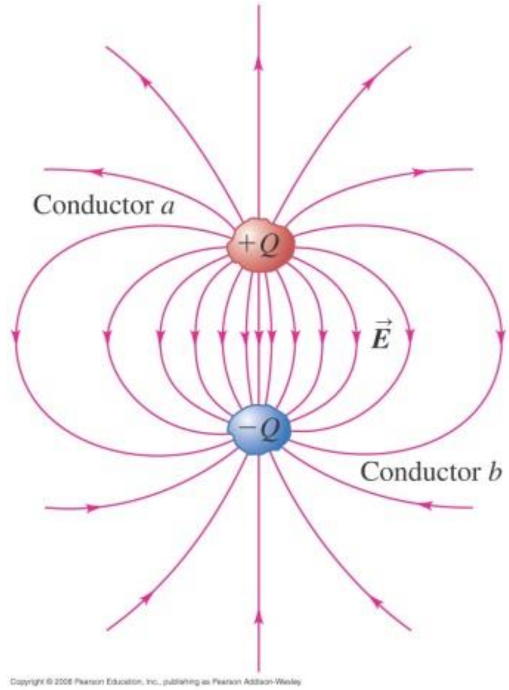
Pre-lecture Notes Section 6: Capacitance

I. Capacitors and Capacitance



Imagine that we have two conductors, of some arbitrary shape, separated in space by an insulating material (for right now, consider the insulating material to be vacuum). Then imagine that we place a charge $+Q$ on one conductor and an equal but opposite charge $-Q$ on the other conductor. Because of the separation of charge, there will be an electric field pointing from the positive conductor to the negative conductor (see picture).

Because there is an electric field pointing from one conductor to the other, there will be an electric potential difference between the two conductors. The potential difference can be labeled as any of the following: $\Delta V \equiv V_{ab} \equiv V_a - V_b = V$ (*warning: we get very lazy with the notation!*), where the positive conductor has potential V_a (*remember that the whole conductor is at the same potential, because it is an equipotential*) and the negative conductor has a lower potential V_b.

It turns out that calculating the potential difference between a pair of arbitrary conductors is no easy task (however, we will be able to do the calculation in a few “simple” cases). However, we can make one observation about this two-conductor system, which is the following: If we increase the charge on the conductors, the electric field between them will increase, and therefore the potential difference between them will increase.

Mathematically, this means that the charge on the conductors is directly proportional to the electric potential difference between the conductors:

$$Q \propto V$$

The constant of proportionality, which will be different for each different set of conductors, is called the **capacitance $\boldsymbol{C}$** of that set of conductors.

$$\boldsymbol{C \equiv \frac{Q}{V}}$$

The capacitance of a pair of conductors is therefore the charge to electric potential difference ratio for that particular pair of conductors. Such a two-conductor system (separated by an insulating material) is called a **capacitor**.

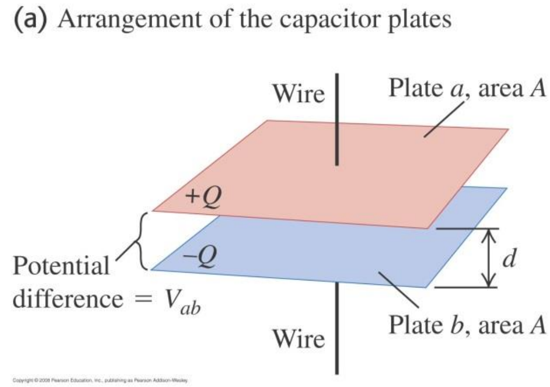
The capacitance of a capacitor is dependent **only** on the geometry of the conductors (i.e. shape, size and separation), not on the amount of charge on each plate.

The SI unit of capacitance is the Farad: 1 F = 1 C/V.

The Parallel-Plate Capacitor

For some simple capacitor arrangements, the capacitance is quite easy to calculate. The simplest is the parallel plate capacitor, shown in the picture to the right. Each plate has an area A and the distance separating the plates is d. We will show in class that the capacitance of a parallel-plate capacitor

$$C_{par\ plates} = \frac{\epsilon_0 A}{d}$$



The capacitance is therefore directly proportional to the area of the plates and inversely proportional to the distance between the plates. We also see that it depends only on the geometric quantities associated with the conductors.

II. Energy Storage

A capacitor can be thought of as a device which stores electrical potential energy. This becomes clear when thinking about the following question: how does one "charge up" the capacitor in the first place? Initially the two conductors are neutral. To charge it, one has to first remove one charge (say an electron) from one of the conductors and then place it on the other conductor. This means that one conductor will have a small positive charge and the other a small negative charge. Then you have to do it with another electron, and another, and another etc… until you have moved some large number of electrons to equal some final total charge Q on the positive conductor and $-Q$ on the negative conductor.

However, starting with the second electron, it will require work for you to move them. Why is this? Because in moving an electron away from the positive side to the negative side you will be "pushing against" the electric force! In fact, the more that are moved, the harder it is to move them, since the electric force increases as the charge builds up.

We would like to know how much energy a given capacitor stores. As we will show in class, the amount of electric potential energy stored by the capacitor is

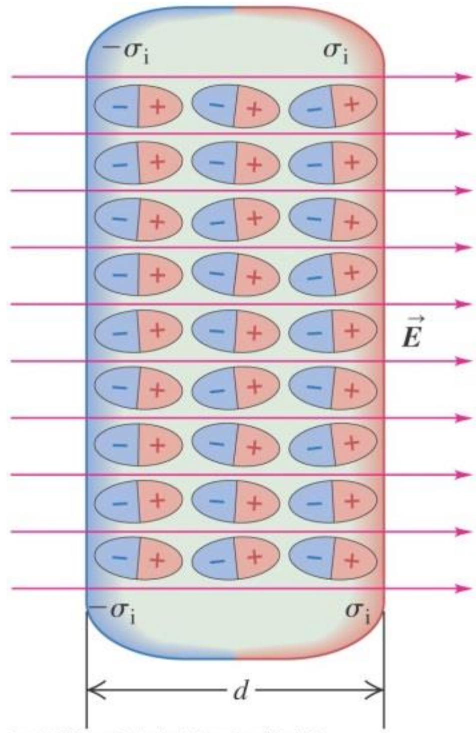
$$U = \frac{1}{2} C V^2$$

For a given electric potential difference between the conductors, the stored energy is directly proportional to the capacitance of the system.

In a practical sense, capacitors are used as devices which can store and then release electrical potential energy. A camera flash bulb is one such example. In this section, we are only concerned with how much energy a capacitor can store. In section 8, we will study the question of how rapidly a given capacitor can store and then release its electrical potential energy.

III. <u>Dielectrics</u>

So far, we have assumed that the space between the conductors in a capacitor is vacuum. What happens when we place a physical insulator between the conductors? This could be air, but is more commonly some other type of insulator like rubber (*note that we would not want to place a conductor between the plates because the charge would just flow across, destroying our attempts to separate the charge*). Such a material is called a **dielectric** material.



In such a scenario, the electric field between the conductors serves to polarize the atoms in the dielectric, as we saw in section 1. This serves to “cancel out” some of the electric field as the electric field produced by the polarized atoms is opposite to the electric field produced by the conductors! (See picture).

The consequence of this is that the electric field in between the plates is less, for a given amount of charge on the conductors, then there would be without the insulating material. We define a constant which is the ratio of the electric field without the dielectric to the electric field with the dielectric. This constant is called the dielectric constant K, and depends on the dielectric material used.

$$\boldsymbol{K \equiv \frac{E_0}{E}}$$

where E is the electric field with the dielectric and E_0 is the electric field if there was only vacuum. Since the electric field with the dielectric is less than in vacuum, the dielectric constant will always be greater than one.

What does this mean for the capacitance of the capacitor with the dielectric? Well, since the electric field is less, the electric potential difference is less by the same factor. Using the fact that $C \equiv \frac{Q}{V}$, we see that for a given amount of charge, the capacitance will be greater by the factor K with the dielectric. Thus,

$$\boldsymbol{C = KC_0}$$

where C_0 is the capacitance of the capacitor without the dielectric and C is the capacitance with the dielectric.

In fact, since every electric field formula contains the ratio $\frac{1}{\epsilon_0}$ (fundamentally this is because of the factor $\frac{1}{\epsilon_0}$ present in Gauss' and Coulomb's law), it is useful to define a new quantity, ϵ, which is defined as

$$\boldsymbol{\epsilon = K\epsilon_0}$$

This quantity is called the **permittivity** of the dielectric (whereas ϵ_0 is called the permittivity of vacuum, or free space), and it quantifies the amount by which the electric field decreases in the presence of a dielectric.

In terms of calculations, this means that when dealing with dielectrics, all our previous formulas and calculational tools apply, except that you simply must replace ϵ_0 with ϵ! The concept of permittivity will become especially useful when we study some of the properties of light traveling in various materials.