Pre-lecture Section 10: Sources of Magnetic Field

I. Introduction to Sources of Magnetic Field

Based on our findings in section 9, we now know the magnetic force that a magnetic field exerts on a moving test charge. With step 2 filled in, our model can be written as

Step 1. A source *moving* charge q produces a magnetic field $\vec{B} = ???$

Step 2. A test *moving* charge q_0 placed in the magnetic field $\vec{B}$ experiences a magnetic force $\vec{F}_B = q_0\vec{v} \times \vec{B}$

In this section, we hope to understand step 1, the creation of the magnetic field by a moving charge.

II. The Magnetic Field of a Steady Current: The Law of Biot and Savart

In 1820, Biot and Savart discovered the law describing the magnetic field created by a current-carrying wire carrying a steady current. The law of Biot and Savart is as follows:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

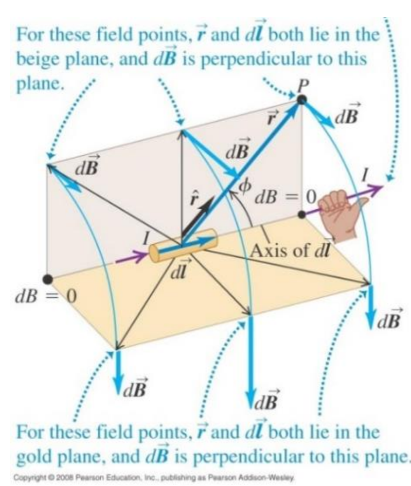
where μ_0 is a constant called the permeability of free space, I is the magnitude of the current, $d\vec{l}$ is the infinitesimal displacement vector in the direction of the current, and $\vec{r}$ is the vector which points from the source point to the field point. μ_0 is a fundamental constant which is equal to $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$.

The law of Biot and Savart is somewhat complicated to use, but we can still learn a few things without doing any calculations. Firstly, we see that the magnitude of the magnetic field depends on

1. the amount of moving charges (current) flowing ($B \propto I$)
2. the distance from the wire (encoded by r; B is inversely proportional to r)

Since the law involves a cross product, the magnetic field will always point perpendicular both to the direction of the moving charges (encoded by $d\vec{l}$) as well as the direction of the $\vec{r}$ vector. As with any cross product, we must use the right hand rule, summarized in the steps below:

- Point your thumb points in the direction of the current ($d\vec{l}$)
- Point your fingers point from the source point to the field point (along the $\vec{r}$ vector).
- Curl your fingers by 90 degrees to find the direction of the magnetic field (see picture).



III. The Magnetic Field of Two Important Current Configurations

Magnetic Field of an Infinite Wire

Our first important current configuration is an infinite, straight current-carrying wire. The steps are tedious, but the answer for the magnetic field magnitude is relatively simple:

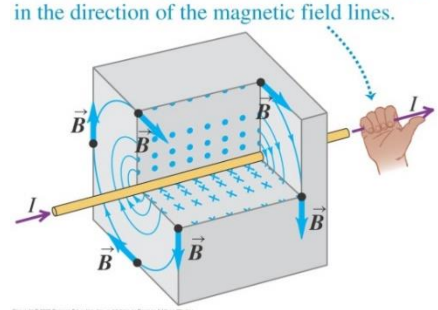
$$B_{inf\ wire} = \frac{\mu_0 I}{2\pi r}$$

We see that similar to the electric field of an infinite wire, the magnitude is inversely proportional to the distance from the wire.

This is just the magnitude, but we also need to be able to find the direction of the field. Using our right hand rule method from above, we can summarize the direction of the magnetic field from the straight wire by using the RHR in the following way:

Point your thumb in the direction of the current. Curl your fingers around the wire. This is the direction of the field, as it circles around the wire in complete loops. See figure.

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.

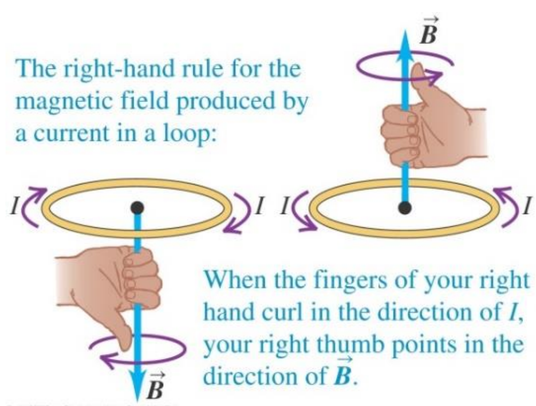


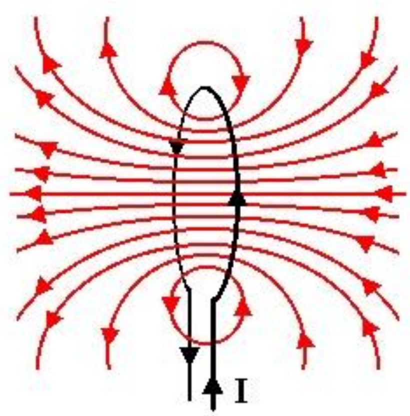
Magnetic Field of a Circular Loop

We next consider a very different shape of wire, that of a circular loop, whose magnetic field can also be determined via the Biot-Savart law. The most important takeaway is not the formula, but the direction of the loop's magnetic field, which also comes from our general RHR method.

RHR for a Current Loop:

Curl your fingers in the direction that the current is oriented. Your thumb will then point in the direction of the magnetic field along the axis of the loop. The field lines then circle back around along the outside of the loop, always forming complete loops. See the two figures below.





IV. <u>Ampere's Law</u>

In electrostatics, we learned that we can gain information about the electric field in two ways:

1. Use Coulomb's law and the law of superposition to find the electric field produced by some distribution of stationary electric charges.
2. Use Gauss' law: The electric flux through a closed surface is directly proportional to the total electric charge enclosed by that surface.

The law of Biot and Savart is much like Coulomb's law; it tells us how to find the magnetic field by integrating over an infinite number of infinitesimal current lengths $d\vec{l}$.

We would like something similar to Gauss' law, but we know the magnetic flux through a closed surface is zero. It turns out that the analog to Gauss' law comes not from integrating over a closed surface, but instead over a closed path (also called a "line" or a "curve"). Such an integral is called a ***<u>line integral</u>***. A closed line integral of the magnetic field $\vec{B}$ is denoted by

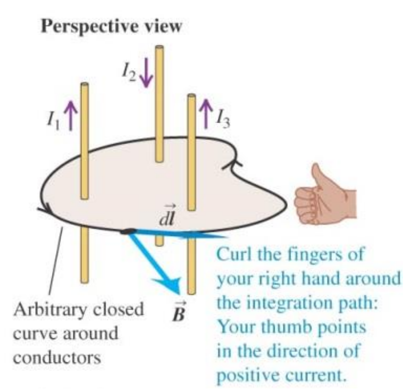
$$\oint \vec{B} \cdot d\vec{l}$$

where $d\vec{l}$ denotes the infinitesimal displacement along the path. This is the basis for Ampere's Law, which says the following:

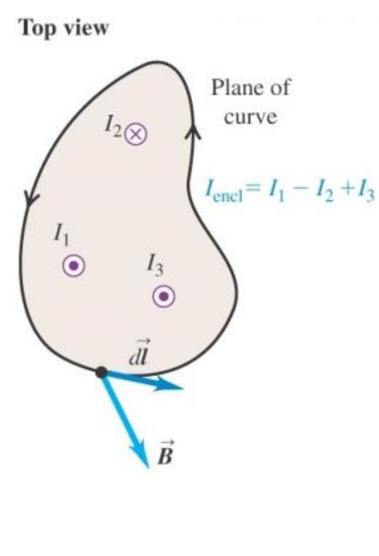
Ampere's Law: The line integral of the magnetic field around a closed path is directly proportional to the net current enclosed by the path. The constant of proportionality is μ_0.

Mathematically, the statement is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$



For instance, in the figures to the right, we see that there are three currents enclosed by the chosen closed curve. From Ampere's law, we know that the line integral of the magnetic field over the closed path is exactly equal to μ_0 times the net current enclosed, $I_1 - I_2 + I_3$. (*The negative sign comes from the fact that the second current is going opposite the direction of the right hand rule circled around the path, as shown in the caption to the "Perspective view" figure*).

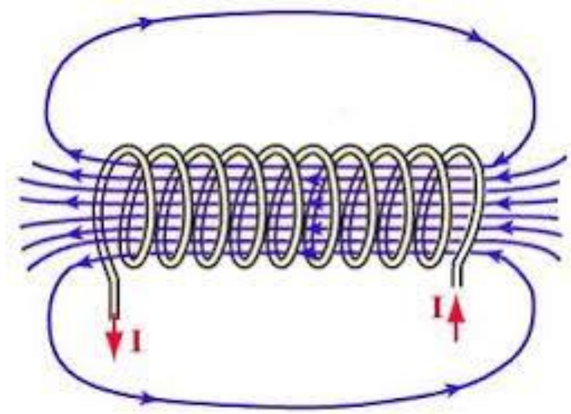


As we will see in class, Ampere's law allows a very simple way to find the magnetic field of a number of current distributions, including that of an infinite, straight wire.

Magnetic Field of a Solenoid

There is one other configuration of current which is of great importance for practical use, and whose magnetic field can be determined most easily by use of Ampere's law. This is the configuration of a solenoid.

A solenoid is a wire that has been wrapped in circles many times consecutively, forming a cylindrical shape, as in the picture to the right. An ideal solenoid is one in which the "cylinder" has an infinite length and each winding of wire is completely adjacent to the neighboring winding, so that the windings are completely perpendicular to the axis of the cylinder.



Using Ampere's law, the magnetic field of an ideal solenoid is (*we will prove in class*)

$$B = 0 \quad \text{outside the solenoid}$$

$$B = \mu_0 n I \quad \text{inside the solenoid}$$

where $n \equiv \frac{N}{L}$ is the number of turns N, or windings, per length L of the solenoid.

The direction of the magnetic field can be found in exactly the same way as for the circular loop; the direction of the field is along the axis of the solenoid and points in the direction specified by the RHR (see picture).

This result is important for two reasons.

- The magnetic field outside the ideal solenoid is zero. A solenoid therefore is a device which creates a magnetic field, but only in a certain region (namely, inside the solenoid). The magnetic field outside the solenoid completely cancels out.
- The magnetic field inside the ideal solenoid is uniform, as there is no dependence on position in the formula for the magnetic field, and the direction is always along the axis, no matter where you are located.

Therefore, the solenoid is the magnetic analog to the two parallel plate set-up in electrostatics. Just as the parallel plates produce a uniform electric field inside and zero electric field outside, the solenoid produces a uniform magnetic field inside and zero magnetic field outside. Of course, in both cases, the set-ups are idealized. In the parallel plate case, the plates must be infinite in size, and in the solenoid case, the solenoid must have infinite length and no space between the windings. However, in many cases, the actual physical situation is close enough to the ideal to provide an accurate approximation.