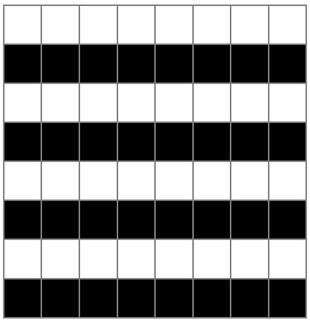
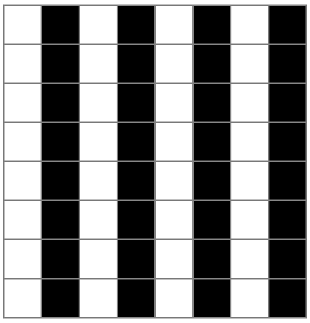
ELEC S347F Multimedia Technologies,
Spring Term, 2021
Tutorial 03 Solution

Question 1: Consider the below six luminance blocks A, B, C, D, E and F.

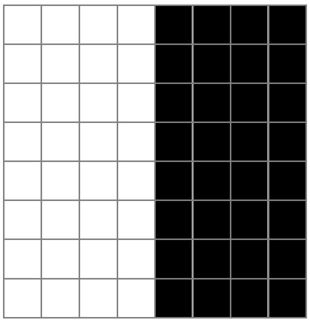
A



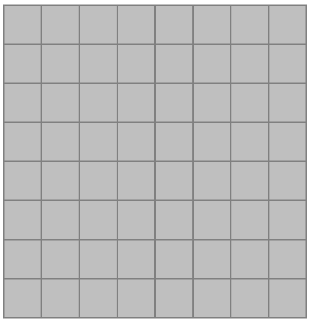
B



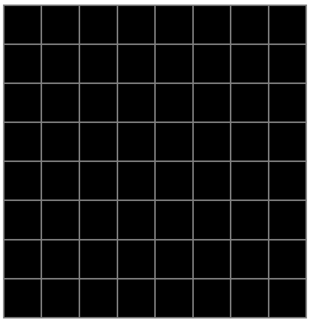
C



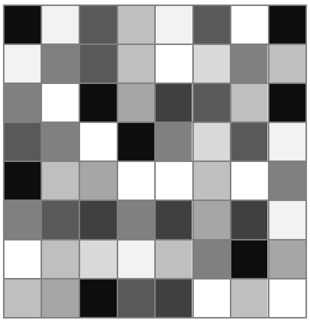
D



E



F



Determine which of the below matrix best represents each of the above block.

(i)

1020	924	0	-324	0	216	0	-183
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(ii)

769	-21	41	45	18	-69	50	-82
36	-50	-32	-43	-71	126	-86	-139
7	-77	113	-82	183	-27	-59	76
-22	-106	-79	76	-48	32	148	-134
-63	-6	-152	-26	31	-97	37	105
-25	-81	46	62	-23	-66	75	-143
-79	86	-144	95	-154	45	-36	-23
-29	-63	-57	-44	44	93	-66	102

(iii)

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(iv)

1020	0	0	0	0	0	0	0
-183	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
-216	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
-324	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
-924	0	0	0	0	0	0	0

(v)

1020	183	0	216	0	324	0	924
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(vi)

1020	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

A: (iv) The block has intensity change in vertical direction only. There is no horizontal variation. So only the first column's AC component are non-zeros.

B: (v) The block has intensity change in horizontal direction only. There is no vertical variation. So only the first row's AC components are non-zeros.

C: (i) Similar as Block B, Block C has intensity change in horizontal direction only. But compared with B, C has fewer changes. So the magnitude of the high frequency coefficients are smaller than that of B.

D: (vi) The block is of pure intensity. It has no variation in all directions. So it just has a DC component only and no any AC components.

E: (iii) Similar as Block D, Block E has no variation in all directions. But the base intensity are darker than D. So the value of the DC coefficient is smaller value than that of E.

F: (ii) The block has sharp and random color changes in all directions. It has many high frequency AC components.

Question 2: Consider a color represented in YUV model where (Y, U, V) = (0.5, 0, 0).

(a) What is the corresponding value of the color in the RGB model?
(b) What is the corresponding value of the color in the YCbCr model?
(c) What is the corresponding value of the color in the CMYK model?
(d) A printer only has yellow, cyan and black paints. Explain whether the printer can create the pure red color or not?
Note: refer to the equation in the lecture notes

(a) U = B – Y = 0 => B = Y = 0.5
V = R – Y = 0 => R = Y = 0.5
Y = 0.299R + 0.587G + 0.114B => 0.587Y = 0.587G => G = Y = 0.5
(R, G, B) = (0.5, 0.5, 0.5)

(b) Y is the same (0.5)
Cb = U/2 + 0.5 = (B-Y)/2 + 0.5 = 0.5
Cr = V/1.6 + 0.5 = (R-Y)/1.6 + 0.5 = 0.5
(Y, Cb, Cr) = (0.5, 0.5, 0.5)

(c) C' = 1 – R = 0.5
M' = 1 – G = 0.5
Y' = 1 – B = 0.5
K = min(C', M', Y') = 0.5
C = C' – K = 0
M = C' – K = 0

Y = Y' – K = 0

(C, M, Y, K) = (0, 0, 0, 0.5)

(d) For pure red color, R = 1, G = B = 0

C' = 1 – R = 0

M' = 1 – G = 1

Y' = 1 – B = 1

K = min(C', M', Y') = 0

C = C' – K = 0

M = C' – K = 1

Y = Y' – K = 1

(C, M, Y, K) = (0, 1, 1, 0)

That means the print needs M and Y paints to create the pure color. So the answer is NO.

Question 3: Suppose the DC coefficients of the first few blocks of an image are listed below:

35	..	..
..	..	..
..	..	..

29	..	..
..	..	..
..	..	..

34	..	..
..	..	..
..	..	..

37	..	..
..	..	..
..	..	..

29	..	..
..	..	..
..	..	..

(a) DC coefficients are encoded differently from AC coefficients. Explain why DC coefficients are DPCM-coded.

(b) Find the DPCM-coded DC coefficients.

(c) Represent the answer of (b) in symbol form.

(d) Find the final Huffman encoded bit string (and length) for the DC coefficients

(a) DC coefficient represent the average value of the block. In general, DC coefficients across adjacent blocks are changed gradually. So it is good to use the previous DC coefficient to predict the current DC value.

(b) DPCM-coded DC Coefficients = {35, -6, 5, 3, -8}

(c) Each VALUE in (b) is represented by (SIZE, AMPLITUDE)

VALUE	SIZE (bits)	AMPLITUDE	AMPLITUDE (-VE)
35	6	$35=10\ 0011_2$	
6	3	$6=110_2$	$-6=001_2$
5	3	$5=101_2$	
3	2	$3=11_2$	
8	4	$8=1000_2$	$-8=0111_2$

Symbols = {(6_{Huff}, 100011), (3_{Huff}, 001), (3_{Huff}, 101), (2_{Huff}, 11), (4_{Huff}, 0111)}

(d) Huffman Table (steps omitted here)

SIZE Symbol	Count	Huffman Codeword
3_{Huff}	2	1
6_{Huff}	1	01
2_{Huff}	1	000
4_{Huff}	1	001

Encoded result = 01100011 1001 1101 00011 0010111 (28 bits in total)

Comment: the computed Huffman table is not unique as you may obtain another result if in the first step combining symbol "6_{Huff}" with symbol "2_{Huff}". However, the resultant length of the Huffman encoding result is always the same.

Question 4: Consider the below matrix which describes the DCT result of a luminance block.

1135.2	380.2	0.3	11.4	-7.6	6.1	-13.7	14.9
680.1	-2.2	-5.3	14.5	-6.0	7.3	8.4	-15.2
377.6	5.1	14.4	4.2	8.4	-4.4	-14.6	8.6
-736.4	-11.4	-12.2	0.7	10.2	2.6	-3.1	-10.1
-390.2	2.5	0.5	9.2	-15.5	7.8	15.1	13.3
370.4	8.7	-4.7	-5.3	13.1	-3.9	8.8	7.2
12.3	-7.5	10.9	0.5	-11.0	1.1	5.4	0.0
-14.1	-13.2	-11.6	9.2	8.2	6.0	0.2	-1.5

(a) Determine the matrix after quantization for quality factor q = 50.

(b) Find the RMSE distortion of the quantized block in (a).

(c) Find the zig-zag scanning result of the AC coefficients for the block in (a).

(d) Find the run-length coding result of (c).

(e) Represent the answer of (d) in triple form.

(f) Find the final Huffman encoded bit string (and length) for the AC coefficients.

Note: Root mean square error (RMSE) = $\sqrt{\frac{\sum_{i=1}^{8}\sum_{j=1}^{8}\left(Y(i,j)-X(i,j)\right)^2}{64}}$

Given: The AC coefficients are encoded as follows:

1. First perform a zig-zag scan to generate the run-length encoded (RLE) pairs for each block.
2. The RLE pairs (run-length, value) are then transformed to triples form (run-length, size, amplitude) where run-length is represented using 4 bits and size is represented using 4 bits.

3. Run-length and size form a byte-sized symbol which is then Huffman coded.
4. Amplitude is represented using one's complement scheme for negative value.

(a) Q' = (100-50)/50 x Q = Q

Fq(u, v) = round(F(u,v)/Q(I,j))

71	35	0	1	0	0	0	0
57	0	0	1	0	0	0	0
27	0	1	0	0	0	0	0
-53	-1	-1	0	0	0	0	0
-22	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

(b) After decompression, the restored block (Y) is

1136	385	0	16	0	0	0	0
684	0	0	19	0	0	0	0
378	0	16	0	0	0	0	0
-742	-17	-22	0	0	0	0	0
-396	0	0	0	0	0	0	0
360	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

The error of each coefficient (Y-X):

0.8	4.8	-0.3	4.6	7.6	-6.1	13.7	-14.9
3.9	2.2	5.3	4.5	6	-7.3	-8.4	15.2
0.4	-5.1	1.6	-4.2	-8.4	4.4	14.6	-8.6
-5.6	-5.6	-9.8	-0.7	-10.2	-2.6	3.1	10.1
-5.8	-2.5	-0.5	-9.2	15.5	-7.8	-15.1	-13.3
-10.4	-8.7	4.7	5.3	-13.1	3.9	-8.8	-7.2
-12.3	7.5	-10.9	-0.5	11	-1.1	-5.4	0
14.1	13.2	11.6	-9.2	-8.2	-6	-0.2	1.5

Root mean square error (RMSE) = $\sqrt{\frac{\sum_{i=1}^{8}\sum_{j=1}^{8}\left(Y(i,j)-X(i,j)\right)^2}{64}}$ = 8.26 (3 sig. fig.)

(c) AC Zig-Zag Scan = {35, 57, 27, 0, 0, 1, 0, 0, -53, -22, -1, 1, 1, 0, 0, 0, 0, -1, 0, 15, 0, .., 0}

(d) The AC coefficients encoded by RLE are represented in this form: (RUN-LENGTH, VALUE)

RLE = {(0, 35), (0, 57), (0, 27), (2, 1), (2, -53), (0, -22), (0, -1), (0, 1), (0, 1), (4, -1), (1, 15), (0, 0)}

(e) Similar to DC encoding, each VALUE is represented by (SIZE, AMPLITUDE)

VALUE	SIZE	AMPLITUDE	AMPLITUDE(-VE)
35	6=0110_2	35=$10\ 0011_2$	
57	6=0110_2	57=$11\ 1001_2$	
27	5=0101_2	27= $1\ 1011_2$	
1	1=0001_2	1= 1_2	-1= 0_2
53	6=0110_2	53=$11\ 0101_2$	-53=$00\ 1010_2$
22	5=0101_2	22= $1\ 0110_2$	-22= $0\ 1001_2$
15	4=0100_2	15= 1111_2	
0	0=0000_2	0= *	

The AC coefficients in triple form: (RUN-LENGTH (4-bit), SIZE (4-bit), AMPLITUDE (var-bit))

Triple Form = {(0000, 0110, 10 0011), (0000, 0110, 11 1001), (0000, 0101, 1 1011), (0010, 0001, 1), (0010, 0110, 00 1010), (0000, 0101, 0 1001), (0000, 0001, 0), (0000, 0001, 1), (0000, 0001, 1), (0100, 0001, 0), (0001, 0100, 1111), (0000, 0000, *)}

(f) Huffman Table (steps omitted here, refer to class recording)

Symbol	Count	Huffman Codeword
0000 0110	2	001
0000 0101	2	010
0010 0001	1	011
0010 0110	1	110
0000 0001	3	10
0100 0001	1	111
0001 0100	1	0000
0000 0000	1	0001

Huffman Encoded Bit String = {(001, 10 0011), (001, 11 1001), (010, 1 1011), (011, 1), (110, 00 1010), (010, 0 1001), (10, 0), (10 1), (10, 1), (111, 0), (0000, 1111), (0001)}

(72 bits in total)

Comment: the Huffman coding result is not unique. You may obtain another result. However, the resultant length of the Huffman encoding result is always the same.