Pre-lecture Section 9:

Magnetic Field and Magnetic Forces

I. <u>**Introduction to Magnetic Field and Magnetic Forces**</u>

We are now switching gears, so to speak, away from electric phenomena and into the study of magnetic phenomena. Before we begin, I would like to re-summarize our basic understanding of the electric field and force, as shown in our two-step model.

<u>Electric Field Two-Step Model:</u>

Step 1. A source charge q produces an electric field $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$

- The total electric field at a given point is the vector sum of the electric fields at that point due to all source charges.

Step 2. A test charge q_0 experiences an electric force $\vec{F}_E = q_0\vec{E}_{tot}$

We have studied a variety of topics regarding the electric field and force (Gauss' Law, the idea of electric potential) as well as applications of the electric field and force (capacitors, direct-current circuits etc…) but the entirety of the subject can be boiled down into the two step process above.

Notice that, in the electric field and force model, an electric charge is required both to create an electric field and to feel an electric force.

It turns out that the magnetic field and force model shares some similarities with the electric field and force. The major difference is that instead of simply requiring an electric charge, a magnetic field can only be created by a <u>***moving electric charge***</u>, and the magnetic force can only be felt by a <u>***moving electric charge***</u>.

We will find that the magnetic field and force can be broken down into a similar two-step process. We could imagine that the two-step process looks something like the following:

Step 1. A source *moving* charge q produces a magnetic field $\vec{B} = ???$

- The total magnetic field at a given point is the vector sum of the magnetic fields at that point due to all source moving charges.

Step 2. A test *moving* charge q_0 experiences a magnetic force $\vec{F}_B = ???$

We therefore know what the basic steps should look like, but we still need to "fill in", so to speak, the expressions for how a moving charge creates a magnetic field and also how a moving charge feels a magnetic force.

In this section, we will briefly discuss the magnetic field, but our main concern in the section is to answer the following question: <u>*Assuming we know what the magnetic field is, what force does it exert on*</u>

a moving test charge? In other words, we are going to be focusing on step 2 of our two-step process. In the following section (section 10) we will focus on step 1.

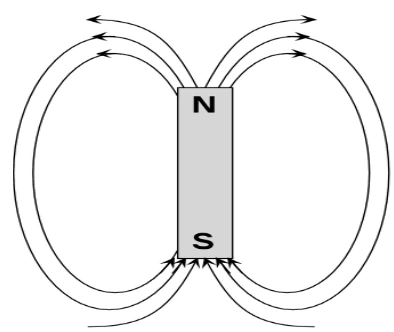
II. The Magnetic Field

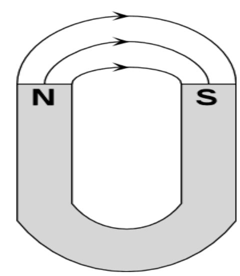
One very important fact about the magnetic field is that, like the electric field, the magnetic field is a vector field. At each point in space, the magnetic field has both a magnitude (which can be qualitatively seen as the density of the field lines at a given point in space) and a direction.

As was discussed earlier, we are not going to say much about the origin, or creation, of the magnetic field in this section (that will wait for the next section). Instead we will simply be content to acknowledge that a magnetic field can be created by an object called, appropriately enough, a magnet. Only certain materials (most notably iron) can create magnetic fields, and in this section we will imagine using these materials to create magnetic fields (in this next section we will discuss how they create magnetic fields). Given that these materials create magnetic fields, we would like to discuss a few properties of the magnetic field that these magnets create.

Properties of Magnetic Fields

Firstly, each magnet is said to have a *north pole* and a *south pole*. The magnetic field of a magnet always points out of the north pole and into the south pole. A few examples are shown in the picture *(the first object is called a "bar magnet" and the second is called a "horseshoe magnet")*. In fact, the Earth itself is a (somewhat weak) magnet whose magnetic field looks quite similar to a bar magnet.

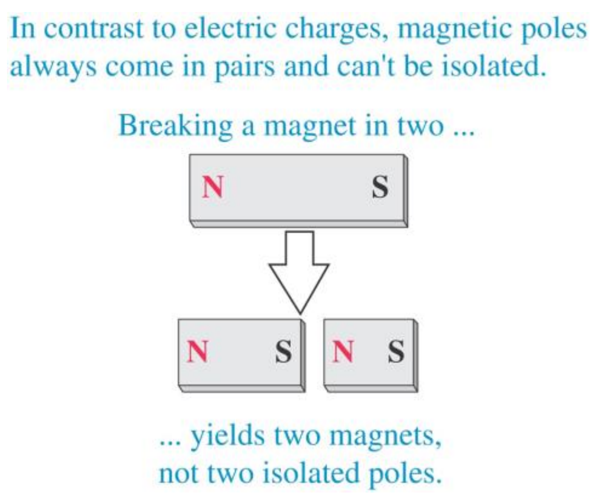




From experiment, scientists have deduced the following facts about the magnetic field:

1. A magnet cannot contain *only* a north pole or a south pole. In other words, the north and south poles can never be separated from each other. Every magnet contains both a north and a south pole.

   If you do break a magnet in an attempt to try and "separate" the poles, you simply obtain two magnets, each with a north and a south pole. See the picture to the right.



2. Unlike electric field lines, magnetic field lines have no "beginning" or "end". In other words, magnetic field lines always form complete loops. A picture of the full magnetic field of a bar magnet is shown in the figure below. Notice that the

magnetic field lines do not begin or end anywhere, but simply form complete loops inside the magnet and out.

3. Finally, we would like to define and calculate the magnetic flux. As with the electric flux, the magnetic flux is defined as the integral of the magnetic field over a particular surface.

$$\Phi_B \equiv \int \vec{B} \cdot d\vec{A}$$

As with the electric field, an interesting surface to consider is a closed surface.
Unlike the electric field, however, magnetic field lines have no beginning and no end. This means that the flux through a closed surface will always be zero; whatever field lines come in must also go out!!! This result is called "Gauss' Law for Magnetism".

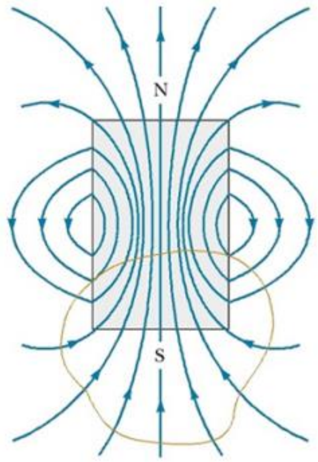


Figure 30.22 The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through the closed surface (dashed red line) surrounding one of the poles (or any other closed surface) is zero.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

All of these facts about the magnetic field are related. In fact, they are all completely equivalent (i.e. starting with one of the rules you can prove the other two)!

The Unit of Magnetic Field

The SI unit for the magnitude of the magnetic field is the Tesla (T). As we will see in a moment, a Tesla is given in terms of our more familiar units by

$$1\ \mathrm{T} \equiv 1\ \mathrm{N/(A{\cdot}m)}$$

Another common unit for the magnetic field strength is the Gauss (G), where $1\ \mathrm{G} \equiv 10^{-4}\ \mathrm{T}$.

The magnetic field of the Earth on the surface is on average about 0.5 G, or 5×10^{-5} T.

III. The Magnetic Force

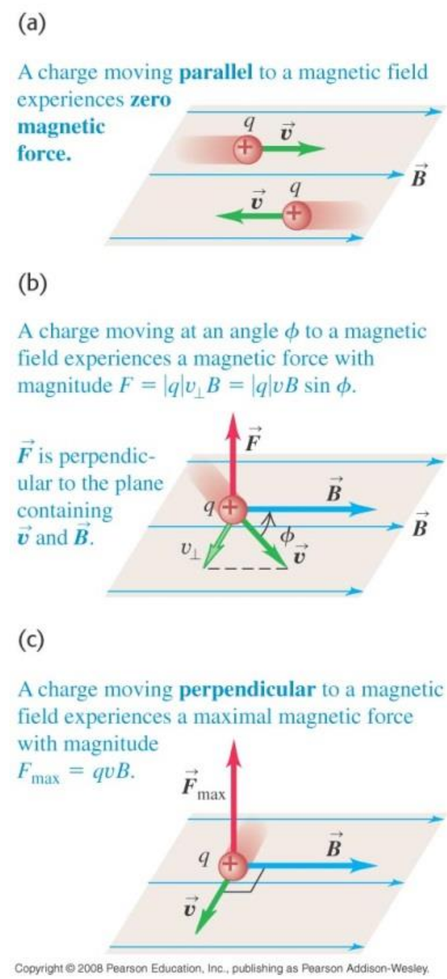
Now that we have established a few facts about the magnetic field, we would like to describe the force that a magnetic field exerts on a moving electric charge. The exact nature of this force was discovered via experiment and formalized in the late 19th century, and is given by

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

where q is the charge feeling the force (test charge), $\vec{v}$ is the velocity of the charge and $\vec{B}$ is the magnetic field at the location of the charge.

To make the units of this equation work properly (i.e. to give the force in Newtons), the SI unit of magnetic field must be N/A·m, as stated previously.

Notice that the force involves a cross product between the velocity and magnetic field. This means that the direction of the force will always be perpendicular to both the velocity and magnetic field! Thus the magnetic force is not an attractive or repulsive force. It is more accurate to say that it is a "deflecting" force.



As defined by the cross product, the magnitude of the force is given by

$$F_B = qvB \sin \varphi$$

and the direction of the force is given by the Right Hand Rule.

The "sin φ" term in the cross-product tells us that the magnitude of the force is greatest when the velocity and magnetic field are perpendicular to each other, and in fact that the force is zero if the velocity and magnetic field are parallel (or anti-parallel)! See picture to the right.

One convenient way to use the right hand rule to find the direction is the following:

If your thumb points in the direction of $\vec{v}$ and your index finger points in the direction of $\vec{B}$, then your middle finger will point in the direction of $\vec{F}$ on a positive charge. If the charge is negative, the force points in the opposite direction of the middle finger! Try using the right hand rule to verify pictures (b) and (c)!

The Magnetic Force and Work

One seemingly strange fact about the magnetic field is that it is not capable of doing work. In Phys 2425, you learned that a force cannot do work on an object if the force acting on the object and the displacement of the object are perpendicular. The magnetic force exhibits exactly this property, since the magnetic force always points perpendicular to the instantaneous velocity (and thus the instantaneous displacement) of the test charge.

The consequence of this (using the work-kinetic energy theorem) is that the magnetic force by itself cannot cause a particle to speed up or slow down, but instead can only cause a particle to change direction!

We will discuss much more about the magnetic force and its consequences for particle motion in class.