Pre-lecture Notes Section 3:

Gauss' Law

I. Introduction to Gauss' Law

In this section, we will learn of an alternative way to think about, and to calculate, the electric field produced by a given system of charges. In a theoretical sense, Gauss' Law is a major stepping stone to a deep understanding of both electric and magnetic fields and their behavior. In a practical sense, Gauss' Law often gives us a way to solve for the electric field in scenarios where it would be extraordinarily difficult to do so using the Coulomb's law two-step method of the previous section.

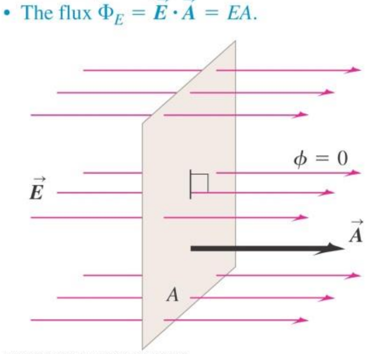
II. Electric Flux

To understand Gauss' law, we first need to understand the concept of electric flux, which relies crucially on the idea of electric field lines. Conceptually, electric flux can be thought of as a measure of the number of electric field lines passing through a given surface in space. Notice that the concept of electric flux relies on the surface that is being used. One important fact is that this surface does not have to actually physically exist! We often imagine a surface existing, and then ask how many field lines are passing through the surface. Such an imaginary surface is often called a "**Gaussian surface**".

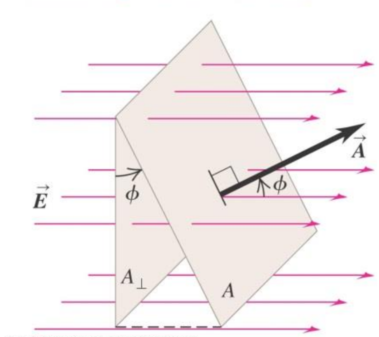
The next question we might ask is the following: how do we quantify this notion of "amount of electric field passing through a surface? To start with, we note three factors which should affect the answer.

1. The electric field strength: The stronger the electric field, the denser the lines, so more lines pass through the surface and the greater the flux.
2. The area of the surface: The more surface area, the more space for the electric field lines to pass through and the greater the flux.
3. Finally, we note one final factor: Regarding the two pictures to the right, in which case is there more electric flux passing through the surface? Even though the electric field and surface area is the same in both pictures, the answer is scenario a), because of the way the surface is angled relative to the electric field lines. Therefore the angle of the surface relative to the electric field matters.

(a) Surface is face-on to electric field:
- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.



(b) Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.



To quantify this third factor, we define an infinitestimal **area vector** $d\vec{A}$, which is a vector pointing normal to an infinitesimal patch of area on the surface. You can see the area vector in the pictures to the right. The magnitude of the area vector is the surface area of the surface. With this definition, all three factors above are taken into account in the following formula for the electric flux Φ_E.

$$\Phi_E \equiv \int \vec{E} \cdot d\vec{A}$$

The integral is over all of the little $d\vec{A}$ patches on the surface. Techincally, because you are integrating over a two-dimensional surface, this is called a "double integral" (which is a concept discussed in detail in calculus 3). However, in cases of interest to us, the integral will become much simpler, and the double integral will be easy to evaluate and the electric flux will take on a much simpler form. See the first pre-lecture video for an explanation of this simplification.

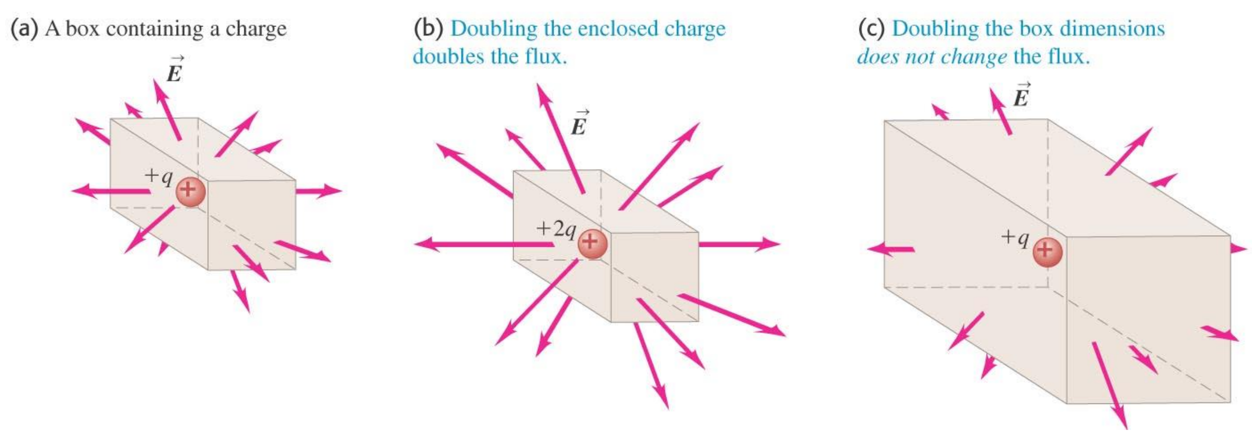
III. <u>Flux Through Closed Surfaces</u>

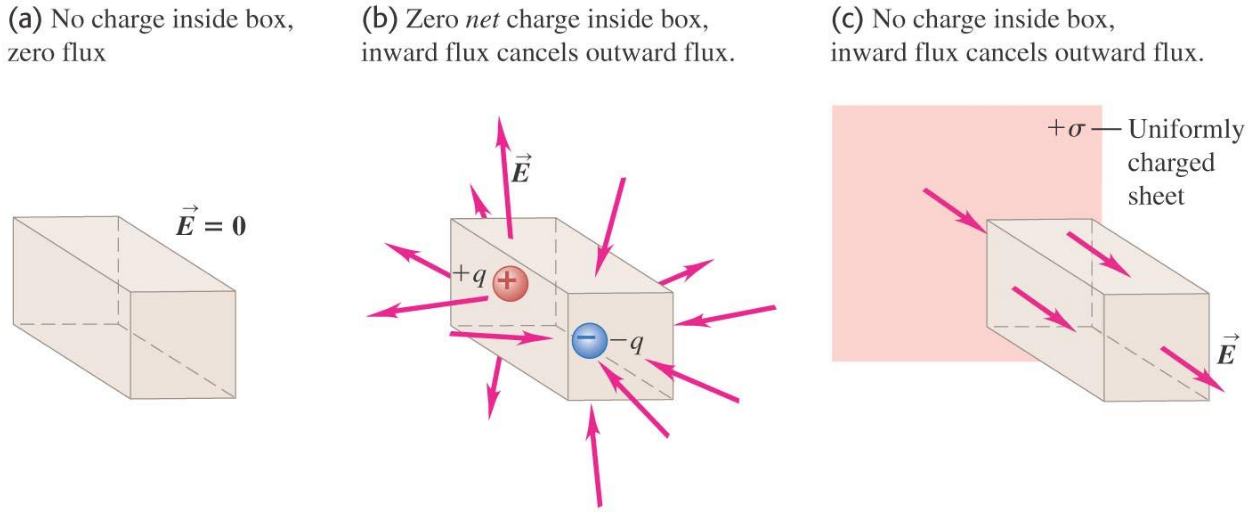
Now we wish to focus strictly on the electric flux through <u>*closed surfaces*</u> (i.e. surfaces with no boundary). Some examples of closed surfaces are cubes, spheres and cylinders, although a general closed surface will not necessarily be a nice symmetric shape.

The following observations about electric flux through closed surfaces lay the groundwork for Gauss' law:

1. The total electric flux through a closed surface is directly proportional to the net electric charge enclosed by the surface. In other words, the more charge inside the surface, the more electric field lines produced by the charge and the greater the flux through the surface.
2. The volume and shape of the closed surface does not affect the total amount of flux passing through the surface.
3. Electric charges placed <u>*outside*</u> of the closed surface produce zero net electric flux through the surface. This is because if the charge is outside of the surface, the electric field lines enter and then leave the surface, producing zero net flux.

All of these observations can be deduced from looking at the following images.





These observations relate a defined quantity, the electric flux, to the amount of charge inside a closed surface. That relation, which is conceptually the essence of Gauss' law, is the only physical statement in this section. Our next step is to make this mathematical!

IV. <u>Gauss' Law</u>

The following is the statement of Gauss' law:

The total electric flux passing through <u>*any closed surface*</u> is proportional to the net electric charge enclosed by the surface. The proportionality constant is $^{1}/_{\epsilon_0}$.

Mathematically, this reads

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

where the "circle" on the integral sign means that the flux must be computed through a <u>*closed*</u> surface. The presence of the constant $^{1}/_{\epsilon_0}$ is necessary so that Gauss' law yields the same answer for the electric field of a point charge as does Coulomb's law. This equivalence is shown in class.

In a practical sense, Gauss' law is extremely useful for calculating electric fields when the system of charges is in some way symmetric. This is because in those cases the integral for the electric flux can be calculated relatively easily. We do several examples and activities in class.