Pre-lecture Notes Section 5: Electric Potential

I. <u>Introduction to Electric Potential</u>

In section 2, we introduced the concept of the electric field. The conceptual difference between the electric <u>force</u> and the electric <u>field</u> is that a force is caused by an interaction between two charges, whereas a field is something produced by one source charge. The field then interacts with the second charge (the test charge) to produce a force on that test charge. Recall the electric field model:

Step 1. A source charge q produces an electric field $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

- The total electric field at a given point is the vector sum of the electric field at that point due to all source charges.

Step 2. A test charge q_0 experiences an electric force $\vec{F}_E = q_0\vec{E}_{tot}$

We would like to introduce a similar two-step method for computing the electric potential energy of a system of charges. In other words, we would like to have a two-step process in which a single point charge creates something analogous to the electric field, and this interacts with the second point charge to "create" potential energy. One fact which actually makes this simpler than the force/field concept is the fact that energy is a scalar. Therefore instead of creating a vector field, the source charge will create a <u>*scalar field*</u>!

This scalar field is called the electric potential field, which is usually shortened to <u>**electric potential**</u>, and often finally just to <u>**potential**</u>. It is related to, but not the same as, the potential energy. As we go through this section, be careful that you understand the difference between the two names!

Remembering that we defined the electric field as the electric force that a test charge feels per unit charge $\left(\vec{E} \equiv \frac{\vec{F}_E}{q_0}\right)$, we define the electric potential V to be the electric potential energy per unit test charge.

$$\boldsymbol{V \equiv \frac{U_E}{q_0}}$$

Using this definition, we can create a two-step process for electric potential energy:

Step 1. A source charge q produces an electric potential $\boldsymbol{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$

- The total electric potential at a given point is the scalar(!) sum of the electric potential at that point due to all source charges.

Step 2. When a test charge q_0 is moved from one point a to another point b, the system undergoes a change in electric potential energy equal to $\Delta U = q_0\Delta V$, where ΔV is the difference in electric potential between the two points.

The SI unit of electric potential is the Volt: 1 V = 1 J/C.

II. Relation Between the Electric Potential and Electric Field

As we have seen in the previous section, the change in potential energy is related to the force for any conservative force. The change in potential energy between points a and b

$$-\Delta U = -(U_b - U_a) = U_a - U_b = \int_a^b \vec{F} \cdot d\vec{l}$$

Since $\vec{F} = q_0\vec{E}$ and $U = q_0V$, we can easily determine a relation between V and $\vec{E}$.

$$\boldsymbol{V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}}$$

This relation allows one to find the electric potential difference of a configuration of charges if one knows the electric field of that configuration of charges. Some electric fields that we know include the electric field of a point charge, the electric field of infinite parallel plates, a sphere etc…

In many cases, it is actually easier to find the electric potential of a system of charges than to find the electric field, both theoretically and experimentally (as we will see in our labs). So we need to be able to find the electric field if we know the electric potential. In other words, we need to "invert" the above relation between field and potential. Calculus provides a way to do this. If the potential is the integral of the field, then the field must be the derivative of the potential!! However, since we are working with integrals of vectors, this derivative is not so straightforward. It turns out that such a derivative is called a gradient. The electric field can be found from the electric potential in terms of the negative of the gradient of the electric potential function:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

If the electric potential is radial (i.e. only involves separation distance r instead of Cartesian coordinates x, y, z), then the gradient becomes

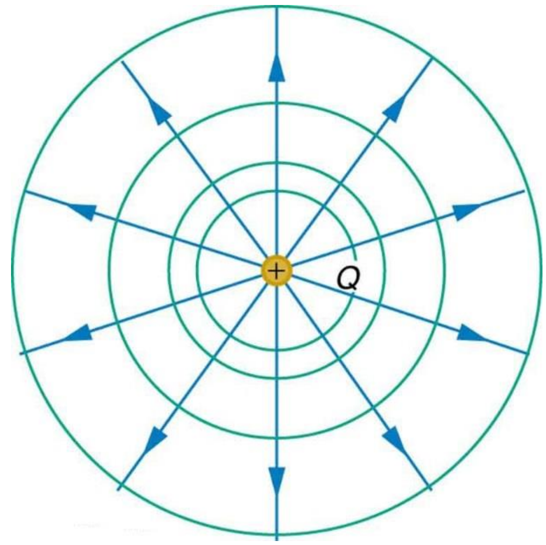
$$\boldsymbol{\vec{E} = -\frac{dV}{dr}\hat{r}}$$

We will show how to use these expressions in class.

One important note about the gradient is that it always points in the direction of greatest increase in the function. Therefore, with the negative sign, **the electric field always points in the direction of greatest decrease in the potential**. This should be familiar, as a given force always points in the direction of greatest decrease in its potential energy!

III. Equipotential Surfaces

Surrounding any charge distribution, it is useful to plot so-called "Equipotential Surfaces". These are surfaces (on a 2D paper they look like curves) on which the electric potential is constant. For instance, for a single point charge, the electric potential is a function only of the distance from the source charge. Therefore, the equipotential surfaces are spheres (on a 2D page, circles) which surround the point charge.



The equipotential lines, as well as the electric field lines, for a positive point charge are shown in the picture to the right.

The reason that equipotential lines are so useful is that they give us information about the shape, direction, and strength of the electric field that the charge(s) produces. We will discuss each in turn.

1. **Shape of the Electric Field Lines**: Equipotential lines are *always* perpendicular to electric field lines. This follows from the fact that the electric field is related to the gradient of the electric potential.

   For instance, for the point charge, if we only knew the shape of the equipotential lines but not the field lines, we could draw the field lines simply by drawing lines perpendicular to the equipotential surfaces. The only lines which are everywhere perpendicular to circles are lines which point radially out from the center of the circles. This is yet another "proof" of how the electric field lines look due to a point charge.

2. **Direction of the Electric Field**: The key point here is that electric field lines always point in the direction of decreasing electric potential. In the previous picture, they do not show the value of the equipotential surfaces, but since it is a positive point charge, it is not difficult to determine (from the formula $V_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$), that the value of the equipotentials decrease as you move further from the charge.

   For instance, the innermost circle shown in the picture may be 10 V, then the next circle 9 V, then the next 8 V etc… Therefore, the direction of decreasing potential is away from the charge, and thus the electric field must point away from the charge!

3. **Magnitude of the Electric Field**: The key point here is that the electric field is equal to the negative rate of change of the electric potential with respect to position,

$$\vec{E} = -\frac{dV}{dr}\hat{r}$$

As far as the magnitude is concerned, the negative sign is not relevant, but the derivative is. The more quickly the potential is changing with respect to position, the greater the relative electric field strength at that point.

Now, you might ask why this analysis is useful, since we already know the electric field of a point charge. The key is that in a practical, laboratory sense, measuring the electric field around a charge distribution is not easy. However, measuring the electric potential is very easy!

Therefore, it we have a charge distribution and we do not know its electric field, we simply measure the potential at various points in space and use the measurements to draw the equipotentials. From those equipotentials we can gain information about the shape, direction and strength of the electric field as described above. We will see how this works ourselves in the laboratory at the end of this section.

Electric Potential of a Conductor

One consequence of the relation between the electric potential and electric field is the following fact, which is very useful in a practical sense, as we will see later:

The electric potential in a conductor must be the same at all points in and on the conductor! In other words, a conductor is always an equipotential.

How do we know this? In section 3 we showed that the electric field in a conductor must be equal to zero. Since the electric field is the derivative of the electric potential, we therefore conclude that the electric potential in a conductor must be a constant function (only a constant function has a derivative of zero!). Therefore, the conductor has the same electric potential everywhere. The conductor is thus an equipotential.