Pre-lecture Notes Section 2:

The Electric Field

I. <u>The Electric Field</u>

How do two objects, which are not touching, exert forces on each other? In other words, how does a long-range force work, whether it is the gravitational force or the electrical force? One way to think about answering this is to invoke the concept of a ***field***. Michael Faraday, who we'll learn a lot more about in future sections, was the first to postulate the concept of an electric field.

Before we start to learn about the electric field, let's first start by discussing the meaning of the word "field". In physics, a field is simply a particular quantity which is assigned to every point in space. There are two kinds of fields of interest to us; scalar fields and vector fields.

A scalar field with which you are familiar (but probably did not know was a scalar field!) is temperature. Watch the weather report and you will notice that there is a temperature (which is a scalar) at every point in space (*of course, in reality they do not literally show the temperature at every point in space, but there is a temperature at every point regardless of how many points they choose to show*). Therefore temperature is a scalar field. Mathematically, we could write the temperature field as $T(x, y, z, t)$. In other words, the temperature is a function of the x, y and z coordinates as well as the time t (*assuming that, as in actual weather, the temperature at a given point in space changes with time*).

A vector field is a field which assigns a vector (magnitude and direction!) to each point in space. A vector field that you are familiar with in your everyday life is wind velocity. At each point in space, there is a particular wind velocity vector. For instance, at a particular point in Houston the wind velocity might be 12 mph pointing 22 degrees to the southeast, whereas at a point in Dallas the wind velocity may be 9 mph pointing 67 degrees to the northwest. At every point in space, there is a vector denoting wind velocity. Therefore wind velocity is a vector field. As with temperature, we can write the wind velocity as $\vec{v}_{wind}(x, y, z, t)$.

An electric field is also a vector field. In a certain sense, it is much more abstract than temperature or wind velocity, although it is much more fundamental and thus easier to calculate and predict mathematically! In the previous section we detailed Coulomb's law, by which a long-range interaction produces a force between two charges. Thinking about the force in that way is what I will call a "one-step" process. The one step is simply that the force is exerted between the two charges. We can calculate the same force using the electric field in what I will call a "two-step" process. The two-step process is as follows:

Step 1. A charge q, which we call the **<u>source</u>**, produces an electric field (a vector field) $\vec{E}$ in the surrounding space. This means that at every point in space there is an electric field vector, which has been created by the charge q.

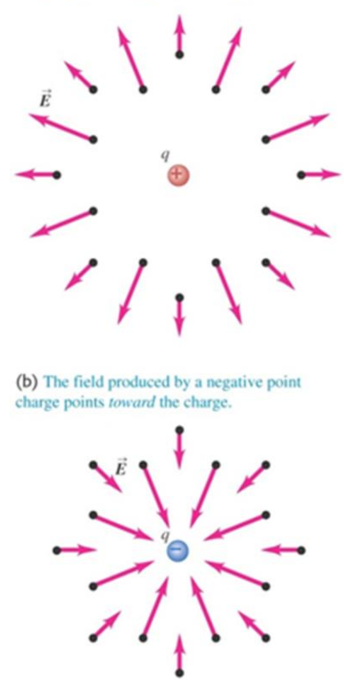
Step 2. Another charge q_0, which we call the **test**, placed at some point P, feels an electric force $\vec{F}_E$ due to the electric field $\vec{E}$ produced by the charge q.

One question we might ask is the following: how does one determine if there is an electric field at a given region in space? To answer, think about the same question with regards to wind velocity. In other words, how do we know what the wind velocity is at a point in space? Simply put, we place an object at that point in space (perhaps a piece of grass) and see how the object reacts (if it is accelerated or not, how large the acceleration is, and in which direction the acceleration points). In fact, the electric field is defined as the electric force that a test charge feels per unit test charge.

$$\vec{E} \equiv \frac{\vec{F}_E}{q_0}$$

Thus, with this definition and Coulomb's law, the magnitude of the electric field from a point source charge is

$$E = \frac{|q|}{4\pi\epsilon_0}\frac{1}{r^2}$$



(a) The field produced by a positive point charge points *away from* the charge.

(b) The field produced by a negative point charge points *toward* the charge.

Direction of the Electric Field

The electric field created by a point charge must always point either away from the charge (for a positive charge) or towards the point charge (for a negative charge) as shown in the pictures to the right. This is so the electric force on a test charge will point in the correct direction.

The problem with this is that the direction is not constant for all points surrounding the source charge. For instance, if you are directly above a positive source charge, the electric field vector will point up (in the $\hat{j}$ direction), whereas if you are directly to the left of a positive source charge the electric field vector will point to the left (in the $-\hat{\imath}$ direction). To find the direction of the electric field at a given point unambiguously, we define two new vectors.

We call the position of the source the "source point", P_{source}. This may or may not be the origin of your coordinate system. The vector $\vec{r}_{source}$ is defined to be the vector which points from the origin of your coordinate system to the source point (i.e. the position vector of the source charge).

$$\vec{r}_{source} \equiv \text{position vector of the source charge}$$

We call the location where we want to determine the electric field the "field point", P_{field}. The vector $\vec{r}_{field}$ is defined as the vector which points from the origin of your coordinate system to the field point (i.e. the position vector of the field point).

$$\vec{r}_{field} \equiv \text{position vector of the field point}$$

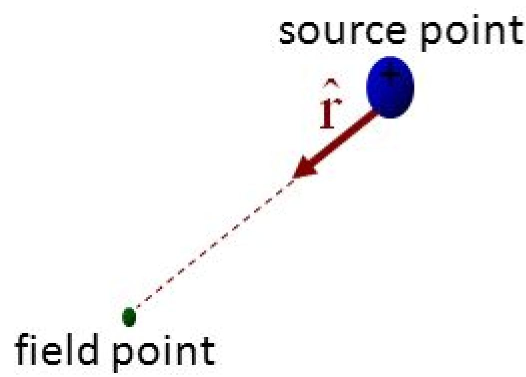
Finally, we define the separation vector $\vec{r}$ as follows:

$$\vec{r} \equiv \vec{r}_{field} - \vec{r}_{source}$$

This vector is actually the vector which points from the source point to the field point (can be easily verified by vector subtraction). Therefore, not only does it point in the correct direction, but its magnitude is the distance between the source and field point! Therefore, the electric field produced by a source point charge, taking into account ***both magnitude and direction***, is

$$\boldsymbol{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

where r is the magnitude of the separation vector, and $\hat{r}$ is the unit vector pointing from the source point to the field point (i.e. the unit vector pointing in the direction of the separation vector). One finds this unit vector with the operation $\hat{r} \equiv \frac{\vec{r}}{r}$.



See the picture to the right. The source charge is at the source point, and the vector $\hat{r}$ points from the source point to the field point, which is the point at which we want to know the electric field.

Summary of the Electric Field Model

In summary, the two-step model to find the electric force on a test charge due to some source charges is:

Step 1. The **source charge** $\boldsymbol{q}$ at source point P_{source}, produces an electric field $\boldsymbol{\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$ in the surrounding space.

Step 2. The **test charge** $\boldsymbol{q_0}$, placed at the field point P_{field}, feels a force $\boldsymbol{\vec{F}_E = q_0\vec{E}}$.

In class, we will investigate how the electric field two-step model exactly reproduces the simple “like charges repel, opposites attract” rule.

It is useful to summarize by comparing and contrasting the electric field with the electric force.

Property	**Number of objects required**	**Variation with distance**
Electric Force	Requires an interaction: two objects	Force between two charges decreases with the square of the distance separating them
Electric Field	Does not require an interaction: one object	Field due to one charge decreases as the square of the distance between the source and the field point

II. Electric Field Lines

Previously, to get a "feel" for a given electric field, we have sketched a few of the electric field vectors at various locations in space (see the pictures for positive and negative charges). However, there is often a nicer and more intuitive way to represent a field pictorially, which is to connect the arrows to form **field lines**.

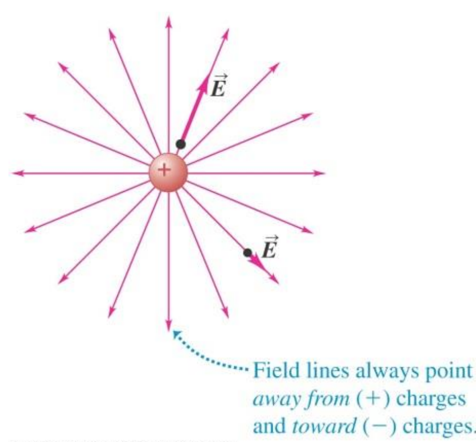
Direction of Field from Field Lines

The direction tangent to the field line curve at any location is the direction of the electric field at that location.

Strength of Field from Field Lines

The relative density of field lines in a given region yields the relative strength of the electric field in that region.

(a) A single positive charge



Examples of Field Lines

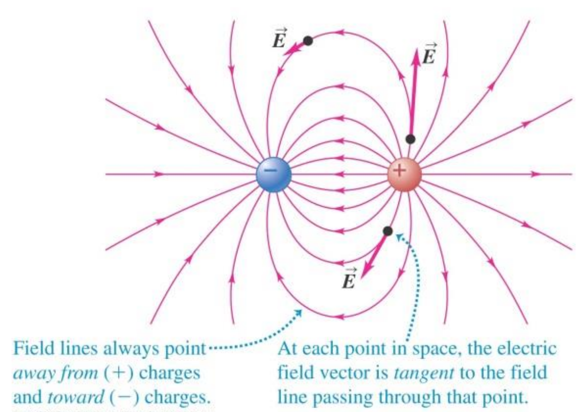
Point Charge

For instance, in the picture to the right we see the field lines produced by a single positive charge. The field lines are all pointing away from the charge, which tells us that the direction of the electric field is away from the charge at all points, which we already know to be true. Furthermore, the density of lines (lines per area) is greater nearer to the charge. This tells us (as we already know) that the electric field is strongest closest to the charge.

Dipole

In more complicated systems, the ability to draw field lines is very helpful as a tool to "picture" the electric field. As an example, consider two charges of opposite sign but equal magnitude, placed some distance apart. Such a system is called a **dipole**. The field lines for a dipole are shown to the right.

(b) Two equal and opposite charges (a dipole)



The tangent line at each point tells you the direction of the total electric field at that point. In addition, the density of lines tells you the relative strength. For instance, in the vicinity of each charge, as well as directly in between the two charges, the electric field is large. On the other hand, far from the two charges, the lines become more spread out, indicating a weaker electric field.

The concept of electric field lines will become especially crucial in section 3 and beyond.