Pre-lecture Notes Section 17:

Interference

I. Introduction and Review of Interference

This section serves as an introduction to the field of **physical optics**. Physical optics denotes the group of phenomena involving light which depends crucially on the wave nature of light. This is in contrast to geometrical optics from the previous chapter, in which the phenomena (formation of images via reflection and refraction) only depended on the direction and speed of the waves (i.e. the ray model), and not on the wavelength or frequency of the waves.

As with all waves, when two light waves overlap in space, they interfere with each other. This phenomenon only happens for waves, and thus the wave properties are crucial in determining the outcome of this interference. The interference can be either constructive (the two waves reinforce each other perfectly), destructive (the two waves cancel each other perfectly) or partial (the two waves reinforce or cancel each other only partially).

II. Coherent Waves and the Conditions for Interference

In Physics 2425 (chapters 15 and 16 in the textbook), we learned how to predict the type of interference that will occur when two waves overlap, assuming

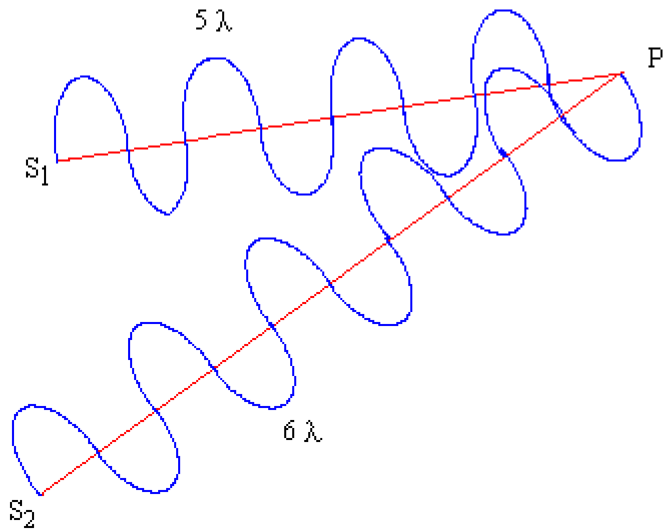
1. The two wave sources produce waves of exactly the same, single frequency. With regards to light, this means that both wave sources must emit identical frequency monochromatic (single "color") light.
2. The waves have a constant phase relationship. This means that each wave must be produced by a source which is oscillating in the same direction at the same time (i.e. the oscillations cannot be random, such as the light coming from an incandescent light bulb).

Two sources which emit light with these two properties are said to be **coherent** wave sources which produce coherent waves. With regards to sound waves, a pair of coherent sources can be constructed by connecting two speakers to the same frequency generator producing a single frequency. With regards to light waves, we will see that a pair of coherent sources can be established by illuminating two small slits with monochromatic light, such as that coming from a laser. Each of the slits will act as a source, and since the light from each slit originated from the same source (the laser), the frequency and phase of the sources will be identical.

To predict to the type of interference that will occur at a given location in space, the key quantity to know is the **path length difference** of the two sources to the observation location. If r_1 is the distance from source 1 to the observation location, and r_2 is the distance from source 2 to the observation location, then the path length difference is defined as

$$\text{Path length difference} \equiv \Delta r = r_2 - r_1$$

For instance, in the figure below, S_1 and S_2 are the wave sources, and the path length difference at the point P is $6\lambda - 5\lambda = \lambda$.

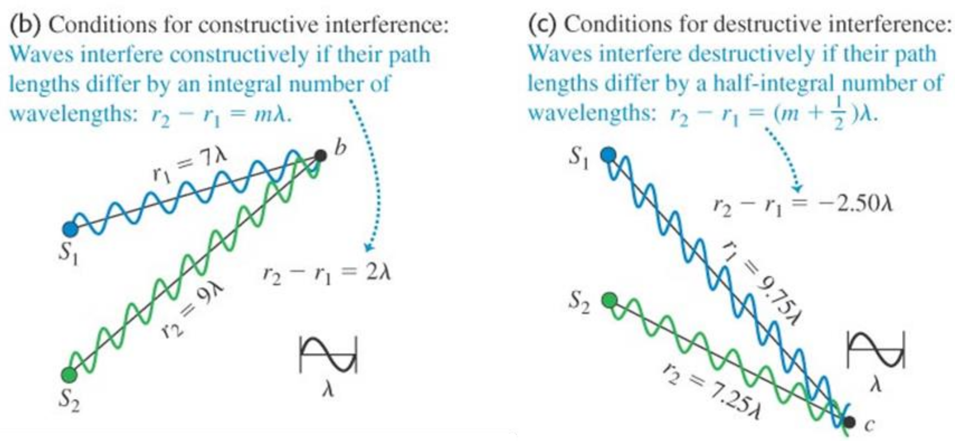


From a simple analysis of the wave equations of the two waves (or equivalently a pair of graphs of the waves, see pictues below), we find the following conditions for constructive intereference and destructive interference, again assuming that the waves are coherent and in phase:

$\Delta r = m\lambda$ $\quad (m = 0, 1, 2, 3, \ldots)$ $\quad$ Constructive Interference

$\Delta r = \left(m + \frac{1}{2}\right)\lambda$ $\quad (m = 0, 1, 2, 3, \ldots)$ $\quad$ Destructive Interference

In words, if the path length difference is <u>*any*</u> whole number of wavelengths (including zero!) then the waves intefere constructively. If the path length difference is <u>*any*</u> half number of wavelengths, then the waves interfere destructively. Finally, if the path length difference is neither, then the waves will exhibit partial interference.



III. Two-Slit Interference and the "Far Field" Approximation

We would like to apply our interference conditions to the famous two-slit experiment, in which a source of monochromatic light is directed at two narrow slits separated from each other by a small distance.

Thomas Young first did a similar experiment in the very early 1800's, and it provided the most compelling evidence that light behaves as a wave (this was well before Maxwell showed that light was an *electromagnetic* wave).

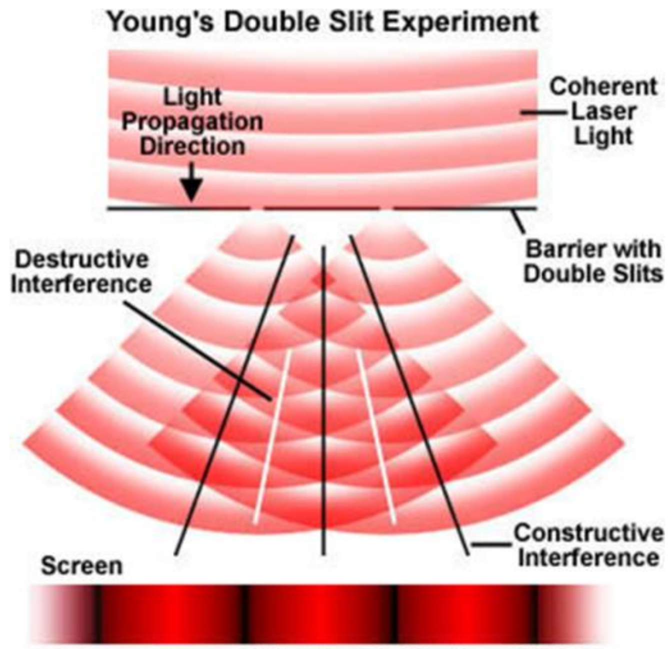


Figure 4

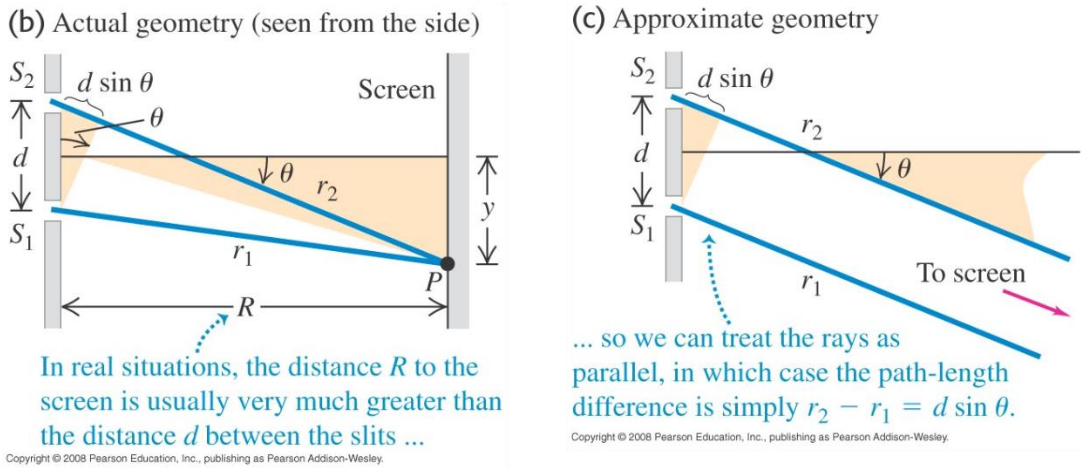
Each slit will end up being a source of coherent light of some given frequency and wavelength. The two waves will interfere, and upon being projected on a screen, will form a series of bright (constructive) locations and dark (destructive) locations. The figure to the right depicts this process.

As with any type of wave interference, the conditions for constructive and destructive interference are the same as those given in the previous section. However, since the wavelength of visible light is very small (on everyday scales) the interference pattern only becomes clear when the distance separating the slits is very small. This makes it very hard to measure the path length difference Δr to the observation point on the screen! However, we can use a geometric approach to derive an approximate expression for Δr which is easy to measure and use.

To see when this approximation is valid, we first define d, the distance between the slits, and also R, the distance from the point located directly in between the slits to the observation point (the screen). If $d \ll R$, then it can be geometrically proven that

$$\Delta r \approx d \sin\theta$$

where θ is the angle from the center line between the two slits to the location of interest on the screen. This is often called the "far field" approximation. The approximation becomes exact when the distance to the screen is infinite, so that the ray from slit 1 is exactly parallel to the ray from slit 2. See the picture below for the geometric diagram of the situation.



This approximation means that, if our condition $d \ll R$ is realized, then we can re-write our interference conditions (substituting $d \sin\theta \approx \Delta r$) as

$d \sin\theta = m\lambda$ $\quad (m = 0, 1, 2, 3, \dots)$ $\quad$ Constructive Interference

$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $\quad (m = 0, 1, 2, 3, \dots)$ $\quad$ Destructive Interference

Each integer m represents a different bright or dark spot, and the far field approximation allows one to predict the angles at which these bright and dark spots will appear, given known values of d and λ.

IV. Application: Diffraction Gratings

One of many applications of this phenomenon is a diffraction grating.

Instead of two equally spaced slits, let us imagine that we have a barrier with many equally spaced slits. Such a device is called a diffraction grating. It turns out that the conditions for constructive interference do not change from the two-slit case. However, the intensity at the bright spot locations becomes greater, and each bright spot becomes less smeared out (more sharply peaked). The physical reason for this is the phenomenon of diffraction, which is another wave interference phenomenon that we will not cover. The consequence is that a diffraction grating produces a series of distinct (not smeared out) bright spots.

The diffraction grating is very useful in determining the frequency spectrum of polychromatic (multiple frequency) light. This is because, due to the different wavelengths of each color, the bright spots for each color will be at a different angle from the diffraction grating. Since the grating produces distinct spots, it is easy to determine which frequencies “make up” the light in question by simply looking through the grating and finding the angles of the bright spots for the various colors. We will see this phenomenon in action in class!