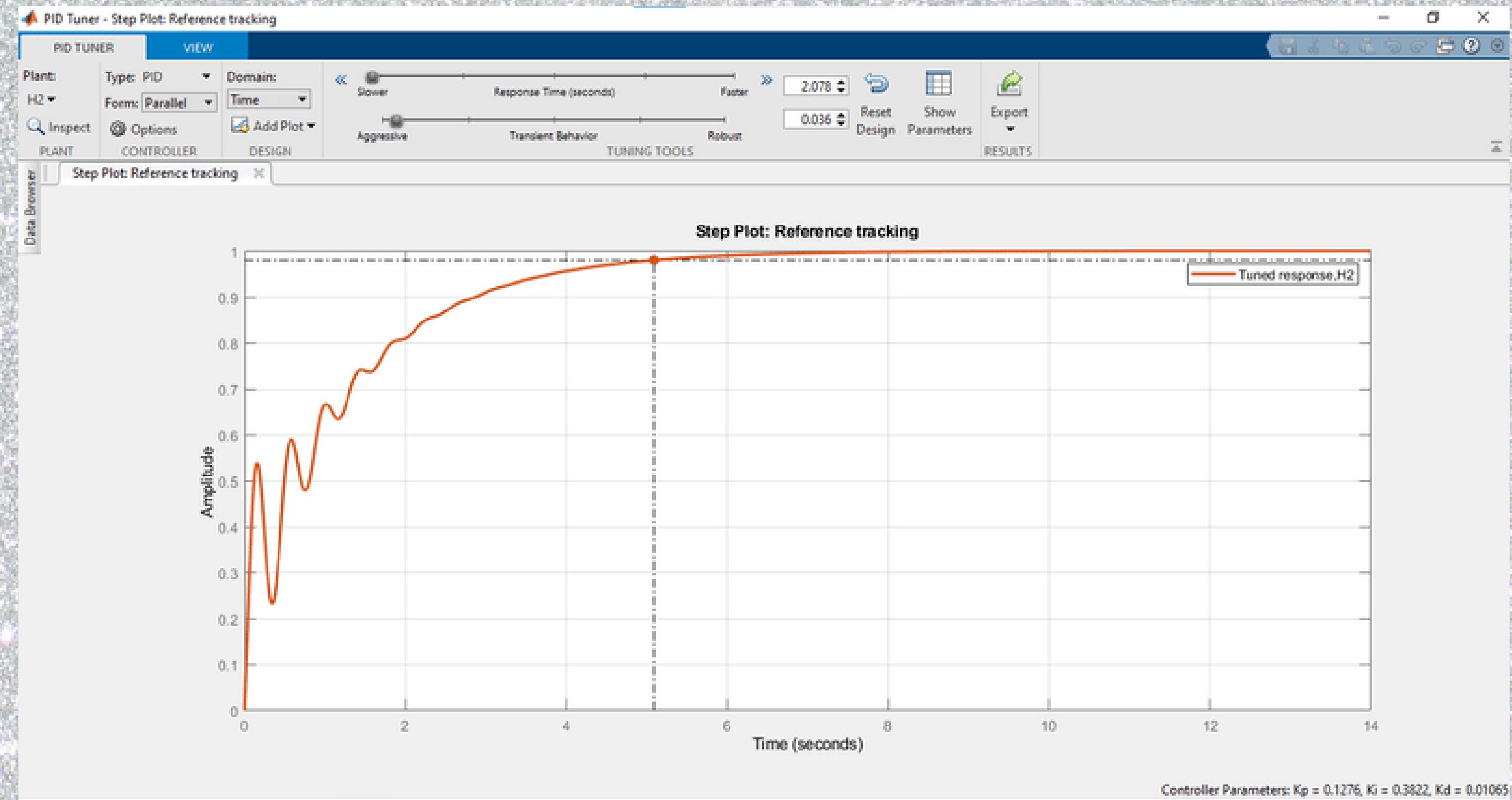
$$\text{OVERSHOOT} = 11.6\%$$

$$\text{RISE TIME} = 0.044 \text{ s}$$

$$\text{SSE} = 0\%$$



ESTADO DEL SISTEMA



ESTADO DEL SISTEMA

Parámetros de control

$$K_p = 0.1276$$

$$K_i = 0.3822$$

$$K_d = 0.01065$$

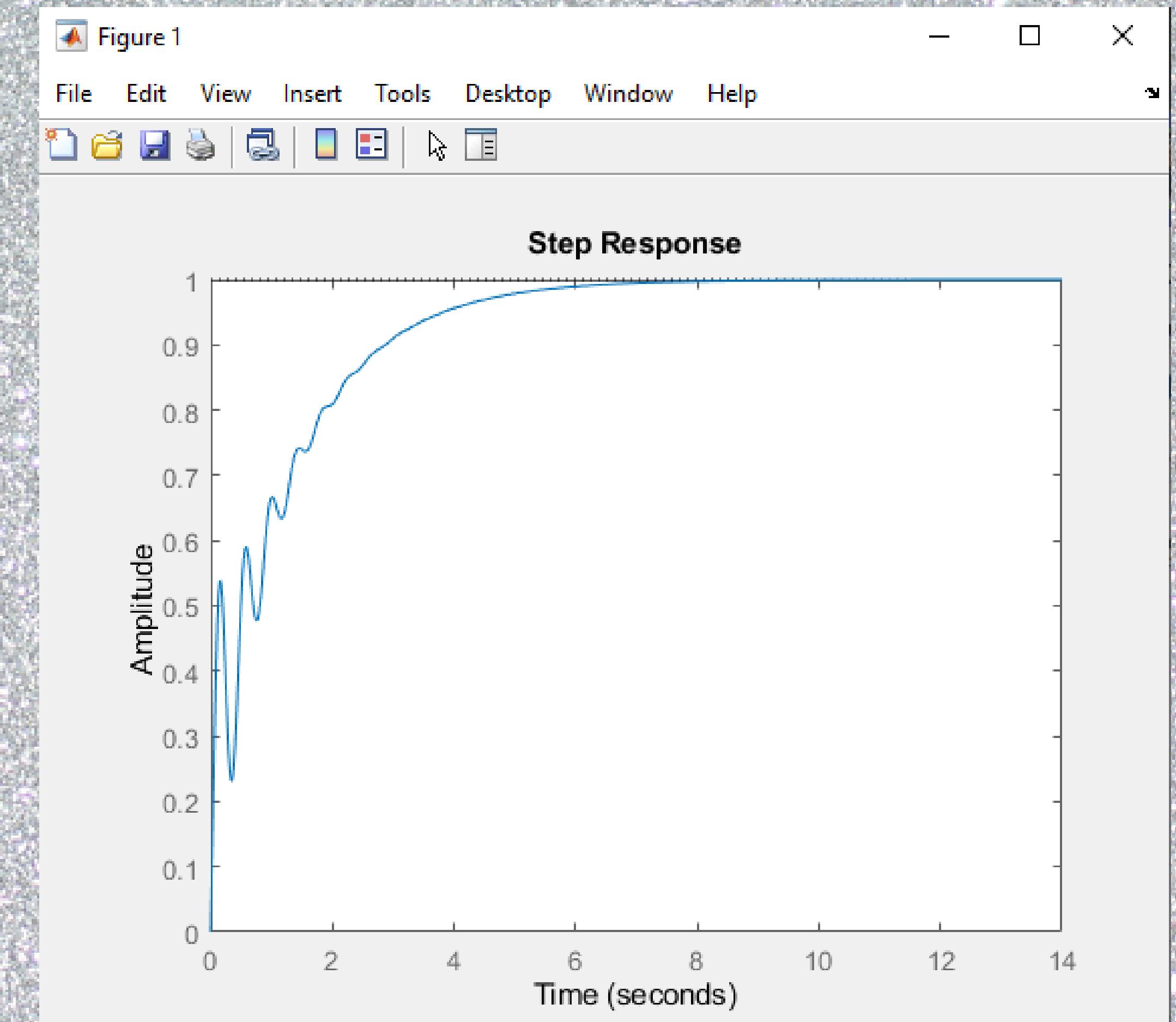
Desempeño

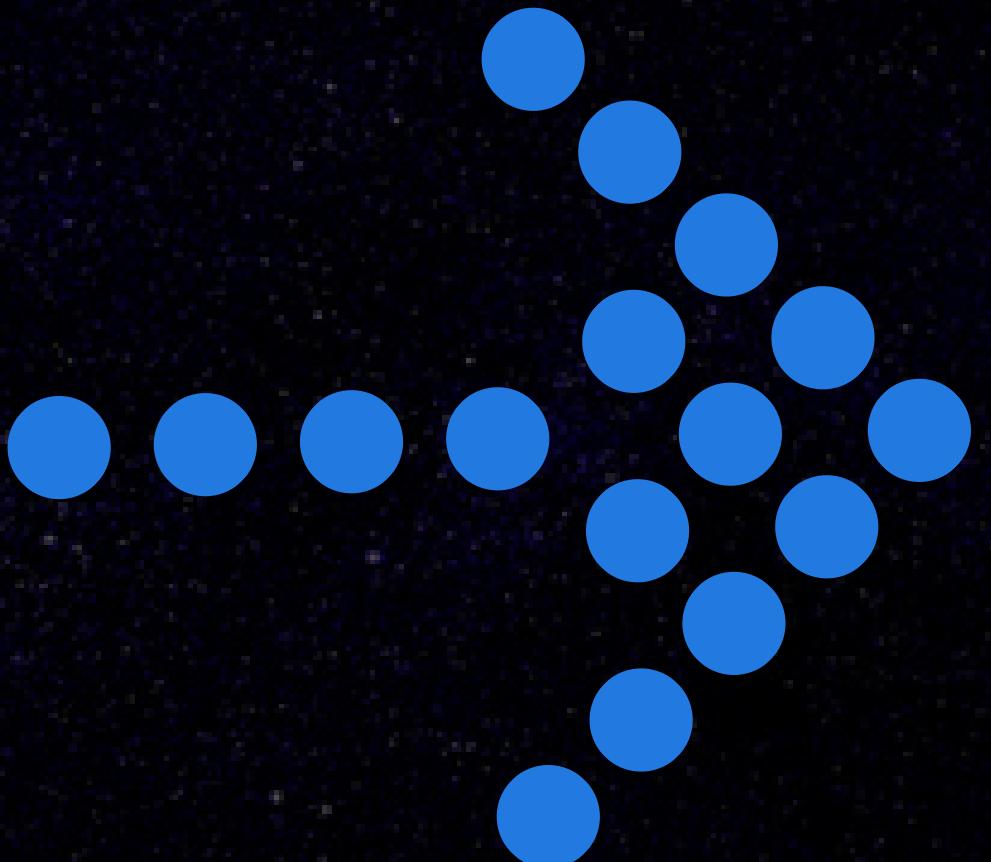
$$\text{SETTLING TIME} = 5.1 \text{ s}$$

$$\text{OVERSHOOT} = 0\%$$

$$\text{RISE TIME} = 2.85 \text{ s}$$

$$\text{SSE} = 0\%$$

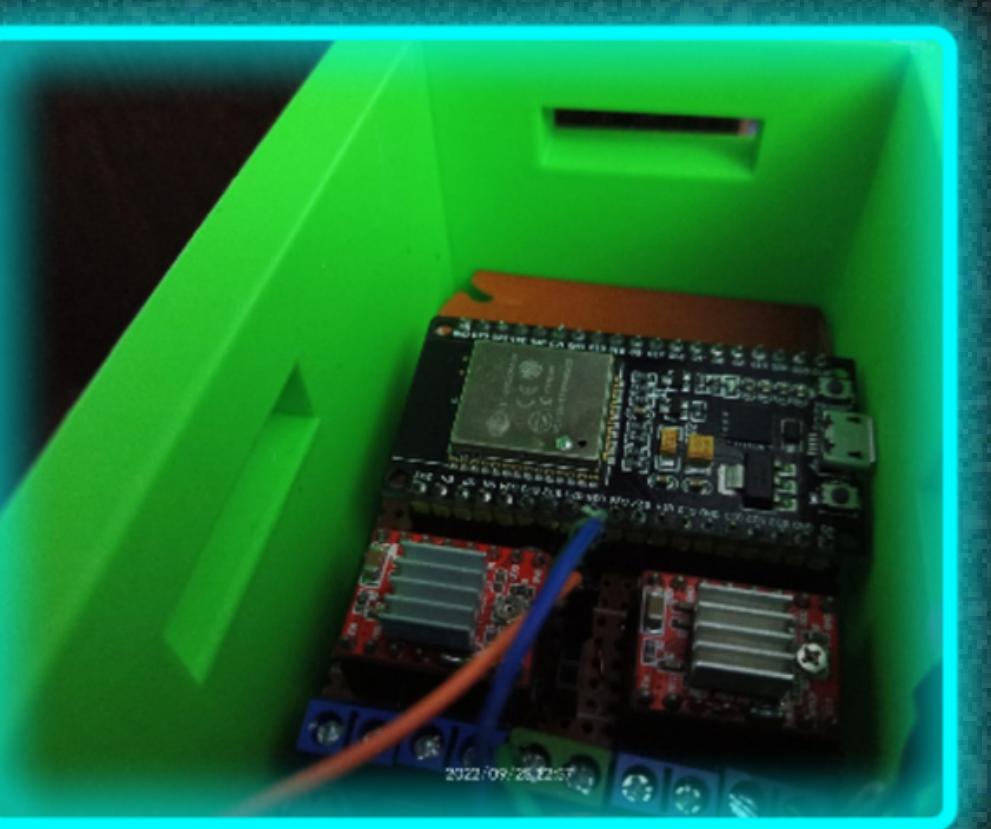
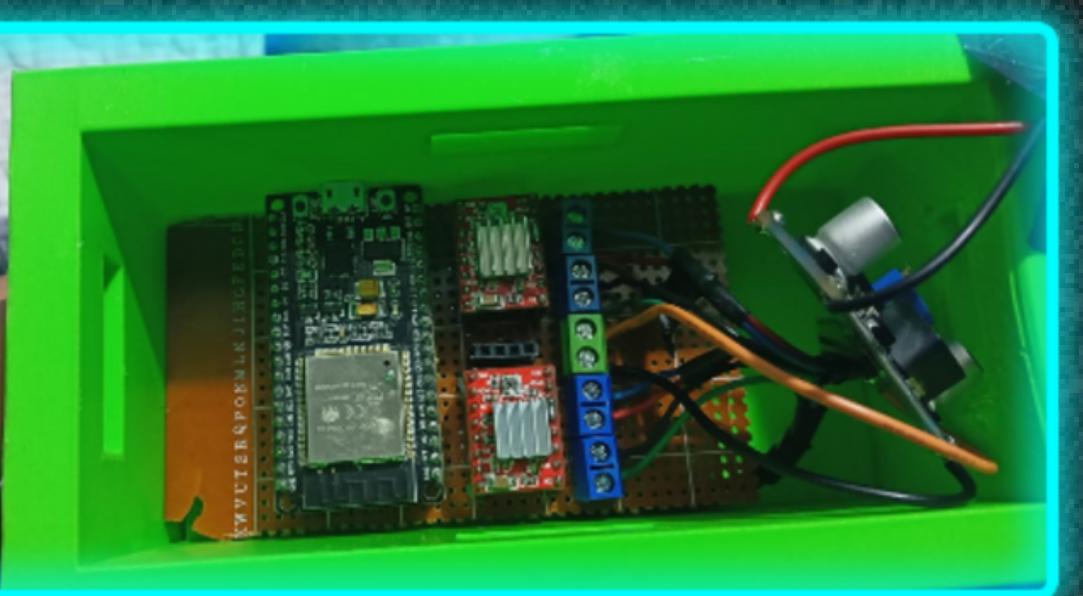


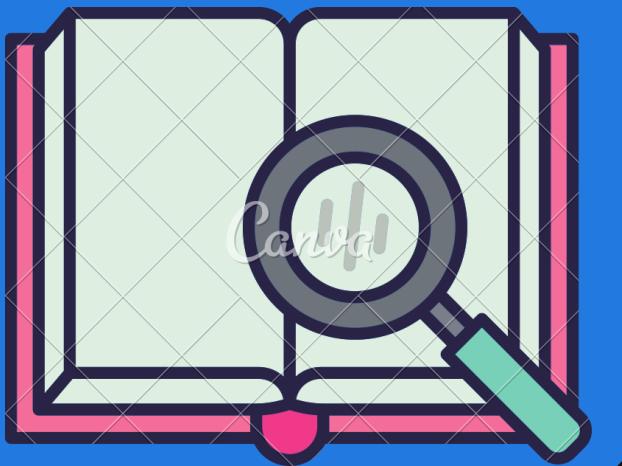


IMPLEMENTACIÓN DEL CONTROL PID



VIDEOS:





BIBLIOGRAFÍA

PID CONTROL

<https://drive.google.com/file/d/1HBY6u7q3DIyGf8npcriYDcEqIls7Xq-DS/view?usp=sharing>
<https://drive.google.com/file/d/1rWt8OJfALZ3hAgXhkbrf4hXow-xgHp6r/view?usp=sharing>

ESTADO DEL ARTE

<https://drive.google.com/drive/folders/11u83Fj0OHO8jXftq8-SaTluw9mk1gILCB?usp=sharing>

THAAN
YOO