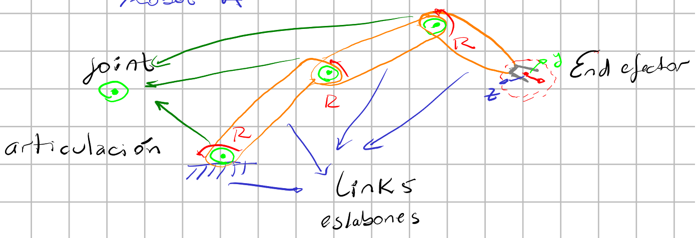
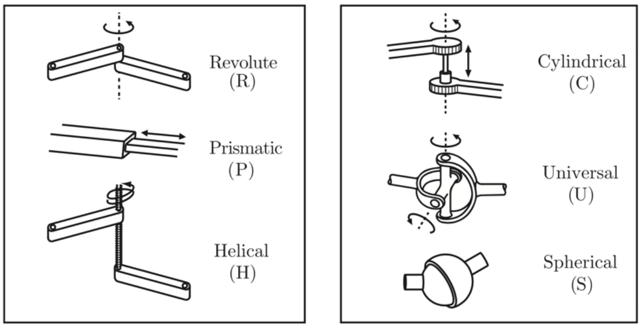
Robot A





Joint type	dof f
Revolute (R)	1
Prismatic (P)	1
Helical (H)	1
Cylindrical (C)	2
Universal (U)	2
Spherical (S)	3

Grübler's Formula

$$\text{dof} = m(N-1-J) + \sum_{i=1}^{J} f_i$$

f_i : dof of joint i

N : # links

J : # joints

m : 3 if plane, 6 if space

Robot A : $m = 3$, $J = 3$, $N = 4$

$$dof = 3(4-1-3) + \sum_{i=1}^{J} f_i$$

$$dof = 3(0) + (1+1+1) = 3$$

any robot like A :

plane case : $m = 3$, $J = x$, $N = J+1$, $f_i = 1$, $\sum f_i = J$

$$dof = 3(\cancel{J+1} - \cancel{1} - \cancel{J}) + \sum f_i$$

$$dof = 3(0) + J$$

$$\boxed{dof = J}$$

space : $m = 6$

$$\boxed{dof = \sum f_i}$$

Another robot :

$m = 3$, $J = 4$, $N = 4$

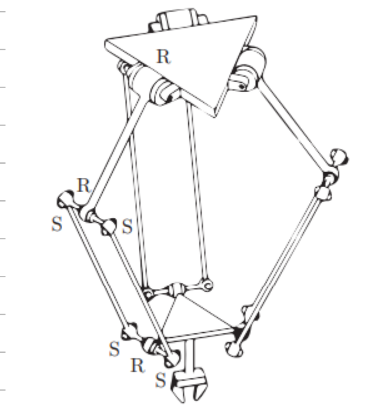
$$dof = m(N-1-J) + \sum f_i$$

$$dof = 3(4-1-4) + 4$$

$$dof = 3(-1) + 4$$

$$\boxed{dof = 1}$$

Delta Robot



$m = 6$

$N = 5 \times 3 + 2$

$J = 7 \times 3$

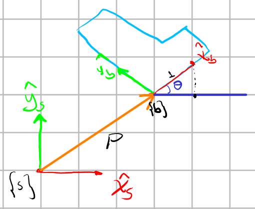
$$dof = m(N-1-J) + \sum f_i$$

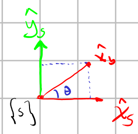
$$dof = 6(17-1-21) + (9R + 12S)$$

$$dof = 6(-5) + (9 \times 1 + 12 \times 3)$$

$$dof = -30 + 9 + 36$$

$$\boxed{dof = 15}$$

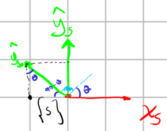




$\hat{x}_b = (\cos\theta, \sin\theta)$

$$\hat{x}_b = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \cos\theta\,\hat{x}_s + \sin\hat{y}_s$$

$$r = \begin{bmatrix} r_x \\ r_y \end{bmatrix}$$



$\hat{y}_b = (-\sin\theta, \cos\theta)$

$$\hat{y}_b = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} = -\sin\theta\,\hat{x}_s + \cos\theta\,\hat{y}_s$$

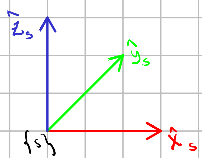
$p = (3,2) = (p_x, p_y)$

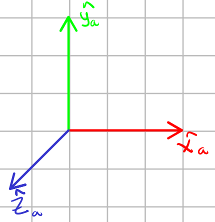
$$\begin{bmatrix} \hat{x}_b \\ \hat{y}_b \end{bmatrix} = [\hat{x}_s \quad \hat{y}_s]\, R \qquad R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

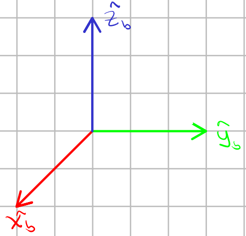
$p = p_x \hat{x}_s + p_y \hat{y}_s$

$p = p_x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + p_y \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

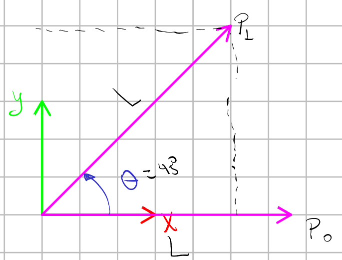
$p = \begin{bmatrix} p_x \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ p_y \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$







$$\hat{x}_a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{y}_a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \hat{z}_a = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \hat{x}_b = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \hat{y}_b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{z}_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

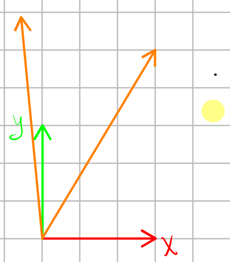


$\|P_0\| = \|P_1\|$

$P_1 = \left(\frac{L}{\sqrt{2}}, \frac{L}{\sqrt{2}}\right)$

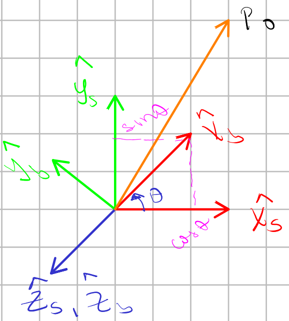
$$\begin{bmatrix} P_{1x} \\ P_{1y} \end{bmatrix} = R \begin{bmatrix} P_{0x} \\ P_{0y} \end{bmatrix}$$

$$\begin{bmatrix} P_{1x} \\ P_{1y} \end{bmatrix} = R \begin{bmatrix} L \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} P_{1x} \\ P_{1y} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} L \\ 0 \end{bmatrix}$$

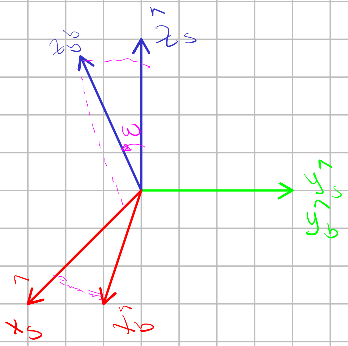
$$\begin{bmatrix} P_{1x} \\ P_{1y} \end{bmatrix} = \begin{bmatrix} L\cos\theta \\ L\sin\theta \end{bmatrix}$$



$$P_0 = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$$

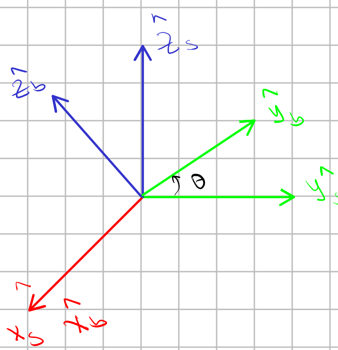
$$\hat{x}_b = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \quad \hat{y}_b = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix} \quad \hat{z}_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



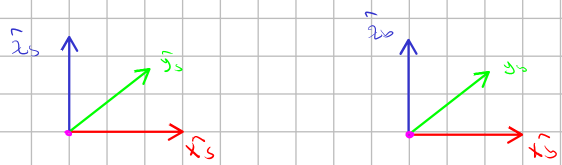
$$\hat{x}_b = \begin{bmatrix} \cos\omega \\ 0 \\ -\sin\omega \end{bmatrix} \quad \hat{y}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \hat{z}_b = \begin{bmatrix} \sin\omega \\ 0 \\ \cos\omega \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\omega & 0 & \sin\omega \\ 0 & 1 & 0 \\ -\sin\omega & 0 & \cos\omega \end{bmatrix}$$

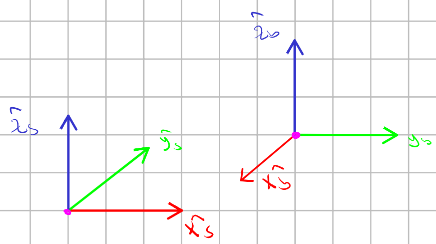


$$\hat{x}_b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad y_b = \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix} \quad z_b = \begin{bmatrix} 0 \\ -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$



$$\hat{x}_b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{y}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad z_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad p = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$



$$\hat{x}_b = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \quad \hat{y}_b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad z_b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad p = \begin{bmatrix} 6 \\ 0 \\ 2 \end{bmatrix}$$

$$Rot(\omega,\theta) = R \qquad Trans(p) \quad \begin{matrix} p_x \\ p_y \\ p_z \end{matrix}$$

$$T = Rot * Trans \qquad R \quad p = \mathbb{R}^3$$

$$T = \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbb{R}^{3\times3} * \mathbb{R}^{3\times1} = \mathbb{R}^{3\times1} \iff \mathbb{R}^{3\times3} \qquad \begin{matrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 0 & p_z \end{matrix}$$

$$\iff R$$

$$T_a = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R P

$$T_a T_b = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

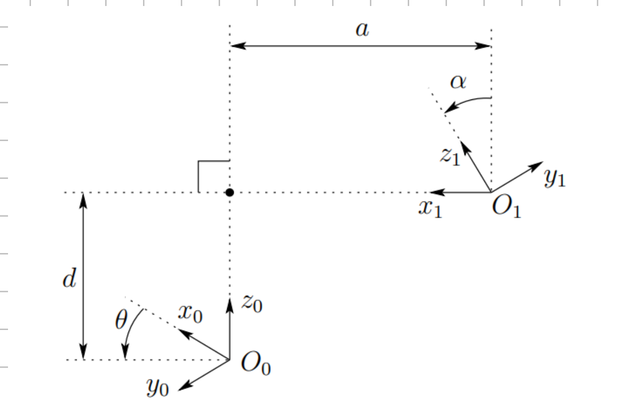
Donavit Hartenberg Representation

$$A_i = R_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, R_{x,\alpha_i}$$

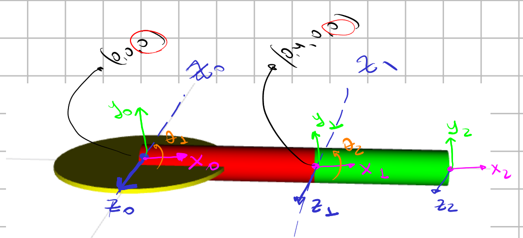
$C_\theta = \cos\theta$

$S_\theta = \sin\theta$

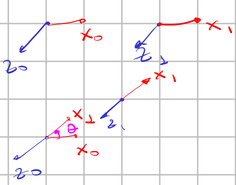
$$A_i = \begin{bmatrix} C_\theta & -S_\theta & 0 & 0 \\ S_\theta & C_\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_{\alpha_i} & -S_{\alpha_i} & 0 \\ 0 & S_{\alpha_i} & C_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

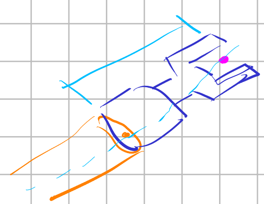


$$A_i = R_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, R_{x,\alpha_i}$$



N	θ_i	d_i	a_i	α_i
1	θ_1	0	0,4	0
2	θ_2	0	0,3	0





Step 1: Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step 2: Establish the base frame. Set the origin anywhere on the z_0-axis. The x_0 and y_0 axes are chosen conveniently to form a right-hand frame.

For $i = 1, \ldots, n-1$, perform Steps 3 to 5.

Step 3: Locate the origin O_i where the common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate O_i at this intersection. If z_i and z_{i-1} are parallel, locate O_i in any convenient position along z_i.

Step 4: Establish x_i along the common normal between z_{i-1} and z_i through O_i, or in the direction normal to the $z_{i-1} - z_i$ plane if z_{i-1} and z_i intersect.

Step 5: Establish y_i to complete a right-hand frame.

Step 6: Establish the end-effector frame $o_n x_n y_n z_n$. Assuming the n-th joint is revolute, set $z_n = \boldsymbol{a}$ along the direction z_{n-1}. Establish the origin O_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = \boldsymbol{s}$ in the direction of the gripper closure and set $x_n = \boldsymbol{n}$ as $\boldsymbol{s} \times \boldsymbol{a}$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-hand frame.

Step 7: Create a table of link parameters $a_i, d_i, \alpha_i, \theta_i$.

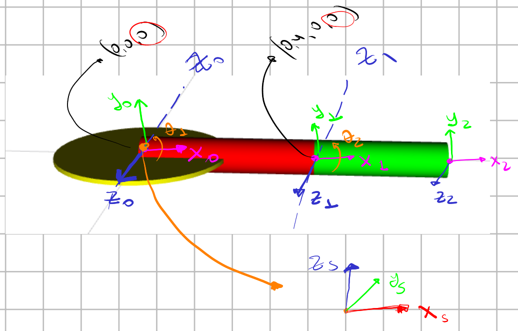
$a_i =$ distance along x_i from O_i to the intersection of the x_i and z_{i-1} axes.

$d_i =$ distance along z_{i-1} from O_{i-1} to the intersection of the x_i and z_{i-1} axes. d_i is variable if joint i is prismatic.

$\alpha_i =$ the angle between z_{i-1} and z_i measured about x_i (see Figure 3.3).

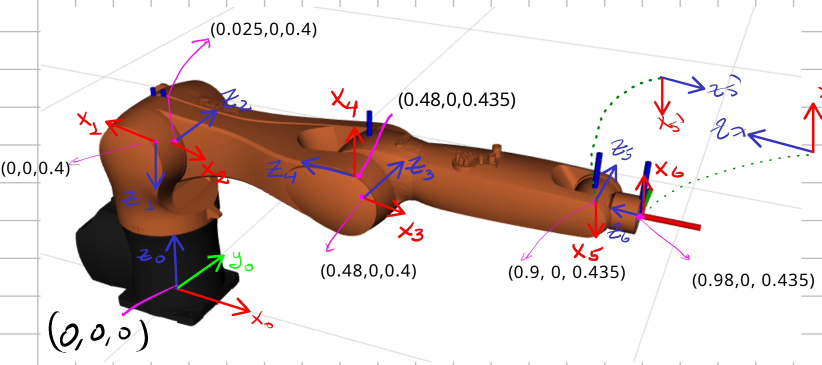
$\theta_i =$ the angle between x_{i-1} and x_i measured about z_{i-1} (see Figure 3.3). θ_i is variable if joint i is revolute.

Step 8: Form the homogeneous transformation matrices A_i by substituting the above parameters into (3.10).



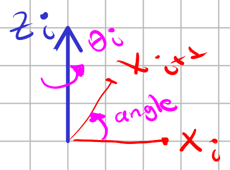
i	θ_i	d_i	a_i	α_i
0	0	0	0	$\pi/2$
1	θ_1	0	0,4	0
2	θ_2	0	0,3	0

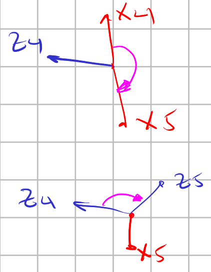
l_2



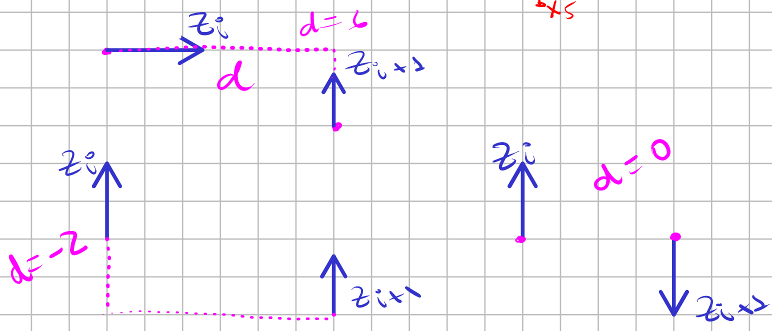
	θ	d	a	α
0 – 1	pi	0,4	0	pi
1 – 2	θ_1+pi	0	0.025	pi/2
2 – 3	θ_2	0	0.455	0
3 – 4	θ_3–pi/2	0	0.035	pi/2
4 – 5	θ_4+pi	–0.42	0	pi/2
5 – 5p	θ_5	0	0	pi/2
5p – 6	pi	0.08	0	pi
6 – 7	θ_6	0	0	0

θ

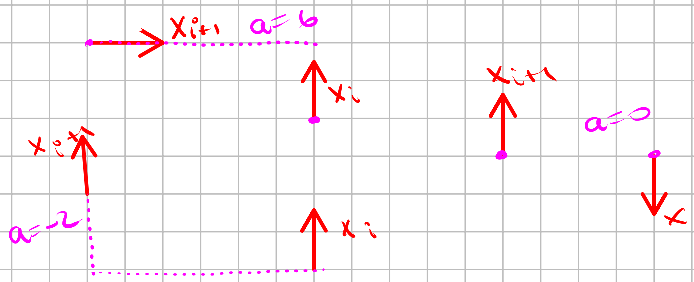




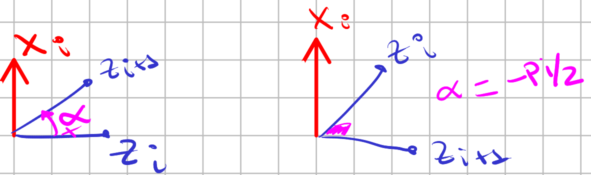
d

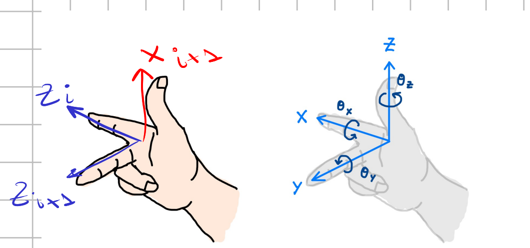


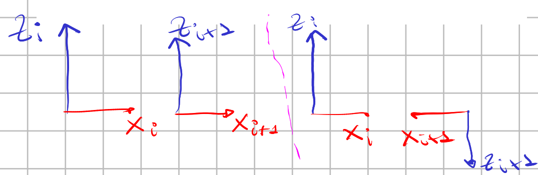
a

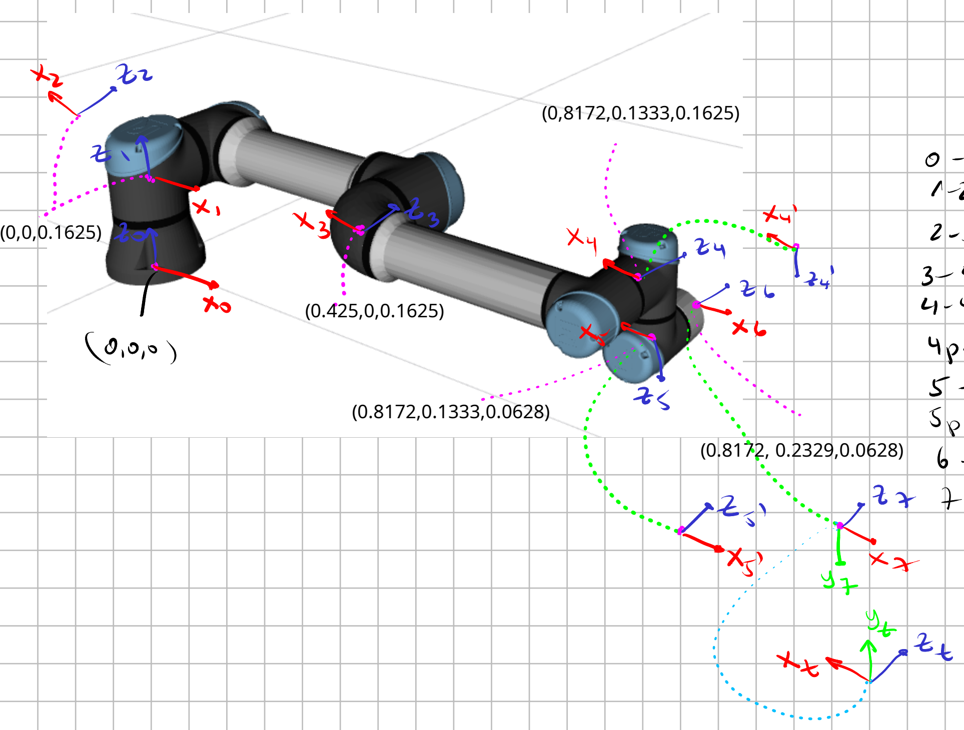


α







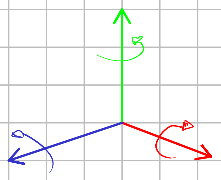


	θ	d	a	α
0-1	0	0.1625	0	0
1-2	θ_1+pi	0	0	pi/2
2-3	θ_2	0	-0.425	0
3-4	θ_3	0.1333	-0.3922	0
4-4p	θ_4	0	0	pi/2
4p-5	0	0.0997	0	0
5-5p	θ_5+pi	0	0	pi/2
5p-6	0	0.0996	0	0
6-7	θ_6	0	0	0
7-t	pi	0	0	0

$$R_t = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{matrix} \overset{-1}{\cancel{\cos\theta}} & \overset{0}{\cancel{\sin\theta}} & 0 \\ \underset{0}{\cancel{\sin\theta}} & \overset{-1}{\cancel{\cos\theta}} & 0 \\ 0 & 0 & 1 \end{matrix}$$

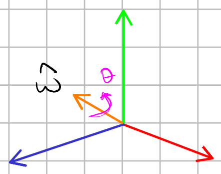
$\theta = pi$



$$R_{\hat{x}}(\theta) \to R(\hat{\omega}, \theta) \qquad R_{\hat{x}}(\theta) = R([1,0,0], \theta)$$

$$R_{\hat{y}}(\theta) \qquad R_{\hat{y}}(\theta) = R([0,1,0], \theta)$$

$$R_{\hat{z}}(\theta) \qquad R_{\hat{z}}(\theta) = R([0,0,1], \theta)$$

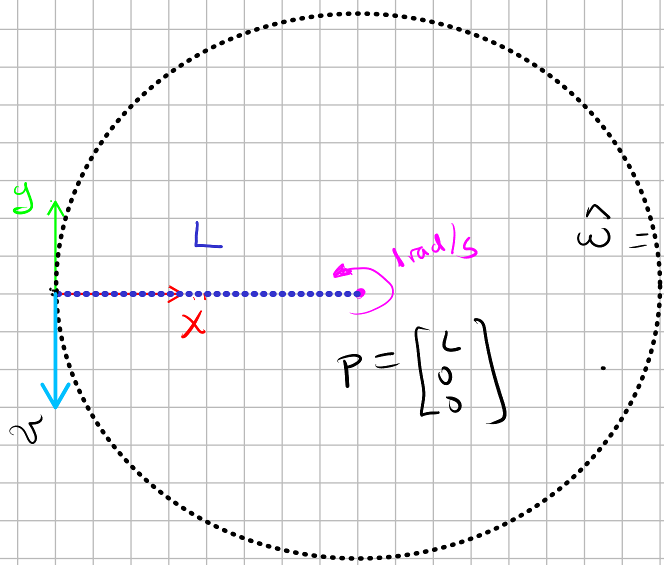


$$R(\hat{\omega}, \theta) = I + \sin\theta [\hat{\omega}] + (1 - \cos\theta)[\hat{\omega}]^2$$

$$R_x(\theta) = I + \sin\theta \left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right] + (1 - \cos\theta) \left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right]^2$$

Skew - Symetric Matrix

$[\hat{\omega}] =$



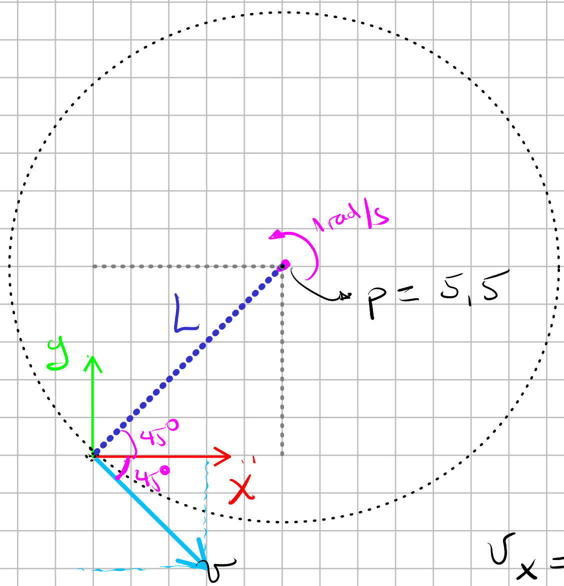
$$\hat{\omega} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$v_x = 0$

$v_y = -L$

$v_z = 0$

$$v = f(\hat{\omega}, P)$$



$$\hat{\omega} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$v_x = 5$

$v_y = -5$

$v_z = 0$

$$v_x = |v|_x \cos(45^\circ) = 5\cancel{\sqrt{2}} \cdot 2/\cancel{\sqrt{2}} = 5$$

$$v_y = -|v|_x \sin(45^\circ) = -5\cancel{\sqrt{2}} \cdot 2/\cancel{\sqrt{2}} = -5$$

$$|v| = L = 5\sqrt{2}$$

$$[\hat{\omega}] = \begin{bmatrix} 0 & -\omega_2 & \omega_1 \\ \omega_2 & 0 & -\omega_0 \\ -\omega_1 & \omega_0 & 0 \end{bmatrix} \qquad \hat{\omega} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$v = -[\hat{\omega}] \times p \qquad -\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

$$-\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$$

Rodrigues

$$R(\hat{\omega}, \theta) = I + \sin\theta[\hat{\omega}] + (1-\cos\theta)[\hat{\omega}]^2$$

$$R_x(\theta) = I + \sin\theta\left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right] + (1-\cos\theta)\left[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right]^2$$

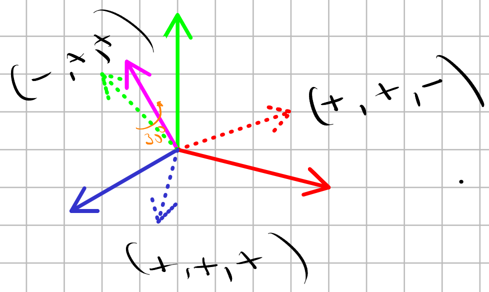
$$R_x(\theta) = I + \sin\theta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_x(\theta) = I + \sin\theta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -S_\theta \\ 0 & S_\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & C_\theta - 1 & 0 \\ 0 & 0 & C_\theta - 1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\theta & -S_\theta \\ 0 & S_\theta & C_\theta \end{bmatrix}$$

R.subs({theta:0.524,w0:0,w1:0.866,w2:0.5})



Inverse Kinematic

$\theta_0, \theta_1 \ldots \theta_n = f(x, y, z)$

$\theta_0, \theta_1 \ldots \theta_n = f(x, y, z, roll, pitch, yaw)$

$= f(x, y, z, x, y, z, w)$

$= f(x, y, z, R)$

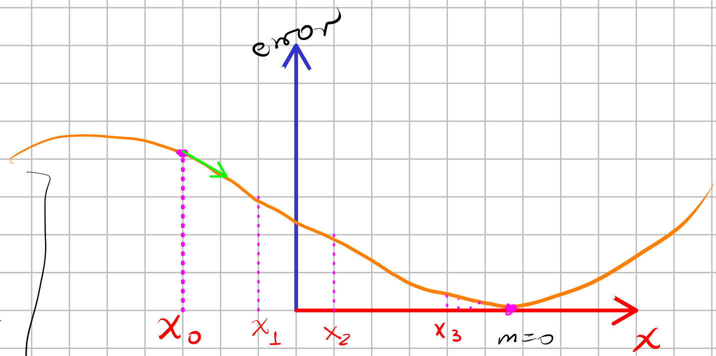
$x, y, z =$ target

$f(\theta) = x_i, y_i, z_i$

target $- f(\theta) = 0$

target $- T(\theta) = 0$

target $- f(x) =$ error



$$J = \begin{bmatrix} \frac{\partial px}{\partial \theta_1} & \frac{\partial px}{\partial \theta_2} & \cdots & \frac{\partial px}{\partial \theta_n} \\ \frac{\partial py}{\partial \theta_1} & \frac{\partial py}{\partial \theta_2} & \cdots & \frac{\partial px}{\partial \theta_n} \\ \frac{\partial pz}{\partial \theta_n} & \frac{\partial pz}{\partial \theta_2} & \cdots & \frac{\partial pz}{\partial \theta_n} \end{bmatrix}$$

$m = \frac{d f(x_0)}{dx}$

$x_{i+1} = x_i + \frac{df(x_i)}{dx} * e$

$f(\theta_1, \theta_2) = \frac{\partial f(\theta_1, \theta_2)}{\partial \theta_1}, \frac{\partial f(\theta_1, \theta_2)}{\partial \theta_2}$

$J(P, \theta) = J_{(px, py, pz, [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5])}$

$J = R^{3 \times 5}$ $\quad \theta = R^{1 \times 5}$

$y = a \cdot x$

$\frac{y}{a} = x$

$y a^{-1} = x$

$R^{3 \times 5} * R^{5 \times 3} = R^{3 \times 3}$

$J \times e = R^{1 \times 5}$

$R^{3 \times 5} \times R^{1 \times 3} = R^{1 \times 5}$

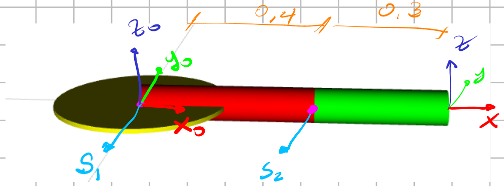
$J^{-1} \Rightarrow$ pseudo inverse $= J^T (J J^T)^{-1}$

$$A A^{-1} = I$$

$$J^{3\times 5}\left(J^{5\times 3}\left(J^{3\times 3}\right)^{-1}\right)$$

$$J^{3\times 5}\left(J^{5\times 3} J^{3\times 3}\right) = J^{3\times 5} J^{5\times 3} = J^{3\times 3} = I$$

Power of Exponentials (PoE)



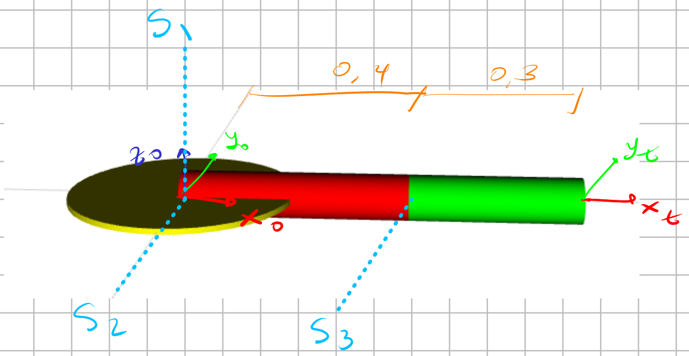
$$M = \begin{bmatrix} 1 & 0 & 0 & 0{,}7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = T_1 T_2 T_3 \dots T_n M = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]} \dots e^{[S_n]} M \qquad S = \begin{bmatrix} \omega \\ v \end{bmatrix}$$

$S_1:$ $\hat{\omega} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $\quad q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$T = e^{[S_1]\theta_1} e^{[S_2]\theta_2} M$$

$S_2:$ $\hat{\omega} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $\quad q = \begin{bmatrix} 0{,}4 \\ 0 \\ 0 \end{bmatrix}$



$$M = \begin{bmatrix} 1 & 0 & 0 & 0{,}7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$S_1:$ $\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\qquad S_2: \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $\quad q = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$S_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $\quad q = \begin{bmatrix} 0{,}4 \\ 0 \\ 0 \end{bmatrix}$

displacing a frame {b}:

$$T = \begin{bmatrix} \mathrm{Rot}(\hat{\omega}, \theta) & p \\ 0 & 1 \end{bmatrix}$$

$T_{sb'} = TT_{sb}$: rotate θ about $\hat{\omega}_s = \hat{\omega}$ (moves {b} origin), translate p in {s}

$T_{sb''} = T_{sb}T$: translate p in {b}, rotate θ about $\hat{\omega}$ in new body frame

"unit" screw axis is $\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$,

where either (i) $\|\omega\| = 1$ or (ii) $\omega = 0$ and $\|v\| = 1$

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for a screw axis $\{q, \hat{s}, h\}$ with finite h,

$$\mathcal{S} = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \hat{s} \\ -\hat{s} \times q + h\hat{s} \end{bmatrix}$$

---

twist is $\mathcal{V} = \mathcal{S}\dot{\theta}$

for $\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} \in \mathbb{R}^6$,

$$[\mathcal{V}] = \begin{bmatrix} [\omega] & v \\ 0 & 0 \end{bmatrix} \in se(3)$$

(the pair (ω, v) can be a twist $\mathcal{V}$ or a "unit" screw axis $\mathcal{S}$, depending on the context)

$$\exp : [\hat{\omega}]\theta \in so(3) \to R \in SO(3)$$

$$R = \mathrm{Rot}(\hat{\omega}, \theta) = e^{[\hat{\omega}]\theta} =$$

$$I + \sin\theta[\hat{\omega}] + (1 - \cos\theta)[\hat{\omega}]^2$$

$$\exp : [\mathcal{S}]\theta \in se(3) \to T \in SE(3)$$

$$T = e^{[\mathcal{S}]\theta} = \begin{bmatrix} e^{[\omega]\theta} & * \\ 0 & 1 \end{bmatrix}$$

where $* =$

$$(I\theta + (1 - \cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)v$$