Business 2400 – Decision Modeling

Assignment 3, 2021 Winter

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Instructions Questions 1 and 2 are to be done by hand, showing all the steps needed to derive the answers. [Of course, you could use LINGO or the Excel Solver to verify your answers, but this is not to be submitted.] Use Word or pdf for this question; if making the solutions with cursive writing, please be neat.

For Questions 3 and 4, a computer-based sensitivity analysis needs to be performed. For each question, on LINGO you need a file for the model, the solution report, and the range report. For Excel, one file with nine tabs can be used; for each of the three questions, there is the model, the Answer Report, and the Sensitivity Report.

Use Word or pdf to give the answers to the sensitivity questions. For each give part, write a couple of sentences; don't just submit a numerical answer.

1. (10 marks) Suppose that the objective of a two-variable optimization model is to maximize $14x_1 + 18x_2$, and that we find by graphing that the binding constraints are:

$$\begin{array}{ll} (2) & 7x_1 + 3x_2 \leq 516 \\ (5) & 5x_1 + 9x_2 \leq 780 \end{array}$$

   Based on the above, the optimal solution is at $x_1 = 48$, $x_2 = 60$, and OFV = 1752.

   (a) For the objective function coefficients, find the allowable increase and decrease for each coefficient (based on one-at-a-time changes).

   (b) Suppose that the right-hand side of (2) is changed to $516 + \Delta b_2$. Find expressions for the values of x_1, x_2, and OFV as a function of Δb_2, and from the latter state the shadow price of this constraint. [Do not worry about the allowable range.]

2. (a) (45 marks) Solve the following model graphically, using a 50 by 50 grid.

$$\begin{array}{rrcrcr}
\text{maximize} & 4x_1 & + & 9x_2 & & \\
\text{subject to} & & & & & \\
(1) & 15x_1 & + & 3x_2 & \leq & 450 \\
(2) & 40x_1 & + & 10x_2 & \geq & 400 \\
(3) & 10x_1 & + & 8x_2 & \leq & 420 \\
(4) & 4x_1 & + & 20x_2 & \leq & 588 \\
 & & & & & \\
 & x_1 & , & x_2 & \geq & 0
\end{array}$$

   (b) Perform a sensitivity analysis for each of the objective function coefficients.

   (c) Perform a sensitivity analysis for the right-hand-side values for each of the non-binding constraints.

   (d) Perform a sensitivity analysis for the right-hand-side values for each of the two binding constraints.

   (e) For each of the two binding constraints, find the change to each variable as function of the right-hand side value, and from these relationships calculate the two shadow prices.

3. (20 marks) Every operating day an oil refinery requires at least 2100 kilograms (kg) of chemical A, 2780 kg of chemical B, and 1500 kg of chemical C. They externally purchase two types of bags which contain these chemicals. Each type 1 bag contains 17.5 kg of chemical A, 10 kg of B, and 2.5 kg of C. Each type 2 bag contains 8.75 kg of chemical A, 14 kg of B, and 15 kg of C. A type 1 bag costs \$140, while a type 2 bag costs \$200. Storage limitations mean that no more than 290 bags (of both types combined) can be purchased each day. They wish to minimize the daily cost of buying these bags. (Because of the repetitive nature of the operations, the number of bags need not be integer.)

   We define X_1 and X_2 to be respectively the number of type 1 and 2 bags purchased each day.

$$\begin{array}{rrcr}
\text{minimize} & 140X_1 + 200X_2 & & \\
\text{subject to} & & & \\
\text{Chemical A} & 17.5X_1 + 8.75X_2 & \geq & 2100 \\
\text{Chemical B} & 10X_1 + 14X_2 & \geq & 2780 \\
\text{Chemical C} & 2.5X_1 + 15X_2 & \geq & 1500 \\
\text{Storage} & X_1 + X_2 & \leq & 290 \\
 & & & \\
 & X_1, X_2 & \geq & 0
\end{array}$$

(a) Use either LINGO or the Excel Solver to create the reports needed for sensitivity analysis.

(b) What happens to the OFV in the following scenarios? (each is independent of the others)

(i) The storage capacity falls by 30 bags.

(ii) The price per bag of Type 1 falls by $90.

(iii) The price per bag of Type 2 falls by $10.

(iv) The price per bag of Type 1 falls by $40 while the price per bag of Type 2 increases by $380.

(v) The company could decrease what they have to buy of Chemical B by making up to 500 kgs of Chemical B in its own operations at a cost of $5/kg.

(vi) The company could decrease what they have to buy of Chemical C by making up to 300 kgs of Chemical C in its own operations at a cost of $0.40/kg.

4. (25 marks) A paint blending operation buys paint in five colours. These are white, black, and the three primary colours of red, blue, and yellow. There are five vats in which to receive these colours from the supplier; the white vat and the black vat each has a capacity of 5000 litres. Each primary colour vat has a capacity of 25,000 litres. All paint regardless of colour is purchased at a price of $9 per litre.

Each product is made as a specified percentage of the five basic paints. Here is the list of their products:

	Percentages of the Five Basic Paints				
Name of the Product	White	Black	Red	Blue	Yellow
Candy Cane	20	0	70	10	0
Hunter Green	0	15	0	25	60
Passion Purple	0	10	50	40	0
Olive Green	0	0	5	20	75
Sunset Orange	5	0	50	0	45
Turquoise	0	0	0	85	15

There are also six vats for the products. For each of the six products, there is a dedicated 15,000 litre capacity vat. All these products are sold at a price of $12 per litre.

Assume that no paint is lost in the blending process.

(a) Give the algebraic model for this situation. (Hint: only 11 variables are needed.)

(b) Solve this model in LINGO or the Excel Solver, and obtain the sensitivity report.

(c) The plant management could seek some funds which could be used to expand the capacity of one or more of the 11 vats. Spreading out such costs over time would work out to a price of about $2.00 per litre to expand any of the 11 vats. Based on the information which could be gleaned from the sensitivity report(s) alone, and considering several possibilities, what would you recommend to management?