**1. (10 marks) Suppose that the objective of a two-variable optimization model is to maximize 14x1 + 18x2, and that we find by graphing that the binding constraints are:**

(2) 7x1 +3x2 ≤ 516

(5) 5x1 +9x2 ≤ 780

Based on the above, the optimal solution is at x1 = 48, x2 = 60, and OFV = 1752.

1. **For the objective function coefficients, find the allowable increase and decrease for each coefficient (based on one-at-a-time changes).**

Step 1

First, we determine the coefficient of x

Let OF = ax1 + 18x2

=>x2=-a/18+OF/18

We can calculate the line along the points at x1=48 and x2=60 to find out where its slope lies.

Slope of the line Z

-x2=-7x1/3+516/3

Boundaries of the Slope

5/9<=14/6<=7/3

Constraints derived with the boundaries

5/9b<=14<=7b/3

Left Side

5b/9<=14

b<=126/5

Right Side

14<=7b/3

b<=42/7

Therefore, the Boundary domains are as follows-;

42/7<=b<=126/5

Step 2: calculate the allowable Increase and Decrease

Allowable Increase =126/5-18=(126-90)/5=36/5

Allowable Decrease=18-42/7=(126-42)/7=84/7=12

Hence a can vary between 36/5 and 12

Step 3: Determine the coefficient of x2

Let

OF=14x1+bx2

X2=-14x1/b + OF/b

Slope -14/b is the Absolute Value

Again, this lined from the equation must lie between the slope of the two constraint lines

5/9<=14/b<=7/3

X2=-7/3x2+172

Slope of 7/3 absolute value

Slope of the line 3

X2=-5x1/9+780/9

Slope of the assumed annotated line

X2=-ax1/18+OF/18

Range of the values

X2=-ax1/18+OF/18

Slope of -a/18 absolute value

Range

5/9<=a/18<=7/3

90/9<=a<=126/3

10<=a<=42

Hence a can vary between 10 and 42

**(b) Suppose that the right-hand side of (2) is changed to 516+∆b2**

**. Find expressions for the values of x1, x2, and OFV as a function of ∆b2**

**and from the latter state the shadow price of this constraint. [Do not worry about the allowable range.]**

Step 1: Model the problem

-x1+3x2<=516+∆b2

=>x2<=-7x1/3+516/3+∆b2/3

=>x2<=7x1/3+172+∆b2/3

Our equation (2)

5x1+9x2<=780

X2<=-5x1/9+780/9

Step 2: Substitute the Equations

Equating both sides equations we get

-7x1/3+172+∆b2/3=-5x1/9+780/9

-7x1/3+5x1/9=780/9-172-+∆b2/3

X1=768/16+3∆b2/3

Now putting this value of x in the equation 2 we get

X2=-5/9(768/16+#+∆b3/16)+780/9

=-3840/144-15∆b3/144+780/9

X2=(8640-15∆b3)/144

But

OFV=14x1+18x2

=14(768/16+3∆b3/16)+18(8640-15∆b3)/144

=(10752+42∆b3)/16+(155520-270∆b3)/144

=(96768+378∆b3+155520-270∆b3)

=(252288+108∆b3)/144

OFV=1752+3/4(∆b3)

**2. (a) (45 marks) Solve the following model graphically, using a 50 by 50 grid.**

MD

maximize 4x1 + 9x2

subject to (1) 15x1 + 3x2 ≤ 450

(2) 40x1 + 10x2 ≥ 400

(3) 10x1 + 8x2 ≤ 420

(4) 4x1 + 20x2 ≤ 588

x1, x2 ≥ 0

Replace the constraints in the model as equations and x1=x and x2=y

maximize (z)=4x + 9y

subject to

(1) 15x + 3y = 450

(2) 40x + 10y = 400

(3) 10x + 8y = 420

(4) 4x + 20y = 588

x, y ≥ 0

Step 1

Part 1

For 15x + 3y = 450

If x=0 then y=150

If y=0 then x=30

Part 2

40x+10y=400

If x=0 then y=40

If y=0 then x=10

Part 3

10x+8y=420

If x=0 then y=52.5

If y=0 then x=42

Part 4

If x=0 then y=29.4

If y=0 then x=14.7

Therefore, the following can be concluded

The line 15x+3y=450 passes through the points (0,150) & (30,0)

The line 40x+10y=400 passes through the points (0,40) & (10,0)

The line 10x+8y=420 passes through the points (0,52.5) & (42,0)

The line 4x+20y=588 passes through the points (0,29.4) & (14.7,0)

Step 2

The feasible region to be found is ABCD

Hence A and B is the point of intersection of the two lines

For A

40x+10y=400

4x+20y=588

Solution through Elimination Method

40x+10y=400

4x+20y=588

Collection of HCF of 20y and 10y is 20y therefore 10y requires a boost of \*2 to make it to terms with HCF

(40x+10y2=400)\*2

4x+20y=588

Functions derived are

80x+20y=800

4x+20y=588

(80x+20y=800)-( 4x+20y=588)

76x=212

X=2.7895

Y=28.84

B)

10x+8y=420

15x+3y=450

Based on the HCF

30x+24y=1260

30x+6y=900

Evaluation

(30x+24y=1260)-(30x+6y=900)

18y=360

y=20

x=26

Hence

A(2.78,28.84) & B(26,20)

Final Step

Corner Points

A(2.78,28.84) ZA=(270.68)

B(26,20) ZB=(284)

C(10.0) ZC=(40)

D(30,0) ZD=(120

X1=x=22.00000

X2=y=25.00000

Objective Value 313

1. **Sensitivity Analysis**

X1=x=22.00000

X2=y=25.00000

Objective Value 313

**(c)**

Hence

A(2.78,28.84) & B(26,20)

Final Step

Corner Points

A(2.78,28.84) ZA=(270.68)

B(26,20) ZB=(284)

C(10.0) ZC=(40)

D(30,0) ZD=(120

**3.(a) Use either LINGO or the Excel Solver to create the reports needed for sensitivity analysis.**

MIN = 140 \* X + 200 \* Y;

!CONSTRAINTS;

!CHEMICAL A;

17.5 \* X + 8.75 \* Y >=2100;

!CHEMICAL B;

10 \* x + 14 \* Y >=2780;

!CHEMICAL C;  
2.5 \* x + 15 \* Y >=1500;  
  
X<=0;

Y<=0;

!Limited Supply;

X + Y<=290;

DISPLAY MODEL

MODEL:

[\_1] MIN= 140 \* X + 200 \* Y;

[\_2] 17.5 \* X + 8.75 \* Y >= 2100;

[\_3] 10 \* X + 14 \* Y >= 2780;

[\_4] 2.5 \* X + 15 \* Y >= 1500;

[\_5] X + Y <= 290;

END

Global optimal solution found.

Objective value: 39200.00

Infeasibilities: 0.000000

Total solver iterations: 2

Elapsed runtime seconds: 0.11

Model Class: LP

Total variables: 2

Nonlinear variables: 0

Integer variables: 0

Total constraints: 5

Nonlinear constraints: 0

Total nonzeros: 10

Nonlinear nonzeros: 0

Variable Value Reduced Cost

X 180.0000 0.000000

Y 70.00000 0.000000

Row Slack or Surplus Dual Price

1 39200.00 -1.000000

2 1662.500 0.000000

3 0.000000 -13.91304

4 0.000000 -0.3478261

5 40.00000 0.000000

DUAL MODEL

MODEL:

MAX= 2100 \* \_2 + 2780 \* \_3 + 1500 \* \_4 + 290 \* \_5;

[X] 17.5 \* \_2 + 10 \* \_3 + 2.5 \* \_4 + \_5 <= 140;

[Y] 8.75 \* \_2 + 14 \* \_3 + 15 \* \_4 + \_5 <= 200;

@BND( -1e+030, \_5, 0);

END

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Current Allowable Allowable

Variable Coefficient Increase Decrease

X1 4.000000 7.250000 2.200000

X2 9.000000 11.00000 5.800000

Righthand Side Ranges:

Current Allowable Allowable

Row RHS Increase Decrease

2 450.0000 INFINITY 45.00000

3 400.0000 730.0000 INFINITY

4 420.0000 26.25000 161.3684

5 588.0000 462.0000 84.00000

6 0.000000 22.00000 INFINITY

7 0.000000 25.00000 INFINITY

(b)

(i)If the storage falls by 30 units the optimal result will not change as according to the sensitivity report there is a surplus of 40 units in the storage readily available

Surplus>Shortage translates to zero change in the optimal solution

(ii)If the price of type 1 falls by $ 90

The new price of the materials will change

New price (Np)

Np=$(140-90)

New Optimal Solution (N)

N=$50\*180+70\*200=23,000

(iii)If the price of type 2 falls by $10

New price (Np)

Np=$(200-10)

=$190

The variable coefficients are

X1=32.22

X2=175.55

Objective Function=32.33\*140+175.55\*190=37866.67

(iv)If the price of type 1 falls by $40

New price (Np)

Np=$(140-40)

=$100

Price of type 2 increases by the new normal by $380, then the new price will be

New price (Np)

N=$(380+200)

=$580

The new optimal solution (N) will be

N=180\*100+70\*580

=58600

**4.(a) Give the algebraic model for this situation. (Hint: only 11 variables are needed.)**

MAX =3 \* X1 + 3 \* X2 + 3 \* X3 + 3 \* X4 + 3 \* X5 + 3 \* X6;

0.2 \* X1 + 0.05 \* X5<=5000

0.5 \* X2 + 0.1\* X3 <=5000

0.7 \* X1 +0.5 \* X3+ 0.05 \* X4+0.5 \* X5 <=25000

0.1 \* X1 +0.25 \* X2+ 0.4 \* X3+0.2 \* X4+0.85 \* X6 <=25000

0.6 \* X2 +0.75 \* X4 +0.45 \* X5+0.15 \* X6 <=25000

X1<=15000

X2<=15000

X3<=15000

X4<=15000

X5<=15000

X6<=15000

X1>=0

X2>=0

X3>=0

X4>=0

X5>=0

X6>=0

Basing on the 11 variables needed we can do away with the non-zero components.

(b)

DISPLAY MODEL

MODEL:

[\_1] MAX= 3 \* X1 + 3 \* X2 + 3 \* X3 + 3 \* X4 + 3 \* X5 + 3 \* X6;

[\_2] 0.2 \* X1 + 0.05 \* X5 <= 5000;

[\_3] 0.5 \* X2 + 0.1 \* X3 <= 5000;

[\_4] 0.7 \* X1 + 0.5 \* X3 + 0.05 \* X4 + 0.5 \* X5 <= 25000;

[\_5] 0.1 \* X1 + 0.25 \* X2 + 0.4 \* X3 + 0.2 \* X4 + 0.85 \* X6 <= 25000;

[\_6] 0.6 \* X2 + 0.75 \* X4 + 0.45 \* X5 + 0.15 \* X6 <= 25000;

[\_7] X1 <= 15000;

[\_8] X2 <= 15000;

[\_9] X3 <= 15000;

[\_10] X4 <= 15000;

[\_11] X5 <= 15000;

[\_12] X6 <= 15000;

END

Global optimal solution found.

Objective value: 240642.9

Infeasibilities: 0.000000

Total solver iterations: 2

Elapsed runtime seconds: 0.09

Model Class: LP

Total variables: 6

Nonlinear variables: 0

Integer variables: 0

Total constraints: 12

Nonlinear constraints: 0

Total nonzeros: 29

Nonlinear nonzeros: 0

Variable Value Reduced Cost

X1 13214.29 0.000000

X2 7000.000 0.000000

X3 15000.00 0.000000

X4 15000.00 0.000000

X5 15000.00 0.000000

X6 15000.00 0.000000

Row Slack or Surplus Dual Price

1 240642.9 1.000000

2 1607.143 0.000000

3 0.000000 6.000000

4 0.000000 4.285714

5 178.5714 0.000000

6 550.0000 0.000000

7 1785.714 0.000000

8 8000.000 0.000000

9 0.000000 0.2571429

10 0.000000 2.785714

11 0.000000 0.8571429

12 0.000000 3.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Current Allowable Allowable

Variable Coefficient Increase Decrease

X1 4.000000 7.250000 2.200000

X2 9.000000 11.00000 5.800000

Righthand Side Ranges:

Current Allowable Allowable

Row RHS Increase Decrease

2 450.0000 INFINITY 45.00000

3 400.0000 730.0000 INFINITY

4 420.0000 26.25000 161.3684

5 588.0000 462.0000 84.00000

6 0.000000 22.00000 INFINITY

7 0.000000 25.00000 INFINITY

(c)

New Prices will be as follows

The production cost will increase by $2

Therefore, total production will be as follows

C=$12-$(9+2)

C=$1

Therefore total profits will be as follows

P =∑ $1\*(Xi)

Global optimal solution found.

Objective value: 80214.29

Infeasibilities: 0.000000

Total solver iterations: 2

Elapsed runtime seconds: 0.08

Model Class: LP

Total variables: 6

Nonlinear variables: 0

Integer variables: 0

Total constraints: 12

Nonlinear constraints: 0

Total nonzeros: 29

Nonlinear nonzeros: 0

Variable Value Reduced Cost

X1 13214.29 0.000000

X2 7000.000 0.000000

X3 15000.00 0.000000

X4 15000.00 0.000000

X5 15000.00 0.000000

X6 15000.00 0.000000

Row Slack or Surplus Dual Price

1 80214.29 1.000000

2 1607.143 0.000000

3 0.000000 2.000000

4 0.000000 1.428571

5 178.5714 0.000000

6 550.0000 0.000000

7 1785.714 0.000000

8 8000.000 0.000000

9 0.000000 0.8571429E-01

10 0.000000 0.9285714

11 0.000000 0.2857143

12 0.000000 1.000000

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Current Allowable Allowable

Variable Coefficient Increase Decrease

X1 4.000000 7.250000 2.200000

X2 9.000000 11.00000 5.800000

Righthand Side Ranges:

Current Allowable Allowable

Row RHS Increase Decrease

2 450.0000 INFINITY 45.00000

3 400.0000 730.0000 INFINITY

4 420.0000 26.25000 161.3684

5 588.0000 462.0000 84.00000

6 0.000000 22.00000 INFINITY

7 0.000000 25.00000 INFINITY

The profit margins will drop to 80214.29 from 240642.9 therefore almost dropping by two thirds.

Normality Induction Capital=240642.9-80214.29

=160,428.61

Therefore, the management will have to raise external capital of 160,428.61 to maintain normality