PG4200: Algorithms And Data Structures

Lesson 08: Graphs

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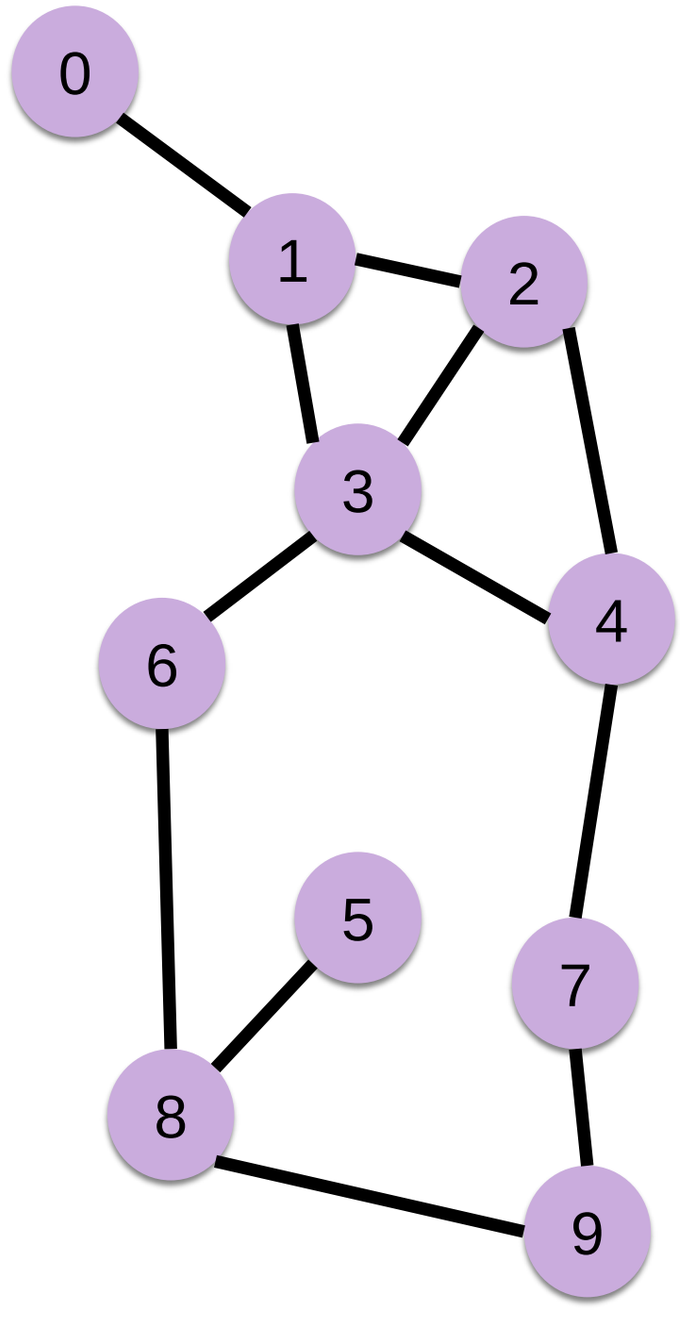
Graphs

- A set of **vertices** connected by **edges**
- **Directed** or **Undirected** graphs
  - If edge from X to Y, implies edge from Y to X?
- Many different problems can be represented with graphs
- Many different algorithms specialized for graphs
- Here just having a very high level view



Oslo Metro

Frognerseteren 1
Voksenkollen
Lillevann
Skogen
Voksenlia
Holmenkollen
Besserud
Midtskogen
Skådalen
Vettakollen
Gulleråsen
Gråkammen
Slemdal
Ris
Gaustad
Vinderen
Steinerud
Frøen

2
Østerås
Lijordet
Eiksmarka
Ekraveien
Røa
Hovseter
Holmen
Makrellbekken

3
Sognsvann
Kringsjå
Holstein
Østhorn
Tåsen
Berg

Vestli 5
Stovner
Rommen
Romsås
Grorud
Ammerud
Kalbakken
Rødtveit
Veitveit
Linderud
Vollebekk
Risløkka
Økern
Hasle

Nydalen 4 5 6
Storo
Sinsen

Ullevål stadion
Forskningsparken
Blindern
Carl Berners plass

1 2 **Ellingsrudåsen**
Furuset
Lindeberg
Tosterud
Haugerud
Tveita

Ensjø
Helsfyr
Brynseng
Hellerud

Smestad
Borgen

Montebello
Ullernåsen
Åsjordet
Bjørnsletta
Jar 6
Ringstabekk
Bekkestua
Gjønnes
Haslum
Avløs
Valler
Gjettum
Hauger
Kolsås

Majorstuen
Nationaltheatret
Stortinget
Jernbanetorget
Grønland
Tøyen

Høyenhall
Manglerud
Ryen
Brattlikollen
Karlsrud
Lambertseter
Munkelia
Bergkrystallen 4

Godila
Skøyenåsen
Oppsal
Ulsrud
Bøler
Borgerud
Skullerud
Mortensrud 3

1 Line and terminus
Station
Closed for renovation
Transfer to tram
Transfer to train

Oslo

Bergen

Add destination

Leave now

OPTIONS

Send directions to your phone

via Rv7 — 7 h 2 min

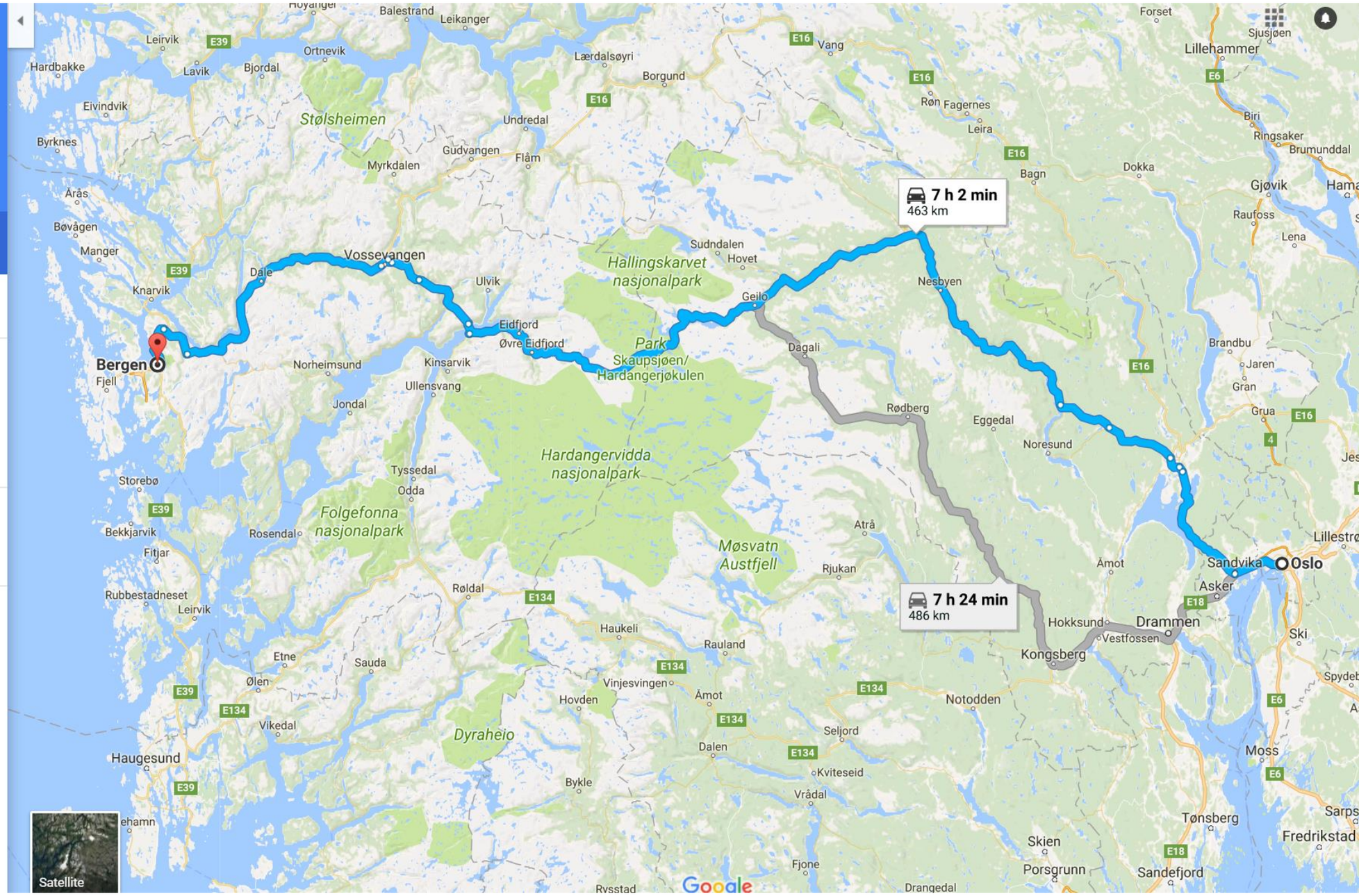
7 h 2 min without traffic — 463 km

This route has tolls.

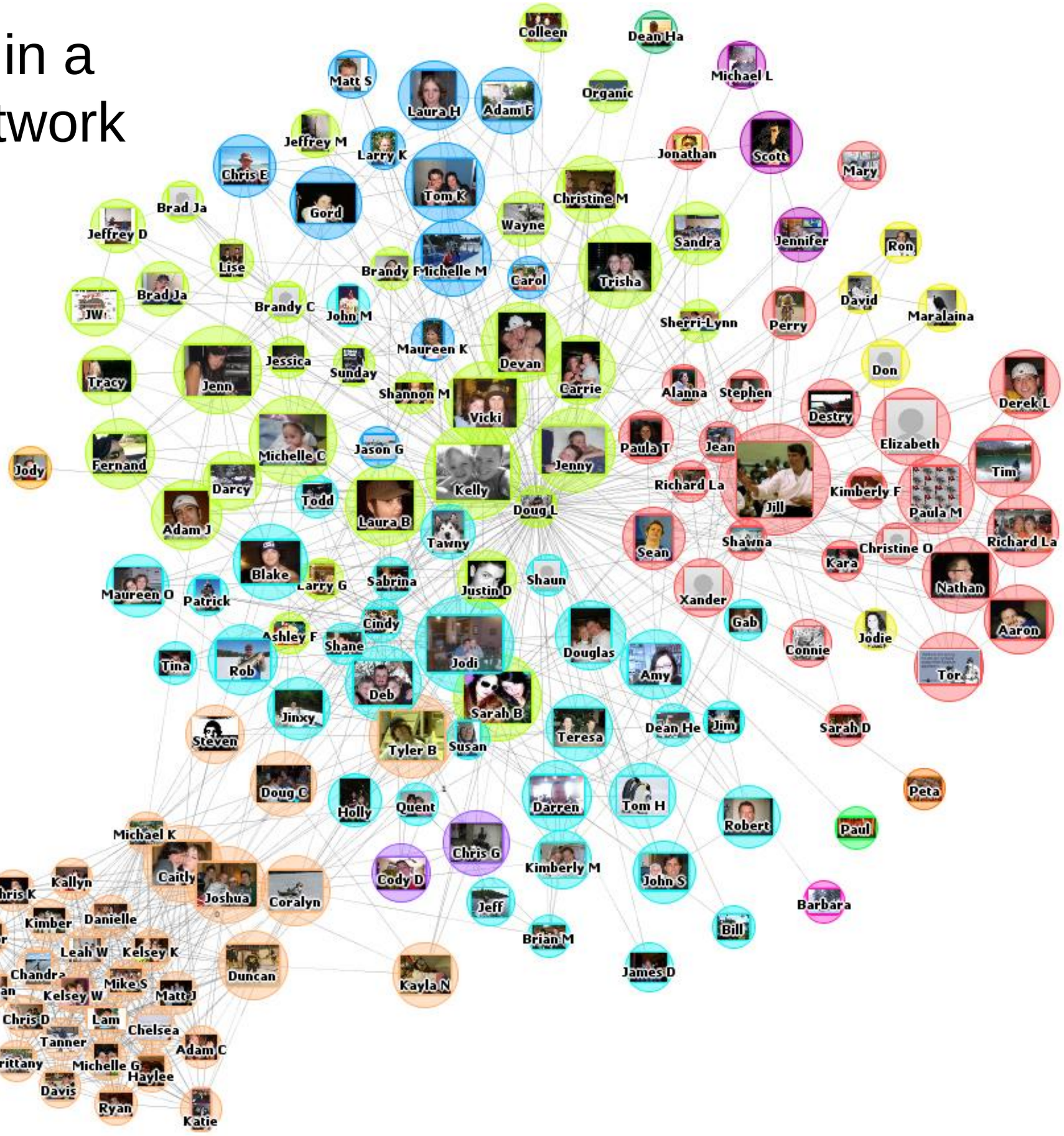
DETAILS

via Fv40 — 7 h 24 min

7 h 24 min without traffic — 486 km



Friends in a social network



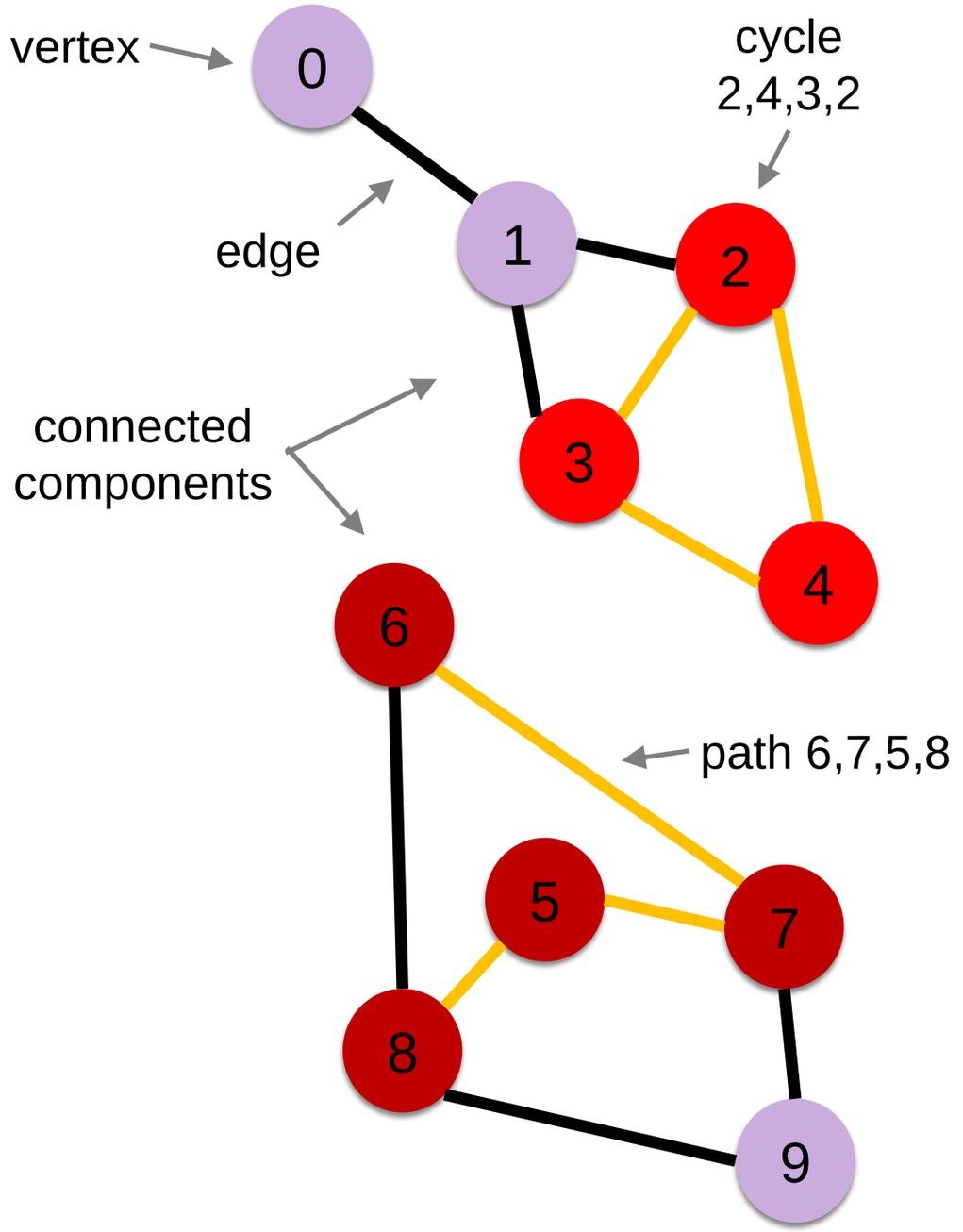
powered by TouchGraph

Machines connected on internet

Note: the picture only show a tiny subset of the whole internet graph…

Terminology

- **Vertex**: a node, for which can use a label to identify it
- **Edge**: connection between 2 nodes
- **Path**: a sequence of connected nodes
- **Cycle**: a path starting and ending on the same node
- Note: in a graph, not all nodes are necessarily connected



How to represent a graph?

- Vertices can be object with state
  - Eg, name of station, city, friend, IP address
- “Map” from vertex X (key) to a collection of vertices (value) reachable from X
- Note: in this way, do not need objects to explicitly represent edges

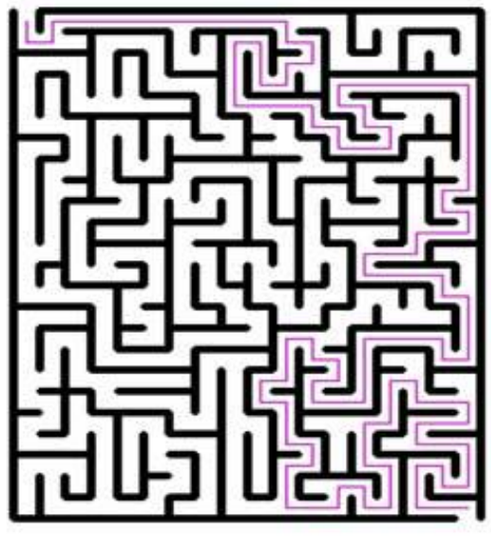
Operations

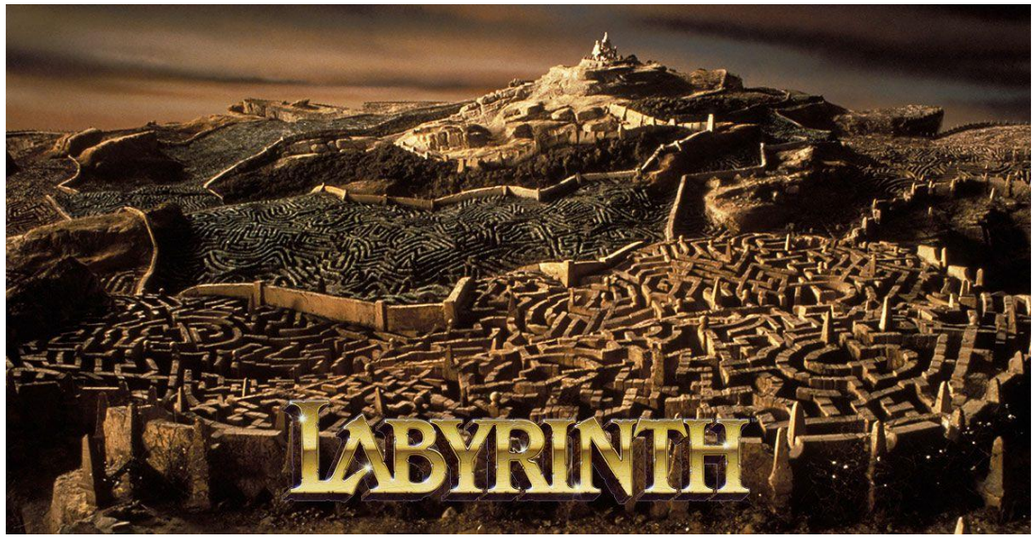
- Given an existing graph, there can be different operations we might need to do

- **Path Finding**: given two vertices (eg, 2 cities), find a path connecting them, and avoiding cycles

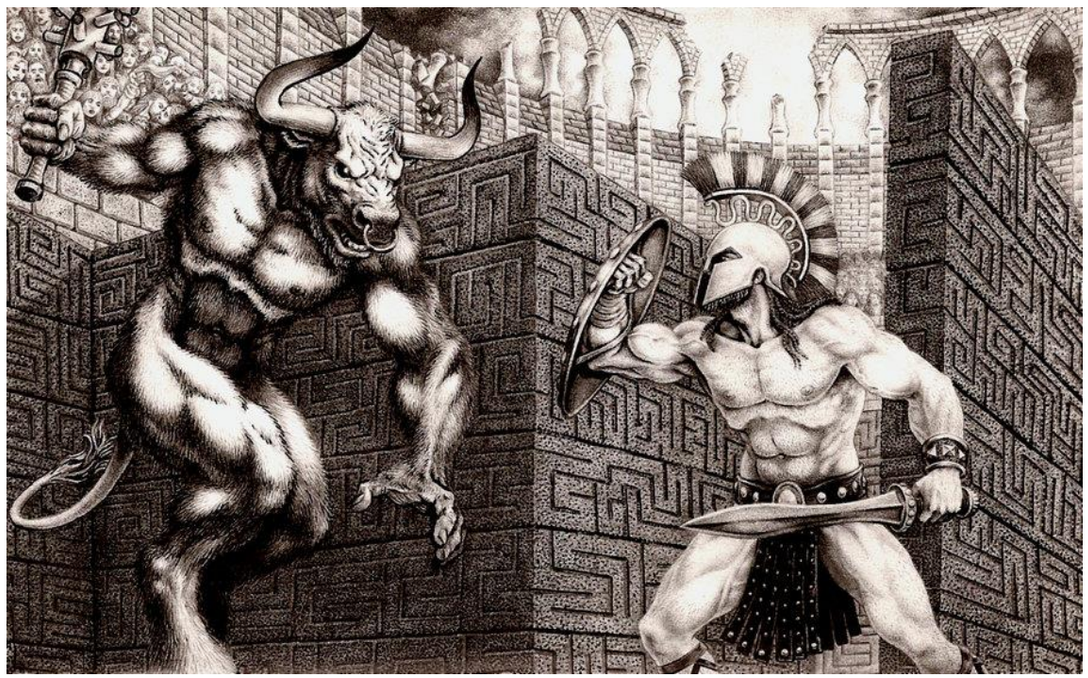
Maze / Labyrinth

- Mazes can be represented with a graph
- Vertices: intersections
- Edges: passages between two intersections
- Find path from starting vertex to the vertex of the exit





Thesus vs. the Minotaur



- Thesus slays the Minotaur at the center of the labyrinth
- Used a thread to trace back the exit

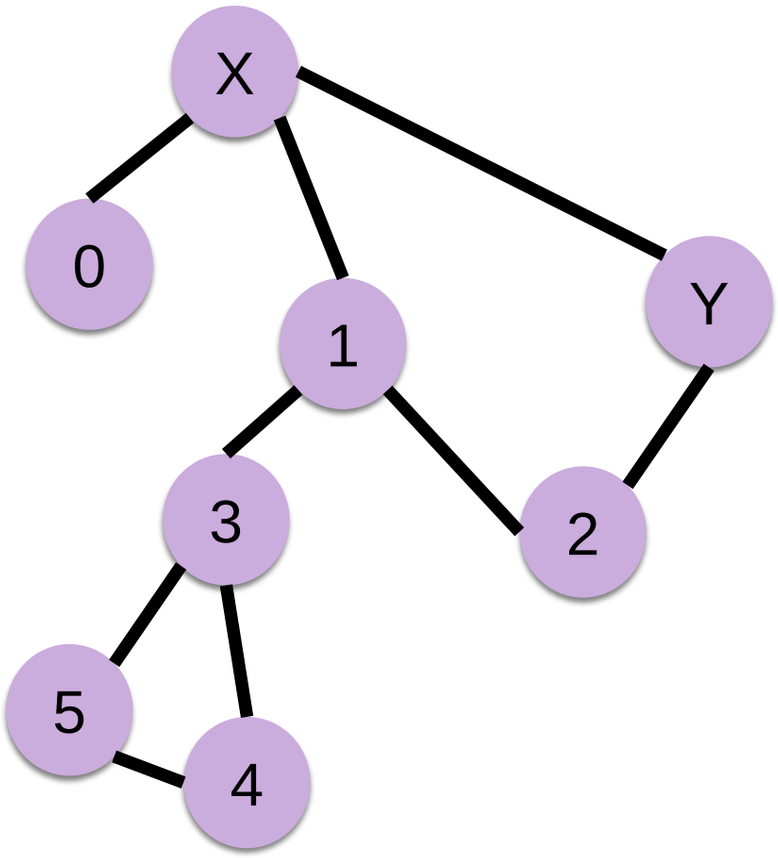
Trémaux's Algorithm

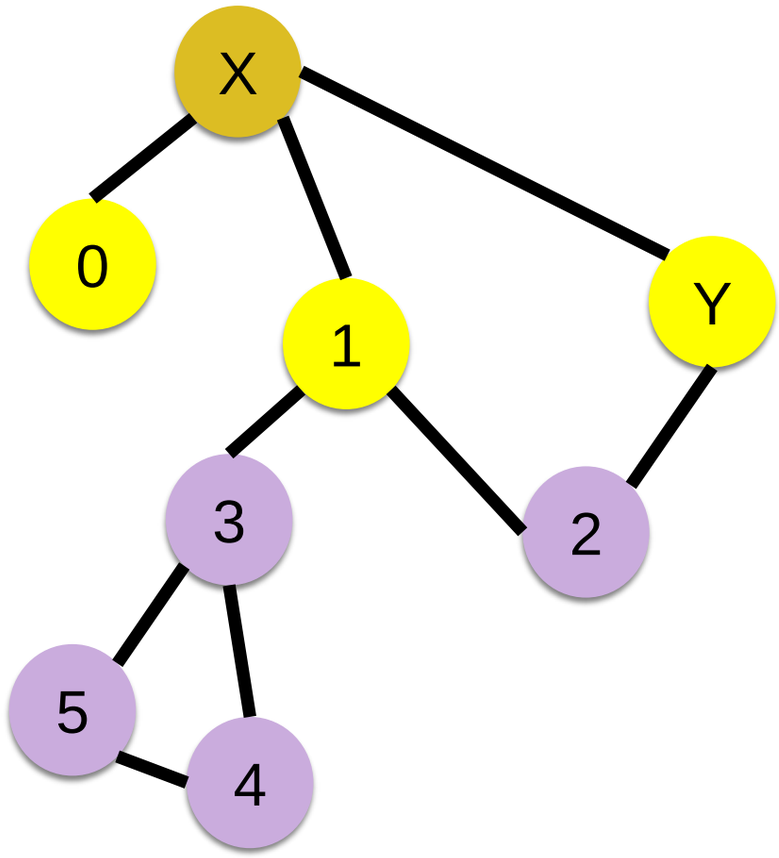
- Charles Pierre Trémaux, 19th-century
- Method that guarantees to find an exit in a maze
  - Note: talking about actual mazes, not computers…
- Trémaux's Algorithm is an instance of what we now call *Depth-First Search* in graphs

Depth-First Search (DFS)

- Try to find a path from vertex X to Y
- Mark current vertex as visited
- *Recursively* look at each connected vertex from the current
  - But skip already marked vertices (eg, by using a set)
- Use stack to represent path from X toward current vertex
  - Push when recursively evaluate a connected vertex
  - Pop when backtrack out of a recursive call

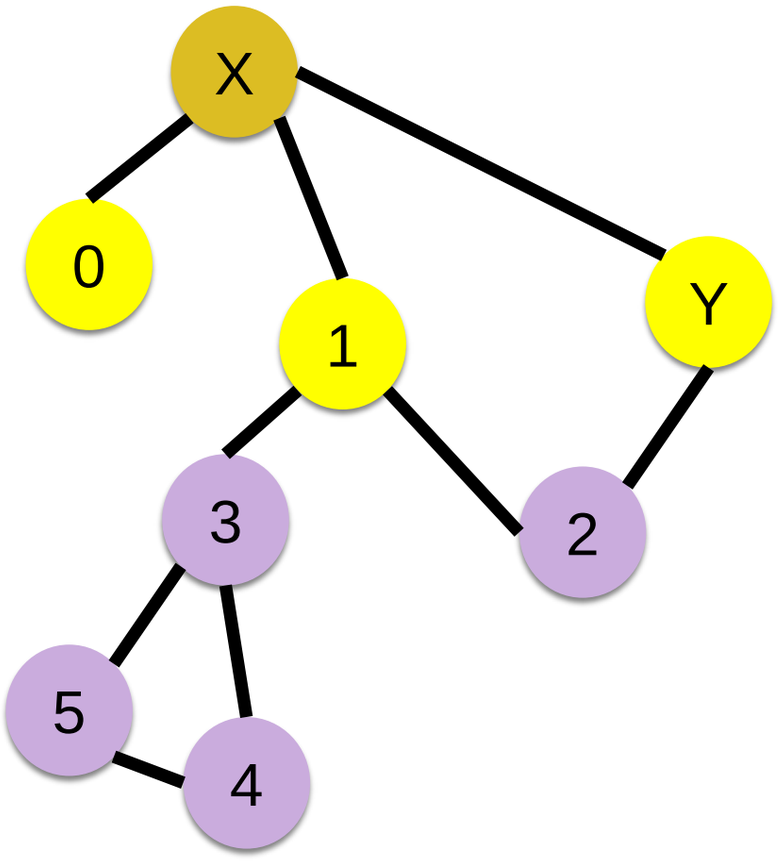
Example

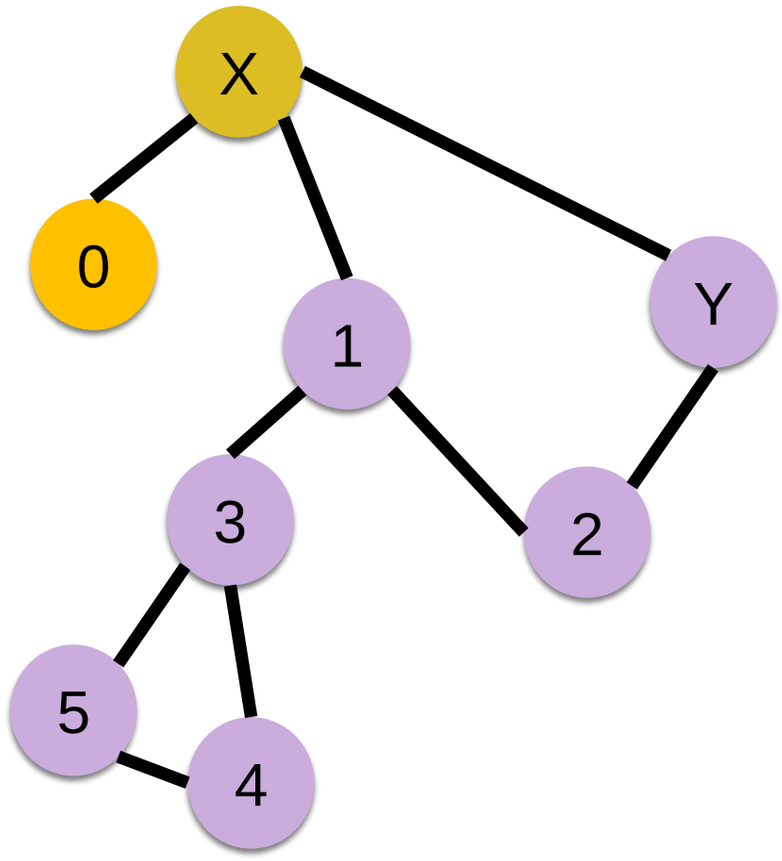




Visited: X
Stack: X
Connected: 0, 1, Y

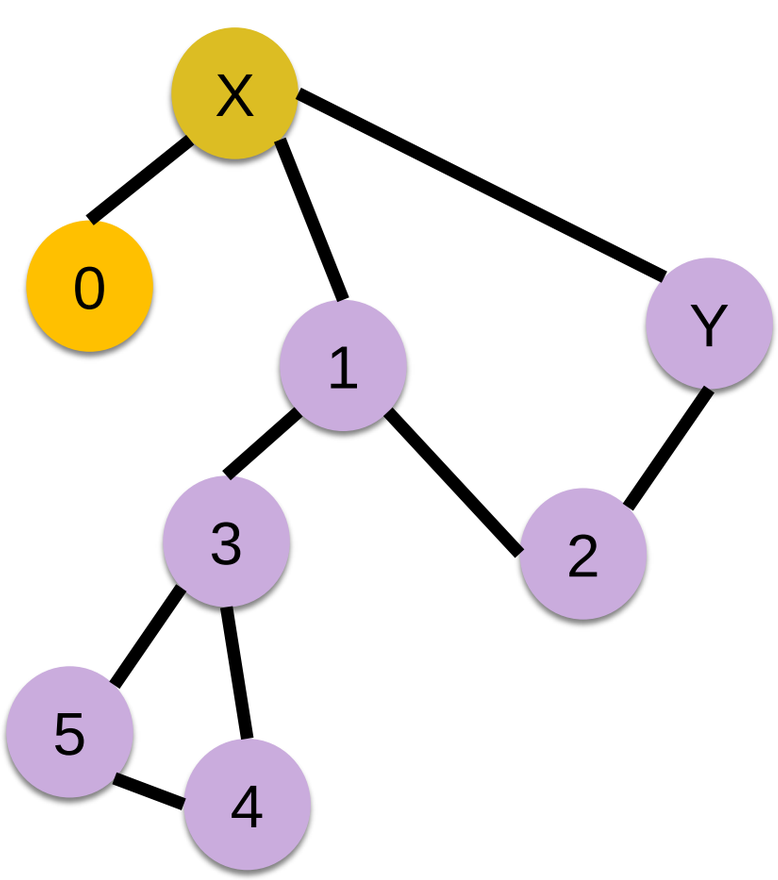
Recursion on 0

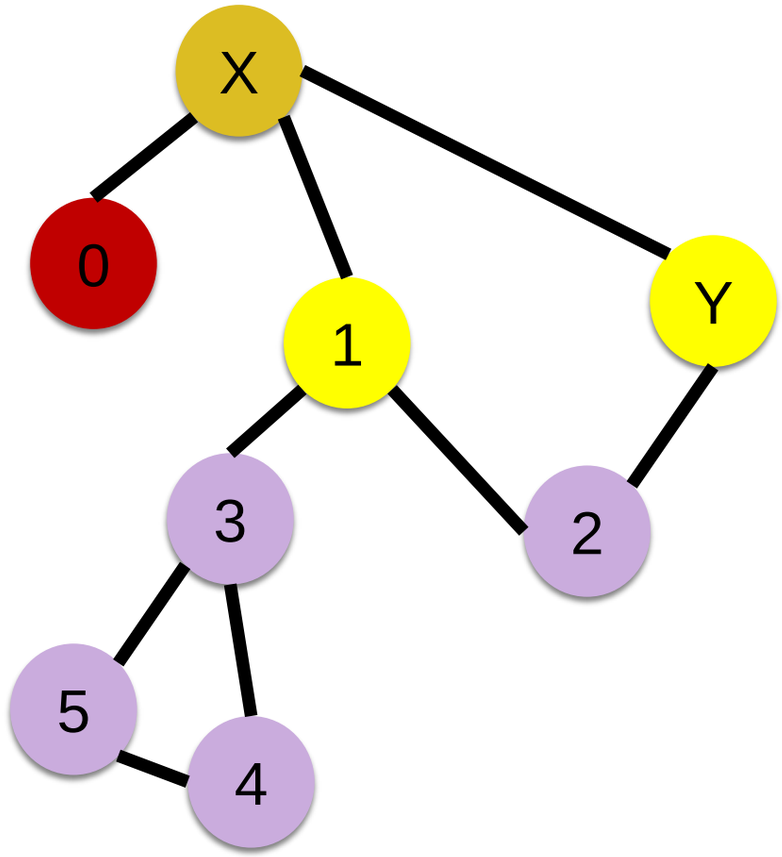




Visited: X, 0
Stack: X, 0
Connected: (X)

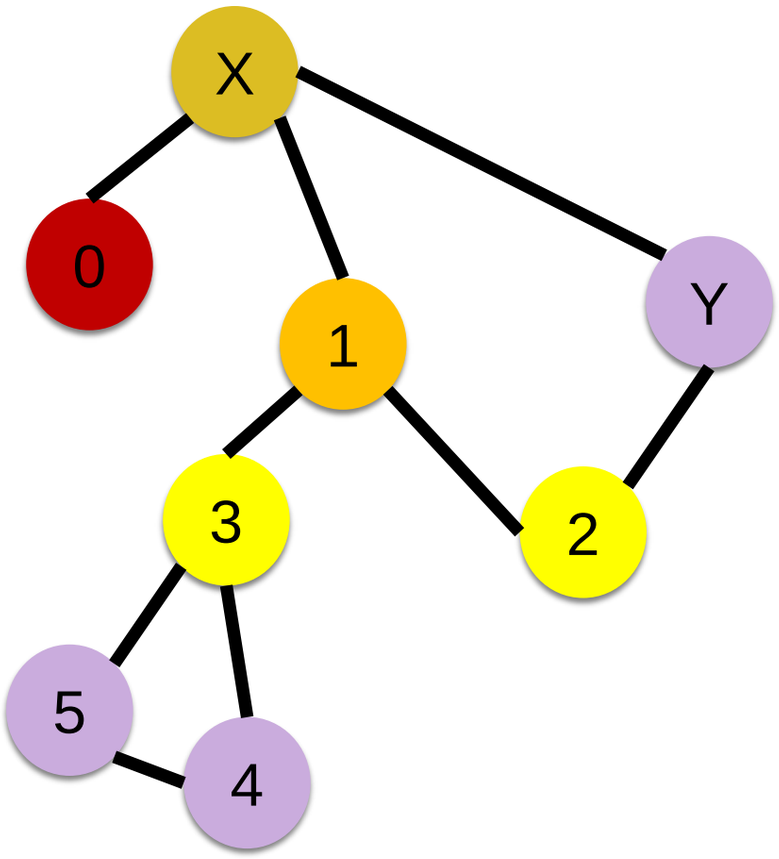
Backtrack to X



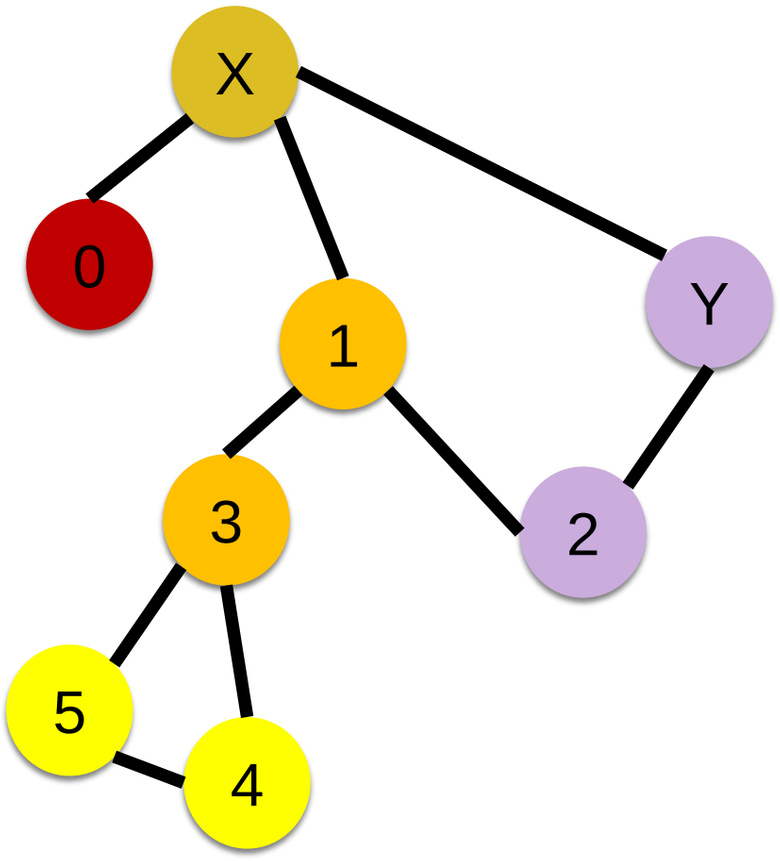


Visited: X, 0
Stack: X
Connected: (0), 1, Y

Evaluating 1 and then 3

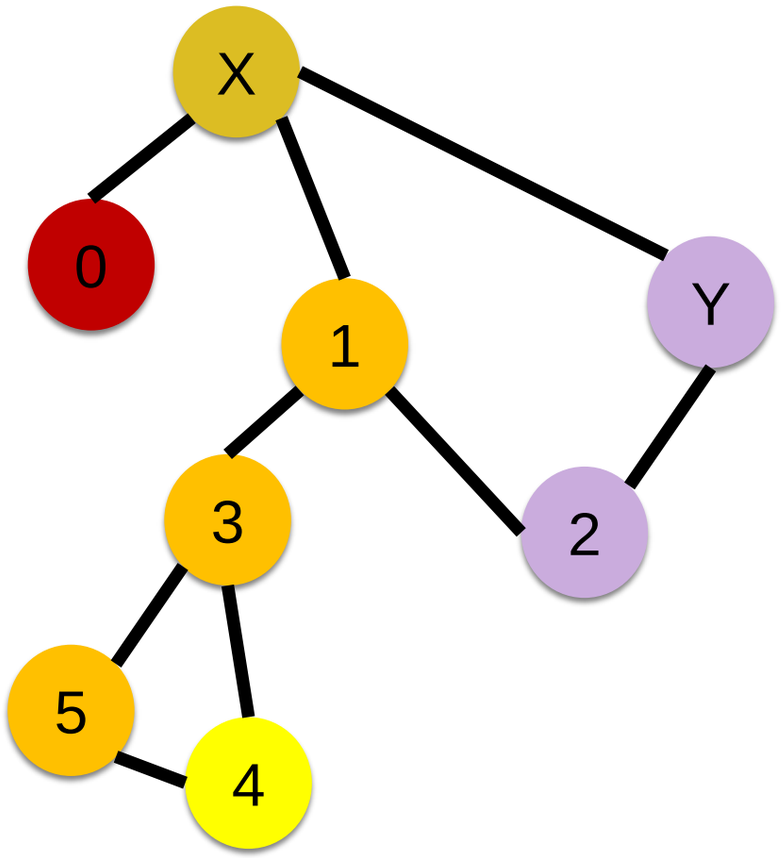


Visited: X, 0, 1
Stack: X, 1
Connected: (X),3, 2

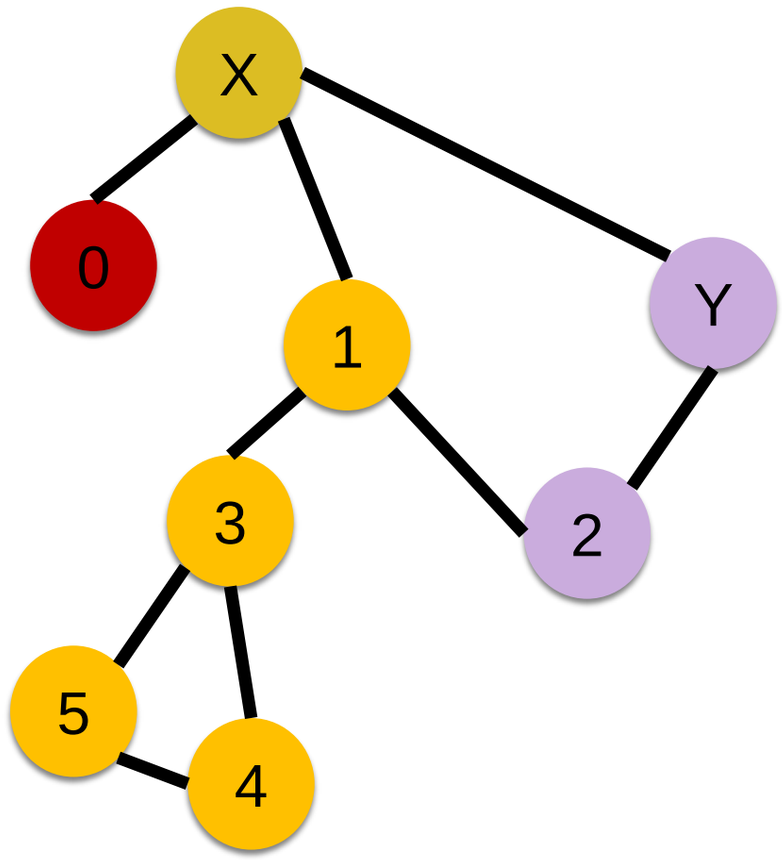


Visited: X, 0, 1, 3
Stack: X, 1, 3
Connected: (1), 4, 5

Evaluating 5 and then 4

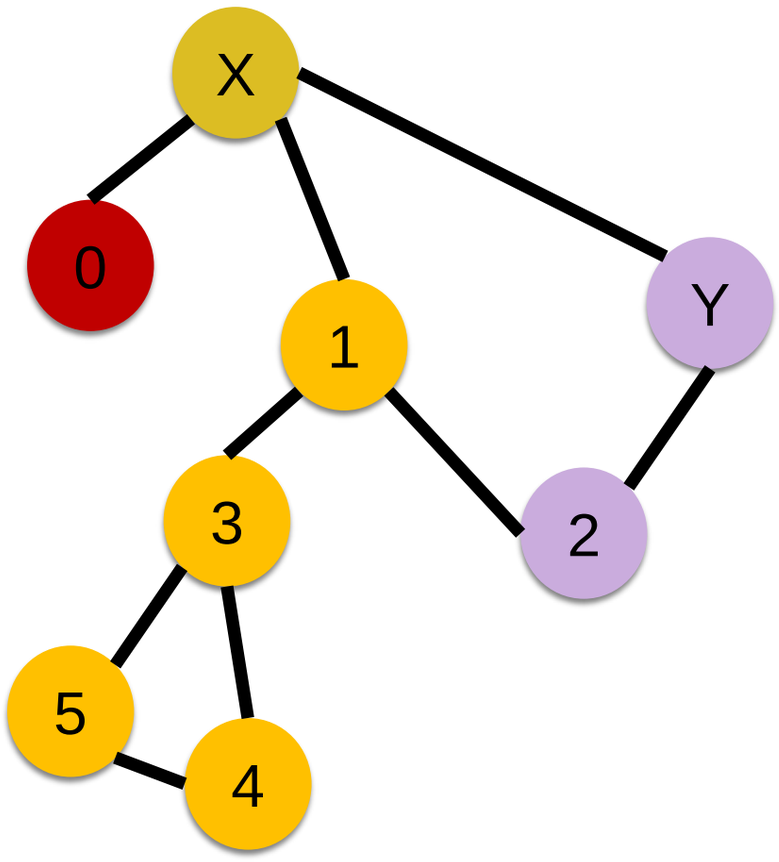


Visited: X, 0, 1, 3, 5
Stack: X, 1, 3, 5
Connected: (3), 4

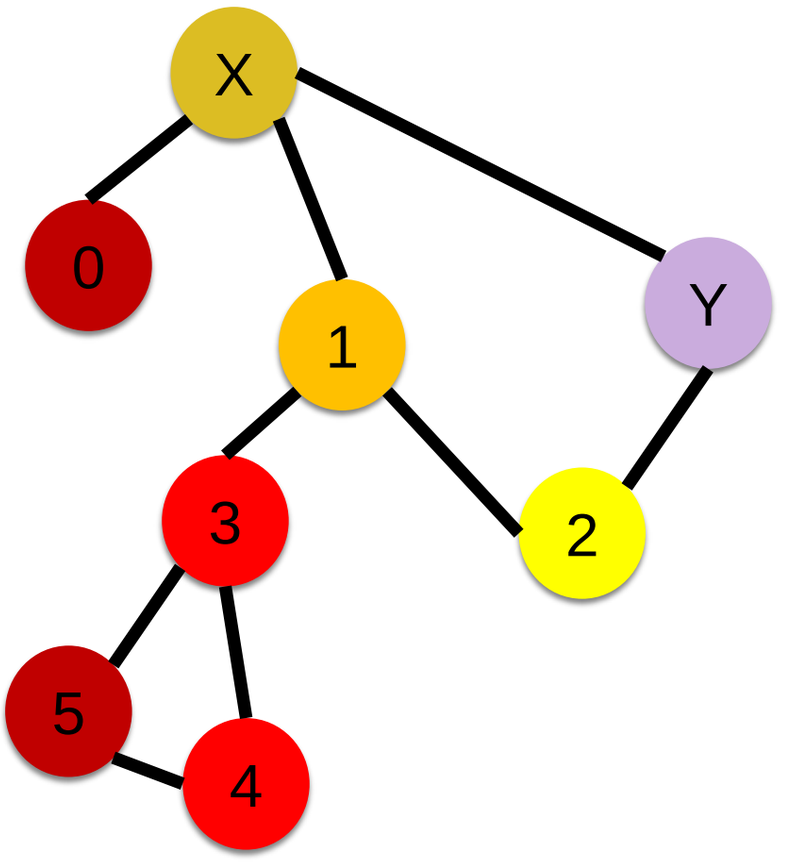


Visited: X, 0, 1, 3, 5, 4
Stack: X, 1, 3, 5, 4
Connected: (3), (5)

Backtracking Till 1

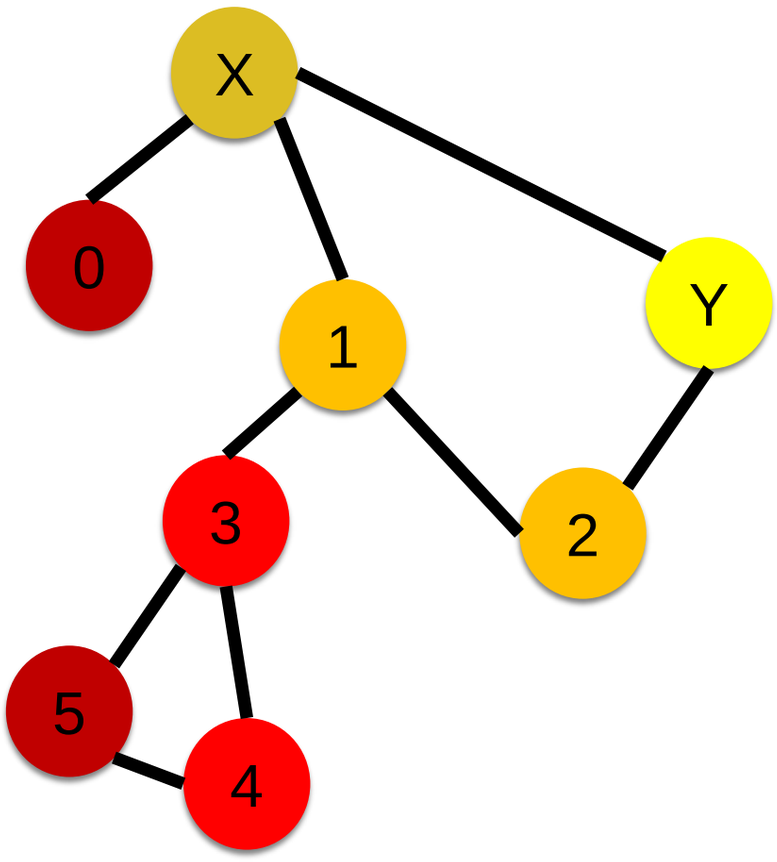


Visited: X, 0, 1, **3, 5**, 4
Stack: X, 1, 3, 5, 4
Connected: **(3), (5)**

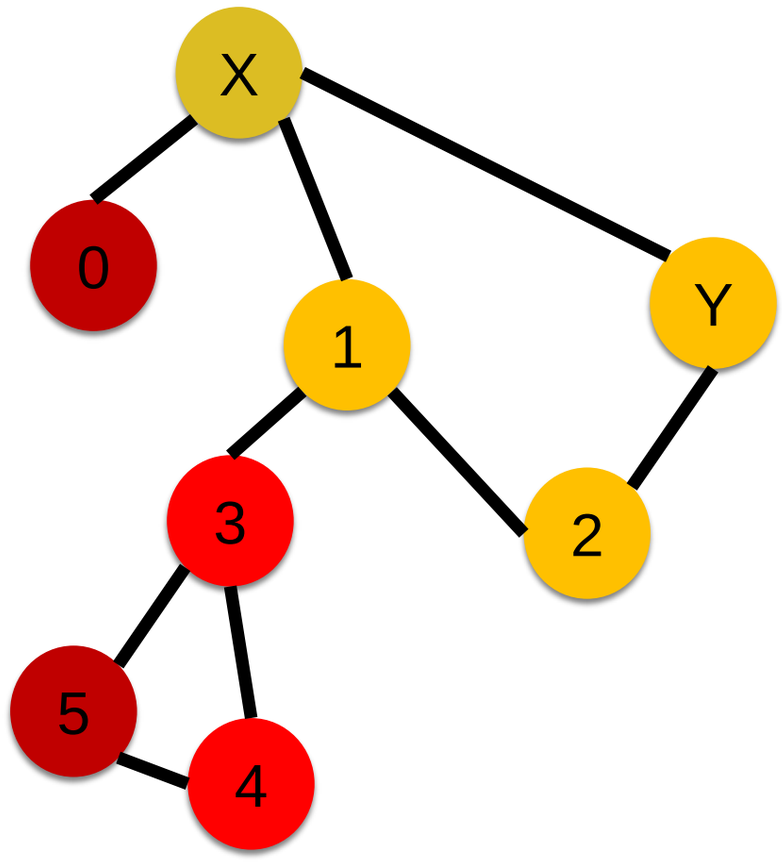


Visited: X, 0, 1, 3, 5, 4
Stack: X, 1
Connected: (X), (3), **2**

Evaluating 2 and 1

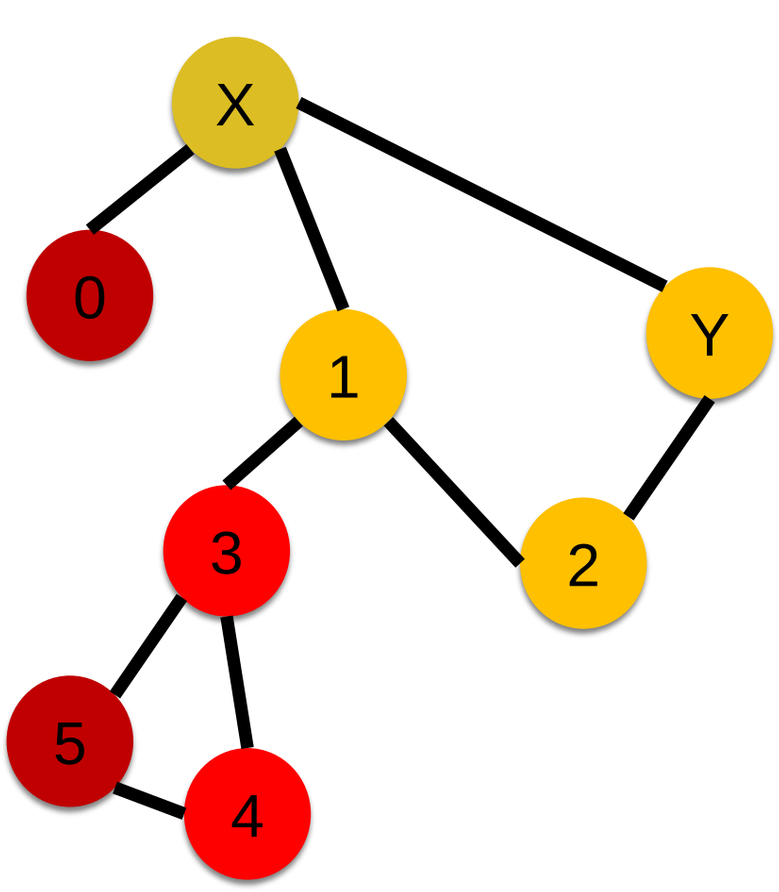


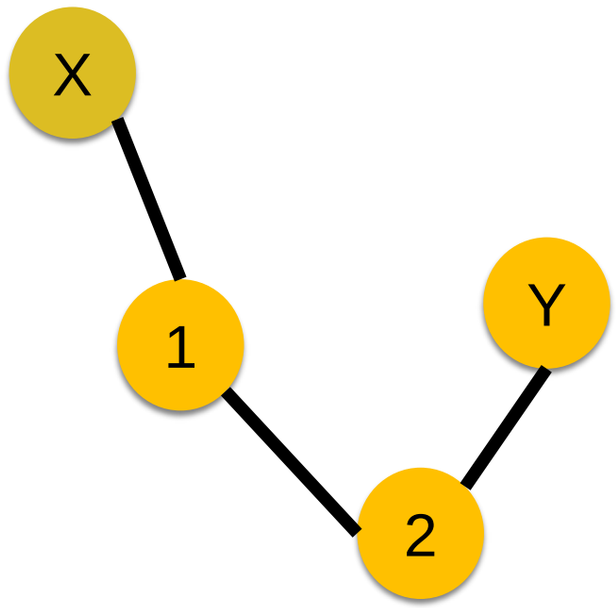
Visited: X, 0, 1, 3, 5, 4, 2
Stack: X, 1, 2
Connected: (1), Y

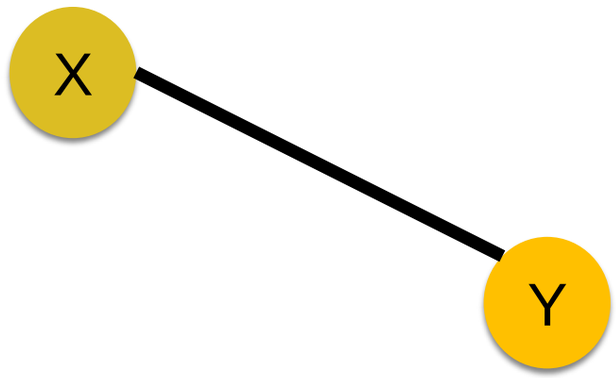


Visited: X, 0, 1, 3, 5, 4, 2, Y
Stack: X, 1, 2, Y
Connected: (X), (2)

Final Path X, 1, 2, Y







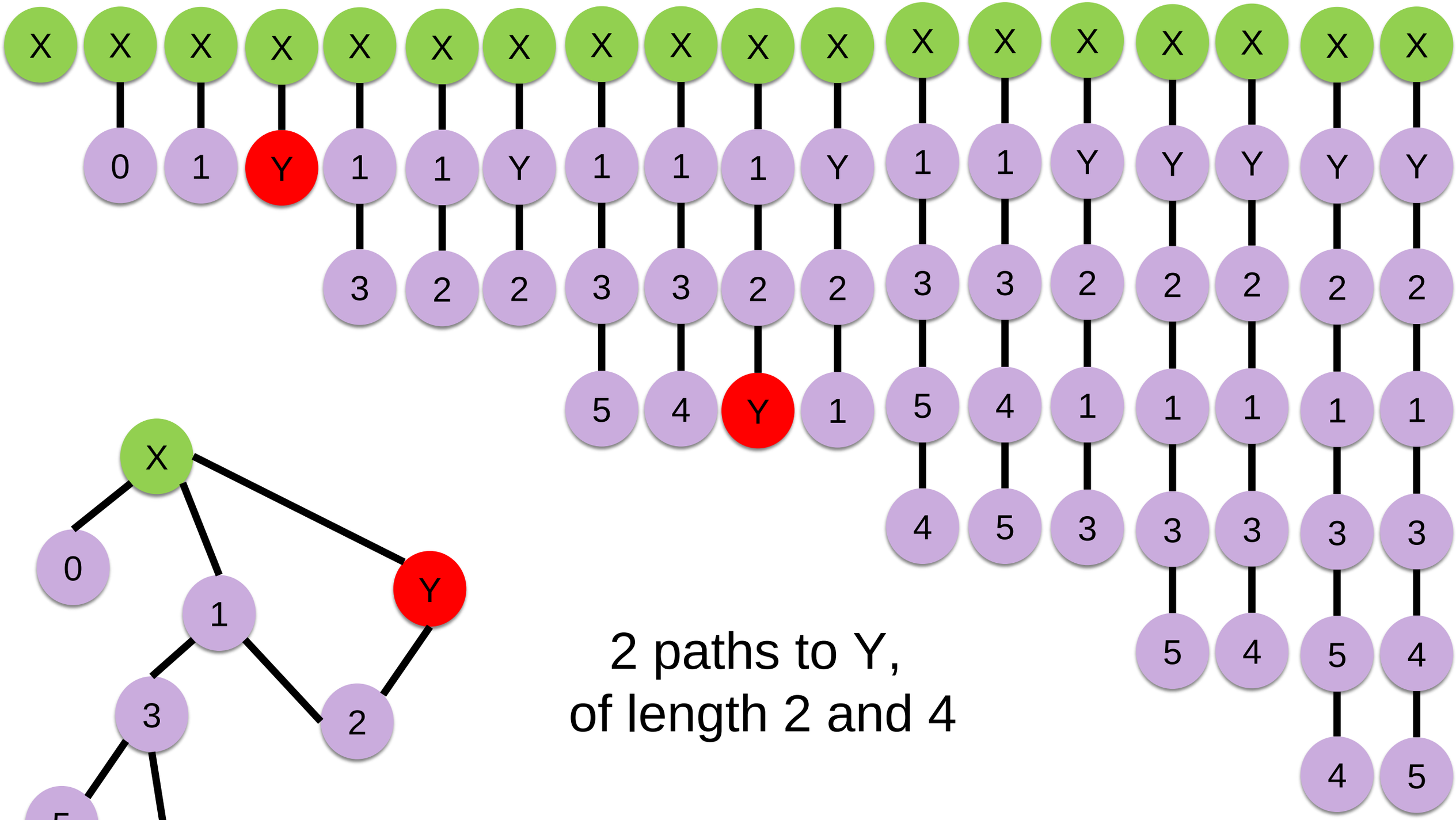
Visited: X, 0, 1, 3, 5, 4, 2, Y
Stack: X, 1, 2, Y
Connected: (X), (2)

We found a path (which is stored in the stack), but it is not necessarily the shortest (which would be X-Y)

Breadth-First Search (BFS)

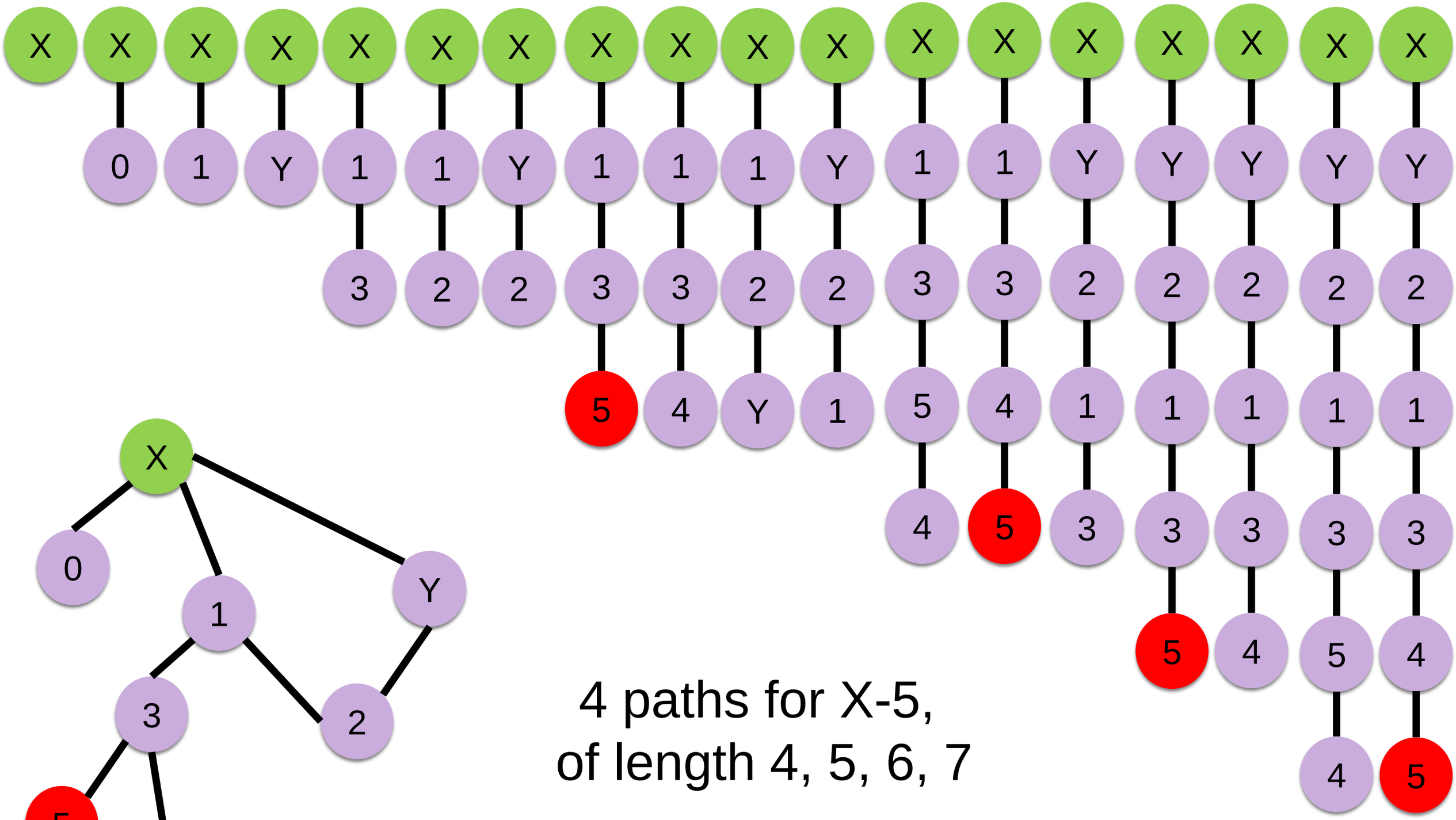
- DFS no guarantee to find shortest path, whereas BFS does
- From starting point X, look at all paths of length 1, then all paths of length 2, then 3, … then N, until found Y or visited whole graph
- Considering paths without cycles

Paths Starting From X



2 paths to Y,
of length 2 and 4

From X to 5



4 paths for X-5,
of length 4, 5, 6, 7

BFS Details

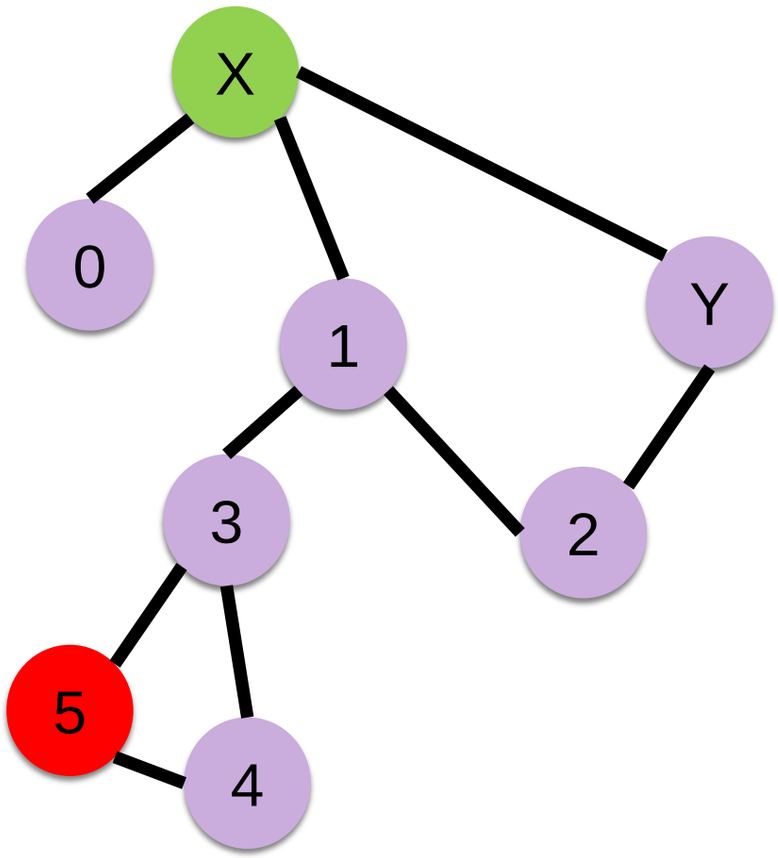
- Need special algorithm to keep track and visit all paths of length N in increasing order

- BFS: use a *queue* of yet to visit vertices

- Pull vertex from queue, add connected vertices to back, if not already visited

- Keep track of which pulled vertex X added a connected vertex Y

Example

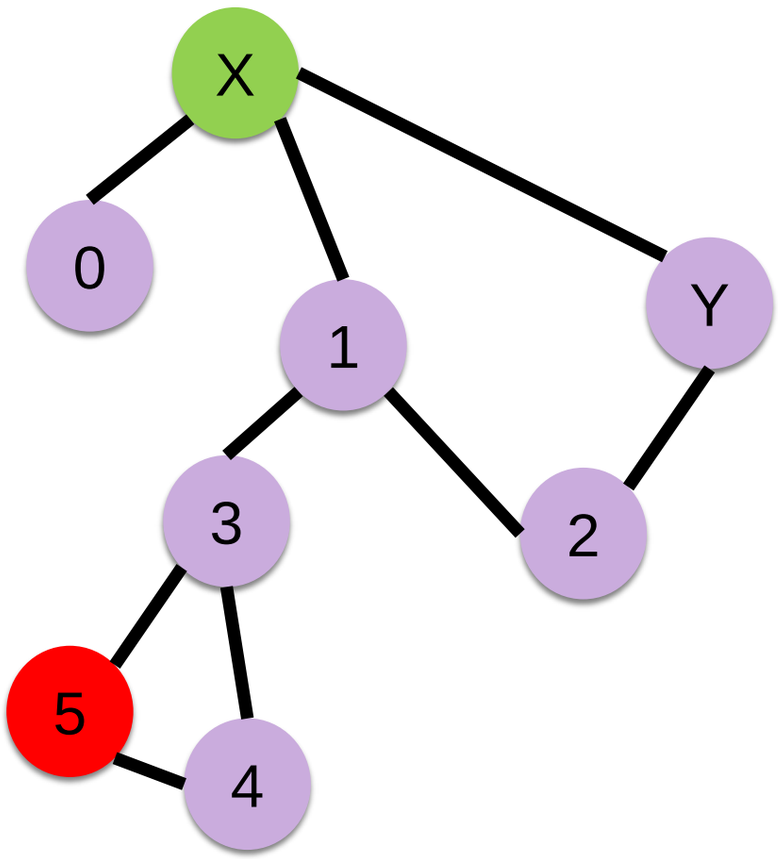


Queue: X

Map:

X	root
Y	null
0	null
1	null
2	null
3	null
4	null
5	null

Cont.



Queue:

0	1	Y

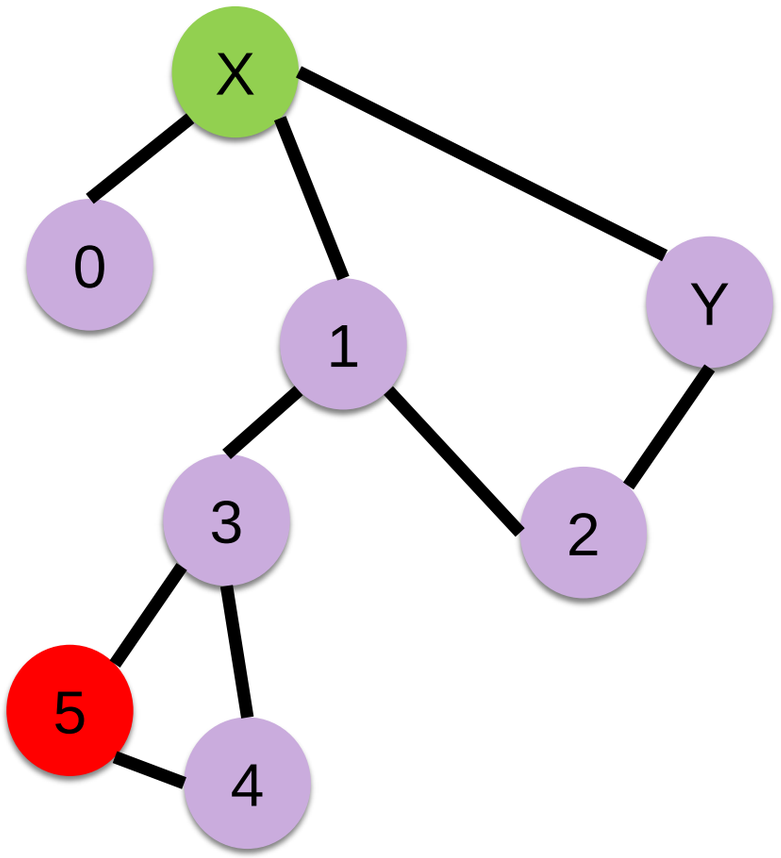
Map:

X	root
Y	X
0	X
1	X
2	null
3	null
4	null
5	null

Pulled X, added 0, 1, and Y.

Those represent all paths of length 2

Cont.



Queue:

Y	3	2

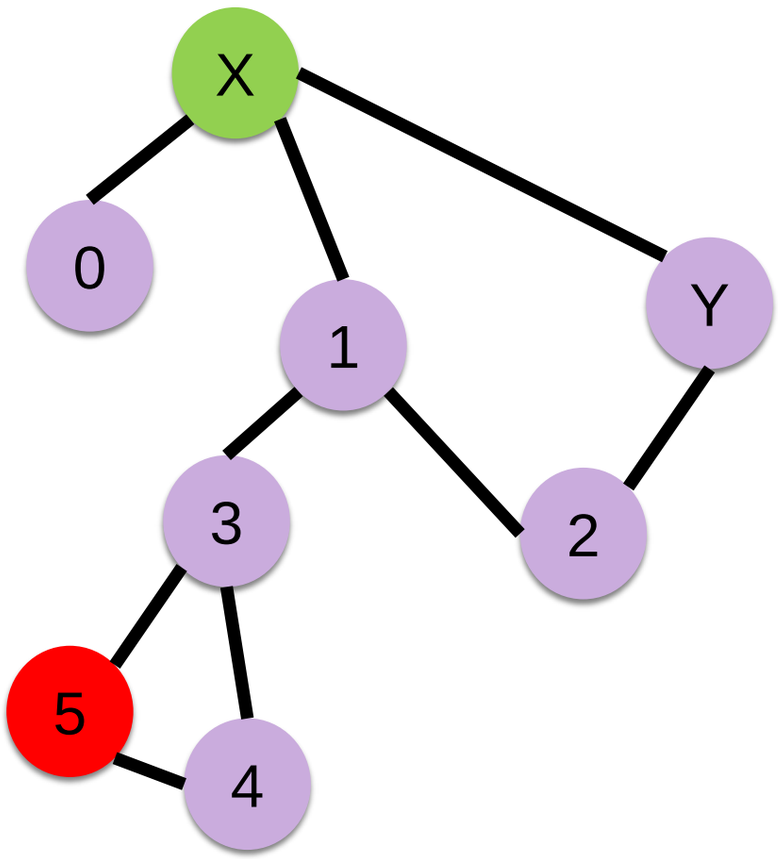
Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	null
5	null

Pulling 0 has no effect, as not adding X back

Pulling 1 results in adding 3 and 2

Cont.



Queue:

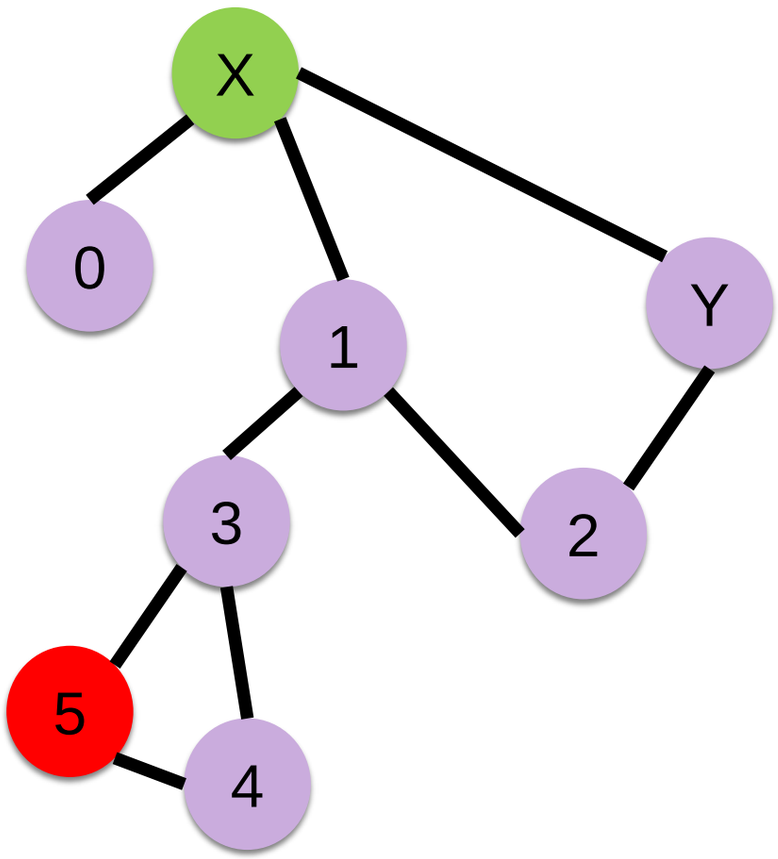
2	3

Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	null
5	null

Pulling Y has no effect, as connected 2 and X have already been handled

Cont.



Queue: 2 | 3

Queue: 3

Queue: 4 | 5

Map:

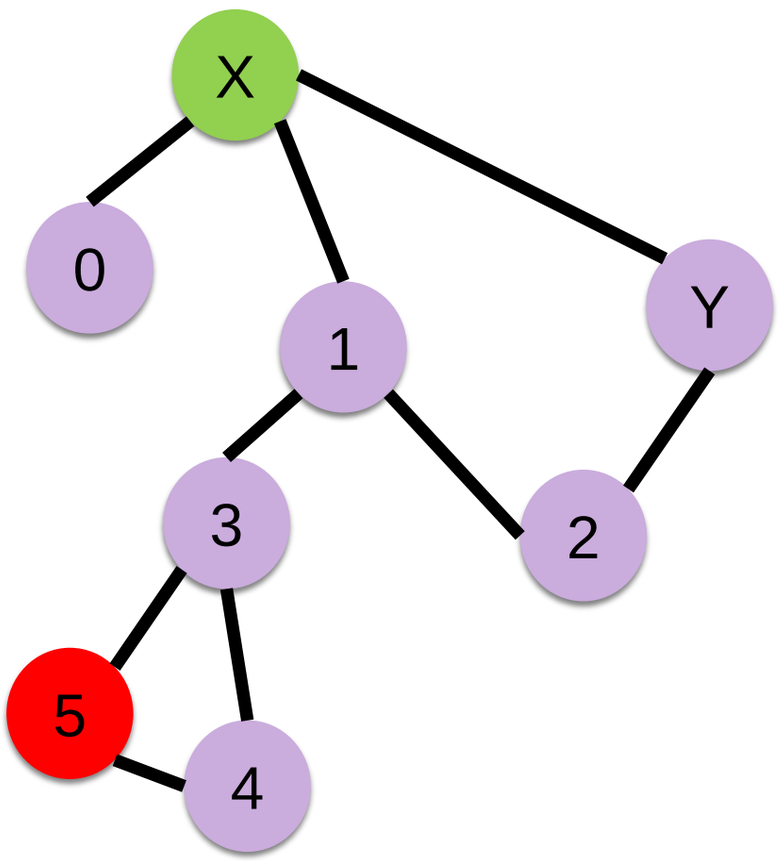
X	root
Y	X
0	X
1	X
2	1
3	1
4	3
5	3

Pulling 2 has no effect (1 and Y already handled).

Pulling 3 leads to push 4 and 5 (but not 1 that has already been handled)

5 is our target

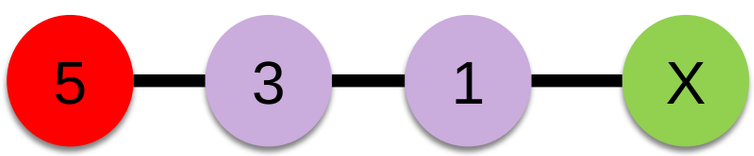
Retrieve Path



Map:

X	root
Y	X
0	X
1	X
2	1
3	1
4	3
5	3

From 5, follow links backward in the map till X



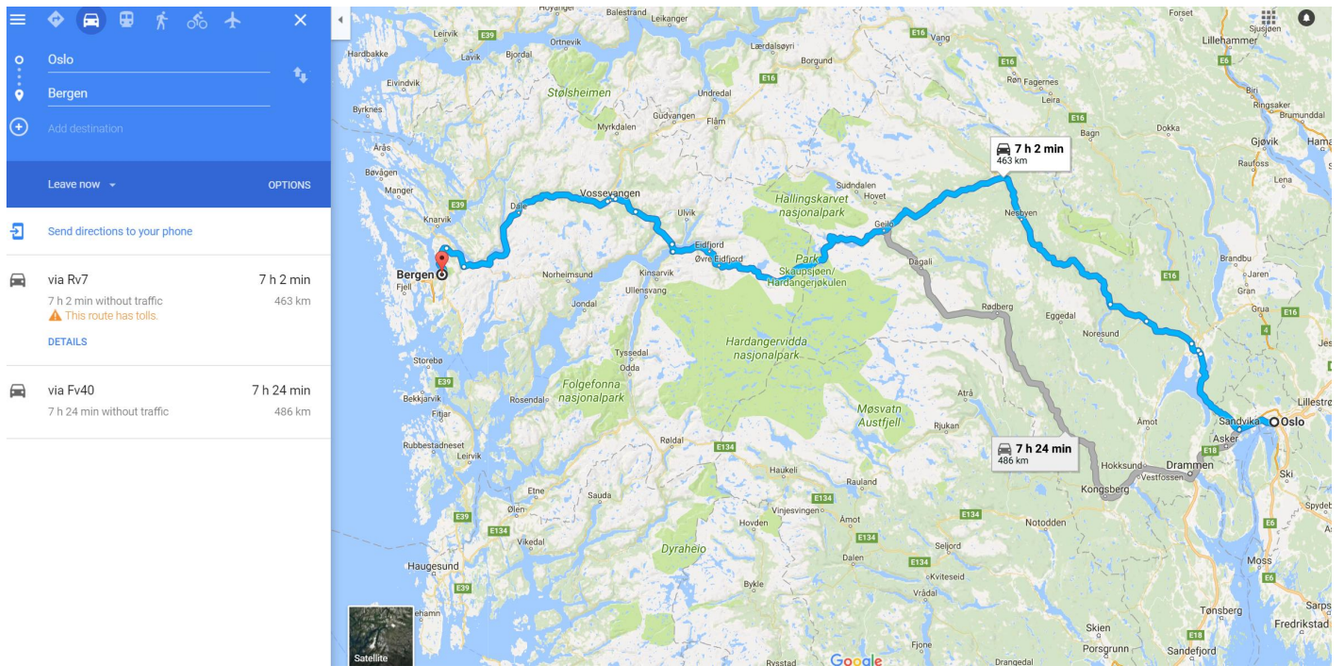
DFS or BFS?

- BFS guarantees to find minimum path, whereas DFS does not

- But BFS is “usually” more expensive, both in terms of time and memory

Weighted Graphs

- You can have graphs where edges have weights
  - Eg, distance between two cities, road tolls, etc.
- Find paths with shortest weight/cost on the traversed edges, even if traversing more vertices



Homework

- Study Book Chapter 4.1
- Study code in the *org.pg4200.les08* package
- Do exercises in *exercises/ex08*
- Extra: do exercises in the book