Recursive Best-First Search

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- Difference between A* and RBFS
  - A* keeps in memory all of the already generated nodes
  - RBFS only keeps the current search path and the sibling nodes along the path
- **RBFS is a linear-space algorithm that expands nodes in best-first order even with a non-monotonic cost function** and generates fewer nodes than iterative deepening with a monotonic cost function
- RBFS Example: ILBFS(Iterative Linear Best-First Search)

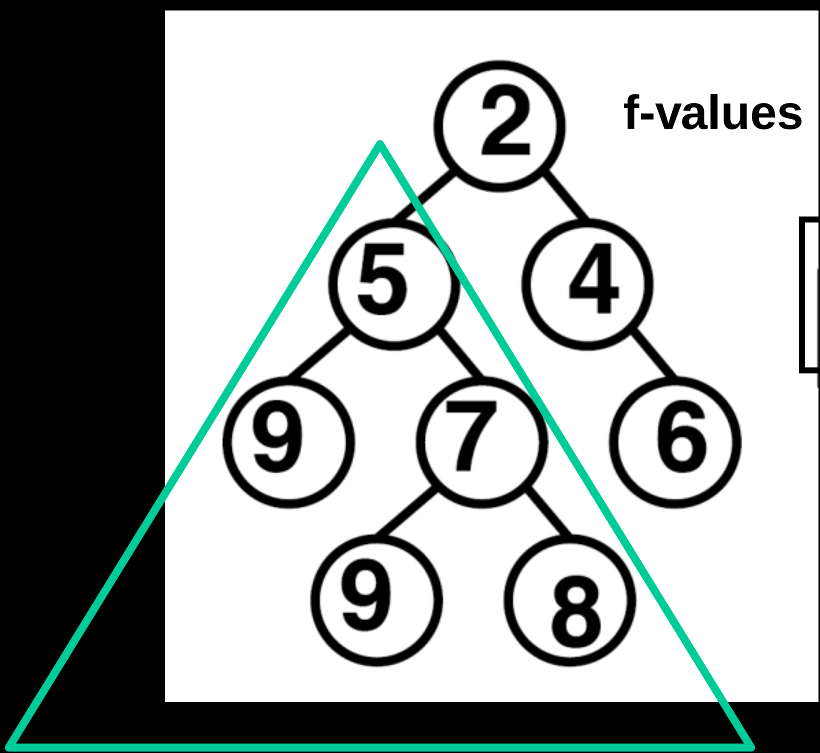
RBFS vs ID

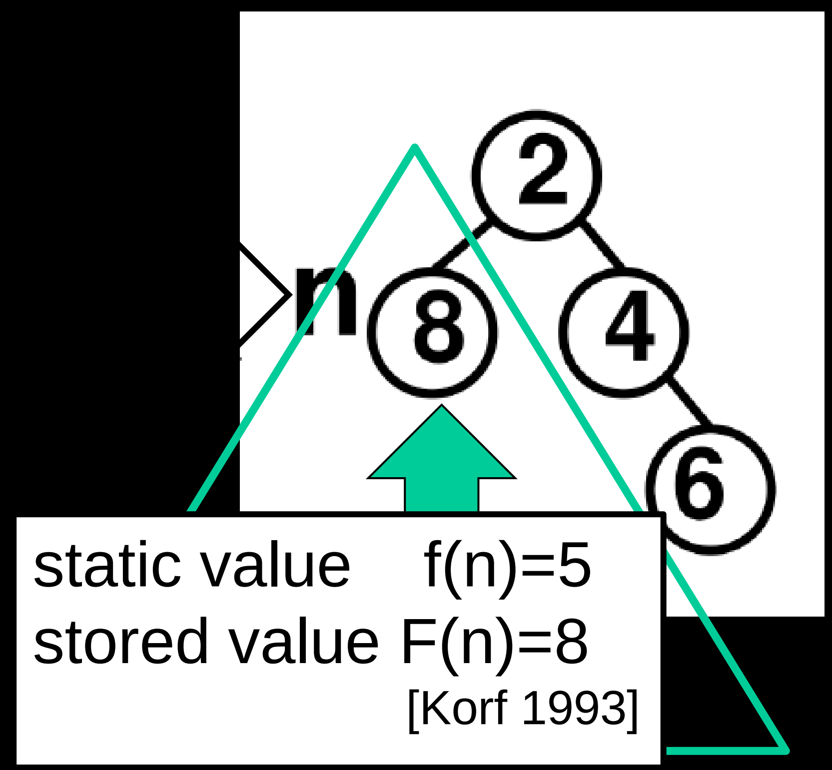
- If the cost function is non-monotonic, the two algorithms are not directly comparable.
- RBFS generate fewer nodes than ID on average. Because RBFS only backtracks to their common ancestor instead of directly to the root as IDA*.

Collapse and Restore macros

The ILBFS – RBFS alogorithms

Collapse macro for best-first search

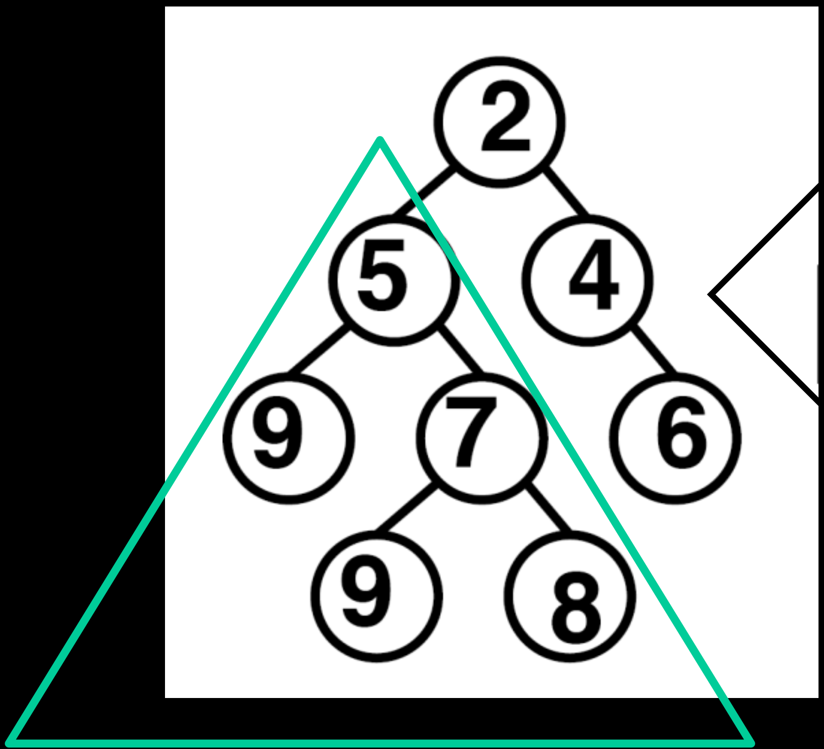


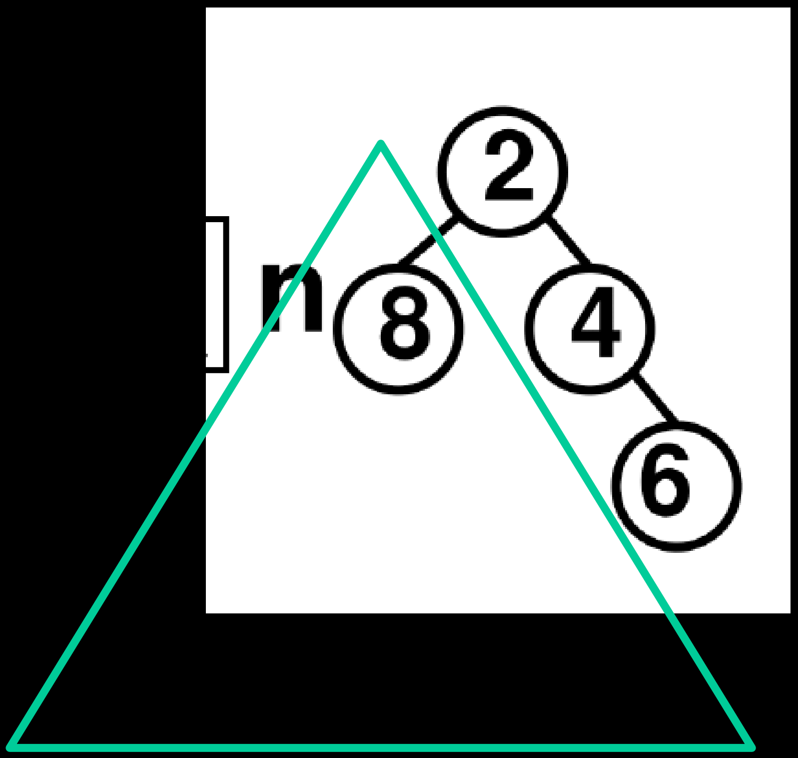


9 9 8 6

8 6

Restore macro





9 9 8 6

8 6

- Collapse is a **lossy** compression

  1) How do we know a node was collapsed?

  2) How do we restore?

<u>Restore is algorithm dependent</u>

If F(n) > f(n) and the f-value is monotonically increasing just perform a bounded DFS by F(N). [Korf 1993]

ILBFS [#1 SoCS-2015]

Iterative linear best-first search

Iterative variant of RBFS [Korf, AIJ 1993]

Algorithm 1: High-level ILBFS

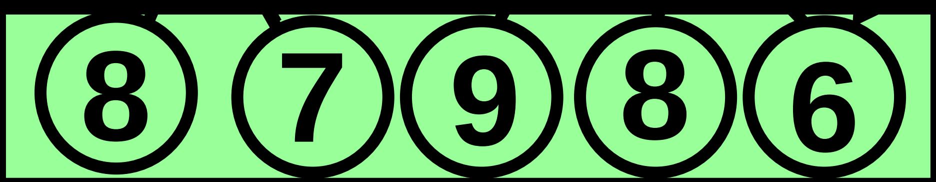
```
Input: Root R
Insert R into OPEN and TREE
oldbest=NULL
while OPEN not empty do
    best=extract_min(OPEN)
    if goal(best) then
        exit
```

Collapse

Restore

```
    foreach child C of best do
        Insert C to OPEN and TREE
    oldbest ← best
```

ILBFS



Principal branch invariant

Store only the branch of the best node and its siblings

ILBFS

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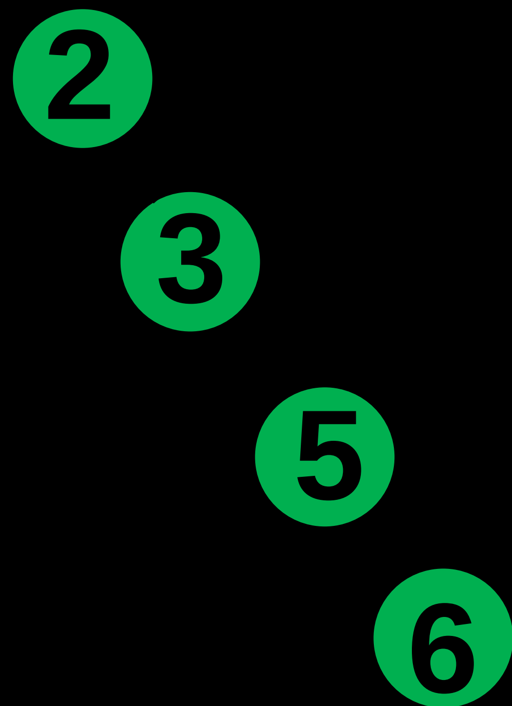
Store only the branch of the best node and its siblings

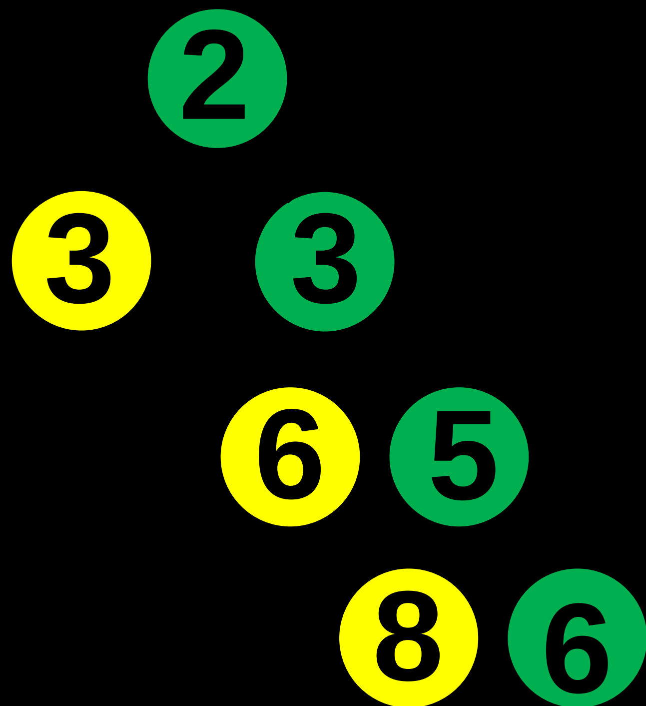
ILBFS



Principal branch invariant

Store only the branch of the best node and its siblings

ILBFS



Principal branch invariant

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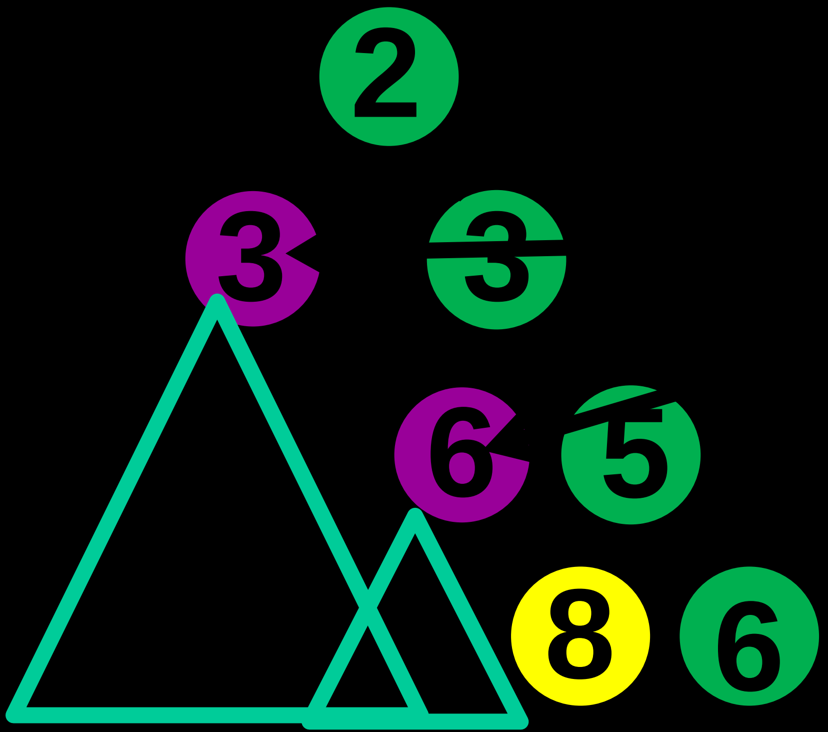
ILBFS



Collapse

Principal branch invariant

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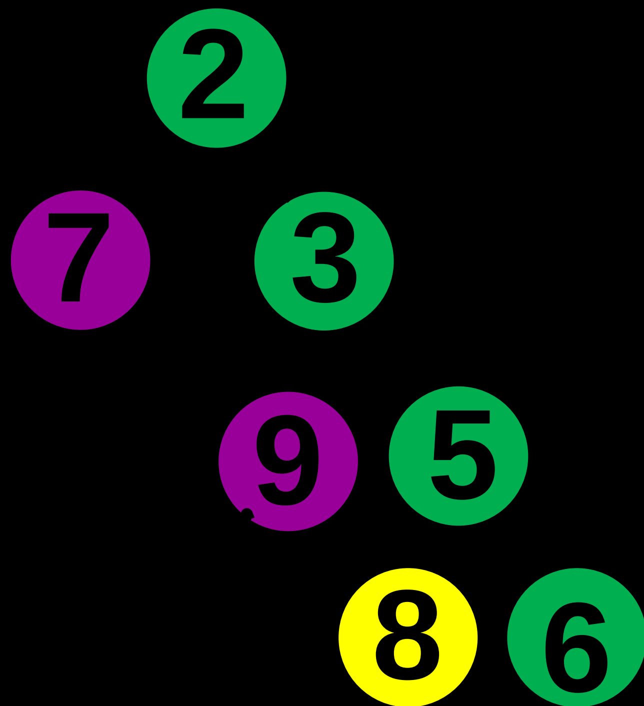
ILBFS



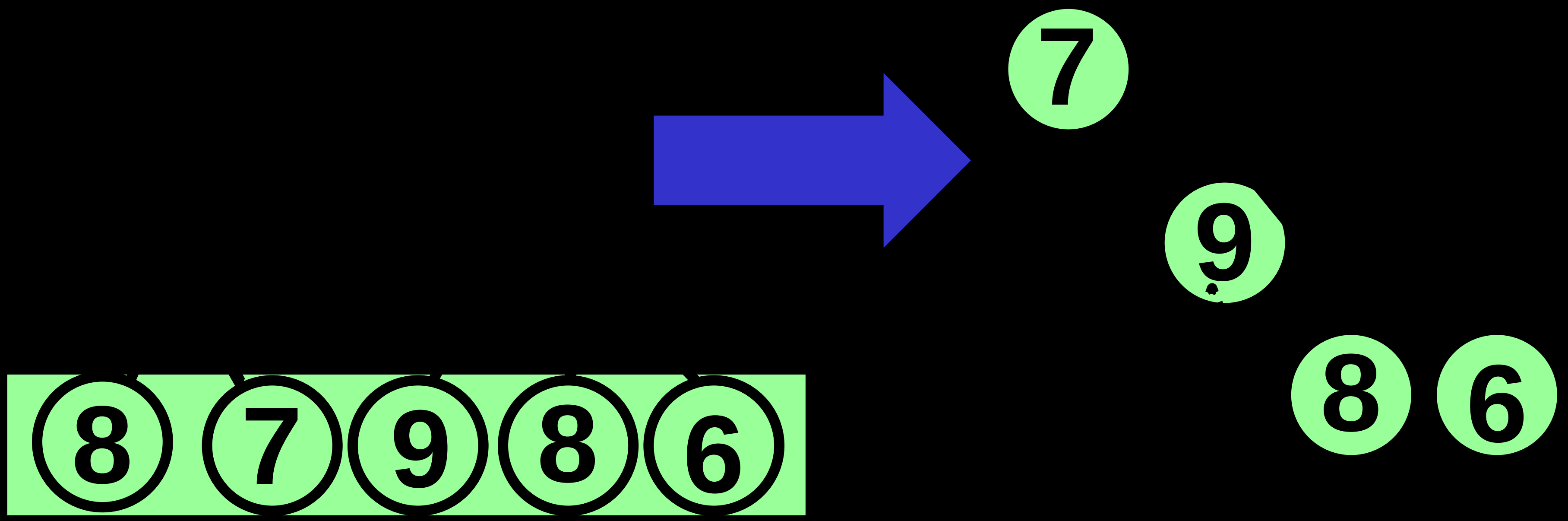
ILBFS

7

9

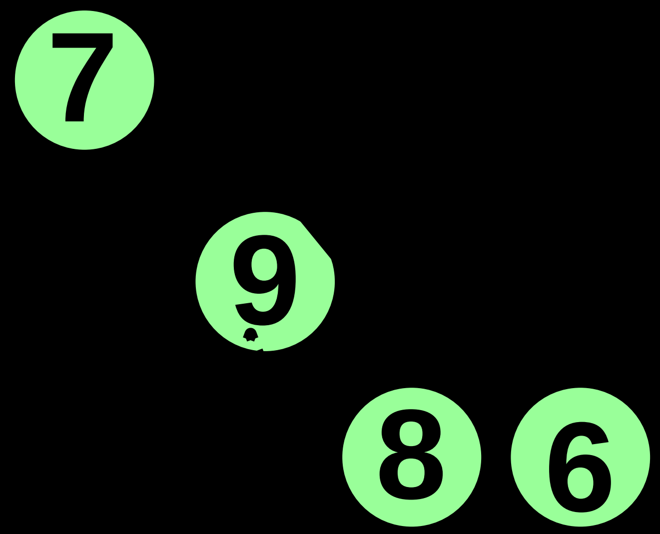
8 6

ILBFS



Principal branch invariant

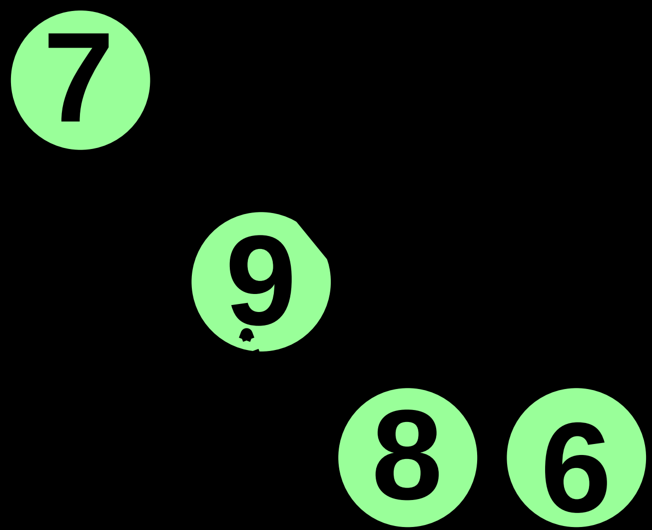
ILBFS



Case 1:

Best is a child of oldbest

ILBFS



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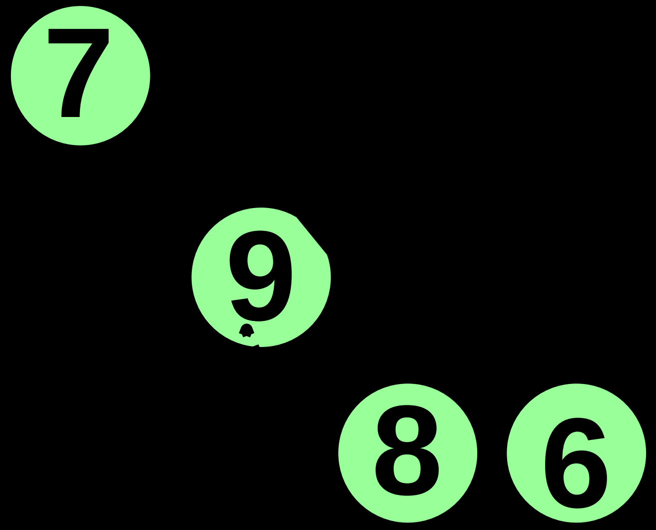

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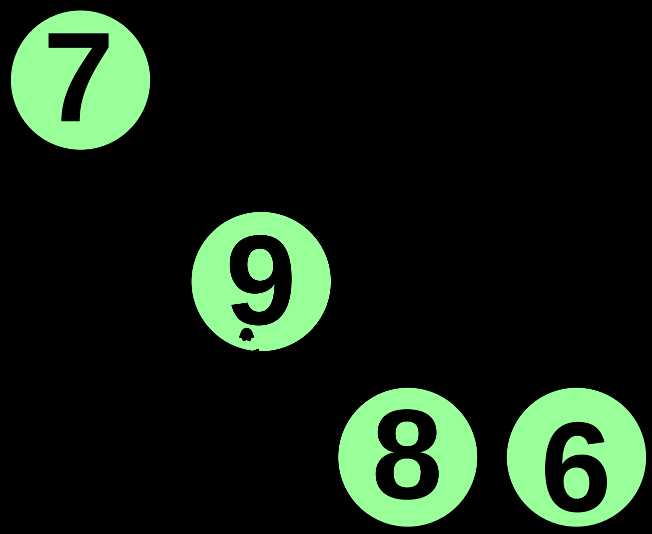

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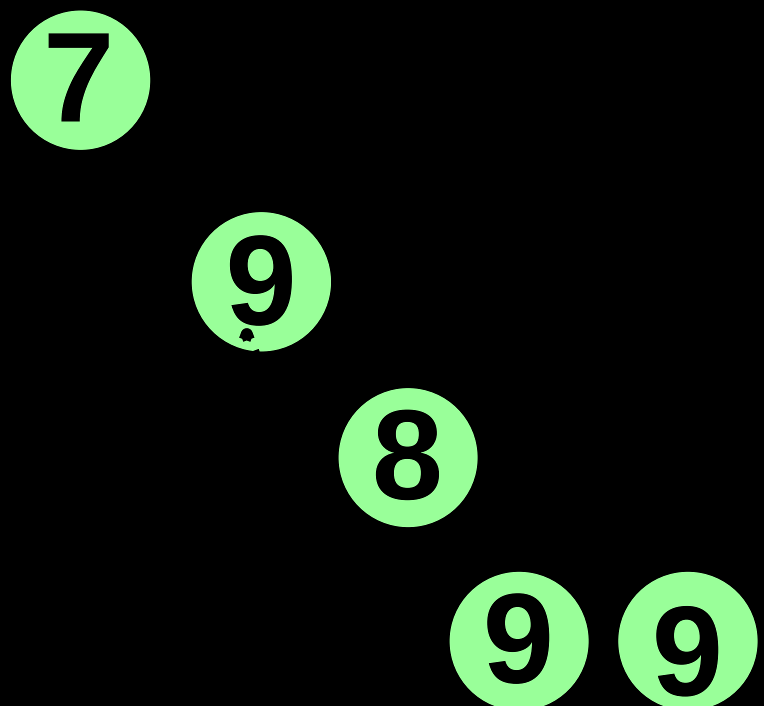

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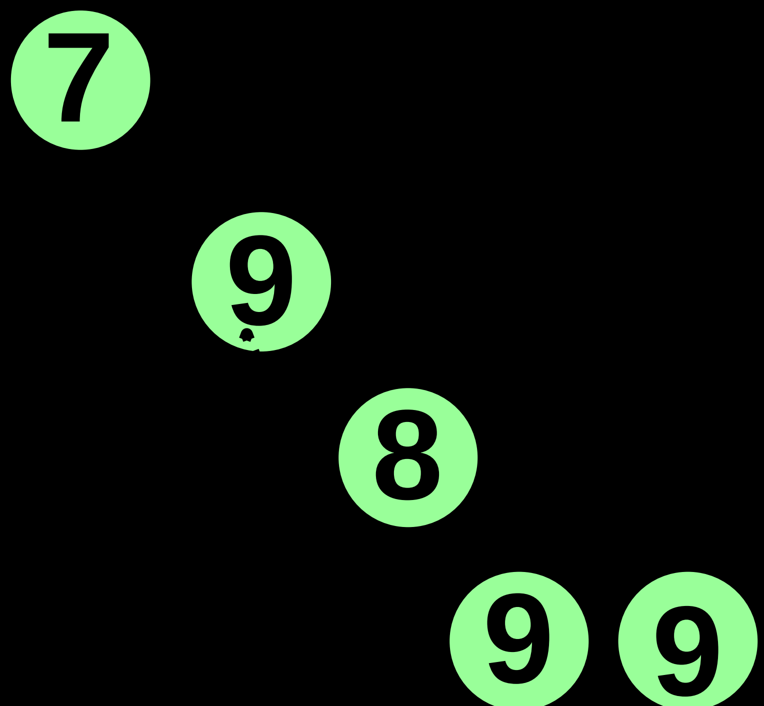
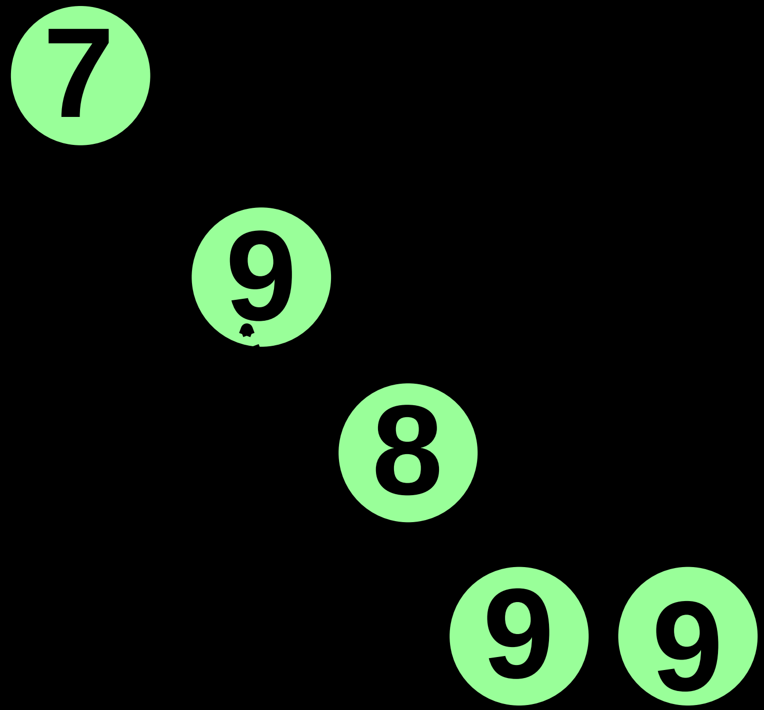
ILBFS



Case 2:

Best is not a child of oldbest

ILBFS



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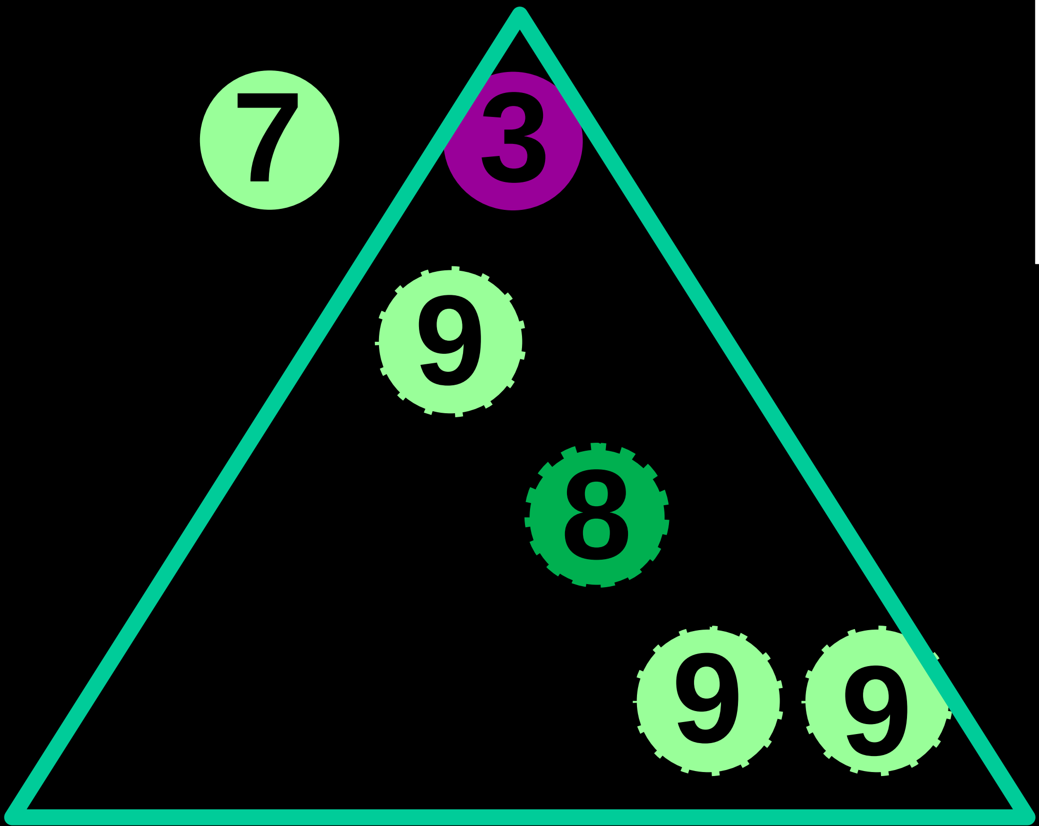
ILBFS

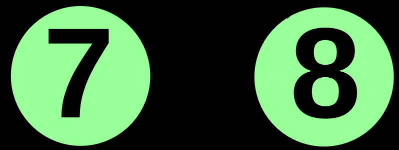


collapse

Case 2:

Best is not a child of oldbest

ILBFS



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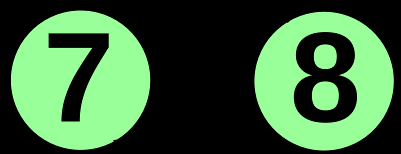
ILBFS



Case 2:

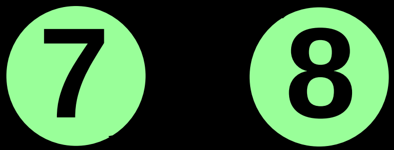
Best is a collapsed node

Restore

f=3, F=7

DFS(7)

ILBFS



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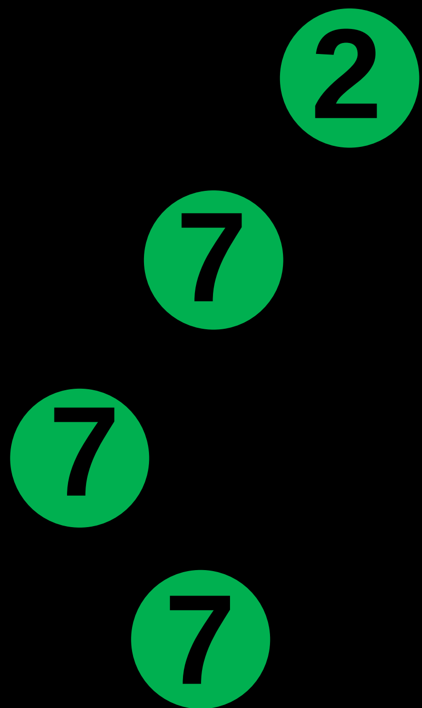
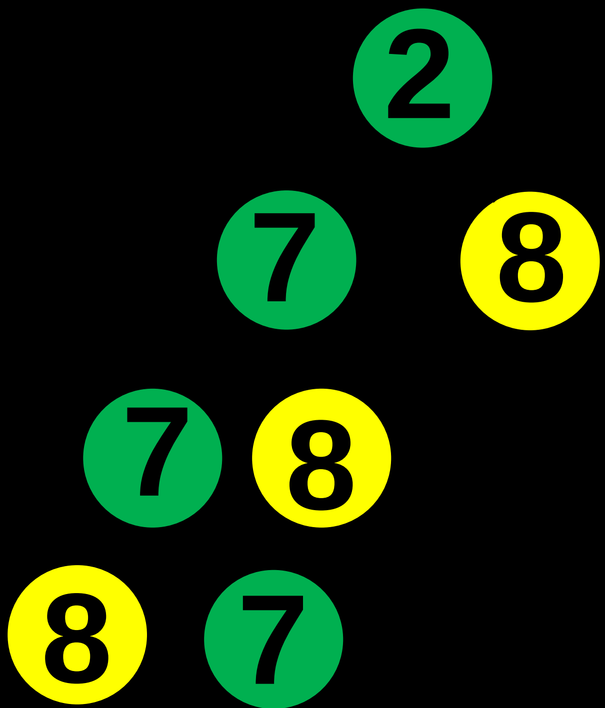


Case 2:

Best is a collapsed node

ILBFS



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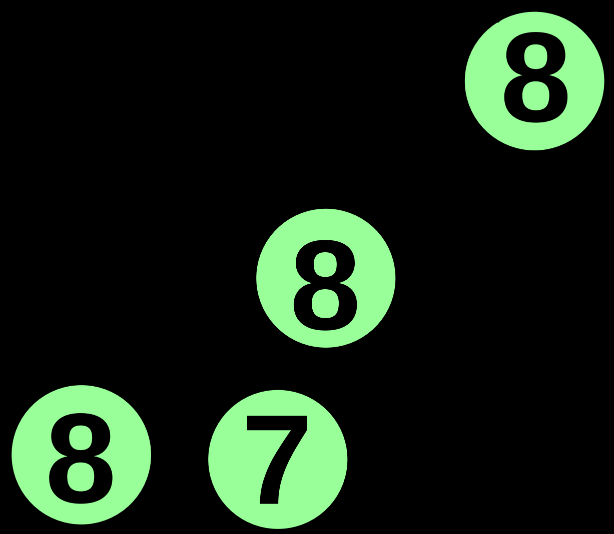
Best is a collapsed node

ILBFS



Case 2:

Best is a collapsed node

Linear-space best-first search

Iterative variant - ILBFS

Algorithm 1: High-level ILBFS

Input: Root R

Insert R into OPEN and TREE
oldbest=NULL
while *OPEN not empty* **do**
  best=extract_min(OPEN)
  if *goal(best)* **then**
    exit
  if *oldbest* $\neq$ *best.parent* **then**
    $B \leftarrow$ sibling of *best* that is ancestor of *oldb*
    collapse(B)
  if *best.C*=*True* **then**
    best $\leftarrow$ restore(*best*)
  foreach *child C of best* **do**
    Insert C to OPEN and TREE
  oldbest $\leftarrow$ *best*

Recursive variant - RBFS

RBFS(n, B)
1. if n is a goal
2. $\quad solution \leftarrow n$; exit()
3. $C \leftarrow expand(n)$
4. if C is empty, return ∞
5. for each child n_i in C
6. $\quad$ if $f(n) < F(n)$ then $F(n_i) \leftarrow max(F(n), f(n_i))$
7. $\quad$ else $F(n_i) \leftarrow f(n_i)$
8. $(n_1, n_2) \leftarrow best_F(C)$
9. while $(F(n_1) \leq B$ and $F(n_1) < \infty)$
10. $\quad F(n_1) \leftarrow$ RBFS$(n_1, min(B, F(n_2)))$
11. $\quad (n_1, n_2) \leftarrow best_F(C)$
12. return $F(n_1)$

Restoring?

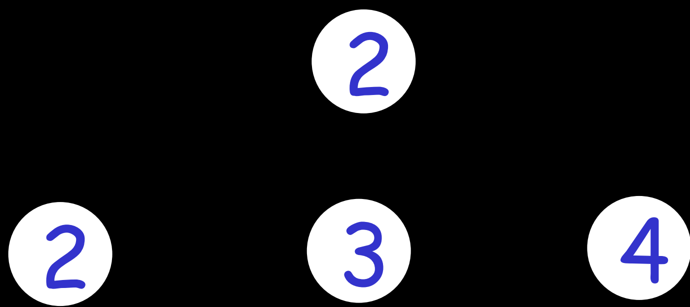
Ordering child nodes

Ordering child nodes
with new F-values

ILBFS/RBFS Summary

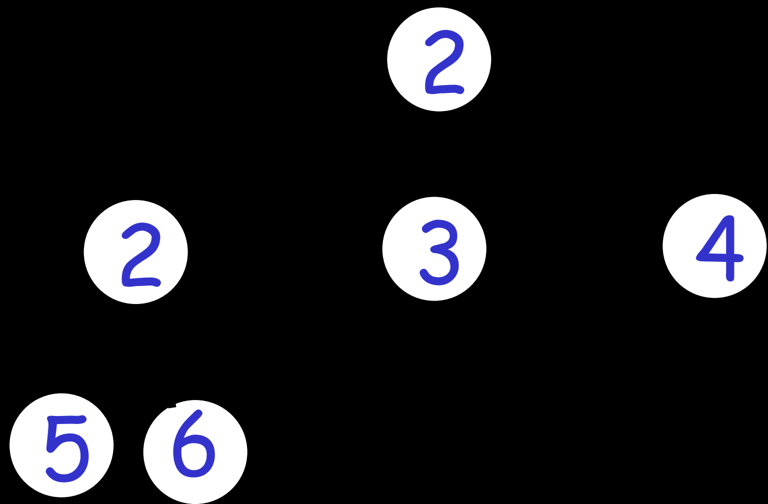
- More efficient than IDA* and still optimal
  - Best-first Search based on next best f-value-threshold (countour); fewer regeneration of nodes
  - Exploit results of search at a specific f-value-threshold by saving next f-value-threshold associated with a node who successors have been explored.
- Like IDA* still suffers from excessive node regeneration
- IDA* and RBFS not good for graphs
- Can’t check for duplicates other than those on current path
- Both are hard to characterize w.r.t. expected time complexity

SMA* (Simplified Memory Bounded A*) [#2:Russell 1992]



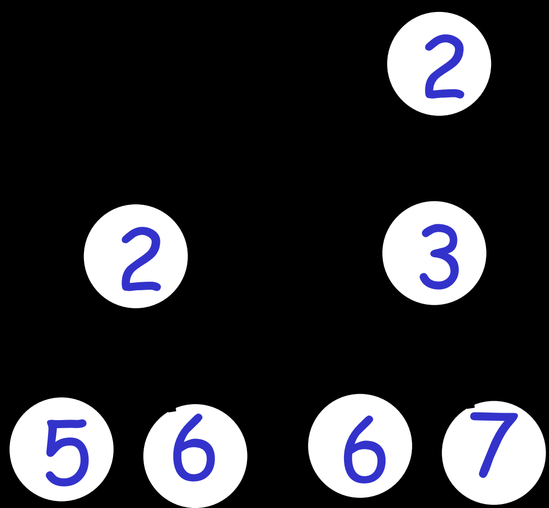
2 3 4

SMA* [Russell 1992]



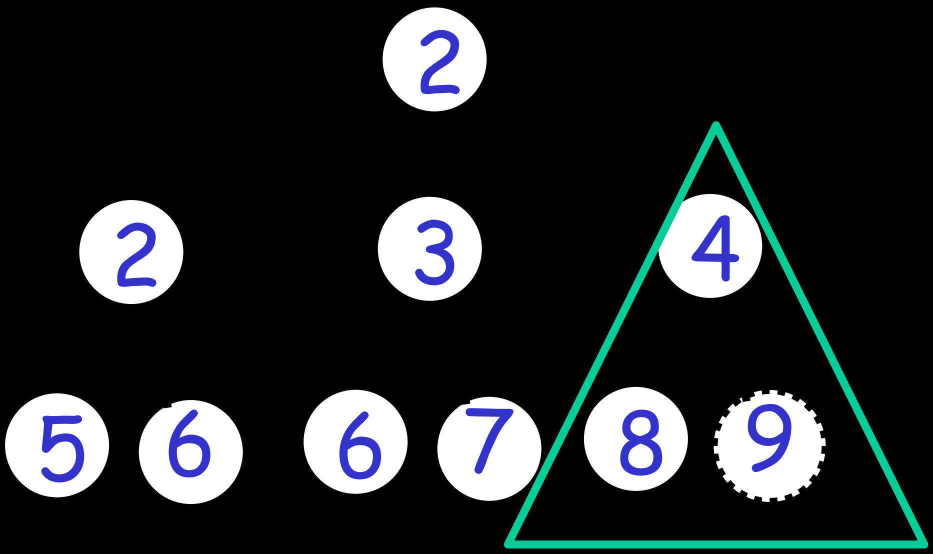
3 4 5 6

SMA* [Russell 1992]



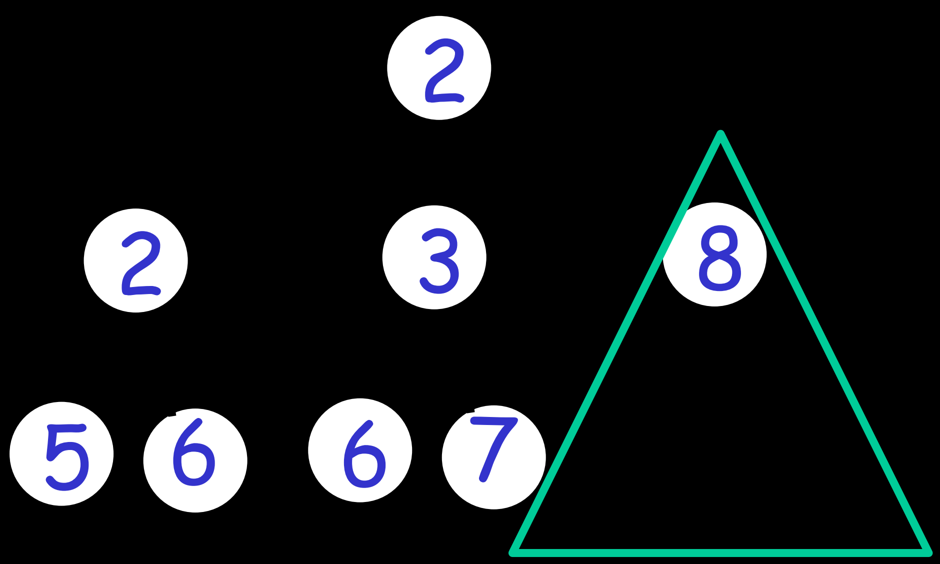
4 5 6 6 7

SMA* [Russell 1992]



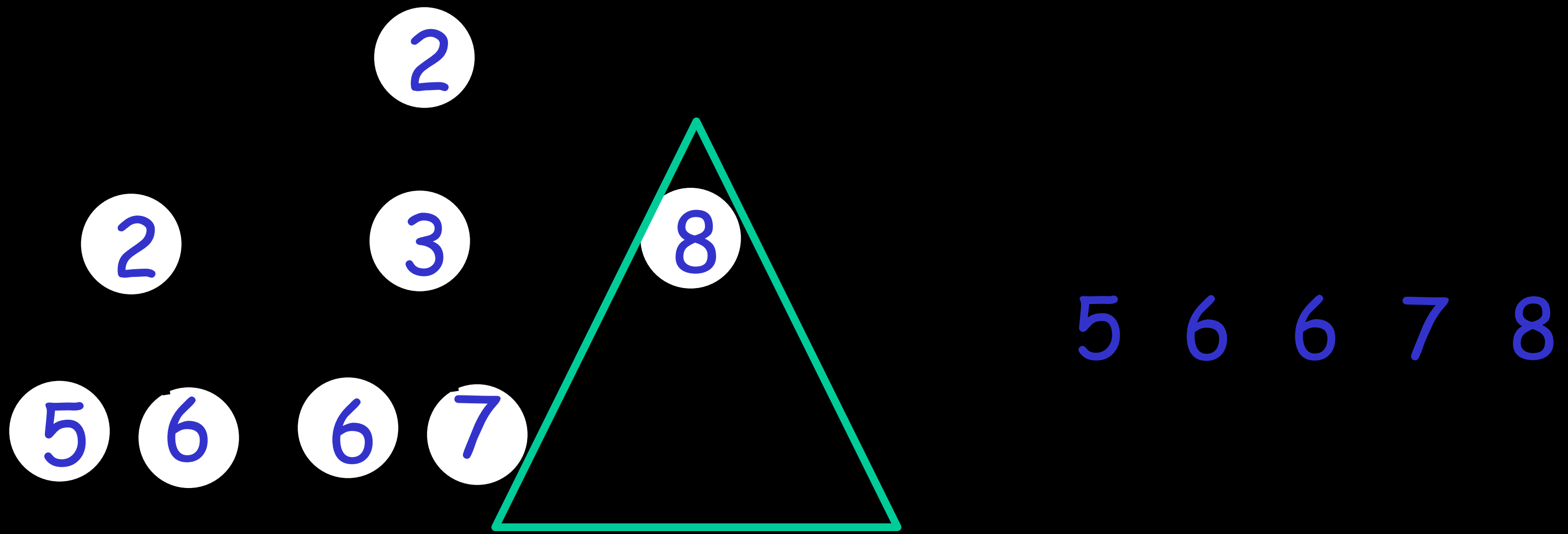
5 6 6 7 8 9

SMA* [Russell 1992]



5 6 6 7 8

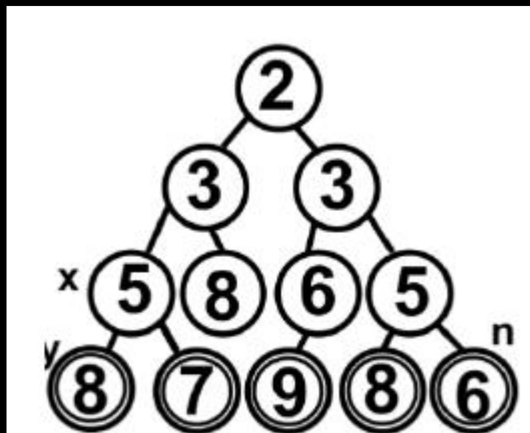
SMA* [Russell 1992]

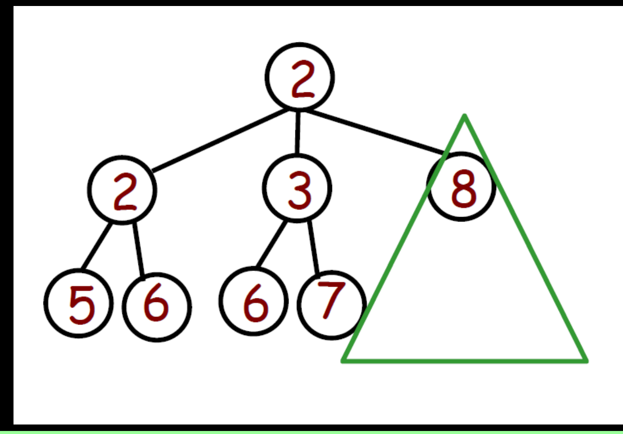


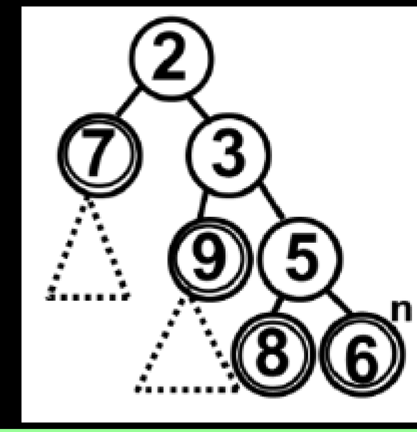
SMA* uses a variant of pathmax for its restore macro

SMA* Summary

- It is complete, provided the available memory is sufficient to store the shallowest solution path.
- It is optimal, if enough memory is available to store the shallowest optimal solution path. Otherwise, it returns the best solution (if any) that can be reached with the available memory.
- Can keep switching back and forth between a set of candidate solution paths, only a few of which can fit in memory (thrashing)
  - **Memory limitations can make a problem intractable wrt time**
- With enough memory for the entire tree, same as A*







memory	b^d	M	$O(d)$
collapse	never	lazily	eagerly

continuum