Pre-lecture Notes Section 8:

Direct-Current Circuits

I. Introduction to Direct-Current Circuits

In this section, we first will analyze circuits with various resistors and batteries in simple configurations, with the goal to be able to predict how much current will flow in the circuit and how much energy is converted from one form to another.

Then, we increase the complexity of circuits to less simple scenarios. In these cases, we must go back to our fundamental principles of conservation of energy and conservation of charge to analyze the circuit. Within circuit analysis, these fundamental principles go by the name of Kirchhoff's Rules.

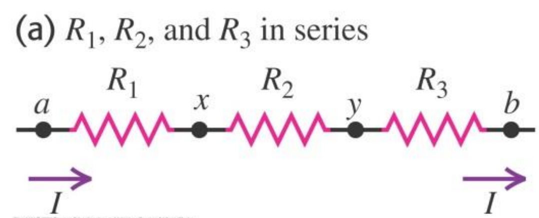
Finally, we will introduce a capacitor into a simple circuit. Due to the nature of the capacitor, the current will not flow at a constant rate, but will instead provide us our first look at a time-varying current in a circuit.

II. Simple Circuits: Resistors in Series and Parallel

In section 6, we saw that we could arrange capacitors in series and parallel. We can do the same with resistors in a circuit.

<u>Resistors in Series</u>

A set of resistors are in series if they lie along the same conducting path, so that the current through each of the resistors is identical. Such is the case with the three resistors shown to the right.



With resistors in series (as with capacitors), it is often useful to define an **equivalent resistance** for the series. The equivalent resistance allows one to imagine that the set of resistors is really just one resistor, which can greatly simplify the analysis of circuit problems. It can be proven that the equivalent resistance of a set of resistors in series is

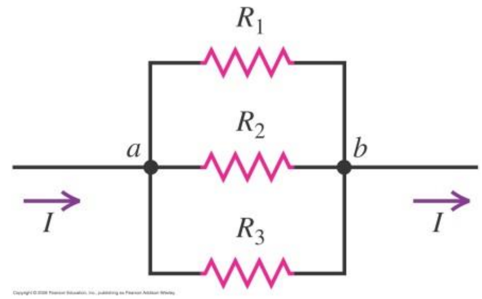
$$\boldsymbol{R_{eq,ser} = R_1 + R_2 + R_3 + \cdots}$$

Notice that the equivalent resistance is greater than any of the individual resistances. This is a sensible result because with a set of resistors in series the electric charge encounters more overall resistance along its single conducing path.

Resistors in Parallel

A set of resistors are in parallel if they all have the same voltage across them. Such is the case with the three resistors shown to the right.

(b) R_1, R_2, and R_3 in parallel



With resistors in parallel (as with capacitors), it is often useful to define an **equivalent resistance** for the set. The equivalent resistance allows one to imagine that the set of resistors is really just one resistor, which can greatly simplify the analysis of circuit problems. It can be proven that the equivalent resistance of a set of resistors in parallel can be found via the equation

$$\frac{\mathbf{1}}{\boldsymbol{R_{eq,par}}} = \frac{\mathbf{1}}{\boldsymbol{R_1}} + \frac{\mathbf{1}}{\boldsymbol{R_2}} + \frac{\mathbf{1}}{\boldsymbol{R_3}} + \cdots$$

Note that the equivalent resistance is <u>*less*</u> than the resistance of any of the individual resistors. (*If you don't see this mathematically, plug in a few arbitrary numbers for resistances to check!*). Why is the resistance of a parallel combination of resistors less than if there was just one resistor? This is because a set of parallel resistors provides multiple paths for the electric charge to flow, which means there is actually less overall resistance to the flow.

In review, it is useful to compare and contrast the properties of resistors in series and parallel.

	Current	Voltage	Equivalent Resistance
Series	Same for all $(I_1 = I_2 = I_3 \cdots)$	Additive $(V_{tot} = V_1 + V_2 + V_3 + \cdots)$	$R_{eq} = R_1 + R_2 + R_3 + \cdots$
Parallel	Additive $(I_{tot} = I_1 + I_2 + I_3 + \cdots)$	Same across all $(V_1 = V_2 = V_3 \cdots)$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

III. Kirchhoff's Rules

In many cases, circuits contain sets of resistors that are neither in series nor in parallel. In these cases, we must go back to our fundamental principles of energy conservation and charge conservation for analysis. In the context of circuits, these fundamental principles are called "Kirchhoff's Rules".

The Junction Rule

Kirchhoff's junction rule expresses conservation of charge. A junction is a point where two (or more) conducting paths meet. The junction rule says that the total amount of electric current flowing into a junction must equal the total amount of electric current flowing out of a junction. If this were not true, the charge could not possibly be conserved, since charge that went into the junction wouldn't come out!

Mathematically, this is expressed as

$$\boldsymbol{I_{tot,in} = I_{tot,out}}$$

The total current flowing into a junction must equal the total current flowing out of the junction.

<u>The Loop Rule</u>

Kirchhoff's loop rule expresses conservation of energy. We have stated before that the voltage (electric potential difference) from one point in a conducting loop back to the same point must be zero. Again, this is due to the conservative nature of the electric force. If an electric charge goes from one point around the loop back to the same point, its potential energy must not have changed. This is Kirchhoff's loop rule.

Mathematically, this is expressed as

$$\boldsymbol{\sum_{loop} V = 0}$$

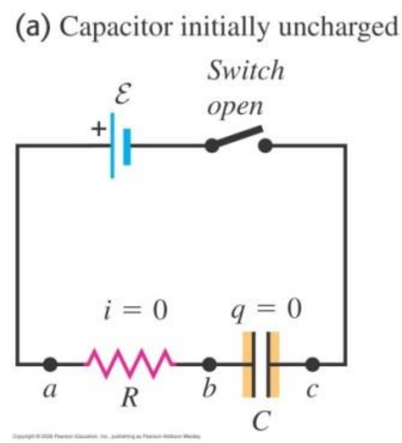
The sum of voltages around any complete loop must be zero.

With both of these rules, we can analyze the properties of any combination of resistors and batteries. A number of examples are done in class and in the text.

IV. <u>Circuits with Capacitors and Resistors (R-C Circuits)</u>

<u>Charging a Capacitor</u>

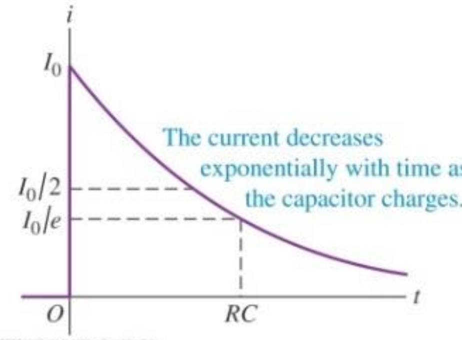
We next consider what happens if we take a simple circuit (one battery and one resistor) and add an (initially uncharged) capacitor. A diagram of such a circuit is shown to the right.



Qualitatively, we can describe what should happen. Once the conducting path is completed (i.e. the switch closed), the battery will begin to push charges through the resistor, and as in a normal circuit, current will flow.

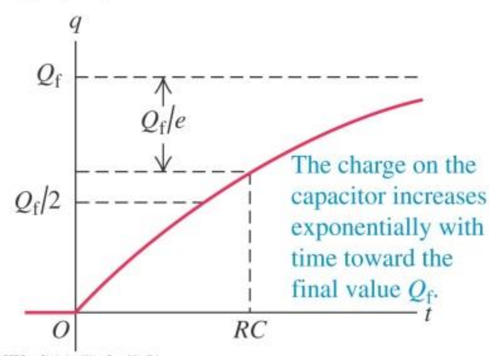
However, these charges will begin to build up on the plates of the capacitors. This will produce an electric field opposing the current produced by the battery, causing the current to decrease. As more charges build up on the capacitor, the electric field across the capacitor will get stronger and stronger, causing the current to decrease to smaller and smaller values. Finally, at some time, the electric field (and thus the voltage) produced by the capacitor will become equal to the voltage produced by the battery. At this point equilibrium is reached and current stops flowing. A graph of this behavior (current vs. time) is shown in figure (a). As can be seen from the graph, the decrease in current is an example of exponential decay.

(a) Graph of current versus time for a charging capacitor



When the current has dropped to zero the capacitor is "fully" charged. A graph of the charge of the capacitor as a function of time is shown in figure (b).

(b) Graph of capacitor charge versus time for a charging capacitor



We now would like to consider the charge on the capacitor and the current in the capacitor quantitatively. We use Kirchhoff's loop rule (shown in class) to find that

$$q(t) = Q_f\left(1 - e^{-t/RC}\right) = C\varepsilon\left(1 - e^{-t/RC}\right)$$

where Q_f is the final charge on the capacitor after it has been "charged up". Since the voltage on the capacitor will be equal to the voltage of the battery at this point, the final charge on the capacitor is $Q_f = C\varepsilon$. (*Note: notice that we use lowercase q to denote the charge when it is changing with time.*)

The current in the circuit as a function of time is simply the time derivative of the charge (remember $i(t) = \frac{dq(t)}{dt}$)

$$i(t) = I_0 e^{-t/RC} = \frac{\varepsilon}{R} e^{-t/RC}$$

where $I_0 = \varepsilon/R$ is the initial current in the circuit. (*Note: notice that we use a lowercase i to denote a current which is changing with time*).

Looking at the equations both for the charge on the capacitor and the current in the circuit, we notice the common exponential factor $e^{-t/RC}$. In fact, as you should verify for yourself, after a time equal to RC has passed, the current has reduced to $e^{-1} = \frac{1}{e} = .368 \ldots$ of its initial value. The time for any quantity to drop to $\frac{1}{e}$ of its initial value is called the **time constant τ** for that system.

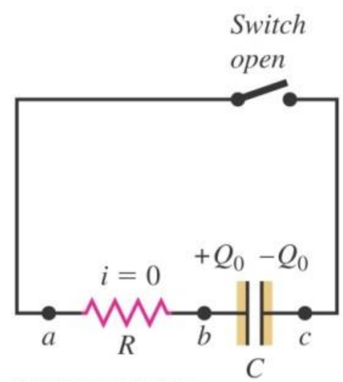
Therefore, the time constant for an RC-circuit is equal to

$$\boldsymbol{\tau = RC}$$

The greater the value of RC, the longer it takes the current to drop to e^{-1} of its initial value. This means that the charge builds up (and the current drops) more slowly for greater values of resistance R and capacitance C.

Discharging a Capacitor

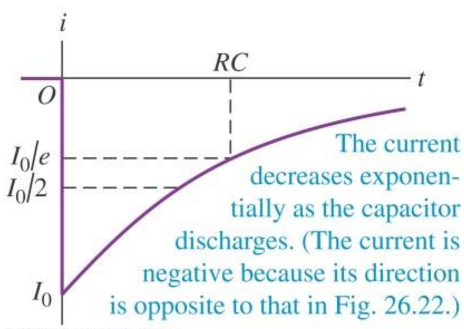
(a) Capacitor initially charged



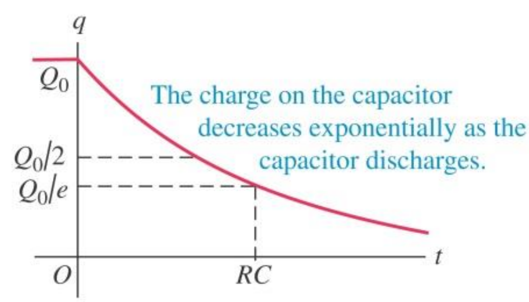
We now ask what happens if we have a fully charged capacitor and we connect its plates with a conducting wire and a resistor. This is shown in the picture to the right.

Below are graphs of the current in the circuit and charge on the capacitor as a function of time.

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time for a discharging capacitor



The mathematical functions describing the decay of charge on the capacitor and current in the circuit are

$$q(t) = Q_0 e^{-t/RC}$$

where Q_0 is the initial charge on the capacitor and

$$i(t) = -\frac{dq(t)}{dt} = e^{-t/RC}$$

As you can see, the current and charge both contain the exponential factor $e^{-t/RC}$. Therefore, we see that the time constant for the discharging of a capacitor is the same as for the charging capacitor. Since both the charging and discharging capacitor have the same time constant, we simply say that the time constant for an R-C circuit is $\tau = RC$.

The charging and discharging of a capacitor is our first experience with circuits that exhibit time-varying currents. In later sections, after we study the magnetic force, we will encounter yet more complex examples of time-varying currents.