Pre-lecture Notes Section 14:

Electromagnetic Waves

I. <u>Review of Wave Mechanics</u>

In this section, we will be studying electromagnetic waves. However, before we set out to understand what electromagnetic waves are, how they are produced, and what properties they have, it will be useful to review a bit of the wave mechanics discussed in Physics 2425 (specifically chapter 15 of the text).

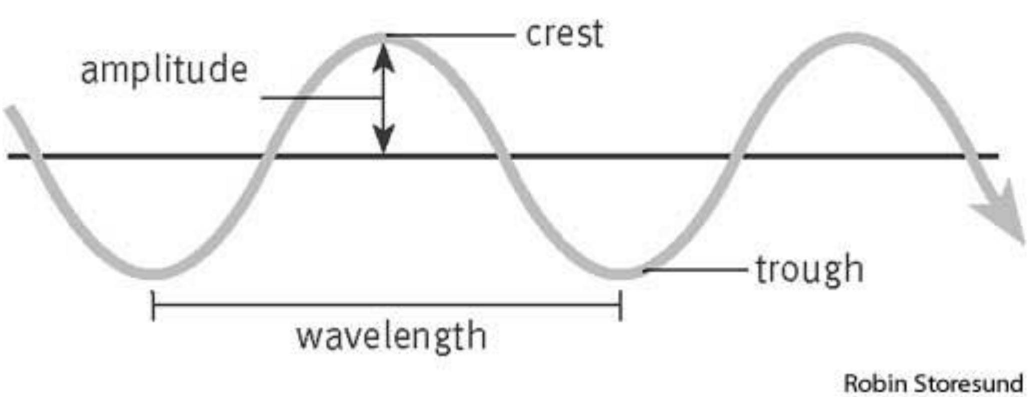
The concept of a wave is somewhat difficult to define. We can, however, describe a wave as a disturbance which propagates both in space and in time.

One of the key features of a wave is that it is periodic in time. This means that at a given location in space, the wave repeats itself over and over again as time goes forward. <u>The ***time*** required for the wave to repeat one cycle in time is known as the ***period T*** of the wave</u>.

A wave is also periodic in space. This means that at a given time, the wave repeats itself over and over again as you move along the wave in space. <u>The ***distance*** required for the wave to repeat once in space is known as the ***wavelength λ*** of the wave</u>.

<u>The Simple Harmonic Wave Function</u>

The simplest type of wave disturbance is one which has a sinusoidal shape both in space and in time. A sinusoidal disturbance must have fixed wavelength and period, as well as fixed amplitude. Such a disturbance is shown in the picture below. (*Note that this is a graph of the wave at a single time; therefore, this picture shows the wave disturbance only as a function of space i.e. it is a "snapshot"*). This is called a **<u>Simple Harmonic Wave</u>**.



How should we describe this wave mathematically? First of all, our mathematical description must be both a function of time and space. It also should be periodic in both of those variables. Clearly, a cosine or a sine is the correct function to describe this wave. We choose the cosine by convention.

Since the wave disturbance repeats itself in space once every wavelength, and in time once every period, the function should repeat in the same manner. Therefore our function for a wave propagating in the x-direction must be

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x \pm \frac{2\pi}{T}t\right)$$

This is called the **Simple Harmonic Wave Function**. Not all wave disturbances have the Simple Harmonic form, but this form is the easiest to work with, so we will do so in this section.

The variable y is the displacement of the wave medium from equilibrium, and it depends on position x and time t.

The "$\pm$" denotes the direction of travel of the wave. If the wave is traveling in a direction such that x is increasing with time, then the negative must be chosen. Conversely, if the wave is traveling in a direction such that x is decreasing with time, then the positive must be chosen.

Finally, for mathematical convenience we define the following constants related to time:

Frequency: $f \equiv \frac{1}{T}$ and angular frequency: $\omega \equiv 2\pi f$

and a constant related to space:

wave number: $k \equiv \frac{2\pi}{\lambda}$

The mathematical convenience is that we can now express our simple harmonic wave function as

$$y(x,t) = A\cos(kx \pm \omega t)$$

Finally, we would like to note that there is a relationship between the speed of the wave, the period of the wave, and the wavelength of the wave. For if it takes a time T for the wave to repeat itself in time, and a distance λ to repeat itself in space, then the wave must propagate with a speed of

$$v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$

The last equality, $v = \frac{\omega}{k}$, is a common way to write the wave speed, and comes simply from substituting the wave number and angular frequency for the wavelength and frequency.

II. <u>Electromagnetic Waves</u>

In 1865, James Clerk Maxwell showed, using his four equations (which we now call Maxwell's equations; see section 11) that electric and magnetic fields can propagate as waves!

Specifically, he showed that the electric and magnetic fields obey the functions

$$E(x,t) = E\cos(kx \pm \omega t) \qquad\qquad B(x,t) = B\cos(kx \pm \omega t)$$

where E and B are the amplitudes of the electric and magnetic waves, and k and ω are the wave number and angular frequency of the waves.

More amazingly, by plugging these functions back into Maxwell's equations, one can show that

$$\frac{\omega}{k} = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Recall from above that $\frac{\omega}{k}$ is the wave speed of any wave. Therefore, Maxwell found that the wave speed of the electric and magnetic waves is

$$v = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 2.99 \times 10^8 \text{ m/s}$$

The value of 2.99×10^8 m/s comes directly from plugging in the values of ϵ_0 and μ_0 (plug them in yourself and see!). This was the known value of the speed of light! Therefore, Maxwell had discovered that light consists of electric and magnetic fields propagating as what we now call <u>**electromagnetic waves**</u> (*Note: the speed of some general wave is denoted as v, while the special value of the speed of electromagnetic waves (light) in vacuum is denoted as c*).

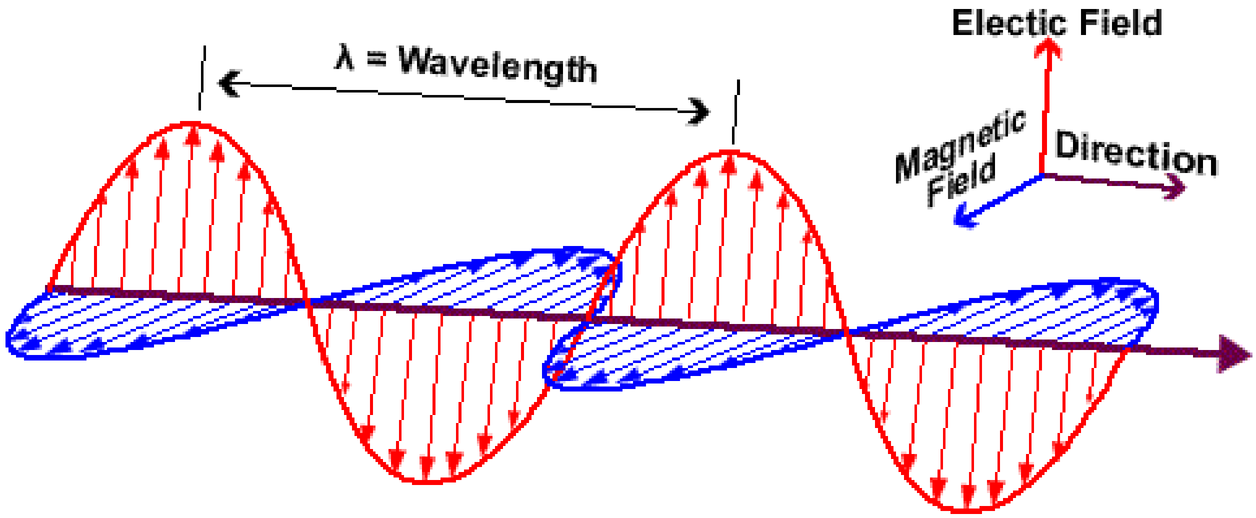
For this amazing theoretical insight, as well as work in other areas of physics, Maxwell is universally regarded as the greatest theoretical physicist of the 19th century.

<u>Properties of Electromagnetic Waves</u>

In addition to predicting the nature of electromagnetic waves as waves in the electric and magnetic fields, Maxwell's equations also yield addition information about the properties of these waves.

1. Maxwell found that the electric and magnetic fields are always perpendicular each other, as well as to the direction of propagation of the wave. Therefore, the electromagnetic wave is always a <u>***transverse wave***</u> (see figure below)

2. Maxwell also showed that the electric and magnetic field magnitudes are related by the speed of the wave in following way: $E = cB$.



See Figure above for a "picture" of a sinusoidal electromagnetic wave: Note that the electric and magnetic fields are perpendicular to each other, and to the direction of propagation of the wave.

The Source of Electromagnetic Waves

Now that we have described an electromagnetic wave, we would like to answer the following question: Where does an electromagnetic wave come from?

The answer is the same as with any other wave. There must be a source which "starts" the oscillation at one location in space. For instance, with water waves on a pond, there must be an oscillating source (for instance a stick bobbing up and down) for the wave to be created. This wave will then propagate outward with a speed v, where the speed is determined by the properties of the water medium itself.

To create electromagnetic wave, therefore, requires an oscillating electric charge! The charge, oscillating with some frequency f, will cause the electric and magnetic field in its region of space to oscillate with the same frequency, and this disturbance will propagate outward at a speed c, which is determined by the electric and magnetic properties of the vacuum! (ϵ_0 and μ_0).

The Electromagnetic Spectrum

Maxwell's discovery also predicted that electromagnetic waves can have any frequency, even frequencies that our eyes cannot detect as light! Maxwell's equations predicted that all of these electromagnetic waves, no matter the frequency, would propagate with the same speed c.

The first experimental discovery of electromagnetic waves outside of the visible frequency range was made by Heinrich Hertz in 1887. He was able to establish the existence of radio waves, which is a type of electromagnetic wave with a frequency much less than that of visible light, and indeed found that radio waves traveled with speed c. This discovery confirmed Maxwell's electromagnetic wave theory.

Since then, scientists have discovered electromagnetic waves over a very large frequency range, both with frequencies below and above visible light. Given that all electromagnetic waves travel with the same wave speed, they are classified by their frequency and wavelength.

For instance, the human eye can detect electromagnetic waves with wavelength between about 400 nm - 700 nm. Therefore waves with wavelengths in this range are known as visible electromagnetic waves, or visible light. The largest wavelength (equivalently lowest frequency) EM waves are called radio waves, and the smallest wavelength (equivalently highest frequency) EM waves are called gamma rays.

See the spectrum below for a full description of the various ranges of frequencies and wavelengths in the EM spectrum.