Pre-lecture Notes Section 4:

Electric Potential Energy

I. Review of Gravitational Potential Energy

The electric force, like the gravitational force, is a conservative force. This means that the work done by the electric force in moving a given charge from one configuration in space to another does not depend on the path that the charge takes to get there. This allows us to define a potential energy function for the electric force.

Let us start by reviewing the gravitational force and gravitational potential energy. In physics 1, you learned that the gravitational force between two point masses is

$$\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r}$$

The negative sign is present because the force is always attractive.

Now, how do we find the gravitational potential energy given this force. In physics 1, you learned that the change in potential energy from some point A to some other point B is the negative of the work done by the force. In turn, the work is given by the integral of the force over the displacement from A to B.

$$\boldsymbol{\Delta U \equiv U_B - U_A = -W_{A\to B} = -\int_A^B \vec{F}\cdot d\vec{r}}$$

From this, we find the gravitational potential energy function:

$$U_g = -G\frac{m_1 m_2}{r}$$

Note that this is a scalar; it has no components or direction.

Now, how do we use this gravitational potential energy? Remember that the work-energy theorem can be written as

$$W_{other} = \Delta K + \Delta U_g$$

This is just the principle of conservation of energy.

In a practical sense, this allows us to solve many problems. Specifically, if there is no non-conservative work done, we can use conservation of energy to solve for speeds and separation distances in various problems.

II. Electric Potential Energy

With the groundwork laid in the previous section, it is not difficult to compute the electric potential energy of two point charges. The electric force between two point charges is

$$\vec{F}_E = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\hat{r}$$

This is almost the exact same form as the gravitational force; therefore the potential energy is easily calculated. The result is that the electric potential energy between two point charges is

$$\boldsymbol{U_e = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}}$$

Again, we can plug this into the work-kinetic energy theorem to get

$$\boldsymbol{W_{other} = \Delta K + \Delta U_e}$$

As with the gravitational potential energy, this allows us to solve many problems involving the behavior of charged particles.

III. Work and Electric Potential Energy

A question we will ask in this section, but even more frequently later in the course, is the following: How much work must an external force do to move a given system of electric charges from one configuration to another?

As an example, let us say we have two positive charges some distance away from each other, being held at rest. We want to know how much work an external force would need to do to move those charges closer to each other, still at rest.

We can find our answer to any question such as this from the work-energy theorem in the form of

$$W_{ext} = \Delta U_e + \Delta K$$

Since our system starts and ends at rest, the change in kinetic energy is zero. Thus, the amount of external work required is exactly equal to the change in the potential energy of the system!

So, for instance, let's say we start with a system of charges in configuration a and we want to move the charges to configuration b. To find the work required, we simply need to find the potential energy of the system in configurations a and b and then take the difference. Mathematically this is equal to:

$$W_{ext,a\rightarrow b} = U_b - U_a$$

When you think about it, this is the same thing we have done with gravity in physics 1. Let's say you wanted to know how much work you need to do on a block to move it from a table to a higher shelf. The work required is simply the gravitational potential energy of the block on the shelf minus the gravitational potential energy of the block on the table.

As an example, let's say $U_{g,table} = 5$ J and $U_{g,shelf} = 8$ J. Then the work required to move the block from the table to the shelf would simply be 3 J.

This type of calculation will be used many times throughout the remainder of the course.

IV. Relation Between Electric Force and Electric Potential Energy

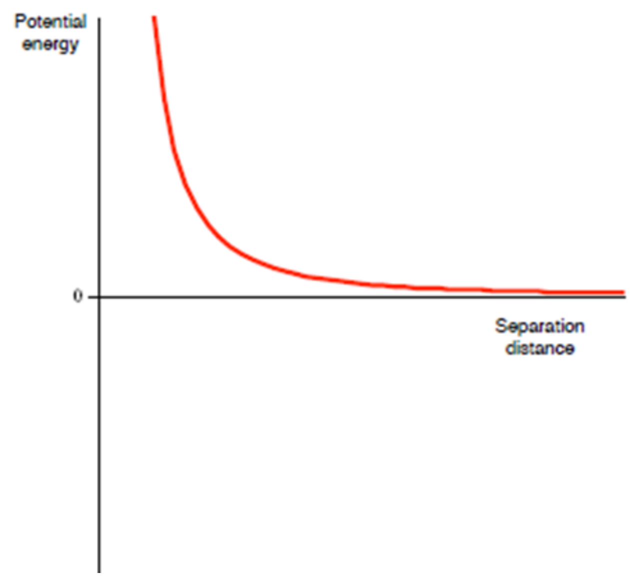
It is important to remember that a potential energy function is always related to a conservative force. In particular, the force is always equal to the negative derivative of the potential energy function with respect to positive (this is the inverse of the integral relation finding change in potential energy from force). In one dimension (using r as our position coordinate) this takes the following form:

$$\vec{\boldsymbol{F}} = -\frac{d\boldsymbol{U}}{d\boldsymbol{r}}\hat{\boldsymbol{r}}$$

In words, this mathematical relationship says the following two important things:

1. The larger the magnitude of the derivative of the potential energy is, the greater the magnitude of the force
2. **The force always points in the direction of decreasing potential energy**.

For instance, let's look at a graph of potential energy vs. separation distance r for two positive point charges. The graph is proportional to $1/r$ and thus is shown to the right. What you should notice about this graph is that



1. The magnitude of the slope is greater as the separation distance gets smaller. Therefore, the electric force between point charges should get greater as the charges get closer (we already knew this!).

2. The potential energy decreases as the separation distance gets larger. There, the electric force should point in such a direction such as to make the separation distance larger. In other words, this force is repulsive (we already knew this too!).