Cost Effective Demonstration of Wave-Particle Duality on a Table Top

Johnny Allain-Labon, Chong Keat Gea, Steven Vuong, Georges Ajaka,
Benji Berczi, Kelvin Fang, Alex Stock, Mohit Motwani

11/1/18, Lab 1, Department of Physics and Astronomy, University College London, London WC1E 6BT
Board member: Professor Ryan Nichol

Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

CONTENTS

I. Introduction

An age-old and much investigated field of research in science has been into the nature of light. For centuries, scientists have endeavoured to understand its nature and fundamental properties. At the turn of the 20th century, light was widely believed to behave as a wave, but the discovery of phenomena such as the photoelectric effect called this theory into question. Eventually, it was agreed on that light exhibited both wave-like and particle like behaviour, a phenomenon known as wave-particle duality. The interpretation of this quantum mechanical behaviour, however, is still open to debate. The commonly accepted interpretation, known as the Copenhagen Interpretation, was first proposed by Niels Bohr and Werner Heisenberg in 1927. It pushed forward the idea that a physical system would only have definite properties upon measurement; prior to measurement it was only possible to measure probabilities of obtaining a given measurement. An alternative interpretation, the De Broglie Bohm theory, was pushed forwards in 1952. It is deterministic in nature; it claims that even prior to observation there exists a driving equation or 'pilot wave' whose properties determine measurements. In 2005, research by *Couder et al.* [1] suggested that droplets bouncing on a surface could display quantum mechanical behaviour as proposed by the pilot wave theory. Indeed this macro system was a remarkably good analogy for the pilot wave theory; waves induced by a driving droplet quite closely match a driving wave-equation determining the results of measurements.

A. Aims

To this end, the purpose of this project is to develop a cost-effective table top version of the experiment used by *Couder et al.*,demonstrating the various different behaviours of bouncing droplets. Such motion can include simple oscillations, motion parallel to the liquid surface and double slit diffraction of droplet generated waves. It is intended to be paired with a computer based visualisation that can display effects outside the bounds of this apparatus. The overarching goal will be to develop a prototype that can be used for outreach purposes, allowing students to the grasp the non-intuitive nature of quantum mechanics.

B. Objectives

These above aims raise the following core project objectives:

1) Development of table top apparatus:
    a) To create a working first prototype capable of displaying basic motion, such as bouncing and walking droplets
    b) To develop an effective method by which the rapid motion of droplet oscillations can be recorded.
    c) To further improve the prototype, and demonstrate multiple droplet motion (such as droplets repelling and coalescing) and phenomena such as double slit diffraction
2) Development of an in-depth visualisation simulation:
    a) To identify and implement the mathematical equations necessary for displaying single droplet motion in Java.
    b) To extend the capabilities of the program in order to cover aspects of quantum mechanics not displayable by the experimental apparatus. Such effects may include double slit diffraction and 'tunneling' effects, where the droplet escapes over the edge of the apparatus.

3) Creating an iteration of the initial prototype usable in college/undergraduate courses as a demonstration tool.
   a) To create a standalone version of the simulation program.
   b) To create a simple to use version of the experimental apparatus, for third-party users.
   c) To write a business plan for implementation in future.
   d) To explore the possibility of testing the apparatus with undergraduates and A-level students.

II. Theory

A. *Droplet motion*

The basis of this investigation lies on the motion of moving droplets on a liquid surface. In 1978 it was reported by J. Walker [2] that droplets vibrating at a certain frequency upon a soap-solution surface would not immediately coalesce, but could maintain motion for up to 18 minutes. Couder *et al.* further researched this phenomenon in 2005. Using a silicon oil surface(with no soap-substrate) they discovered that it was possible to create a an oscillating droplet at certain frequencies, and even walking droplets that moved parallel to the surface. The walking droplets were created by simply vibrating the liquid surface vertically at a frequency determined by Equation 1:

$$\frac{\gamma_m^B}{g} \approx 1 \tag{1}$$

Where g is the acceleration due to gravity, γ_m^B is the specific value of co-efficient γ_m of Equation 2:

$$\gamma = \gamma_m cos(2\pi f_0 t) \tag{2}$$

Equation 2 determines γ, which is the vertical acceleration of the fluid, f_0 is the driving frequency of the liquid and t is time. The droplets bounce due to a remarkably simple mechanism. Upon collision of a droplet with the liquid, there exists a thin layer of air trapped between the two surfaces. Normally, the droplet will coalesce with a surface in a matter of seconds, as this air is forced away. But above a threshold value of γ_m, the droplet will instead bounce, as the air film retains its integrity, forcing the droplet upwards. By specifying a threshold vertical acceleration, it is also ensured that the air film will have a chance to replenish after each subsequent bounce, allowing the motion to be periodic over a large period of time. The regimes in which different types of motion occur are described in Figure 1

Furthermore, droplets can also exhibit motion parallel to the droplet surface. Such droplets are labelled walkers, and like bouncing droplets, they exist in a certain $\frac{\gamma_m}{g}$ regime, just below the onset of the Faraday instability. The Faraday instability can be defined as the point at which a surface becomes spontaneously wavy. Just before its onset, a bouncing droplet has a period of motion twice that of its driving oscillation. It thus generates a damped travelling Faraday wave at each bounce where Faraday waves are standing waves generated by a vibrating liquid. The subsequent landing then occurs on the reverse slope of this previously generated Faraday wave. This landing generates another travelling Faraday wave, and causes the droplet to perform a parabolic bounce, causing motion parallel to the surface. An intuitive way to understand this motion is to imagine jumping on a trampoline. Each time you land on the trampoline, a wave is generated. If you land again anywhere but at the centre of this wave, you spring off again, and your motion has a slight horizontal component, allowing you to travel around the trampoline. As the wave

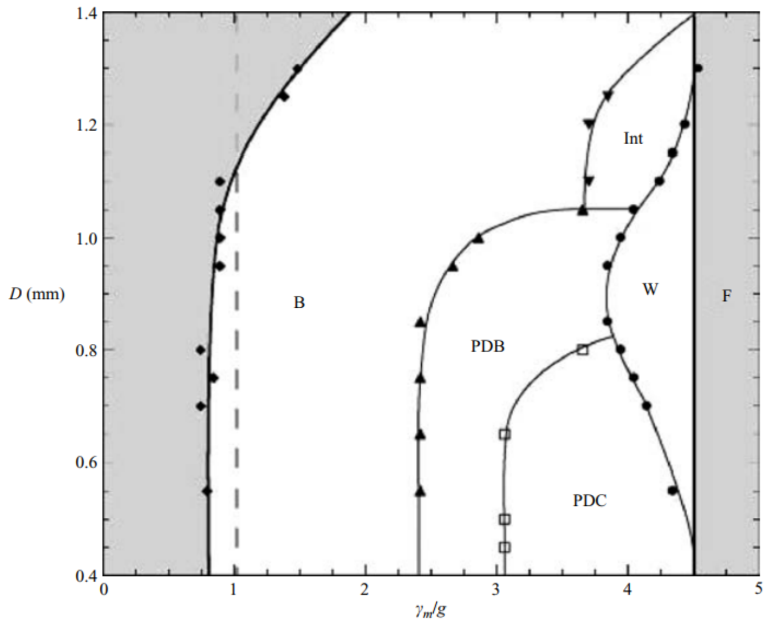


Fig. 1: A graph of droplet Diameter against $\frac{\gamma_m}{g}$ for an oscillating droplet in a vibrating liquid. It illustrates the different vertical acceleration regimes: Bouncing(B), Walking(W), Faraday instability(F), Period Doubling (PDB), transition from Periodic to chaotic behaviour(PDC) and Intermittent behaviour(Int).

you generate when landing travels, you constantly land on its side, therefore there is a constant horizontal motion, allowing you to 'walk' around the trampoline. This motion is illustrated in Figure 2:

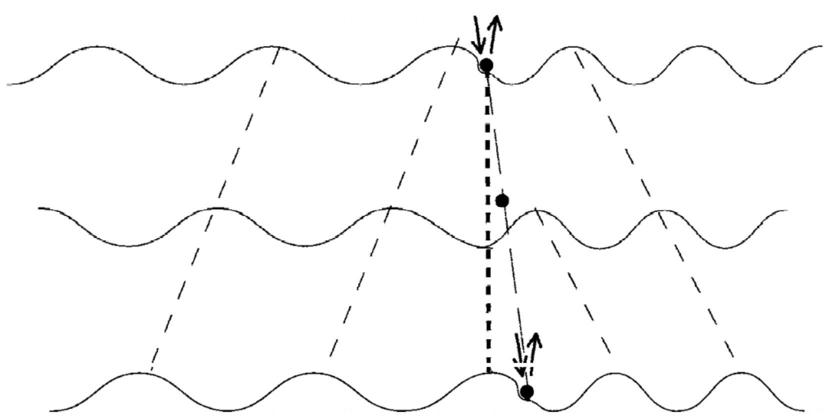


Fig. 2: An illustration of a walking droplet's motion for subsequent bounces. The movement of droplet is exaggerated for visual purposes. In reality, wave amplitude decays with distance from droplet and is not necessarily constant; it is modulated by the driving force

B. *Pilot Wave Theory*

Pilot Wave Theory, also known as the De Broglie-Bohm theory, is an alternative deterministic interpretation to quantum mechanics.

It proposes that a 'hidden equation' is partly responsible for determining the measurements of a system, and that this measurement result exists prior to measurement. For a single particle case, this guiding equation is specified as Equation 3:

$$\frac{d\mathbf{Q}}{dt}(t) = \frac{\hbar}{m}\mathrm{Im}\left(\frac{\nabla\psi}{\psi}\right)(\mathbf{Q}, t) \tag{3}$$

Where Q is the 'configuration space', ψ is the particle wave-function, m is the particle mass and $\hbar$ is the reduced Planck constant.

This is unlike the Copenhagen interpretation where, as an example, the velocity of a particle is only known after measurement. Pilot Wave Theory also satisfies nonlocality, or the idea that a system of particles separated by a large distance can instantaneously know of each others state. As the pilot wave equation and Schrodinger's equation jointly specify the velocity of a particle according to the position of other particles in the defined 'universe', there is a necessary inter-dependency that is not local to the particle's position.

Surprisingly enough, on a macroscopic level, Pilot Wave Theory is very well described by the motion of droplets on a vibrating surface, and the subsequent Faraday waves. In the following discussion, quantum mechanical effects and their analogies to this droplet system will be introduced:

III. Initial Apparatus

The initial version of this apparatus is based off previous run experiments by *Harris et al.*. It consists of a 250 W, 8 Ohms High Powered Woofer With Aramid Fibre Cone, serving as the source of vibrations, mounted in a custom made ply-wood frame. The speakers volume, and thus the amplitude of the vibrations was altered through either a a signal generator or a signal generating app on a phone. In order to ensure that vibration amplitude was sufficiently large, an amplifier[specify type] was used. This amplifier also allowed for bass and treble modification.

A piece of plywood with a 6.5 inch diameter, that fit perfectly on the rim of the speaker.Graph paper was then cut into shape and placed on the wooden surface. The graph paper was attached to the wood surface using double sided tape. The wood surface was secured to the speaker using a combination of fold-back clips and double sided tape. A [] inch petri dish was filled with 50 cSt silicon oil. The petri dish was attached to the graph paper using double sided tape carefully placed on the edges of the dish, to ensure that the remains of tape did not coat its lower surface, which interfered with images. The droplet motion was observed using a high-speed camera capable of recording at 1,000 fps; the recording area was illuminated using two spotlights.

A. Box creation

To be filled in with the help of Gea.

B. Droplet Creation

A key aspect of this experiment involves creating a droplet of consistent size, in order to allow for various behavioural regimes to be consistently recreated. While previous such experiments use a 'trial-and-error' approach to such matters, due to our lack or ready access to strong light sources and suitable cameras, a more consistent approach was necessary.

An initial attempt was made using a needle, or segment of wire, dipped this rapidly into the liquid surface. As this approach was unsuccessful, an attempt was made to flick the top of the surface with the tool tip; a semi-reliable way of creating a droplet as it required a few attempts.It was then discovered that, by rapidly plunging and removing a segment of laser-cut plywood into the liquid surface, it was possible to create multiple droplets simultaneously [NEED WOOD IMAGE PLS]. An important point to note from 1 was that the regimes in which walking and bouncing droplets occur was dependent on droplet diameter. Therefore, while this approach was certainly successful, refinement was necessary in order to create droplets of consistent sizes. A range of smaller droplet sizes were obtained by reverting back to the needle where necessary

C. Signal Generation

Initially, an IsoTech[TYPE NECESSARY] signal generator was chosen to produce an input signal for the speaker. However, it was soon discovered that the signal generator did not produce a signal of enough power to create bouncing droplets. Therefore an amp, and a signal generator with a gain was obtained[model no].

It was discovered that the frequencies utilised (50Hz-80Hz) were low enough that the amplifier considered them to be in the bass regime. Therefore altering the treble had little effect on the vibrations produced, while changes in bass were quite significant. This was used to help increase the vertical acceleration of the droplet to the critical value necessary for walking behaviour.

Several weeks into experimentation, it was discovered that the amp was not grounded, while the amplified signal generator was. Therefore, both devices did not have a common ground voltage, leading to an improperly functioning amp, and a reduced signal amplitude. Subsequently, it was decided to use a signal generating amp on a phone to generate the driving frequency, with an amp to boost the signal.

D. Liquid Choice

It was discovered that liquid viscosity choice had a significant impact on droplet formation and behaviour. Initially, a 1000cSt silicon oil was used in this experiment, and the results were understandably non-conclusive. The high viscosity liquid resulted in droplets of an extremely large size, so much so that the droplets did not bounce, but established themselves on the surface for several seconds before coalescing with the surface, as predicted b[INSERT REFERENCE TO COUDER PAPER]. Therefore, a 50cSt oil was utilised instead. It is important to note that high viscosity liquids are usable in this experiment. Indeed[REFERENCE HERE] utilised such liquids to great success in their experiments. However, such liquids require vertical accelerations that the given apparatus is incapable of producing. A suitable replacement may be a vibration exciter mounted on a sturdier frame.

A further aim of this project is to develop a commercial version of the apparatus. Doing so would require a more affordable liquid, that would be more widely available outside of a laboratory, for easy replacement. One such option is to utilise liquid soap diluted in water 2, but these droplets have a small maximum lifetime of 18 minutes, only after a long period of operation. Secondly, vegetable oils are also an option. These options were investigated further later.

E. Recording Droplet Motion

Key to this experiment was recording the vertical motion of the droplet, which was too fast to be observed by eye. Initially, an attempt was made to use slow motion recording on phone cameras, but as the droplets oscillated at 50-80 hz, this provided 3-4 frames per droplet oscillation, which was unacceptably low for analysis purposes. Attempts were made to obtain a Phantom high speed camera, but as the rental was valued at 20 a day, and it was thus decided to use this option once the setup was refined, and all other possible options were exhausted.

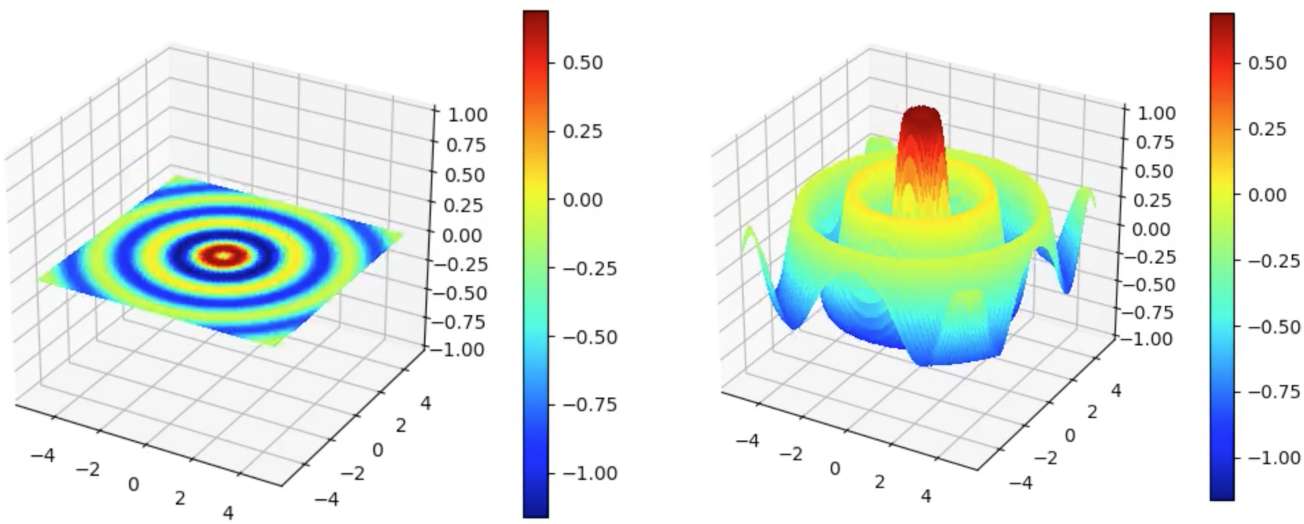
[Andrew?] Fortunately, for our initial experimentation, a PhD student from the mechanical engineering department kindly provided us with access to a 1000fps, handheld, high speed camera. He also provided us with two spotlights, vital in generating the intense illumination necessary for high speed photography. The camera and spotlight setup is as illustrated in figure []. Observing wave motion with this setup was difficult for two reasons. Firstly, the liquid was transparent, making it hard to discern where the waves formed exactly, even with slow-motion footage. This played into the second factor, which was the lack of a macro-lens for the camera. While the camera was suitable for recording the droplet to analyse its size and motion, it could not be brought to focus on the waves themselves, due to the limitations of the lens. As the camera did not allow for the mounting of such a lens, it was determined that another option, such as a phantom, would be necessary to observe wave motion.

IV. Simulation

A. Initial work in Python

Initially, work undertaken by the simulation group consisted of finding suitable mechanical descriptions of the droplet's motion, and implementing those in code. A simplified model was taken from [3], where the droplet was treated as stationary in the x-y plane and moving in z according to (4). Here, h_0 is the maximum height of the drop, ω_0 is the driving frequency of the system, r is the displacement of the drop from the centre in polar co-ordinates and c is the speed of the wave. J_0 is a Bessel function of the first kind. With ω_0, r and c set arbitrarily to 1, wave motion was demonstrated using an animation framework from [4]. The results of this are shown in Figure 3. Unfortunately, the Python language used to generate this proved too slow to be usable as a live demonstration, so any simulations generated using this method would need to be exported to a movie file and played back later. This was deemed an unacceptable solution, and so Java was chosen as a more efficient language to use in future.

$$h = -h_0 \cos(\omega_0 t) J_0(\omega_0 r/c) \tag{4}$$



(a) Wave motion at $h = 0$

(b) Wave motion at $h = h_0$

Fig. 3: Simulation output generated with Python and Matplotlib. (a) represents the initial state of the wavefield, while (b) represents the state of the wave-field when the "droplet" at the centre is at its maximum height

B. Rapid prototyping in Matlab

Matlab was first chosen to test implementations of mathematical concepts found in our research, as it has a wide variety of built in libraries, such as for graphing software and advanced mathematical processes. Initially, equations of motion for the droplet were taken from [5], where (5) defines the wave-field of the particle and (6) defines the bath height over time. To implement these, a grid object was constructed and populated with x-y coordinates. A point in the middle of the grid was then selected as the starting position for a droplet, which was used as the centre of modelling for the first interaction.

$$m\vec{x}'' + D\vec{x}' + k\vec{x} = -mg\nabla h(\vec{x}, t)) \tag{5}$$

$$h(\mathbf{x}, t) = \sum_{n=-\infty}^{\lfloor t/T_F \rfloor} A\mathbf{J}_0(k_F|\mathbf{x} - \mathbf{x}_p(nT_F)|)e^{-(t-nT_F)/(T_F M_e)} \tag{6}$$

Once a centre had been chosen, the wave-field generated by the droplet at that point was calculated at time $t = 0$. Assuming that the droplet bounces in phase with the forcing frequency, the droplet next interacts with the bath at time $t = T_F$, where $T_F = 2/\omega$, with ω representing the forcing frequency of oscillation. The evolution of the wave-field during this time period was calculated from the Bessel function $\mathbf{J}_0$, and so the droplet experiences a (quasi-instantaneous) acceleration proportional to the gradient of the wave-field at the droplet's position. A new wave-field is calculated at the droplet's new position following this acceleration. This process is repeated, with the n most recent wave-fields added to the current wave-field, where n represents the number of threads used to calculate the wave-field. Each iteration of this process was represented with a frame in the animation, which contained a colour-coded (x, y, z, t) point for the entire grid, as shown in Figure 4.

In an ideal case, the number of recent wave-fields held in memory depends only on M_e, the memory parameter of the system. However, due to computational limitations only the 8 most recent impacts were used in the calculation of the wave-field, as this was the number of threads that could be processed in parallel. To work around this issue, the possibility of generating the data using a faster language like Java was investigated. However, Matlab remains the optimal solution for displaying and animating these wave-fields, due to the heavy optimisation offered by the built in graphics libraries.

C. Constructing a GUI in Java

In parallel to this work in Matlab, a Java simulation was developed, which aimed to construct a pixel grid which could be used to represent a wave-field. Objects representing a given data point and a ”frame” of these data points were constructed, and populated with amplitudes using (4). These amplitudes were then displayed in 2D by assigning them to an opacity scale, with 100% opacity representing the maximum possible height and 100% transparency representing the minimum possible height. For a 40,000 pixel frame running over 10 seconds, this process took approximately 9 seconds, but the animation process after this ran in real time. Figure 5 shows a still image of this GUI taken when the ”droplet” was at a minimum height of $-h_0$. This animation was a success, but at higher resolutions, latency when drawing the pixels to the screen caused it to lag, suggesting a need to either run multiple drawing tasks in parallel, or to display the droplet motion in an alternative way.

D. Probabilistic prediction of position

Having demonstrated the possibility of displaying wave-functions in Java, displaying an object representing a droplet was the next task. Here, the assumption was that the wave (described by Equation 7) represented a probability density function. Therefore, for a given (and ideally infinitesimal) pixel n of area simulation/matlab.png

Fig. 4: The wave-field of a single droplet after multiple iterations; COLOUR SCALE TBC

dA, the probability P_n of finding the particle at pixel n was calculated from that function. The height (z_n) of each pixel within that section was calculated, with P_n/z_n representing the probability of the particle being at that pixel. The sum of probabilities for this section, $Z = \sum_n P_n$, defined the normalisation value of the thread. A random number generator then generated a number R such that $0 \leq R \leq 1$, which was multiplied by Z to give a relative random number. The program then looped back over all the pixels in the section, and repeatedly subtracts P_n from RZ until $RZ < 0$. The first pixel where this condition is satisfied is determined to be the location of the droplet. This process could then be multi-threaded to improve computational efficiency.

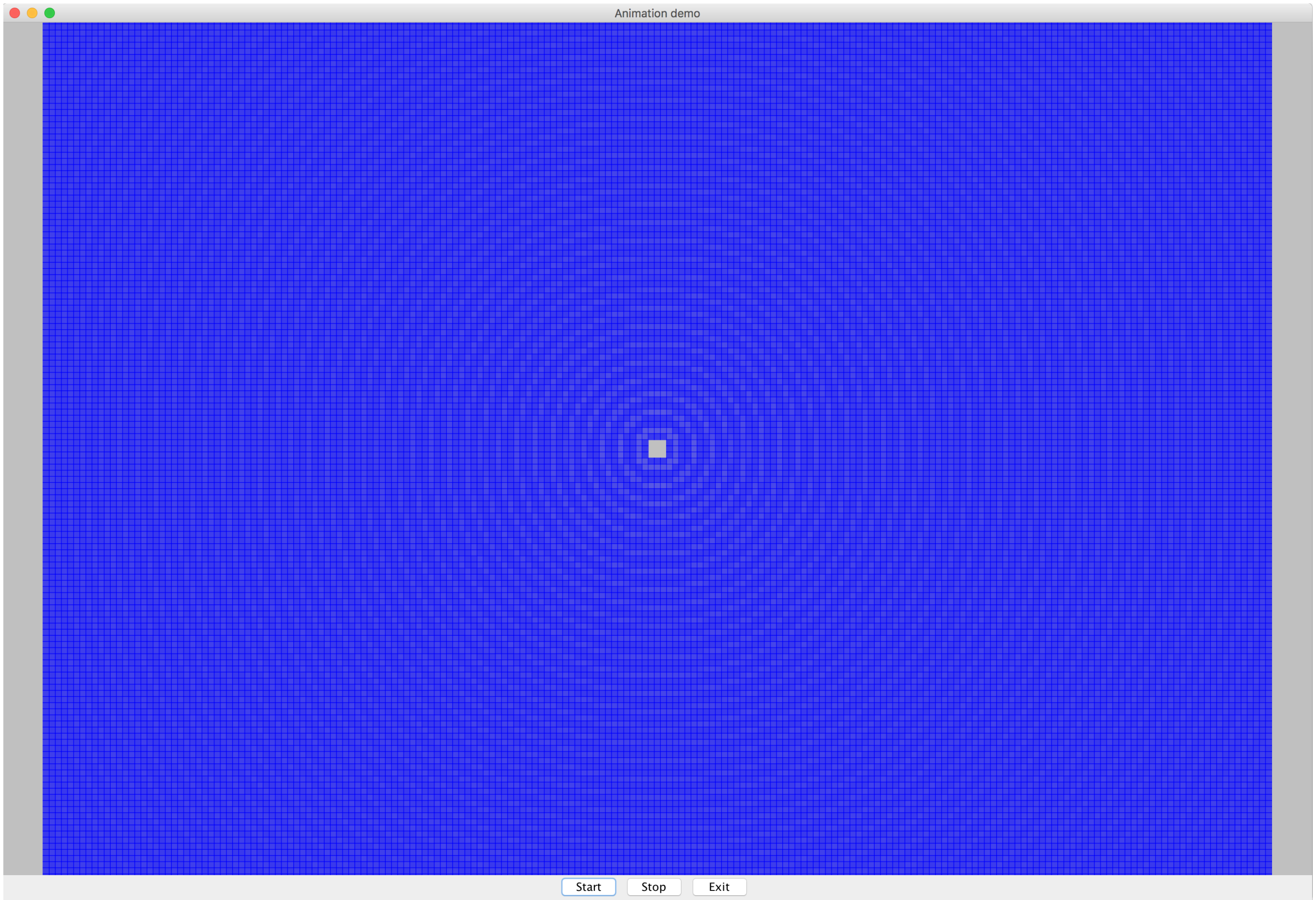


Fig. 5: A basic Java GUI, here showing the droplet at its minimum height of $-h_0$

The application of this to our project was that once the droplet is found at a given pixel, the distance between that pixel and the next pixel representing the location of the droplet is used to calculate the velocity v of the particle, assuming the droplet moves to the new pixel in the space of one period $T = 1/f$. The wave-function was updated with the new velocity, once a Lorentz transform was accounted for. This whole process is repeated to find the trajectory of the pixel.

$$h = -\cos(\omega t) J_0(\frac{\omega}{c}|\vec{r_f} - \vec{r_i}|) \tag{7}$$

Although this process was successful, all it ended up proving was that a random position generator works. It does not accurately simulate the position of droplets created in our experiment. Therefore, we proceeded to calculate the equations of motion and the wave-function of the droplet at each point.

E. Theoretical motion of the particle

Equations of motion for the droplet were taken from [5], which showed that the surface wave produced on the nth impact of a particle at postion r_n and time T_n on a surface oscillating vertically at ω above the Faraday threshold is given by (8). Here, $h(\vec{r}, t)$ is the height of the wave at $\vec{r}$, J_0 is a Bessel function of the first kind, and wave number k satisfies the relation $J_0\left(k\vec{r_0}\right) = 0$. $\vec{r_0}$ is a numerical cutoff parameter, while the amplitude of the wave, A is material specific and determined by the system parameters used.

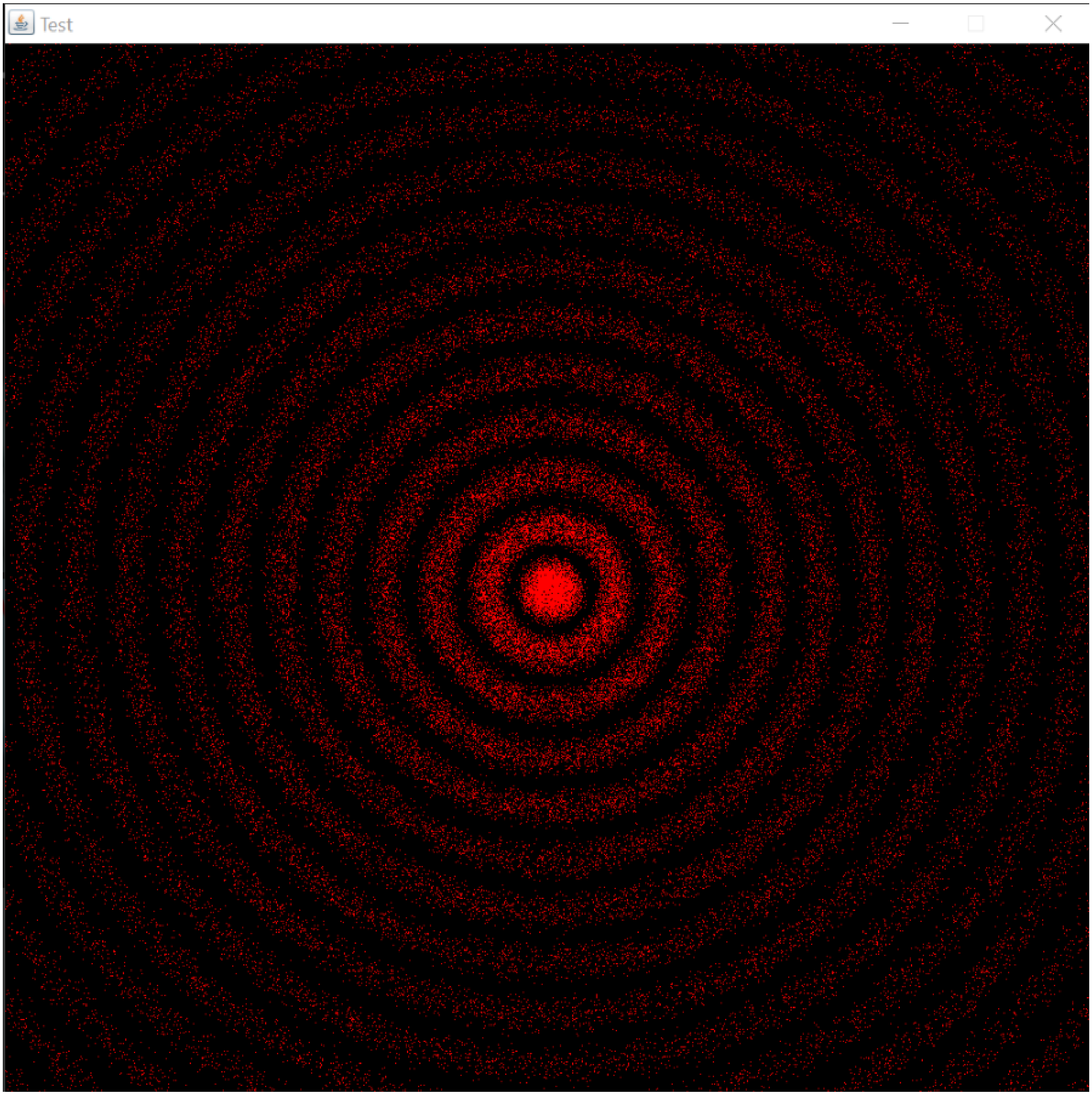


Fig. 6: Results of the probabilistic prediction of the droplet position, where each dot represents the droplet being present at this point

The exponential decay term at the end of the equation describes the wave decaying over time at a rate determined by the memory M_e and period of particle T_f.

$$h_n(\vec{r}, t) = AJ_o\left(k\left|\vec{r} - \vec{r_n}\right|\right)e^{-\frac{(t-T_n)}{T_f M_e}} \tag{8}$$

The overall surface wave can thus be described by the sum of all waves produced by every prior impact. Assuming the particle only interacts with the surface once every period (when the particle is at its lowest point), the overall wave equation $h(\vec{x},t)$ is thus given by (9).

$$h(\vec{r},t)=\sum_{n=-\infty}^{\frac{t}{T_f}} AJ_o\left(k\left|\vec{r}-\vec{r_n}\right|\right)e^{-\frac{\left(t-nT_f\right)}{T_f\times Me}} \tag{9}$$

Furthermore, we know that the surface wave needs to consider relativistic effects. The coordinate transform on the wave formed by the n^{th} impact of a particle at some velocity v is as follows:

$$h_n(\vec{r},t)=A_0cos\left(\omega_0 t-\frac{\gamma^2\omega_0 v}{c^2}\right)J_0\left(k\left|(\vec{r}-\vec{r_n})''\right|\right)e^{-\frac{\left(t-nT_f\right)}{T_f\times Me}} \tag{10}$$

Where:

$$\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{11}$$

$$(\vec{r}-\vec{r_n})''=\gamma^2(\vec{r_v}-\vec{v}t)+\gamma\vec{r_\perp} \tag{12}$$

$\vec{r_v}$ and $\vec{r_\perp}$ are the components of $(\vec{r}-\vec{r_n})$ in the direction of velocity and perpendicular to velocity respectively.

F. Modelling assumptions and simulation mechanics

The following assumptions were made during the simulation:

1) The particle and surface waves produce are oscillating in phase.
2) The particle and surface only interacting within a small time frame, T_i of the particles' lowest point.
3) Average force exerted by the particle over T_i is given by an effective wave force F_b. F_b depends on material parameters and the mass of the particle. It has a maximum magnitude equal to the weight of the particle.
4) Impulse applied to the particle in the vertical direction perpendicular to the surface is assumed to be negligible and the particle continues oscillating vertically at the same frequency and amplitude.
5) Particle experiences no damping force
6) The overall wave equation $h(\vec{r},t)$ is dominated by the waves due to the last N bounces only. Waves formed longer than N bounces ago are removed from the overall wave equation. This simplifies the computation of the infinite sum within the overall wave equation. In the java simulation, by integrating multi-threading methods into our calculation, we were able to efficiently perform calculations with $N=300$.

Following these modelling assumptions, the parameters of the system were set as follows [**?**]:

- Particle mass, $m_p = 2.6\text{e}-7$ kg
- Driving Frequency $\omega = 80$ Hz
- Period of particle, $T_f=\frac{2}{\omega}$ s
- Gravitational acceleration, $g=-9.81$ m/s
- Effective force, $F_b = 1.3174e-6$ N
- Wave number, $k=1250$
- Amplitude, A = $\frac{F_b}{m_p kg}$ m

- N = 300

The simulation starts by initiating a number of droplets on the surface, which vertically oscillate and form waves. On impact, each wave is added to an ArrayList of 2-D waveforms. To determine the height at any point, the height of all waves at that location in the ArrayList was summed, according to the superposition principle.

From the assumptions made above, it can be shown that the change in velocity of the particle in the direction parallel to the surface is given by:

$$\Delta\vec{v} = \frac{T_i F_b}{m_p} \times \frac{dh(\vec{x},t)}{d\vec{x}} \tag{13}$$

Where m_p is the mass of the particle and the last term is the vector gradient of the wave.

In 1-D, the gradient at a point $x = x_0$ is given by:

$$\left.\frac{dy}{dx}\right|_{x_0} = \lim_{x\to 0} \frac{y(x+\delta x) - y(x-\delta x)}{2\delta x} \tag{14}$$

Using a Taylor expansion about x_0, it can be shown that for $\delta x \neq 0$:

$$\left.\frac{dy}{dx}\right|_{x_0} = \frac{y(x+\delta x) - y(x-\delta x)}{2\delta x} - O(\delta x) \tag{15}$$

Where $O(\delta x)$ is the residual given by:

$$O(\delta x) = \sum_{n=2}^{\infty} \left(\frac{y^{(n)}(x_0)}{n! \times (2\delta x)} \left[(x + \delta x - x_0)^n - (x - \delta x - x_0)^n \right] \right) \tag{16}$$

The 2-D vector gradient of $h(\vec{r},t)$ was determined by first finding the 1-D gradient in the x-direction by using $\delta\vec{r} = (\delta x, 0)$, followed by that in the y-direction by using $\delta\vec{r} = (0, \delta y)$. The 1-D gradients in each direction corresponds to the component of the 2-D vector gradient in their respective directions.

In our computational model, $|\delta\vec{r}| = 1 \times 10^{-18}$ was used. The resolution of the Bessel functions used prevented the use of any value smaller than this. Attempts using $|\delta\vec{r}| = 1 \times 10^{-19}$ resulted in gradients calculated to either have magnitude 0 or magnitude ≈ 30, which does not match the shape of the first order Bessel function.

The wave velocity was calculated as ≈ 0.201. The perturbation velocity was on the order of 0.001, corresponding to $\gamma = 1.00001$. For simplicity of simulation, the Lorentz effects were assumed to be negligible and left out of calculation.

G. Simulation in the high- and low-memory regimes

The goal of the first simulation was to show that walking occurs as a result of the high memory regime. The difference between the high memory and low memory regimes is the rate of decay of the waves created by each impact on the surface. In the high memory regime, the waves decay slowly, and so waves created in the distant past can still strongly influence the particle trajectory. However, in the low memory regime, the waves decay rapidly, so only waves created in the recent past can significantly influence the particle's trajectory.

To test this, two simulations were run, one at Me = 150 and one at Me = 15. In both simulations, N = 300 was used. The particle was allowed to first bounce on the spot to build up 300 waves in the overall

(a) Graph of displacement in the x-direction over time

(b) The particle velocity fluctuates chaotically before entering a stable state ≈ 11s after perturbation

.

(c) The overall surface wave after 7.5s; the wave-field has a similar shape to a 2-D harmonic potential well

(d) The wave-field at 14.5s, just prior to the particle breaking through the potential barrier and travelling

.

(e) Following multiple interactions with wave peaks, the particle decelerates and reforms a potential well

.

(f) The stable wave-field at 19.5s, now displaced by $0.01m$.

.

Fig. 7: Results of the simulation in the high-memory regime at different time periods

wave equation, corresponding to t = 7.5s. The particle was then perturbed by spontaneously changing its velocity to $\vec{v} = (0.0005, 0)$. The results of the high and the low memory regime simulations are shown in Figures 7 and **??** respectively.

We see from Figure 7b that the particle velocity is chaotic. This causes the particle to oscillate around two centres of displacement, initially the start position, and later about the point $(0.01, 0)$. We also observe in Figure 7c that, prior to perturbation, the wave-field is similar in shape to a 2-D harmonic potential well, of width $\lambda/2$ and centred about the initial position of the particle. Upon perturbation, the particle oscillates, as expected for such a system. However, with each impact at positions away from the start position, new waveforms are added to the overall wave-function at these points. The waves formed at this point superpose over the 2-D harmonic potential well, flattening it along the x-axis. After approximately 7s, the peaks at $\lambda/2$ away from the start position along the x-axis were low enough that the particle could leave the well as shown in Figure 7d.

The particle is now free to "walk" outside the initial potential well.

The overall surface wave at 7.5s is shown in figure 1(c). The wavefield is observed to be similar in shape to the 2-D harmonic potential well within half wavelength about the particles' start position. At this point, the particle is perturbed. As expected for a particle oscillating in a 2-D harmonic potential, the particle enters oscillatory motion. However, with each impact at positions away from the start position, new waveforms are added to the overall wavefunction at these points. The wave forms at this point superpose over the 2-D harmonic potential well flattening it along the x-axis. after approximately 7s, the peaks at half wavelength away from the start position along the x-axis were low enought that the particle could traverse it and leave the well about the start position. The overall surface wave at this point is shown in figure 1(d). As the particle carries on in its trajectory away from the start position, it interacts with other smaller peaks causing it to slow down. As the particle slows down, the most recently formed waves are centered closer and closer together eventually reforming yet another potential well approximately about point (0.01,0). The overall surface wave at this point is shown in figure 1(e). The particle then starts oscillating about this point for approximately 4s before the same mechanisms that allowed it to break through the first potential well occurs and the particle leaves the new formed well. Following this the particle enters a steady state and continues travelling with a stable velocity of (-0.043,0) ms^{-1}. The particle has thus started "walking".

The high regime simulation was repeated for pertubation velocity with magnitude between 0.0001 ms^{-1} and 0.002 ms^{-1} at intervals of 0.0001 ms^{-1} to ensure that the above result of achieving walking was not due to specifically selected variables. In all cases the particle entered a steady ”walking” state with velocities ranging form magnitude 0.00859 to 0.102. The final steady state velocities of each particle and their corresponding perturbation velocity can be foudn in Annex A. No observable trend was found relating the steady ”walking” state velocity and the perturbation velocity.

// details of what each figure is 1 a) displacement against time. (b) Velocity against time. (c) waveform at 7.5s -¿ waveform before pertubation (d) wave form at 14.5s (frame 580) -¿ waveform when particle breaks the first peak at have wavelength away from original centre (e) waveform at 16.5s (frame 660) particle slows down after multiple interaction with peaks and reforms a potential well (f) waveform at 19.5s (frame 780)

REFERENCES

[1] Y. Couder, S. Protière, E. Fort, and A. Boudaoud, “Dynamical phenomena: Walking and orbiting droplets,” *Nature*, vol. 437, p. 208, 2005.

[2] J. Walker, “The amateur scientist: Drops of liquid can be made to float on the liquid. what enables them to do so?” *Scientific American*, vol. 238, pp. 151–159, 1978.

[3] R. Brady and R. Anderson, “Why bouncing droplets are a pretty good model of quantum mechanics,” *arXiv preprint arXiv:1401.4356*, 2014.

[4] The beginner programmer. (2015, 7) The wave equation. Accessed 21/2/18. [Online]. Available: http://firsttimeprogrammer.blogspot.co.uk/2015/07/the-wave-equation-2d-and-3d.html

[5] A. U. Oza, R. R. Rosales, and J. W. Bush, “A trajectory equation for walking droplets: hydrodynamic pilot-wave theory,” *Journal of Fluid Mechanics*, vol. 737, pp. 552–570, 2013.

V. Appendix A: Minutes 11/1/18

Meeting Time: 4pm Thursday 11th Jan
Meeting Location: Massey Group Study Pod, 3rd Floor, Science Library

Attendance
Chair: Chong Keat Gea CKG
Vice-Chair: Kelvin Fang KF
Secretary: Johnny Allain-Labon JAL
Treasurer: Mohit Motwani MM
Georges Ajaka GA
Course Coordinator Point of Contact: Steven Vuong SV
Benji Berczi BB
Alex Stock AS
Prof. Ryan Nichol RN
Present: CKG KF JAL MM GA SV BB AS RN

1) **Project Outline**
   a) Existing equipment: none, constructing prototype from scratch
   b) Prototype - start as simple as possible e.g. petri dish on loudspeaker e.g. https://www.youtube.com/watch?v=WIyTZDHuarQ. Move on to more complex behaviour if possible
   c) Simulation Programme - want to demonstrate quantum behaviours of the bouncing oil drops if experimental verification is problematic + add missing features of experiment.

2) **Aims & Objectives**
   a) Construct prototype:
      i) Drop bouncing
      ii) Stable drop bouncing over several seconds
      iii) Getting the drops to walk
      iv) Drop interaction with boundaries
      v) Two or more drops
      vi) Drops orbiting
      vii) Double slit interference
      viii) Tunnelling
   b) Simulation: demonstrate same effects on a computer
      i) Start with 2D plot
      ii) Look to animate with 3D movement
   c) Aim: outreach tool for demonstrating quantum effects on real life scale
      i) Ideally interactive i.e. can add new droplets
      ii) Replicable by teachers

d) Stretch aim: video demonstrating + different containers/parameters e.g. frequency - bounce to a song?

3) **Assessment Criteria**
   a) Working prototype
   b) Working simulation
   c) Reporting

4) **Deadlines**
   a) 14/1 - Research deadlines - need understanding of scope of project i.e. read papers. Including summary of papers for presentation in formal report. Maintain file "notes and background reading"
   b) 18/1 4pm - Plan for Prototype + Outline for simulation
   c) 12/2 - READING WEEK - Basic simulation + prototype completed
   d) Reading Week - report to other group
   e) 7/3 - prelim deadline for poster being finalised for printing
   f) 16/3 5pm - FINAL REPORT due + critical self-assessment
   g) 21/3 - poster presentation

5) **Areas of Responsibility**
   a) General Time Management - JAL
   b) Prototype Weds 17/1 6pm brainstorm
      i) SV Lead
      ii) KF
      iii) BB
      iv) MM
      v) CKG
   c) Simulation - written in Python - Weds 17/1 6pm brainstorm
      i) GA Lead
      ii) AS
      iii) JAL
   d) Written Reporting - written as we go - MM

6) **Communications Plan**
   a) Google Drive for documents
   b) ShareLatex for written reports
   c) Facebook group for updates, requests for help, anything permanent
   d) FB chat for real time but ephemeral communication

7) **Future Meetings**

a) Thursdays 4pm - regular slot
b) Thursday 18/1 13:30-14:00 subgroup meeting
c) Thursday 18/1 16:00 - KF to book
d) RK availability - to be emailed
e) Lab use - Derick will accommodate people available, 9-5 lab hours with lunch at 1. Other spaces available for building things - Institute of Making - CKG has one

VI. Appendix B: Minutes 18/1/18

Meeting Time: 4pm Thursday 18th Jan
Meeting Location: Massey Group Study Pod, 3rd Floor, Science Library

Attendance
Chair: Chong Keat Gea CKG
Vice-Chair: Kelvin Fang KF
Secretary: Johnny Allain-Labon JAL
Treasurer: Mohit Motwani MM
Georges Ajaka GA
Course Coordinator Point of Contact: Steven Vuong SV
Benji Berczi BB
Alex Stock AS
Present: CKG KF JAL MM GA SV BB AS

1) **Simulation Group GA**
   a) How far into the simulation are we?
      i) 3D isn't really possible - only by pre-rendering - takes 5 mins for a pre-rendered version to load
      ii) Live-simulation done with heat maps
   b) What can we simulate now?
      i) Basic animation of wave equation in Python
   c) Have we reached movement?
      i) Basic movement in Python
   d) Any issues with the coding?
      i) Python too slow to render live
   e) Are there sufficient/ too many people working on the code?
      i) Don't know yet
   f) Who's done what? Are there any collaboration issues?
      i) None yet
   g) Any issues with the code? Buggy, inefficient etc.
      i) None to report
   h) What's next?
      i) Meeting Monday 11am
      ii) Port Python to Java using AWT - JAL
      iii) Equations from paper GA AS
   i) Goals setting for next week
      i) Team GitHub
   j) Who's doing what?

k) Discussion with the rest of the team on possible directions and improvements.

2) **Prototype Group SV**
   a) How far into the prototype are we?
      i) Use Veritasium model as baseline
      ii) Have list of materials
      iii) Pro-level cameras outside our budget → use phones with cameras & timestamps - issues with timing precision
   b) Propose current idea for apparatus
      i) https://drive.google.com/drive/folders/1OnPH_cwJg6xc4uVNFHgWOlpQpYrkzFWK
      ii) Total estimated cost £80
   c) What equipment do we need, how much will it cost us? Budgeting.
      i) Camera needs depend on aims - do we want to demonstrate (less hi-spec) or do we want to verify theory (higher-spec)
      ii) Use IoM for 3D printing, free wood to use in construction
      iii) Light diffuser to illuminate surface + drop more softly
      iv) Polarising filter to protect camera?
   d) Possible concerns raised with the equipment and discussion on solution.
   e) Time frame from getting equipment. When will we get everything and start construction?
      i) 3 working days - can pay more for advanced deliver
      ii) Need equipment by Tuesday 23/1
   f) Any task delegation issues, too many/ not enough people.
   g) What's next?
   h) Goal setting for next week?
   i) Whos doing what?
   j) Discussion with the rest of the team on possible directions and improvements.
      i) Derek - Silicone oil given to us.
         A) He might have a 50W power supply downstairs
         B) Ask about Amazon Packages
      ii) To ask Bernard about:
         A) Petri Dish (If not, take from Chemistry / Biology)
         B) 50W power supply for Subwoofer
         C) 3D Printer
      iii) Things to Consider:
         A) Diffuser: Need a strong lighting source
         B) LEDs?
         C) Powerful lamps perhaps
         D) Polarising Lens perhaps
         E) IoM, check induction times
      iv) Creatables:

A) Lab Script for the entire process

3) **Written Reporting MM**
   a) Minutes from last meeting to ShareLatex
   b) Can start writing theory section now - MM to liaise with GA AS JAL

4) **Project Management JAL**
   a) Zoho Projects - please confirm login details have been received

VII. Appendix C: Minutes 25/1/18

Meeting Time: 4pm Thursday 25th Jan
Meeting Location: Group Study Pod 3, Ground Floor, Science Library

Attendance
Chair: Chong Keat Gea CKG
Vice-Chair: Kelvin Fang KF
Secretary: Johnny Allain-Labon JAL
Treasurer: Mohit Motwani MM
Georges Ajaka GA
Course Coordinator Point of Contact: Steven Vuong SV
Benji Berczi BB
Alex Stock AS
Prof Ryan Nichol RN
Present: CKG KF JAL MM GA SV BB AS
Absent with Apologies: RN

1) **Overall Group Health CKG**
   a) Is everyone ok with what they are working on? (People feeling they aren't contributing enough or doing too much etc.)
   b) Any issues wrt group dynamics etc? No, all going OK.
2) **Simulation Group Update GA**
   a) What has been accomplished in the past week? Were the goals set up last week accomplished? (Last Week Goals: Team github)
      i) Github permissions are tricky
   b) Request for help: equations to implement in code
      i) Possible solution would be to square the wave to generate probability of next position and velocity towards next position can be calculated using time stamp (CKG)
      ii) GUI written but not running yet - error on compilation
      iii) AS - update on equations - need to do integrals by brute force but should work - possibly computationally intensive? And has expected values of coefficients - http://dx.doi.org/10.1063/1.4817612 https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/drops-bouncing-on-a-vibrating-bath/441A614F657E800EA06F6C08674CCE67
   c) Discussion on possible solution
   d) What's next, who does what?
      i) Implement found equations in Java
      ii) Meet Monday to update
   e) Someone to join simulation team eventually?

3) **Prototype Group Update SV**

a) What has been accomplished in the past week? Were the goals set up last week accomplished? (Last Week Goals: ??)
   i) Got prototype working but droplets too large - due to viscosity? Use different oil with lower viscosity
   ii) Smaller droplets accomplished by using a finer point, or using a device to release the drop at a given time
b) Discussion on the different proposed apparatus. To glue or not to glue?
   i) Attempt non-permanent options before gluing - double sided tape,
c) Any more equipment need purchasing?
   i) Need to look at other amps - EEng?
d) What's next, who does what?
   i) Next Weds for labs
   ii) Fix petri dish to speaker - hot glue?
   iii) Decide on oil to use - less viscous - CK & BB
   iv) Speaker stand - CK

4) **Written Reporting Update MM**
   a) Where are we on this? Do you need help?
      i) ShareLatex set up - everyone to test collaboration access please

5) **AOB**
   a) Speakers - driven by a signal generator
   b) Can purchase an amplifier if needed - money available in budget - £125 left

VIII. Appendix D: Minutes 01/2/18

Meeting Time: 4pm Thursday 25th Jan
Meeting Location: First Year Labs

Attendance
Chair: Chong Keat Gea CKG
Vice-Chair: Kelvin Fang KF
Secretary: Johnny Allain-Labon JAL
Treasurer: Mohit Motwani MM
Georges Ajaka GA
Course Coordinator Point of Contact: Steven Vuong SV
Benji Berczi BB
Alex Stock AS
Present: CKG KF JAL MM GA SC BB AS
Absent with Apologies:

1) **Overall Group Health CKG**
   a) Is everyone ok with what they are working on? (People feeling they aren't contributing enough or doing too much etc.)
   b) Any issues wrt group dynamics etc.
   c) Acknowledgement to team sizing issues. Will be discussed again after the updates from each team.
      i) Going to wait on update from simulation team to rearrange

2) **Simulation Group Update GA**
   a) What has been accomplished in the past week? Were the goals set up last week accomplished? (Implement found equations in Java, Meet Monday to update)
   b) Is the GUI working? Yes, CKG to edit
   c) Have the proposed methods been tested / whats the result? None yet - maths still to be implemented
   d) Any issues to raise? (work allocation, number of people, math etc.)
   e) What's next? Goal setting for next week
      i) Improve robustness of simulation
      ii) Implement and add new maths
      iii) Computing power from UCL requires permission from project supervisor

3) **Prototype Group Update SV**
   a) What has been accomplished in the past week? Were the goals set up last week accomplished? (Last Week Goals: Fix petri dish to speaker - hot glue? , Decide on oil to use - less viscous, Speaker stand)

   i) Cameras have been located to use
   ii) Demonstrated multiple bouncing drops & coupling
   iii) Tested in f=50-80Hz
   iv) Still need to glue base down

b) Stage of the prototype?
c) Any issues to raise (work allocation, number of people, what to test next, etc)
d) What's next? Goal setting for next week
   i) Glueing base down
   ii) Record droplet in motion
   iii) Experiment with amplitudes
   iv) Quantify observations
   v) Rest tunnelling

4) **Written Reporting Update MM**
   a) Where are we on this? Do you need help?
      i) No progress without maths and finalised equipment list
   b) Has everyone tested the collab?
      i) JAL can, assume it generalises

5) **Team reallocation CKG**
   a) Discussion on needs of each group.
      i) BB to start work on poster design, collab w MM for copy
   b) Self evaluation on what each person has contributed and areas for improvement.