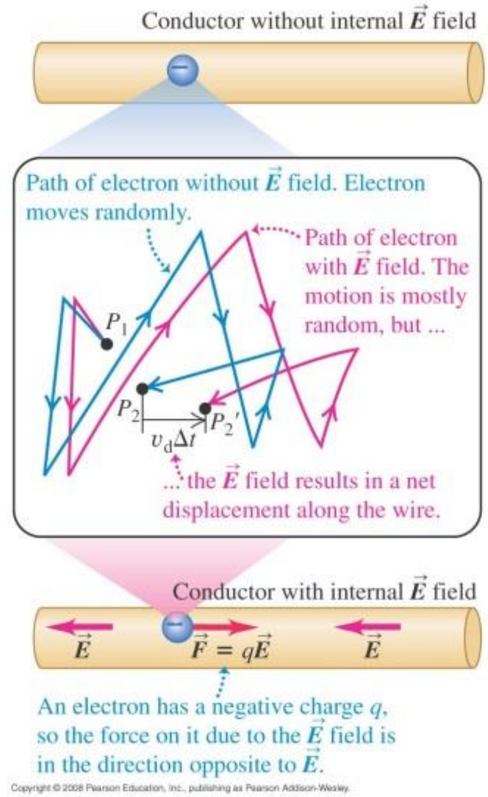
Pre-lecture Notes Section 7: Current and Resistance

I. <u>Electric Current</u>

We are now ready to consider charges in motion.

In a conductor, the outer electrons are not bound to their atomic nuclei, and are therefore free to "roam" around the material. These electrons will be randomly moving about, with an average speed related to the temperature of the material (*higher temperature = greater average speed*).

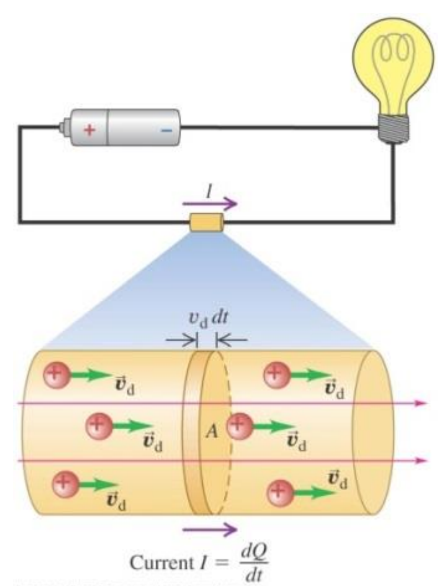


Imagine now that you apply an external electric field to the conductor. What will happen? The electric field will of course produce a force on the electrons (*in the direction opposite to the electric field*). This will cause the electrons, which are still randomly bouncing around, to slightly "drift" in that direction. The electrons will therefore have a **<u>drift velocity</u>** and will start to flow in a direction opposite to the electric field (see picture). This is the basis for an electric current, which is simply a flow of electric charge.

<u>A Quick Note about the Direction of the Current</u>

*As we know, the electrons will flow opposite the direction of the electric field. However, it is conventional to discuss the current as if it were flowing in the <u>same</u> direction as the electric field, necessitating that it be composed of positive charges. Therefore, we define the **<u>conventional current</u>** direction to be in the same direction as the electric field, even if the actual charge carriers (the electrons) are flowing in the opposite of this direction! This convention may seem strange and confusing, but it helps us keep things simple when we start to analyze more complex circuit problems.*

<u>Now Back to Electric Current</u>



The electric current I through a given cross-section of a material is defined as the amount of charge per unit time passing through that cross section. In other words, electric current is the rate of charge flow through a cross-section of material. Mathematically, electric current is defined as

$$\boldsymbol{I \equiv \frac{dQ}{dt}}$$

where dQ is the infinitesimal charge passing through a cross-section of the material in infinitesimal time dt.

The units of current are Amperes: 1 A = 1 C/s.

Now we would like to understand what factors current depends on. Looking at the figure and thinking a bit, it is clear that the current through a section of material depends on four factors.

1. The magnitude of charge $|q|$ of each charge carrier (*in the case of conductors each charge carrier is an electron with magnitude of charge* e). The more charge each charge carrier contains, the more total charge moves through the cross-section in a given time.
2. The drift velocity v_d of the charge carriers. The greater the drift velocity, the more charge carriers pass through the cross-section in a given time.
3. The number density n of charge carriers. The number density is defined as the number of charge carriers per unit volume. The more charge carriers per volume in the material, the more are available to pass through the cross section.
4. The area A of the cross section of the conductor. Clearly, the greater the area, the more space there is for the charge carriers to pass through the cross section.

In fact, the current, as defined above, can be proven to be the product of all four factors:

$$\boldsymbol{I = n|q|v_d A}$$

The current, as defined, is a scalar and not a vector. However, it becomes useful to define something called the **current density**, which is the current per unit cross-sectional area of the conductor. It is a vector, pointing in the direction of the drift velocity.

$$\vec{J} = nq\vec{v}_d$$

The most important fact about the current density is that it is a vector, which we will soon relate to the electric field that we have applied to the conductor.

II. Resistivity and Ohm's Law

Now that we have defined the electric current and current density, we would like to know how the current depends on the electric field that we apply to the material. For one, we know that if we apply no electric field to the material, there will be no current! This means that we should suspect that the current (actually the current density, since it is a vector) be proportional to the electric field that we apply.

$$\vec{J} \propto \vec{E}$$

The greater the electric field that we apply, the greater the force on the charge carriers and the more current that flows!

What should the proportionality constant be? For one, it should somehow depend on the material in question. In other words, if we apply the same electric field to a copper wire as to a piece of glass, we would expect the current flow to be very different. In fact, since copper is a good conductor and glass a good insulator, we would expect that the electric field would cause a large current in the copper and a very small current (if any) in the glass. The proportionality constant is called the **resistivity $\boldsymbol{\rho}$**, which quantifies how much a material resists current flow in the presence of an electric field. The larger the resistivity is, the smaller the current flow is in response to a given electric field.

Mathematically, resistivity is defined as

$$\rho \equiv \frac{E}{J}$$

where E is the magnitude of electric field inside the conductor and J is the magnitude of the current density.

This definition shows that indeed the larger the resistivity, the smaller the current density for a given electric field.

The units of resistivity are V·m/A.

Ohm's Law

One important question is the following: Is the resistivity for a given material constant, or does it change if the applied electric field changes?

It turns out that many different kinds of materials have the property that the resistivity is independent of the electric field applied. In other words, the resistivity is constant, dependent only on the type of material, not on the electric field applies. As another way to see the consequences of this, we re-writing the above equation to find

$$\boldsymbol{J = \frac{1}{\rho} E}$$

If the resistivity is constant for all electric field values, then the current density depends *linearly* on the electric field, so that if, say, the electric field triples, the current density must also triple.

Materials that obey this linear relationship between current density and electric field (or equivalently have a constant resistivity) are said to obey **Ohm's law**. It is important to note that Ohm's law is not true for all materials! There are some materials for whom the relationship between J and E is not linear. Therefore, Ohm's law is not really a law in the sense you are used to, but more like a rule-of-thumb that applies for many common conducting materials.

Materials which obey Ohm's law are called *ohmic* or linear *conductors*. Common metals like copper and silver are ohmic conductors. Materials which do not obey Ohm's law are called *nonohmic* or *nonlinear* conductors. Semiconductors and diodes are devices which do not obey Ohm's law.

Every different ohmic material will have a different (constant) value for resistivity. For instance, we would expect that the resistivity of copper would be much less than for glass (see Table 25.1 in the text for a sense of just how different they are!). The resistivity of an ohmic material, then, is a property of each specific material, independent of the size or shape of that material.

The Familiar Form of Ohm's Law – Introducing Resistance

Ohm's law above is stated in terms of the electric field and current density, but often it is much easier, as we will see in our labs, to measure the electric potential difference (which is related to, but not equal to, the electric field) and the current (which is related to, but not equal to, the current density). Therefore, we would like an alternate expression of Ohm's law that relates current and voltage (electric potential difference).

To find this alternative relationship, imagine a conductor as a thin wire with an electric field applied across it. The magnitude of the voltage between the two ends can be found from the relationship $E = \frac{dV}{dx}$. Upon integrating this with a constant electric field (imagine the two ends as parallel plates), we find that $E = \frac{V}{L}$, where V is the voltage between the two ends and L is the length between the two ends. Also, we know that the current density is defined as $J \equiv \frac{I}{A}$. Combining these two, and plugging into Ohm's law, we can make the transformation

$$J = \frac{1}{\rho}E \quad \rightarrow \quad \frac{I}{A} = \frac{1}{\rho}\frac{V}{L}$$

Rearranging this we find

$$V = \frac{\rho L}{A} I$$

We then define a quantity called the ***resistance*** R of the conductor, where

$$\boldsymbol{R \equiv \frac{\rho L}{A}}$$

The resistance is a function of the type of material (given that it is dependent on the resistivity) but also on the size and shape of the material (given the dependence on length and area).

Finally then, our more "practical" version of Ohm's law is

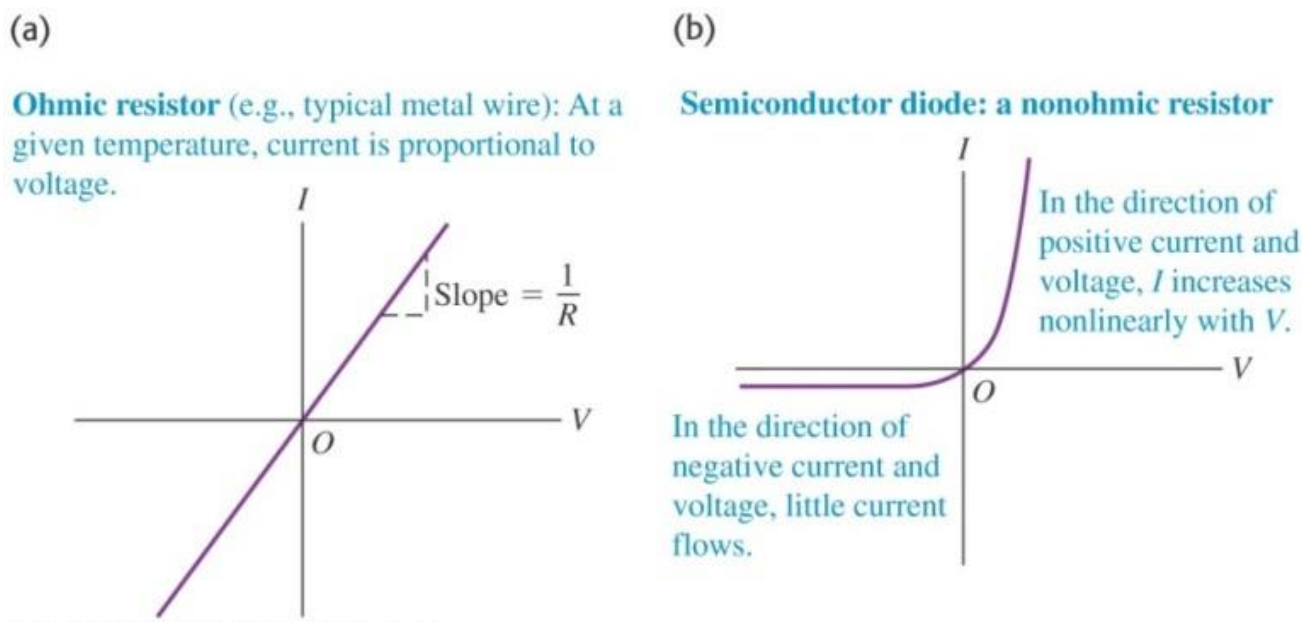
$$\boldsymbol{V = IR}$$

where the resistance is a constant for a given piece of conducting material and depends on the material in question as well as its size and shape.

The units of resistance are Volts/Amp, but this gets its own special name, the Ohm Ω.

Thus 1 Ω = 1V/A.

For a material to obey Ohm's law, the resistance must be constant (not dependent on voltage), so that the voltage and current have a linear relationship. For a nonohmic material, the current and voltage will have a more complicated relationship. In the two graphs, which plot current vs. voltage, note that the first figure depicts an ohmic material whereas the second figure depicts a nonohmic material.



Conductors, being excellent at allowing electrons to flow, generally have extremely low resistances, and in fact, we often ignore their resistance entirely! (*By ignoring the resistance, we are stating that the electric potential difference is zero from one end of the conductor to the other*). On the other hand, materials which have higher resistances are often designed to have a specific value of resistance (this again depends on the type of material as well as the shape and size). Such devices are called **resistors**.

To analyze what a resistor does, we need to look at the atomic level. The resistance to the current occurs because the charges, as they flow, are constantly colliding with other electrons and with atomic nuclei. This causes a gain in random kinetic energy. In other words, the thermal (or internal) energy of the resistor increases, producing heat and/or light. For instance, an incandescent light bulb is an example of a resistor. The electron collisions in the bulb's wire produce significant heat and light. In a resistor, therefore, electrical potential energy (directly related to the electric potential) is converted to thermal energy. The greater the electric potential difference (voltage) between one end of the resistor and the other, the greater the change in electric potential energy and the more energy converted from electrical to thermal. We will further discuss the energies involved in current flow further in class.