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V. Lucic & A. Sepp

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Valuation and hedging of cryptocurrency inverse options

V. LUCIC† and A. SEPP‡*

†Imperial College and Marex, London, UK
‡LGT Bank, Zurich, Switzerland

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Currently, the most liquidly traded options on the crypto underlying are the so-called inverse options. An inverse option contract is quoted and traded in the units of the underlying cryptocurrency. The main economic reason for the popularity of inverse contracts in the crypto exchanges (such as Deribit) is that inverse contracts enable traders to operate without maintaining fiat cash accounts. For the theoretical part, we show that inverse options are just regular vanilla options considered under the martingale measure using the forward of the underlying as the numéraire. This measure requires an adjustment to option delta. For the empirical part, we use Deribit options data of past five years to backtest delta-hedged option strategies. We introduce USD and Coin accounting of trading Profit&Loss (P&L) which is important for designing strategies in crypto options. We show empirically that USD and Coin accounting rules are equivalent when performance is measured is Coin and USD units, respectively. We establish that the risk-premia observed in options on Deribit is negative and significant so that strategies selling volatility are expected to generate positive risk-adjusted performance in the long-term.

Keywords: Inverse options; Perpetual futures; Deribit exchange; Change of numéraire; Cryptocurrencies

JEL Classifications: C02, G12, G23

1. Introduction

Options on digital assets have developed significantly over the recent years, both on the classical exchanges (CME, Deribit) but also within the DeFi space, specific to the digital assets only. Amongst the centralised exchanges (CEX) hosting options on digital assets, Deribit has a majority of the market share, covering 86% of the open interest as of August 2023. One of specific features of trading options on Deribit is that the exchange does not deal with fiat currencies, so that the price and the hedge ratio of the options are expressed in the digital currencies (BTC, ETH) rather than the usual 'payout currency' (in this case USD).

Pricing of options on cryptocurrencies has been addressed in a number of studies, with a particular focus on European vanilla options. From a fundamental point of view, Cao and Celik (2020) propose an equilibrium-based valuation model for options on Bitcoin which takes into account the fundamental factor (the total supply of tokens) and jump risk of price dynamics. Madan *et al.* (2019) apply variance gamma models for pricing and calibration of vanilla options on Bitcoin. Hou *et al.* (2020) apply an affine stochastic volatility model with co-jumps in returns and volatility for vanilla vanilla options on cryptocurrencies. Hilliard and Ngo (2022) apply a jump-diffusion model with stochastic convenience yield for the valuation of vanilla options on Bitcoin. Alexander *et al.* (2023) consider the valuation of crypto inverse options in the Black-Scholes model using FX setup in Garman and Kohlhagen (1983). This covers practically the most important case of the Deribit exchange, and it is closely related to the current work.

For the fundamental part, by extending the earlier note Lucic (2021), we show that pricing and hedging of the inverse options naturally fit into the well-established no-arbitrage

*Corresponding author. Email: artursepp@gmail.com

framework developed some 30 years ago. We show that the delta of inverse options must be adjusted by the premium, which also occurs for some types of FX options. Our paper provides with a thorough derivations and analysis of valuation and delta-hedging of inverse options under a generic model and with an intuition behind using the Black-Scholes model for hedging inverse options. Our inputs are important because inverse options play a vital role in crypto ecosystem. In fact, Deribit exchange and a few providers of crypto data analytics use Blacks futures model (Black 1976) for marking and risk metrics of inverse options. This is the first major contribution of this work.

From the empirical point of view of hedging crypto options†, Alexander and Imeraj (2023) analyse the hedging of Bitcoin inverse options using data from Deribit exchange and utilising a few of skew-related adjustment for Black-Scholes delta. In a similar empirical study, Matic *et al.* (2023) apply various stochastic volatility and jump models to test the effectiveness of hedging Bitcoin options. In both studies, it is not clear whether the authors use the premium adjustment for the inverse option Deltas (which comes directly out of the numéraire change).

We augment these empirical studies by providing a backtest performance of delta-hedged strategies in inverse options on Bitcoin and Ethereum in both USD and coin measures. This is the second main contribution of this work. We note that the investment community in digital assets is split between USD-focused investors, who measure the performance of their digital holdings and strategies relative to stablecoins or USD, and crypto-focused investors, who measure their performance relative to Bitcoin or Etherium. This framework naturally blends with the direct (USD) and inverse (BTC) options. To account for this, we introduce the USD and Coin-based accounting rules for Profit&Loss (P&L in short) of systematic option strategies. We show that, if an investor wants to keep a long bias to cryptocurrencies, she must follow Coin-based accounting. On the other hand, USD-based accounting produces market-neutral exposure over the long-time‡. Our backtest is performed at the industry standard, with realistic transaction costs fully incorporated.

Finally, we contribute to the empirical studies of volatility risk-premia observed in options on Bitcoin and Ether. Using Deribit options data for the past five years, we apply the backtest simulation of systematic roll-based strategies selling and buying option with dynamic delta-hedging. We define carefully the implementation of the delta-hedge using perpetual inverse futures. We show that options feature negative risk-premia (higher for BTC and lower for ETH) indicating that buying and delta-hedging put and call options produce negative returns in the long run§. To our knowledge, our empirical study of systematic strategies for crypto options appears to be first in the existing literature.

Our paper is organised as follows. In Section 2, we apply the arbitrage-based approach for the valuation of inverse options. In Section 3, we derive the delta for hedging inverse options. In Section 4, we consider the inverse perpetual futures and their application for delta-hedging of inverse options. In Section 5, we develop the methodology for the simulation of systematic strategies for inverse options using USD- and Coin-based accounting of P&L. We then apply this methodology for backtesting simulation of typical option strategies and discuss the volatility risk-premia observed in cryptocurrency options on Deribit exchange.

2. Arbitrage-based valuation

We provide a general approach for valuation and hedging of inverse options using martingale-based valuation and the numéraire invariance.

2.1. Securities

We denote the price of the underlying cryptocurrency at time T by S_T, and we assume that the price is quoted in USD. The option underlying could be either the spot or the futures contract. We consider 'regular' call and puts payoffs settled in USD cash at maturity time T, respectively, as follows

$$u^{\text{call}}(S_T) = \max\{S_T - K, 0\}, \quad u^{\text{put}}(S_T) = \max\{K - S_T, 0\} \tag{1}$$

where K is the USD strike price.

The payoffs of 'inverse' call and put options are converted to units of the underlying asset at maturity T as follows

$$\begin{aligned} r^{\text{call}}(S_T) &= \frac{1}{S_T}\max\{S_T - K, 0\}, \\ r^{\text{put}}(S_T) &= \frac{1}{S_T}\max\{K - S_T, 0\}. \end{aligned} \tag{2}$$

Finally we consider the 'regular' forward and futures contracts with payoff given by

$$u^{\text{futures}}(S_T) = S_T - K, \tag{3}$$

where $K = S_0$. The corresponding 'inverse' futures contract has the following payoff in number of units

$$r^{\text{futures}}(S_T) = \frac{1}{S_T}(S_T - K). \tag{4}$$

† We do not address the empirical specification of dynamics for cryptocurrencies and their realised and implied volatilities. While a robust specification of the empirical dynamics may be important, most of market practitioners use market implied volatilities and Black-Scholes model with various adjustments for hedging their options exposures. The overview of empirical dynamics of cryptocurrency volatilities can be found, for example, in Huang *et al.* (2022), Teng and Härdle (2023).

‡ An analogy with the traditional finance would be a strategy for the S&P 500 index: for USD-based accounting, the performance is measured in USD, while for Coin-based (units-based) accounting, the performance is measured in accumulated units of the S&P 500 index.

§ The academic literature defines the volatility risk premia as the spread between realised and implied volatilities, while practitioners define the risk-premia the other way around as the spread between implied and realised volatilities. We stick with the academic definition of the volatility risk-premia, which implies that the observed risk-premia in cryptocurrencies is negative so that performance of strategies buying options is expected to negative. In this content, what practitioners' definition of volatility risk-premia refers to strategies selling options so practitioners' risk-premia is positive.

Deribit supports only (perpetual and termed) futures with inverse payoffs. Some crypto exchanges support both types with most liquidity available for inverse futures. We consider in detail the perpetual inverse futures in Section 4 because this instrument is vital for delta-hedging.

2.2. Martingale valuation

We consider a continuous-time market with a fixed horizon date $T^* > 0$ defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$. We consider the price process S_t adapted to $\{\mathcal{F}_t\}$ which models prices of cryptocurrencies. When dealing with incomplete markets, e.g. dynamics including stochastic volatility or jumps, we fix a martingale measure using specific risk preferences (see for an example Lewis 2000).

We introduce a risk-free rate† $r(t)$ with the value of one unit of the money market account (MMA) $M(T)$ given by

$$M(T) = e^{\int_0^T r(s)\,\mathrm{d}s}. \tag{5}$$

We introduce an equivalent martingale measure $\mathbb{Q}$ corresponding to the numéraire M. Given payoff function $u(S_T)$ at time T, we denote the time-t value of an option yielding this payoff by $V(t, S, \cdot)$, where $\cdot$ stands for additional state variables (for an example, stochastic volatility). By martingale valuation we obtain

$$V(t, S, \cdot) = M(t)\mathbb{E}\left[\frac{1}{M(T)}u(S_T)\middle|\mathcal{F}_t\right], \tag{6}$$

with expectation $\mathbb{E}$ taken under the MMA measure $\mathbb{Q}$. Under the measure $\mathbb{Q}$ the forward price is given by

$$\mathbb{E}[S_T \mid \mathcal{F}_t] = S_t e^{\int_0^T (r(s)-q(s))\,\mathrm{d}s}, \tag{7}$$

where we assume deterministic interest rate $r(t)$ and $q(t)$ is deterministic borrow rate‡ .

Next, we define the inverse martingale measure $\tilde{\mathbb{Q}}$ as the one corresponding to S as numéraire. The option value function, denoted by $\tilde{V}(t, S)$, with the payoff paid in units of S, equals to

$$\tilde{V}(t, S, \cdot) = \tilde{\mathbb{E}}\left[\frac{1}{S_T}u(S_T)\middle|\mathcal{F}_t\right], \tag{8}$$

with expectation $\tilde{\mathbb{E}}$ taken under the inverse martingale measure $\tilde{\mathbb{Q}}$.

† Most DEX-es and CEX-es (such as Deribit) assume zero discount rate when marking their listed options. We would call r as a low-risk opportunity cost available in DeFi with the risk being a potential hack of blockchain technology when deposited and staked assets could be appropriated. Staking of high quality stablecoins in top DeFi protocols would yield $1\% - 2\%$ in the current environment, which is far less than rates on government short-term bonds in traditional markets ($4\% - 5\%$ as of October 2023).

‡ For a CEX, the borrow rate q is the negative of the funding rate reported by the CEX, By the convention of crypto CEX-es, the positive (or negative) funding rate is the rate paid (or received) by traders with long positions. In opposite, the positive (or negative) funding rate is the rate received (or paid) by traders with short positions. For a DeFi protocol, the borrow rate q is the accrued borrow rate.

For future reference, we state the change of numéraire theorem (e.g. Theorem 1 in Geman *et al.* 1995) applied to this setting.

THEOREM 2.1 *Assume a complete market and that both the MMA measure $\mathbb{Q}$ and the inverse measure $\tilde{\mathbb{Q}}$ are equivalent martingale measures. Then the values of options under the MMA measure in equation* (6) *and under the inverse measure are in equation* (8) *satisfy*

$$M(t)\mathbb{E}\left[\frac{1}{M(T)}u(S_T)\middle|\mathcal{F}_t\right] = S_t\tilde{\mathbb{E}}\left[\frac{1}{S_T}u(S_T)\middle|\mathcal{F}_t\right]. \tag{9}$$

We call the MMA measure as USD measure and the inverse measure as BTC measure. Theorem 2.1 establishes the no-arbitrage equality between the value $V(t, S, \cdot)$ of regular European option under the USD measure and the value $\tilde{V}(t, S, \cdot)$ of inverse option under the BTC measure given as follows

$$V(t, S, \cdot) = S_t\tilde{V}(t, S, \cdot) \tag{10}$$

where $\cdot$ includes state variables in addition to S. Thus given a quoted price (on Deribit exchange) of the inverse option, we obtain the price of the regular option by multiplying by the underlying price.

The assumption of the existence of the USD and BTC measures in Theorem 2.1 is readily established under the log-normal Black-Scholes dynamics because these dynamics are strict martingales under both MMA and inverse measures. To apply models with stochastic volatility (SV), we may need to impose certain restrictions on model parameters. For an example, exp-OU SV model is not arbitrage-free when the asset-volatility correlation is positive (see Sepp and Rakhmonov 2023 for details and for the valuation of inverse options under a log-normal stochastic volatility model).

3. Hedging

COROLLARY 3.1 (Delta hedge of inverse options) *Differentiating equation* (10) *with respect to S, we obtain the hedge ratio for the inverse options*

$$\partial_S\tilde{V}(t, S, \cdot) = \frac{1}{S}\left[\partial_S V(t, S, \cdot) - \tilde{V}(t, S, \cdot)\right] \tag{11}$$

Thus, the delta-hedge of the inverse option must be implemented using an inverse perpetual or a termed futures contract with payoff in equation (4) with the units of this inverse contract computed by

$$\tilde{\Delta}(t, S, \cdot) = \Delta(t, S, \cdot) - \frac{V(t, S, \cdot)}{S} \tag{12}$$

where $\Delta(t, S, \cdot)$ is the delta hedge (number of units in the underlying asset or regular futures) of the regular option.

This is essentially the ‘percentage spot Delta’ used in FX options pricing (e.g. Reiswich and Wystup 2010, Clark 2011). We note that Corollary 3.1 presents a model-independent result (for a complete market), which allows to compute the model delta for inverse options using model price and delta for regular options.

The USD value of the options on Deribit exchange is expressed as a multiple of the underlying futures with the same expiry, which implies that the options are priced under the measure corresponding to the futures/forwards as numéraires. Theorem 2.1 applies to that setting by using S_t are the price of the future underlying the option. Then option delta $\tilde{\Delta}(t, S, \cdot)$ in equation (12) specifies the number of contracts on the underlying inverse future for the delta-hedging. Note that when using forward as the numéraire, the option valuation formula gives undiscounted price, as the portfolio is expressed in terms of the forward contract which exchanges cashflows at the option expiry. This is possibly a way for the exchange to avoid dealing with the USD discounting.

The hedging ratios for the inverse options can also be obtained directly from the replicating portfolio for the 'regular option' using the numéraire invariance participle. We provide a detailed account in the next section.

3.1. Black-Scholes model

We provide an alternative derivation of delta in equation (12) and the intuition behind it for Black and Scholes (1973) and Black (1976) model. By the convention of crypto exchanges, traded options are options on term futures contract with same expiry date.

3.1.1. Hedging with the inverse contracts. We note that on the Deribit exchange, the tradable option price V_t^{BTC} is quoted in BTC units. Deribit exchange applies future-based approach for marking option prices and deltas in BTC units using Black (1976) model for options on futures. Let F_t^T be term futures contract settled at time T. We consider the USD-denominated replicating portfolio for a vanilla call option on F_t^T struck at $K > 0$ and expiring at $T > 0$. Option price in USD is given by Black (1976) formula

$$V_t^{USD} = P_t^T \left(F_t^T N(d_1(t)) - KN(d_2(t))\right), \tag{13}$$

where $d_{1/2}(t) = \frac{\ln(F_t^T/K)}{\sigma(K,T)} \pm \frac{1}{2}\sigma(K,T)$, $N(x)$ is the CDF of a standard Normal variable, $\sigma(K,T)$ is the implied volatility for strike K and maturity T, and P_t^T is the price of USD bond maturing at T.

In this setting, the BTC spot value is given by $S_t := P_t^T F_t^T$, so using P_t^T as the numeraire in equation (13), we arrive at the standard formulae for the portfolio and its dynamics in terms of the units of the numeraire:

$$\frac{V_t^{USD}}{P_t^T} = \frac{S_t}{P_t^T} N(d_1(t)) - KN(d_2(t)),$$
$$\mathrm{d}\left(\frac{V_t^{USD}}{P_t^T}\right) = N(d_1(t))\,\mathrm{d}\left(\frac{S_t}{P_t^T}\right).$$

Changing the numeraire to S_t yields

$$\frac{V_t^{USD}}{S_t} = N(d_1(t)) - KN(d_2(t))\frac{P_t^T}{S_t},$$
$$\mathrm{d}\left(\frac{V_t^{USD}}{S_t}\right) = -KN(d_2(t))\,\mathrm{d}\left(\frac{P_t^T}{S_t}\right),$$

hence, as $V_t^{BTC} = V_t^{USD}/S_t$ and $1/F_t^T = P_t^T/S_t$, we have

$$\begin{aligned}\mathrm{d}V_t^{BTC} &= -KN(d_2(t))\,\mathrm{d}\left(\frac{1}{F_t^T}\right)\\ &= \left(F_t^T N(d_1(t)) - \frac{V_t^{USD}}{P_t^T}\right)\mathrm{d}\left(\frac{1}{F_t^T}\right),\end{aligned} \tag{14}$$

where in the last equality we used equation (13). Equation (14) represents the dynamic of the Delta hedge portfolio using the inverse term futures. The incremental P&L of the delta hedge is therefore given by

$$\begin{aligned}P\&L_{t+\delta t}^{BTC} &\equiv \left(\frac{1}{F_{t+\delta t}^T} - \frac{1}{F_t^T}\right)\left(F_t^T N(d_1(t)) - \frac{V_t^{USD}}{P_t^T}\right)\\ &= \frac{F_{t+\delta t}^T - F_t^T}{F_{t+\delta t}^T}\left(V_t^{BTC} - N(d_1(t))\right)\end{aligned} \tag{15}$$

It is important to note that Deribit exchange also displays a non-tradeable USD-nominated value $V_t^{USD\_Deribit} := F_t^T V_t^{BTC}$, which should not be confused with the option USD-nominated spot value $V_t^{USD} := S_t V_t^{BTC} = P_t^T F_t^T V_t^{BTC}$ introduced above.

For future reference we define

$$\Delta_t^{USD} = F_t^T N(d_1(t)) - \frac{V_t^{USD}}{P_t^T}, \tag{16}$$
$$\Delta_t^{BTC} = V_t^{BTC} - N(d_1(t)), \tag{17}$$

where in the last equation we recognise the premium-adjusted Delta from equation (12). We term the premium-adjusted Delta as the Net Delta in line with Deribit conventions. It is remarkable that Net Delta Δ_t^{BTC} does not depend on the discount bond P_t^T because V_t^{BTC} is the quoted price in BTC units which is exogenous† . Net USD Delta Δ_t^{USD} shows the USD amount (cash-delta) of delta exposure to F_t^T.

3.1.2. Illustration. In figure 1 we illustrate the comparison of Black delta, $\Delta = N(d_1(t))$, vs Net Delta, computed using equation (17), for fixed strikes as functions of F_t^T for the at-the-money call (Subplot (A)) and the at-the-money put (Subplot (B))‡ . It is clear that Net Delta under-hedges in-the-money (ITM) calls and over-hedges out-of-the-money (OTM) puts. In an extreme example for a call with strike at zero, Black delta is 1 while the Net delta is zero. In this case, the hedge using Black delta means that the hedged portfolio is USD-neutral, while the hedge using Net delta means that the hedged portfolio is BTC-neutral.

† Deribit has a proprietary model to fit implied Black-Scholes volatilities from bid-ask prices of inverse options using $V_t^{USD} = F_t^T V_t^{BTC}$, where F_t^T is the mark price of the underlying futures, to convert their prices to prices of regular options and to compute respective Black and net deltas.

‡ Github project https://github.com/ArturSepp/StochVolModels provides Python code with these computations.

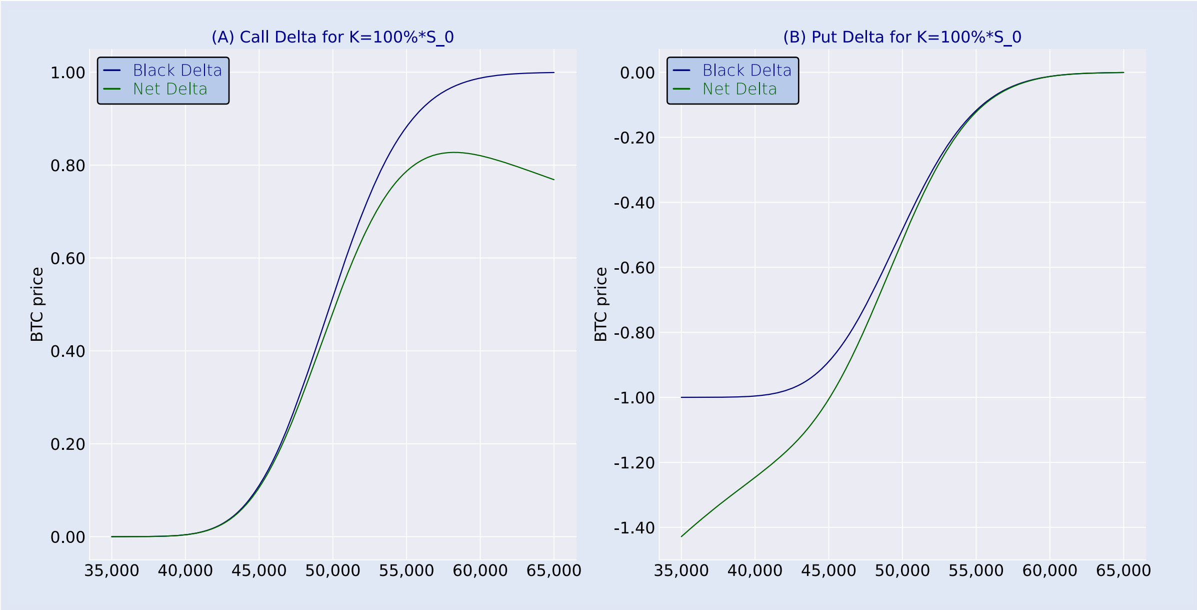

Figure 1. Comparison of Black delta, computed using $\Delta = N(d_1(t))$, vs Net Delta computed using equation (17), for fixed strikes as functions of F_t^T. Subplot (A) shows the delta of ATM call, Subplot (B) shows the delta of ATM put. Model parameters are $F_0^T = 50000$, $\sigma = 0.60$, $T = 7/365$.

In figure 2, we show one-day P&L of delta-hedged short position in ATM call (Subplot (A)) and in ITM call (Subplot (B)). The one day P&L in BTC units is computed by:

$$\mathit{Total\ P\&L}_{\delta t} = \left(V_0^{BTC} - V_{\delta t}^{BTC}\right) - P\&L_{\delta t}, \tag{18}$$

where the hedge P&L $P\&L_{t+\delta t}$ is computed the rhs of equation (15) with $\delta t = 1/365$.

The profile of BTC P&L of the delta-hedged short call is a typical inverted parabola. In the labels, we show breakevens which are the lower and upper % changes in BTC so that the 1-day P&L is zero. For ATM call, the hedge using Net Delta produces a symmetric parabola with the close absolute value of breakeven returns: -3.1% vs 3.1%. While Blacks delta produces an upside bias for delta-hedged position with breakeven returns: -2.5% vs 3.9%. In particular, when using Black delta for ITM call the residual BTC delta is proportional to option premium so the delta-hedged position has an upside potential similar to a digital option, while underperforming on the downside. As a result, hedging using Blacks delta in rising markets is more profitable than hedging using net delta at the expense of higher beta to BTC performance.

4. Inverse perpetual futures

The perpetual futures is a core delta-one product on cryptocurrency exchanges, including the Deribit exchange, because, like a traditional futures, the perpetual (perp in short) futures can be traded on margin with flexible leverage. Unlike a traditional futures, the perpetual futures does not have an expiration date and it is rolled continuously. The carry of the perpetual future is determined by the so-called funding rate, which is designed to keep the price of the futures contract F_t close to the spot price S_t.

The underlying asset of option contract is the termed futures contract with the same settlement as option maturity. However, the market of termed futures is much less liquid than the market of perpetual futures. Here we consider in depth the utilisation of perpetual future for delta hedging of cryptocurrency options using Deribit exchange as the key example. Other crypto exchanges follow similar rules for perpetual futures traded on their platforms.

4.1. Deribit conventions

We consider the method applied by Deribit exchange to compute the mark price of the perpetual future. We then apply this method for P&L computations in both live and historical simulations. The Deribit exchange computes the three market quantities as follows† .

(1) S_t is the spot price (the so-called Deribit index price), which is computed by Deribit and which tracks a basket of cryptocurrency spot prices across other major crypto exchanges.

(2) FP_t is the fair price of the perpetual futures computed as the average of fair impact ask and bid prices as

$$FP_t = \frac{1}{2}\left(\mathit{Fair\ Impact\ Bid}_t + \mathit{Fair\ Impact\ Ask}_t\right) \tag{19}$$

where Fair Impact Bid and Ask Prices are average prices at which 100, 000 USD worth of contracts can be bought and sold, respectively (which are computed from the current order book).

(3) F_t is the mark price of the perpetual futures computed by

$$F_t = S_t + \mathrm{EMA}(FP_t - S_t) \tag{20}$$

† See https://www.deribit.com/kb/deribit-inverse-perpetual for details.

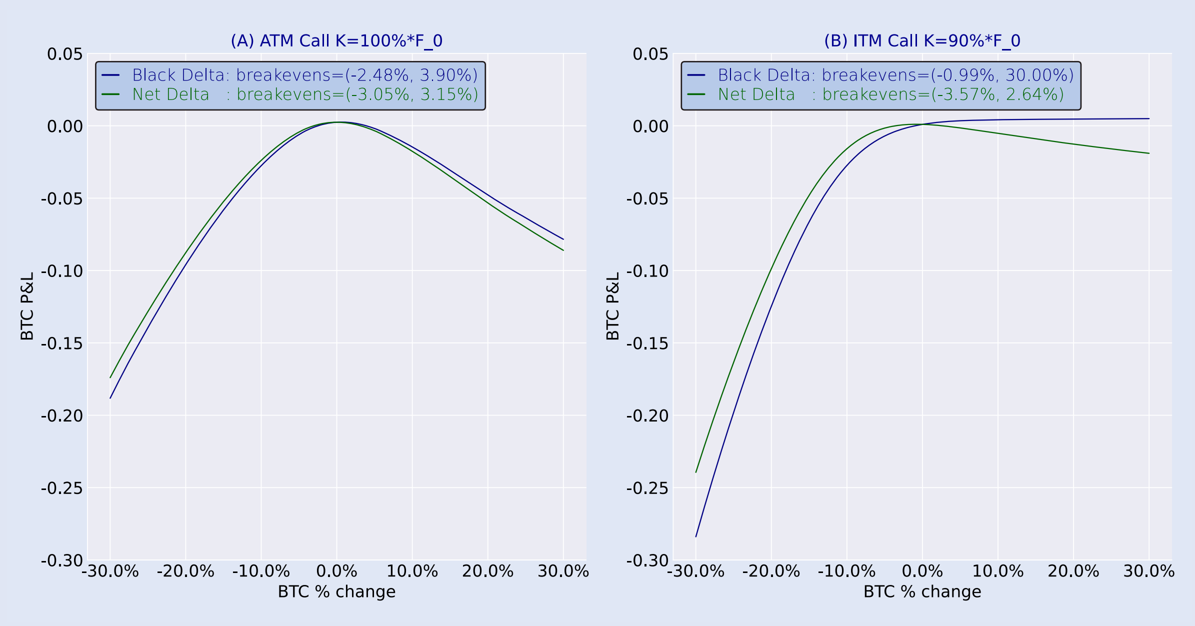

Figure 2. Comparison of 1-day BTC P&L with $\delta t = 1/365$ using Black delta, $\Delta = N(d_1(t))$, vs Net Delta, computed using equation (17), for fixed strikes as functions of S_t. (A) shows delta-hedged P&L of short ATM call with $K = S_0$, (B) shows delta-hedged P&L of short ITM call with $K = 0.9S_0$. Breakevens are the lower and upper % changes in BTC so that the 1-day P&L is zero. Model parameters are $S_0 = 50000$, $\sigma = 0.60$, $T = 7/365$.

where EMA is 30 s exponential moving average.

Once these quantities are computed, the funding rate f_t on Deribit is computed by:

$$f_t = \max\{0.025\%, PremiumRate_t\} + \min\{-0.025\%, PremiumRate_t\} \quad (21)$$

where *PremiumRate* is computed by:

$$PremiumRate_t = \frac{F_t - S_t}{S_t} \quad (22)$$

If the price of perpetual future F_t trades at a premium to the index price S_t (indicating there is a buying pressure) higher than 0.025%, long holders of the contract pay the funding rate f_t to short holders, and vice versa.

Deribit accrues funding fees continuously through the day based on the position exposures using the prevailing funding rate updated every second. The prevailing rate (annualized to a default holding period of 8 h) is reported continuously by Deribit. Account's equity (in BTC and ETH) is credited/debited with accrued funding rate P&L daily at 8.00 AM. As a result, at end of each 24-h period, a trader receives a weighted time average of the funding rate over the past 24 h which is weighted by trader's exposure to BTC or ETH. This convention is reflected in formulas (29) and (30) for our historical backtest.

When a trader opens an inverse futures contract, the notional exposure in USD, denoted by N^{USD}, is fixed at the time of trade and it stays constant during the lifetime of the trade. At the same time, BTC exposure to the perpetual futures is floating as N^{USD}/F_t. In the opposite, for a linear futures contract, the number of BTC units is fixed through the lifetime of the trade, while USD exposure is floating.

The P&L in BTC for an inverse futures contract with notional N^{USD} over the period $[t, t + \delta t]$ is computed by:

$$\text{P\&L}^{BTC}(t + \delta t) = N^{USD}\left(\frac{1}{F_t} - \frac{1}{F_{t+\delta t}}\right) \quad (23)$$

where F_t is the mark price of the perpetual future quoted by Deribit and computed by equation (20). The corresponding P&L in USD is given by

$$\text{P\&L}^{USD}(t + \delta t) = N^{USD}\left(\frac{1}{F_t} - \frac{1}{F_{t+\delta t}}\right) S_{t+\delta t}. \quad (24)$$

The interpretation of equation (24) is that BTC P&L in equation (23) is evaluated using the mark price, which is close to a liquidation price of the position in the perp futures for a relatively small position, while the P&L in USD computed at the prevalent spot rate. As a result, when the position is closed at time $t + \delta t$, the P&L in BTC is determined using mark price $F_{t+\delta t}$ and the P&L in USD is determined by swapping BTC P&L into USD using the index price $S_{t+\delta t}$†.

4.2. *Delta-hedge implementation*

For practical implementation, we consider delta hedge implementation with perpetual futures following Section 3.1. Given a portfolio of K options, we compute aggregated delta Δ^{BTC}_{perp} of the portfolio for delta-hedging using the inverse perp future

† The Deribit exchange supports trading in the spot (although it is much less liquid compared to perps market), so this approach is viable for converting P&L to USD-based accounts. Otherwise, BTC P&L can be transferred to other wallets and exchanges to spot prices in other trading venues.

as follows

$$\Delta^{BTC}_{perp}(t,T) = -\frac{1}{F_t}\sum_{k'} F_t^{T^{k'}} M_{k'}\Delta^{BTC}_{k'}(t,T), \tag{25}$$

where $\Delta^{BTC}_{k}(t,T)$ is Net Delta with respect to the futures contract as defined in equation (17) as marked by Deribit (we do not apply any skew adjustment to the quoted delta and leave this analysis for future research), M_k is the position size, $F_t^{T^k}$ and F_t are the marked prices of the T^k-term and the perpetual inverse futures, respectively.

For the backtest computations, we assume a time-grid $\{t_h\}$ of delta-hedge rebalancing with perpetual inverse future F_t using formula (25), the cumulative P&L over times $\{t_h\}$ is computed by

$$\begin{aligned}
\text{Roll P\&L}^{BTC} &= \sum_{t_h}\Delta^{USD}_{t_h}\left(\frac{1}{F_{t_h}} - \frac{1}{F_{t_h+\delta}}\right)\\
&= \sum_{t_h}\Delta^{BTC}_{t_h}F_{t_h}\left(\frac{1}{F_{t_h}} - \frac{1}{F_{t_h+\delta}}\right)\\
&= \sum_{t_h}\Delta^{BTC}_{t_h}\left(\frac{F_{t_h+\delta} - F_{t_h}}{F_{t_h+\delta}}\right),
\end{aligned} \tag{26}$$

where Δ^{USD}_{t} is computed using equation (25). This roll P&L is augmented with the delta funding costs in BTC which we define in equation (29).

The corresponding Roll P&L in USD terms is computed using equation (26) as follows

$$\text{Roll P\&L}^{USD} = \sum_{t_h}\Delta^{BTC}_{t_h}\left(\frac{F_{t_h+\delta} - F_{t_h}}{F_{t_h+\delta}}\right)S_{t_h} \tag{27}$$

where S_{t_h} is the spot rate. The interpretation is that time-t_h P&L in BTC is swapped to USD using the spot BTC-USD rate S_{t_h}. We further add hedging costs in USD defined in (30).

It is clear that Roll P&L^{BTC} evaluated as the current BTC-USD rate is different from Roll P&L^{USD}:

$$\text{Roll P\&L}^{BTC} \neq \frac{1}{S_t}\text{Roll P\&L}^{USD} \tag{28}$$

4.3. Funding costs

Since the options on Deribit exchange reference termed futures contracts with the same settlement date as the option expiry date, prices of termed futures are applied for computing deltas and settlement prices of listed options. However, termed futures are less liquid than the perpetual future. Moreover for some maturities (typically new maturities of front two weeks), futures are not traded, with Deribit exchange using a linear interpolation for quoting. As a result, for delta-hedging of options, we convert delta exposures to termed futures into the exposure to the perpetual futures, in order to reduce transaction costs for delta-hedging purposes.

We use the average hourly funding rate on hourly basis, $f(t_q)$ to compute the realised costs from t_0 to T as follows

$$\textit{Delta Funding}^{BTC}(t;t_0,T) = -\frac{1}{8}\sum_{t_q\in(t_0,t]} f(t_q)\Delta^{BTC}_{perp}(t_q,T), \tag{29}$$

here $f(t_q)$ is the hourly average of Deribit funding rate reported every second (which is applied for the time period of 8 h so that we normalise it by the factor of 8), t_q is hour timestamps during period $(t_0,T]$, $\Delta^{BTC}_{perp}(t_q)$ is portfolio delta computed using equation (25) on hourly basis. The corresponding USD value of delta funding is

$$\textit{Delta Funding}^{USD}(t;t_0,T) = -\frac{1}{8}\sum_{t_q\in(t_0,t]} S_{t_q}f(t_q)\Delta^{BTC}_{perp}(t_q,T). \tag{30}$$

where we use the spot price S_t to compute the accrued funding in BTC to USD at the prevailing spot exchange rate S_t.

The minus sign in equations (29) and (30) reflects the convention of Deribit (and of all other crypto exchanges) that a long positions pays the funding rate and, in opposite, a short position receives the funding rate.

In top subplots of figure 3, we show the funding rates for BTC and ETH perpetual future as reported by Deribit. We see that BTC and ETH funding rates are positive on average, they tend to spike on the positive side during bull regimes (increased number of long positions need to pay to a smaller number of short positions). In the bottom subplot, we show the cumulative funding costs on holding long one perpetual futures contract and costs on holding a dynamic position for delta-hedging of long and short ATM call option (for brevity the position size is one BTC or ETH for each roll implemented weekly). A delta-hedged position in short ATM call is long delta, so that the accrued funding costs are negative in the long term for this position. In opposite, a delta-hedged strategy selling ATM call option is short delta so that the strategy realises positive funding costs in the long term.

From figure 3, we see that the funding costs accrued for a delta-hedged position can be substantial (delta-hedged strategy selling ATM calls accrues −32% total costs over a four-year period or −8% per annum on average). Thus, funding costs must be taken into account for the historical simulation of option strategies.

4.4. Basis risk

We note that the implementation of hedging delta Δ^{BTC}_{perp} in equation (25) exposes a trader to the basis risk when the carry of a term future contract varies from realized funding costs of the perpetual future contract. This is not an uncommon situation in options trading: for instance, in hedging options on equity indices, the first future is often the instrument of choice due to its superior liquidity (this also introduces the task of rolling the hedging future, which is absent here due to the nature of the perpetual contract). We consider this basis risk in detail.

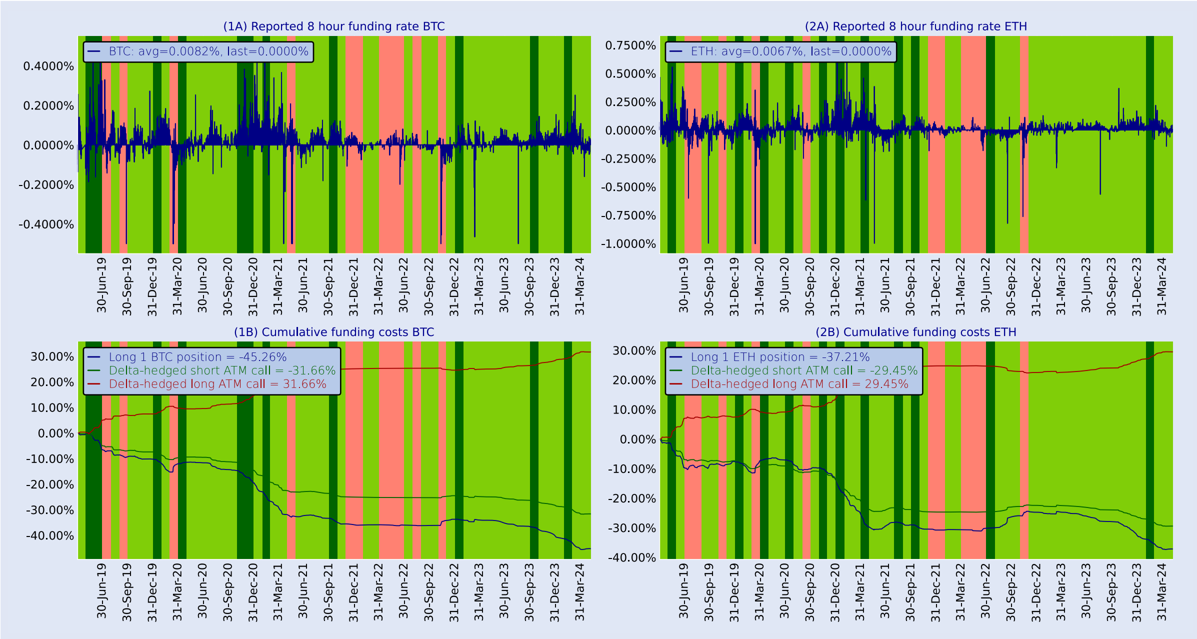

Figure 3. (A) 8 h funding rate reported by Deribit, (B) realised cumulative funding costs for long one coin, and delta-hedged short ATM and long ATM calls. The background colour is obtained by ordering the monthly returns from lowest to highest and the splitting the 16% of worst returns into the 'bear' regime (pink colour), 16% of best returns into the bull regime (dark green colour), and remaining regimes into 'normal' regimes (light green colour).

The dynamics of the time-T expiry Delta hedged option is (c.f. (14) and (16))

$$\mathrm{d}V_t^{BTC} = \Delta_t^{USD}\,\mathrm{d}\left(\frac{1}{F_t^T}\right).$$

The approximate delta hedge implemented in equation (25) is using the perpetual contract F_t:

$$\mathrm{d}\tilde{V}_t^{BTC} = \Delta_t^{USD}\frac{F_t}{F_t^T}\,\mathrm{d}\left(\frac{1}{F_t}\right).$$

We recall that $R_t^T \equiv F_t^T$, $R_t \equiv F_t$. Assuming the one-factor model for the BTC futures curve, we have the following dynamics:

$$F_t^T = e^{-\int_0^t s_u\,\mathrm{d}u}F_t,$$

for some spread process s_t between the carry of the two inverse futures. Thus

$$\mathrm{d}\left(\frac{1}{F_t^T}\right) = \frac{F_t}{F_t^T}\left(\mathrm{d}\left(\frac{1}{F_t}\right) + s_t\,\mathrm{d}t\right),$$

and we have

$$\mathrm{d}\big(V_t - \tilde{V}_t\big) = \Delta_t^{USD}\frac{F_t}{F_t^T}s_t\,\mathrm{d}t = \Delta_t^{USD}s_t e^{\int_0^t s_u\,\mathrm{d}u}\,\mathrm{d}t.$$

It is important that the basis risk scales in δt and the it's sign is related to the sign of the spread, as expected. Empirically, from our experience, the basis risk is compensated by better liquidity of perpetual futures which reduces transaction costs for delta-hedging. We leave an empirical analysis of the trade-off between basis risk and transaction costs for a future study.

5. Backtesting option strategies using deribit data

We consider essential steps for the design and implementation of systematic strategies trading in inverse crypto options. For simulations of systematic option strategies and empirical results, we use Deribit data of all traded options contracts on Bitcoin (BTC) and Ether (ETH) with hourly frequency from 1 April 2019 to 6 May 2024. All data are provided by Tardis data provider. Our approach is generic for all coin-margined trading venues. We use the term Coin as a notation for either BTC or ETH underlying. For clarity, we use notation such as Π_0^{BTC} to denote Coin-measured variables.

5.1. P&L accounting for option strategies

Deribit exchange settles margin requirements and realised trading P&L in Coin so the coin-based accounting is natural when dealing with inverse options. As an alternative, we will consider USD-based accounting with Coin P&L at time t exchanged into USD using BTC-USD (or ETH-USD) spot exchange rate at time t. We show that these accounting rules result in different terminal outcomes and risk profiles, so that, portfolio managers and investors must understand the difference.

We introduce the following accounting methods.

(1) USD-based accounting with the initial deposit/margin funded with USD and with Coin (BTC or ETH) P&L instantly exchanged to USD.
(2) Coin-based accounting with the initial deposit/margin funded with BTC or ETH and with P&L reporting in the same Coin.

For trading in BTC and ETH options at Deribit exchange, a trader needs to fund BTC or ETH, respectively. Thus, USD-based accounting requires depositing Coin collateral and hedging it using perpetual futures and instant swapping of the coin-based P&L into USD. Coin-based accounting is natural for investors who desire to keep the exposure and P&L to BTC or ETH.

We note that a funded USD account could earn interest income. For simplicity, we skip the interest rate from the analysis. The convention of Deribit exchange is to use zero USD and BTC rates for computing mark prices and deltas. In line, we report Sharpe ratios of simulated strategies assuming that the interest rate is zero.

5.1.1. Funded strategies selling options without delta-hedging. We examine some popular strategies in Decentralised Finance (DeFi) which regularly sell options without delta-hedging such as covered call (the strategy is long either BTC or ETH coin and sells at-the-money calls covered by held coins) and put selling (the strategy holds the collateral and sells at-the-money puts against the collateral). Each strategy represents a sequence of non-overlapping rolls (such as weekly or monthly rolls) where options are sold at the inception of the n-th roll and these options are settled using the available collateral at the close of the roll.

Coin accounting

We assume that the strategy is funded with an initial deposit of Π_0^{BTC} coins. The realized P&L in BTC of the n-th roll is given by

$$P\&L^{BTC}(t;t_0^n,T^n) = -N(t_0^n)\left(V^{BTC}(t,T^n) - V^{BTC}(t_0^n,T^n)\right), \tag{31}$$

where t_0^n is the roll inception time, T^n is the maturity time of option in the roll, and t is current time, $N(t_0^n)$ is the amount of BTC contracts sold at the roll inception. For 100% covered strategy, the number of sold contracts equals the amount of coins in the portfolio, $N(t_0^n) = \Pi^{BTC}(t_0^n)$.

We let $\{t_0^n\}$, $n = 1, 2, \ldots$, denote times of the rolls. The cumulative P&L during time period $[t_0^1, t]$ of the strategy realised over n rolls is given by

$$CumP\&L^{BTC}(t) = \sum_{n'=1\ldots n-1} P\&L^{BTC}(T^{n'};t_0^{n'},T^{n'}) + P\&L^{BTC}(t;t_0^n,T^n), \tag{32}$$

where the first term is the cumulative realized P&L of prior $n-1$ rolls and the second term is the mark-to-market P&L of the current roll. For realised P&L of the roll at the end time T^n is computed using the option payoff (settlement) value, $V^{BTC}(T^n,T^n)$, computed by option payoff given spot rate S_{T^n}.

The account value in BTC at time t is the sum of the initial BTC deposit Π_0 and the cumulative P&L

$$\Pi^{BTC}(t) = \Pi_0^{BTC} + CumP\&L^{BTC}(t). \tag{33}$$

The account value in USD is computed using the current BTC-USD rate S_t as follows

$$\Pi^{USD}(t) = S_t\Pi^{BTC}(t). \tag{34}$$

It is important that the initial coin deposit Π_0^{BTC} is a part of P&L attribution for coin-funded strategies when valuing their performance in USD.

USD accounting

Under USD-based accounting, we assume that the initial deposit, denoted by D_0^{USD}, is made in USD and the coin P&L is hedged by swapping realised BTC P&L into USD. Equation (31) for P&L in USD is modified as follows

$$P\&L^{USD}(t;t_0^n,T^n) = -N(t_0^n)\left(S_t V^{BTC}(t,T^n) - S_{t_0^n}V^{BTC}(t_0^n,T^n)\right), \tag{35}$$

where the spot rate $S_{t_0^n}$ is applied to swap option premium at the roll inception and current rate S_t is applied to calculate USD value of the current premium.

The cumulative P&L during time period $[t_0^1, t]$ realised over n rolls is given by

$$CumP\&L^{USD}(t) = \sum_{n'=1\ldots n-1} P\&L^{USD}(T^{n'};t_0^{n'},T^{n'}) + P\&L^{USD}(t;t_0^n,T^n), \tag{36}$$

where for the n-th roll we use the spot price observed at the expiry, S_{T^n}, to convert final payoffs to USD, and we use the spot price at the roll inception, $S_{t_0^n}$, to swap sold premiums to USD.

The account value in USD at time t is the sum of the initial USD deposit D_0^{USD} and the cumulative P&L in equation (36)

$$\Pi^{USD}(t) = D_0^{USD} + CumTotal\ P\&L^{USD}(t), \tag{37}$$

where in line with our assumption we do not consider interest income on USD value of the account. The corresponding account value in measured in Coin is given by:

$$\Pi^{BTC}(t) = \frac{1}{S_t}\Pi^{USD}(t). \tag{38}$$

5.1.2. Margin-based strategies with delta-hedging. We now consider a generic option trading strategy with delta-hedging. At the inception of each roll, the strategy buys and sells a portfolio of options which are held to the close of the roll, which equals options maturity time. We assume that the strategy includes holding of a portfolio with k options (or option legs). The strategy portfolio is delta-hedged regularly through the roll period.

Coin accounting

We assume that the count is funded with the initial coin deposit Π_0^{BTC}. Option P&L of the n-th roll's portfolio is given similarly to equation (31) by

$$Option\ P\&L^{BTC}(t;t_0^n,T^n) = \sum_{k'} N_{k'}(t_0^n)\left(V_{k'}^{BTC}(t,T^n) - V_{k'}^{BTC}(t_0^n,T^n)\right), \tag{39}$$

where $N_k(t_0^n)$ is the number of contracts traded ($N_k(t_0^n) < 0$ if options are sold or $N_k(t_0^n) > 0$ is options are bought).

We use equation (26) to compute the Delta-hedge P&L by

$$Delta\ P\&L^{BTC}(t;t_0^n,T^n)$$

$$= \sum_{t_h} \frac{F_{t_h} - F_{t_{h-1}}}{F_{t_h}} \Delta_{perp}^{BTC}(t_{h-1}, T^n)$$
$$- \eta \sum_{t_h} \left| \Delta_{perp}^{BTC}(t_h, T^n) - \Delta_{perp}^{BTC}(t_{h-1}, T^n) \right|$$
$$+ \mathit{Delta\ Funding}^{BTC}(t; t_0^n, T^n), \quad (40)$$

where t_h are delta re-balancing times with $t_h \in [t_0^n, T^n]$, $\Delta_{perp}^{BTC}(t, T)$ is the position delta in perpetual futures given in equation (25), F_t is the mark price of the perpetual future. The second term is the delta re-balancing costs with η being the trading costs which we assume to be proportional to coin value traded (see Assumption 5.1). The last term $DeltaFunding^{BTC}(t; t_0^n, T^n)$ is the delta funding costs defined in equation (29).

The total P&L of the n-th roll is thus computed by

$$\mathit{Total\ P\&L}^{BTC}(t; t_0^n, T^n) = \mathit{Option\ P\&L}^{BTC}(t; t_0^n, T^n) + \mathit{Delta\ P\&L}^{BTC}(t; t_0^n, T^n). \quad (41)$$

Cumulative P&Ls from option premiums and delta-hedging realised during time period $[t_0^1, t]$ with n rolls are given by

$$\mathit{CumTotalOption\ P\&L}^{BTC}(t) = \sum_{n'=1\ldots n-1} \mathit{Option\ P\&L}^{BTC}(T^{n'}; t_0^{n'}, T^{n'}) + \mathit{Option\ P\&L}^{BTC}(t; t_0^n, T^n)$$
$$\mathit{CumTotalDelta\ P\&L}^{BTC}(t) = \sum_{n'=1\ldots n-1} \mathit{Delta\ P\&L}^{BTC}(T^{n'}; t_0^n, T^n) + \mathit{Delta\ P\&L}^{BTC}(t; t_0^n, T^n), \quad (42)$$

where first terms are realised P&Ls and second terms are P&Ls of the current roll. The cumulative total P&L of the strategy is given by

$$\mathit{CumTotal\ P\&L}^{BTC}(t) = \mathit{CumTotalOption\ P\&L}^{BTC}(t) + \mathit{CumTotalDelta\ P\&L}^{BTC}(t). \quad (43)$$

This decomposition is useful for the P&L explain of an option strategy as we show in Subplot (B1) of figure 6.

The account value in BTC at time t is the sum of the initial BTC deposit Π_0^{BTC} and the cumulative P&L

$$\Pi^{BTC}(t) = \Pi_0^{BTC} + \mathit{CumTotal\ P\&L}^{BTC}(t). \quad (44)$$

The account value in USD is

$$\Pi^{USD}(t) = S_0 \Pi_0^{BTC} + S_t\ \mathit{CumTotal\ P\&L}^{BTC}(t), \quad (45)$$

where S_0 is the value of BTC at the time of initial deposit. It is important that we exclude P&L realised from a change in USD value of the initial deposit so that we use the spot rate S_0. Thus, the value of the margin account reflects only the realized P&L in equation (43) accrued through trading.

USD accounting

We assume that the strategy is funded using the initial USD deposit D_0^{USD} and the coin P&L (which is the native accounting of Deribit exchange) is hedged by swapping realised BTC P&L into USD. Equation (39) for option P&L of the n-th roll is modified as follows

$$\mathit{Option\ P\&L}^{USD}(t; t_0^n, T^n) = \sum_{k'} N_{k'}(t_0^n) \left(S_t V_{k'}^{BTC}(t, T^n) - S_{t_0^n} V_{k'}^{BTC}(t_0^n, T^n) \right), \quad (46)$$

where the value of option premiums at the call inceptions is swapped using the spot rate of $S_{t_0^n}$ and the current value is wrapped using the prevalent rate S_t.

Using equation (27) for conversion of BTC P&L into USD P&L along with equation (40) for delta-hedge P&L is changed as

$$\mathit{Delta\ P\&L}^{USD}(t; t_0^n, T^n) = \sum_{t_h} S_{t_h} \frac{F_{t_h} - F_{t_{h-1}}}{F_{t_h}} \Delta_{perp}^{BTC}(t_{h-1}, T^n)$$
$$- \eta \sum_{t_h} S_{t_h} \left| \Delta_{perp}^{BTC}(t_h, T^n) - \Delta_{perp}^{BTC}(t_{h-1}, T^n) \right|$$
$$+ \mathit{Delta\ Funding}^{USD}(t; t_0, T). \quad (47)$$

Term $DeltaFunding^{USD}(t; t_0^n, T^n)$ is the delta funding costs in USD defined in equation (30).

The total USD P&L of the n-th roll is computed by

$$\mathit{Total\ P\&L}^{USD}(t; t_0^n, T^n) = \mathit{Option\ P\&L}^{USD}(t; t_0^n, T^n) + \mathit{Delta\ P\&L}^{USD}(t; t_0^n, T^n). \quad (48)$$

Cumulative USD P&Ls from option premiums and delta-hedging realised during time period (t_s, t) with n rolls are given by

$$\mathit{CumTotalOption\ P\&L}^{USD}(t) = \sum_{n'=1\ldots n-1} \mathit{Option\ P\&L}^{USD}(T^{n'}; t_0^{n'}, T^{n'}) + \mathit{Option\ P\&L}^{USD}(t; t_0^n, T^n)$$
$$\mathit{CumTotalDelta\ P\&L}^{USD}(t) = \sum_{n'=1\ldots n-1} \mathit{Delta\ P\&L}^{USD}(T^{n'}; t_0^n, T^n) + \mathit{Delta\ P\&L}^{USD}(t; t_0^n, T^n). \quad (49)$$

The cumulative total USD P&L of the strategy realised over n rolls is given by

$$\mathit{CumTotal\ P\&L}^{USD}(t) = \mathit{CumTotalOption\ P\&L}^{USD}(t) + \mathit{CumTotalDelta\ P\&L}^{USD}(t). \quad (50)$$

We show the decomposition of USD P&L explain of in Subplot (B1) of figure 7.

The account value in USD at time t is computed as the sum

$$\Pi^{USD}(t) = D_0^{USD} + CumTotal\ P\&L^{USD}(t), \tag{51}$$

where we ignore the USD funding rates. The account value in BTC is computed using the prevalent spot rate S_t:

$$\Pi^{BTC}(t) = \frac{1}{S_0} D_0^{USD} + \frac{1}{S_t} CumTotal\ P\&L^{USD}(t), \tag{52}$$

where we exclude the performance of the initial deposit in BTC using fixed swap rate S_0 observed at strategy inception.

5.2. Illustrative strategies

To implement historical simulation of option strategies using Deribit data, we make the following assumption regarding costs of delta and option trading, and the delta-hedge execution for our backtest simulations.

ASSUMPTION 5.1 (Transaction Costs) *Transaction cost for delta-hedging using perpetual futures is proportional to traded Coin or USD amount with multiple* η *set to* 5*bp*, $\eta = 0.0005$ *(see equation* (40)*).*

Transaction cost for buying and selling an option contract is proportional to traded premium with charge c ratio set to 50*bp*, $c = 0.005$. *The traded premium is computed as the mid of the bid and ask prices.*

In our opinion, these costs are representative of actual trading costs on Deribit exchange.

ASSUMPTION 5.2 (Delta-hedge execution) *Given time series of option prices and deltas, the portfolio delta is computed using equation* (25) *every hour and the delta hedge is implemented using the last price of the perpetual futures.*

In practical applications portfolios delta would be typically rebalanced within bands, to reduce delta-hedging costs. Optimised hedging produces better risk-adjusted results, but it is subject to trader's risk tolerance. Thus, we do not apply any optimisation in this paper and follow the simple time-based re-balancing for brevity.

5.2.1. Call overwrite strategy. We start with a simple call overwrite strategy which is very popular in DeFi. When using this strategy, an investor deposits an amount of Π_0^{BTC} coins which is used as a collateral for selling at-the-money call options on the same coin. P&L from traded options is settled against the remaining collateral. The strategy regularly rolls a short position in at-the-money call options with sizes equal to the present amount of BTC collateral. Naturally, this strategy is implemented using Coin-based accounting.

In figure 4, we show the performance of the call overwrite strategy that sells at-the-money BTC calls with the maturity of one week with weekly rolls on every Friday. The initial deposit is 1 BTC. As a benchmark, we use long only strategy with holds 1 BTC.

In subplot (A1) we show the coin performance of the strategy using equation (33). When BTC rallies, realized roll P&L is negative so that the amount of BTC held in the strategy drops. In subplot (A2), we show the scatter plot of the daily coin performance vs the daily BTC return. It is clear that the strategy is short call so its beta coin exposure is negative.

In subplot (B1), we show USD performance computed using equation (34). For the long-only strategy, the performance corresponds to the price path of BTC-USD rate S_t. The call overwrite strategy in USD partially benefits from BTC appreciation over time because the margin is held in BTC. In subplot (B2), we show the scatter plot of daily returns for USD performance. Now, because of the BTC margin, the strategy in USD has a positive beta to BTC return.

The performance of the call overwrite strategy in BTC is slightly negative, which indicates that selling call options without delta-hedging may not be a profitable way to harvest the short volatility risk-premia. In comparison with long-only strategy in USD, the call overwrite strategy produces a smaller total return. However, the strategy delivers better risk-adjusted return because the short call reduces the long-only portfolio volatility. As a result, we view the call over-write strategy as a risk-mitigation strategy which reduces the portfolio volatility.

5.2.2. Put selling strategy. Put selling strategy is another popular strategy second to call overwrite strategy. When using this strategy, an investor deposits an amount of Π_0^{BTC} coins which is used as collateral for selling at-the-money put options. We note that put selling account must be a margin account because a short position in inverse put option must be liquidated if the spot drops more than 50% of the strike price (in this case, the price of the put will be worth more than one coin which exceeds the initial margin). Realised option P&L is settled against the collateral. The size of each new roll is equal to the present amount of coin Π_t^{BTC}.

In figure 5, we show the performance of the put selling strategy that sells at-the-money BTC puts with the maturity of one week with weekly rolls on every Friday. The initial deposit is 1 BTC. As a benchmark, we use long-only strategy holding 1 BTC. Subplot (A1) illustrates the coin performance of the strategy using equation (33). When BTC crashes, the outstanding margin drops. Subplot (A2) illustrates the scatter plot of the weekly coin performance vs weekly BTC return. The strategy is short put so its beta coin exposure is positive. In subplots (B1) and (B2) we show USD performance computed using equation (34) and the scatter plot of daily returns for USD performance, respectively. The performance of the long only strategy corresponds to the price path of BTC-USD rate S_t.

The performance of the put selling is slightly negative, which indicates that selling put options without delta-hedging may not be profitable, similarly to selling call options. In comparison with long-only strategy in USD, the short put strategy is leveraged to BTC performance because of long delta from the short put exposure and long delta from the deposit. Thus, we view the put selling strategy as a leveraged long exposure to BTC performance.

5.2.3. Delta-hedged short straddle strategy. We now consider a delta-hedged option strategy which systematically sells at-the-money call and put options through regular rolls.

Options are held to maturity and the portfolio is delta hedged. We simulate weekly rolls with delta hedge executed hourly.

Coin accounting

We take the initial deposit of 1 BTC, for each roll the number of sold calls and puts equals to the account value right prior to the roll time: $N(t_0^n) = \Pi^{BTC}(t_0^n)$.

In figure 6, we show the performance of the short straddle strategy. In subplot (A1) we show the performance of the cumulative P&L of the total, option, and delta parts using equation (42). We see that the cumulative option P&L standalone may become negative so that its volatility and Sharpe ratio cannot be computed. Delta-hedge P&L is negatively correlated with options P&L, so that the delta-hedge strategy has smaller volatility and higher risk-adjusted performance. In the long-term both option and delta-hedge P&L are profitable. In contract to selling ATM puts and calls without hedging, which sell fewer sizes after a drawdown, the delta-hedging mitigates drawdowns of the delta-hedged strategy so that the delta-hedged strategy sells bigger option sizes through market cycles.

In subplot (B1), we show the USD performance of the strategy computed using equation (45). The coin account keeps realised profits in BTC so over time it increases the exposure to BTC and the volatility of the strategy increases in USD terms. In subplots (A2) and (B2), we illustrate the scatter plot of strategy weekly returns in BTC and USD, respectively, versus BTC-USD returns. We see that the strategy is close to be delta-neutral for coin accounting, with its returns profile being short convexity. For USD performance, the strategy NAV increases exposure to BTC through accrued profits so that it produces a positive beta to BTC.

USD accounting

For USD accounting, the set initial USD deposit equal to the price of 1BTC, $D_0 = S_0$. Strategy Coin P&L is swapped into USD P&L at the prevailing spot rate. In figure 7, we show the performance of the same strategy as in figure 6 The number of contracts traded $N(t_0^n)$ is set to the BTC value of the account $\Pi^{USD}(t)/S_t$.

In Subplot (A1) we show the USD P&L using equation (52). We see that, when measured in BTC, the strategy with USD accounting indicates a poorer risk-adjusted performance. The risk-adjusted USD performance of the strategy using USD accounting is similar to the risk-adjusted Coin performance of the same strategy using Coin accounting: with Sharpe ratios of 1.0 vs 1.2, respectively. In Subplots (B1) and (B2) we show BTC and USD performance using equation (52), respectively. In BTC terms the strategy is short BTC. In USD terms the strategy is delta-neutral with a classic profile of short convexity.

5.2.4. Discussion. We conclude with an important conclusion regarding Coin-based and USD-based accounting. Strategies with BTC accounting are suitable for investors who would like to have exposure to BTC (through accrued options and delta-hedge P&L). The performance of delta-hedged strategies is BTC-neutral, but their USD performance is not due to the exposure of realised coin P&L.

Strategies with USD-based accounting are suitable for investors who would like to keep a neutral exposure to BTC. These are strategies with delta-hedging and with USD funding by necessity to remove BTC exposure. Delta-hedged USD performances of these strategies are BTC-neutral by

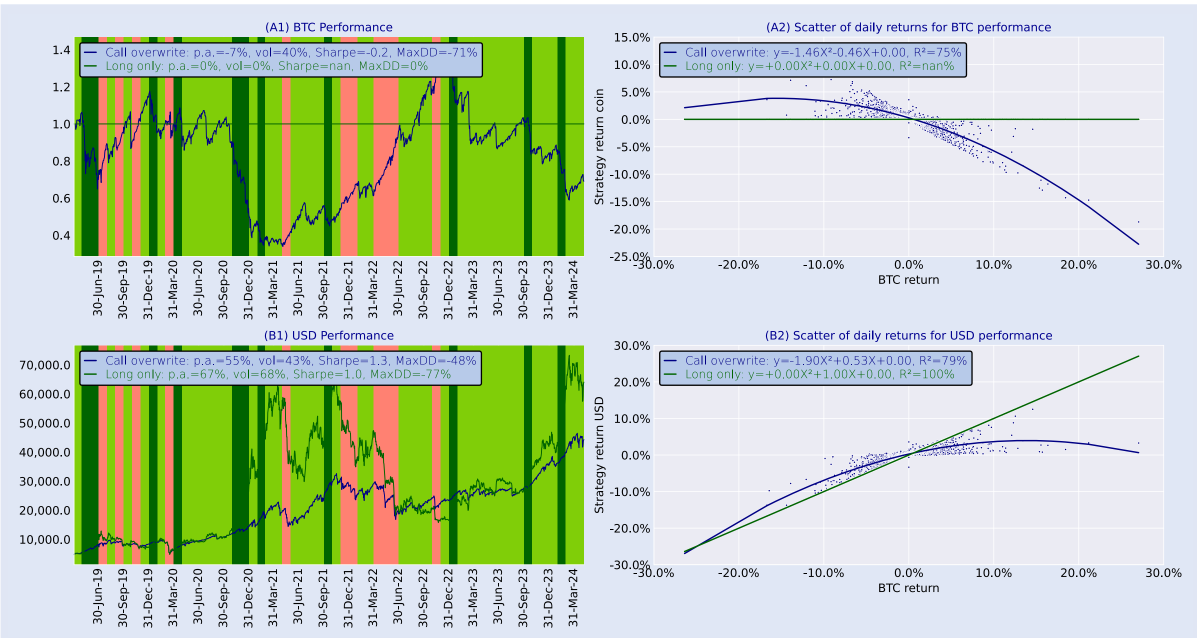

Figure 4. Performances of BTC call option overwrite strategy using Coin accounting and long only strategy holding 1BTC. The simulation time period is from 1 April 2019 to 6 May 2024. Notation in legends of subplots (A1) and (B1): p.a. is annualised return or CAGR, vol is annualised volatility of daily log returns, Sharpe is Sharpe ratio computed using zero risk-free rate, MaxDD is the maximum drawdown; background colours are the same as in figure 3. Notation in legends of subplots (A2) and (B2): the equation is estimated using the quadratic regression with weekly log-returns of the strategy predicted by BTC-USD weekly return, R^2 is the adjusted R-squared of the estimated quadratic regression.

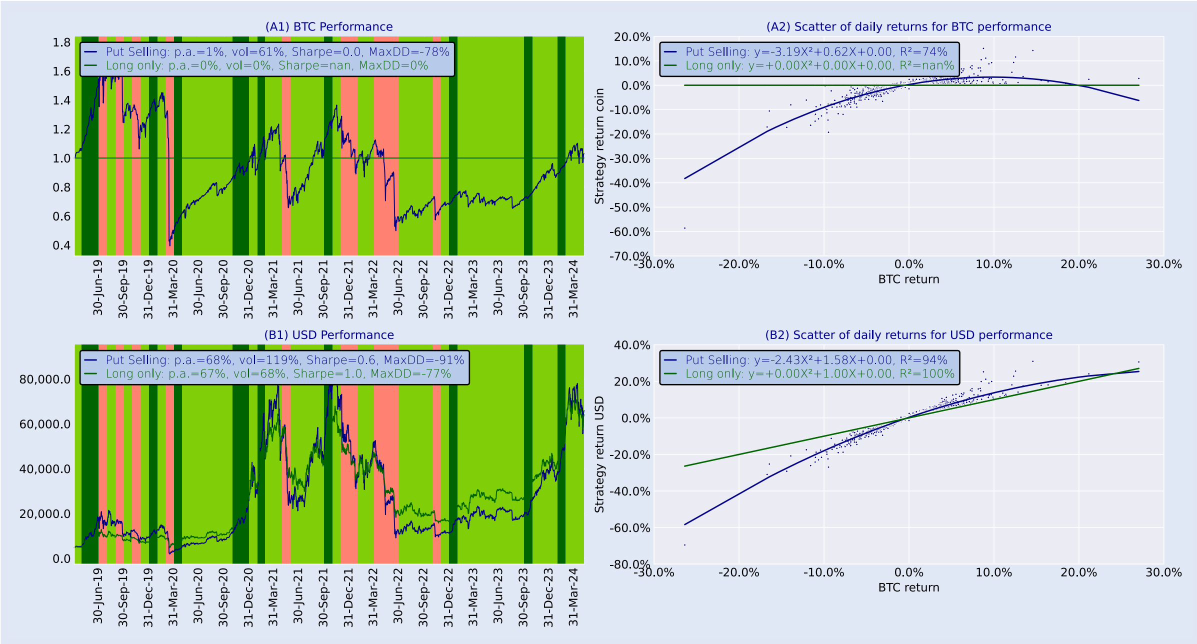

Figure 5. Performances of BTC put selling strategy and BTC long-only strategy. Notations are the same as in figure 4.

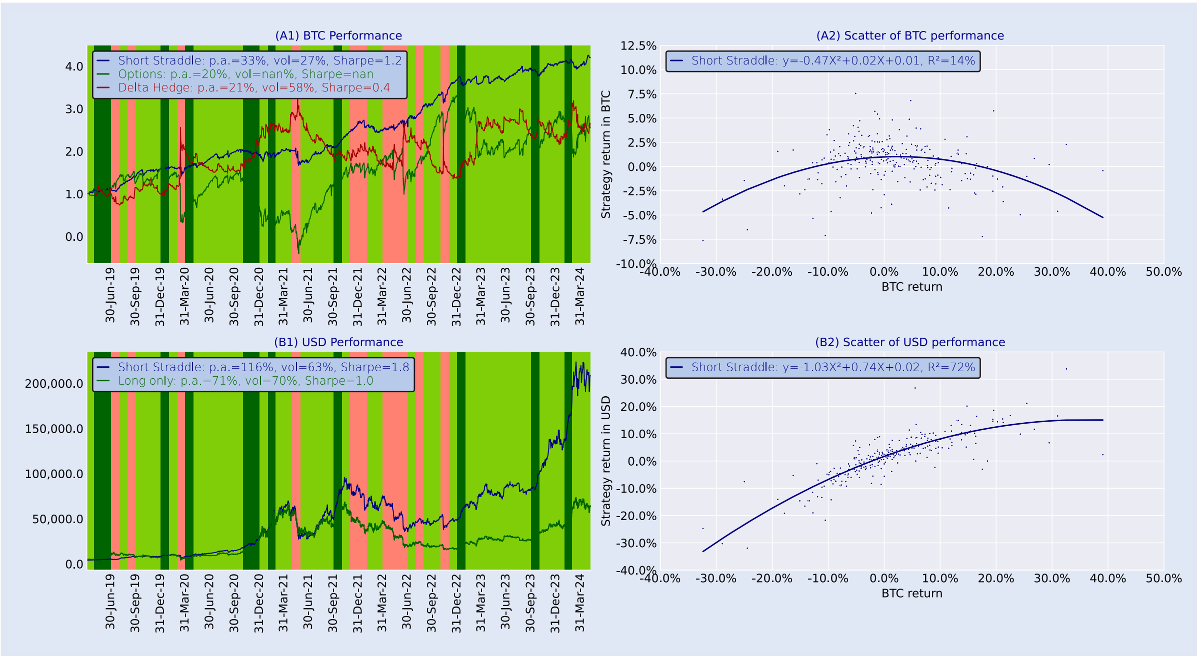

Figure 6. Performance of BTC delta-hedged straddle strategy using Coin accounting. Notations are the same as in figure 4.

construction. However, if BTC out-performances over longer time period, BTC values of realised P&L of these strategies will decline.

Importantly, for a given strategy, the risk-adjusted performance in Coin terms when using Coin accounting is about the same as the risk-adjusted performance in USD terms when using USD accounting. Slight differences may arise from portfolio compounding and trade sizing effects.

5.3. Backtest of systematic delta-hedged strategies

We examine the risk-premia in crypto options traded on Deribit exchange by simulating delta-hedged strategies using Deribit options data. We consider single-leg strategies and multi-leg option strategies. For each strategy, we implement the long and short positions. We implement weekly (Friday every week), monthly (last Friday of a month), and quarterly rolls (last Friday of a quarter). At each roll at 8:00 UTC the existing option position is closed and new position with options expiring on next Friday is opened. For each roll, the number of contracts, or the size, traded is proportional to the current Coin NAV: $N(t) = \Pi^{BTC}(t)$. We note that because the sizing is a path-dependent function of the realized P&L, the performances of long and short positions are not symmetric. We follow the auto-compounding because its most used in practice.

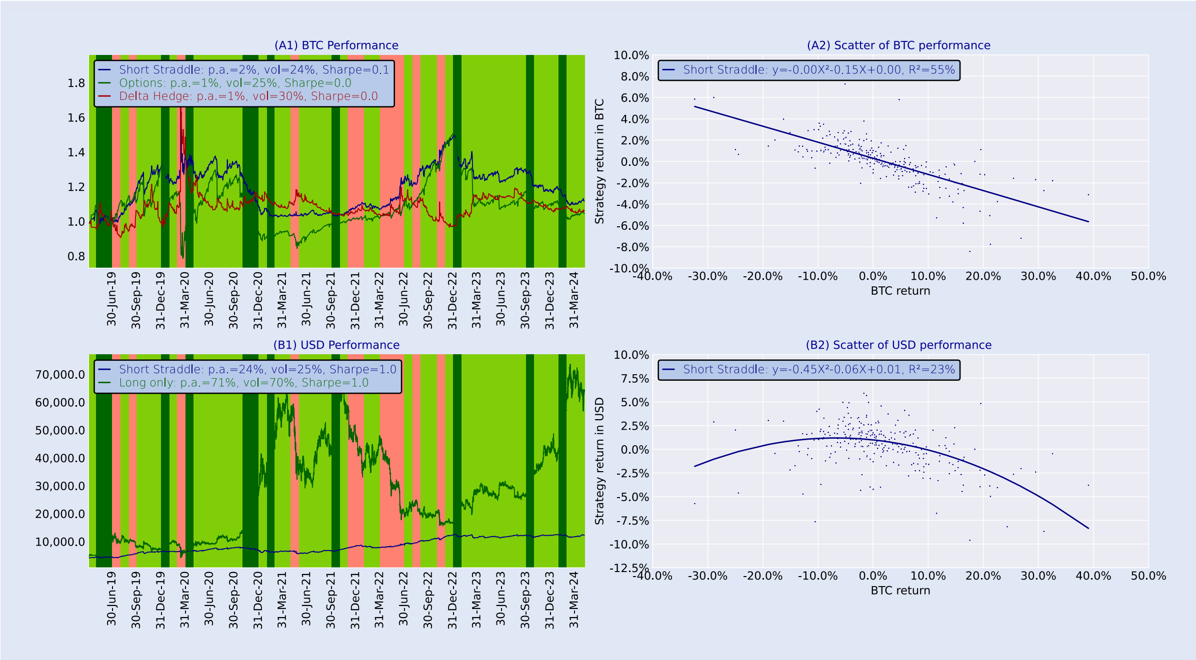

Figure 7. Performance of BTC delta-hedged straddle strategy using USD accounting. Notations are the same as in figure 4.

For computing performance of different strategies, we use Deribit options data from 1 April 2019 to 6 May 2024. We apply assumptions (5.1) and (5.2) in our backtest simulations. When choosing strikes corresponding to given option deltas on the grid, we select the adjacent strike with maximum open interest.

5.3.1. Single leg strategies. Single leg strategies include options with following strikes (NB: we need to decide whether we use Deribit or Blacks delta):

- ATM: calls and puts with at-the-money strikes;
- 25D: options with strikes corresponding to delta of $+0.25$ for calls and -0.25 for puts;
- 10D: options with strikes corresponding to delta of $+0.10$ for calls and -0.10 for puts.

In tables 1 and 2, we show risk-adjusted Coin and USD performances of strategies using Coin-based and USD-based accounting, respectively. We see that, for each strategy, the risk-adjusted Coin performance under Coin-based accounting is close to the risk-adjusted USD performance under USD-based accounting. Both Coin-based and USD-based strategies are market-neutral in the long run as evident by their insignificant beta β to returns of the underlying cryptocurrencies.

It is evident that the risk-premium in Bitcoin options is negative because long option positions are not profitable over a long time period, while short option positions are profitable. Realised Sharpe ratios for shorting ATM, 25D, 10D calls are higher than those for shorting corresponding puts, which indicates extra negative risk-premiums for crypto calls. These risk-premiums for call options on cryptocurrencies may arise from leverage-seeking preferences of crypto investors for obtaining the upside potential. We also see that overall BTC options have higher negative risk-premiums compared to ETH options.

5.3.2. Multi-leg strategies. Single-leg options are either short or long delta, theta, and gamma. Multi-leg strategies allow to create greek-neutral positions so isolate harvesting of the risk-premia in overall volatility or in call or put spreads.

We consider the following multi-leg strategies include the following strategies.

- ATM straddle consists of equal size position in call and put options with at-the-money (ATM) strikes;
- 25D strangle consists of equal size position in call and put options with delta of $+0.25$ and -0.25, respectively;
- 10D strangle consists of equal size position in call and put options with delta of $+0.10$ and -0.10, respectively;
- 25D call spread consists of shorting one unit of call with delta of $+0.50$ and of being long two units of call with $+0.25$ delta;
- 25D put spread consists of shorting one unit of put with -0.50 delta and being long two units of put with delta of -0.25;
- 25D risk reversal consists of shorting one unit of put with -0.25 delta and being long one unit of call with delta of 0.25;
- 25D butterfly ratio consists of long $+0.50$ and -0.50 delta call and put and short -0.25 and -0.25 delta for call and put.

ATM straddle, 25D and 10D strangle strategies are close to being delta-neutral at the inception, they are either long or short gamma, and they produce direct exposure to the volatility risk-premia. Call and put spreads are close to be delta-neutral and they produce exposure to the risk-premia of call skew and put skew, respectively, relative to the ATM volatilities. The long position in the call spread is expected to be profitable if out-of-the-money (OTM) call volatilities

Table 1. Risk-adjusted Coin performances of delta-hedged single-leg option strategies on BTC and ETH using Coin-based accounting for option maturities of one week for period from 1 April 2019 to 6 May 2024.

	Total	P.a.	Vol	Sharpe	MaxDD	Skew	α_{AN}	β	R^2
BTC	908.0%	62.0%	70.4%	0.88	− 77%	− 1.3	0.0%	1.00	100%
Long ATM Call	− 71.8%	− 23.2%	12.4%	− 1.87	− 72%	0.7	− 26.0%	0.00	0%
Long 25D Call	− 56.7%	− 16.0%	9.1%	− 1.76	− 57%	1.8	− 17.1%	− 0.00	0%
Long 10D Call	− 33.3%	− 8.1%	6.3%	− 1.29	− 34%	4.0	− 7.9%	− 0.01	1%
Short ATM Call	125.7%	18.5%	12.4%	1.49	− 8%	− 0.8	17.6%	− 0.00	0%
Short 25D Call	73.3%	12.2%	9.2%	1.32	− 6%	− 1.9	11.8%	0.00	0%
Short 10D Call	31.9%	6.0%	6.5%	0.92	− 7%	− 4.3	5.7%	0.00	0%
Long ATM Put	− 70.2%	− 22.3%	29.4%	− 0.76	− 70%	0.7	− 25.5%	0.01	1%
Long 25D Put	− 54.0%	− 15.0%	25.2%	− 0.59	− 54%	− 0.2	− 15.8%	− 0.00	0%
Long 10D Put	− 35.1%	− 8.6%	21.6%	− 0.40	− 37%	− 0.6	− 8.6%	− 0.00	0%
Short ATM Put	93.7%	14.8%	24.4%	0.61	− 29%	− 1.2	14.9%	− 0.01	1%
Short 25D Put	54.9%	9.6%	20.3%	0.47	− 27%	− 0.3	9.1%	0.00	0%
Short 10D Put	35.1%	6.5%	17.2%	0.38	− 24%	0.1	6.0%	0.00	0%
ETH	1652.5%	81.9%	89.3%	0.92	− 79%	− 1.3	0.0%	1.00	100%
Long ATM Call	− 68.9%	− 21.7%	16.3%	− 1.33	− 70%	0.5	− 24.9%	0.01	0%
Long 25D Call	− 52.3%	− 14.3%	11.7%	− 1.23	− 53%	2.1	− 15.9%	0.01	0%
Long 10D Call	− 33.5%	− 8.2%	6.9%	− 1.18	− 35%	2.7	− 8.5%	0.00	0%
Short ATM Call	96.0%	15.1%	16.5%	0.92	− 14%	− 0.6	15.5%	− 0.01	0%
Short 25D Call	49.7%	8.8%	12.0%	0.73	− 13%	− 2.4	9.8%	− 0.01	1%
Short 10D Call	27.7%	5.2%	7.1%	0.74	− 6%	− 2.9	5.6%	− 0.00	0%
Long ATM Put	− 69.1%	− 21.8%	34.2%	− 0.64	− 69%	2.9	− 23.5%	− 0.01	0%
Long 25D Put	− 43.2%	− 11.1%	28.3%	− 0.39	− 45%	2.7	− 9.8%	− 0.02	2%
Long 10D Put	− 13.4%	− 3.0%	24.7%	− 0.12	− 31%	3.2	− 0.4%	− 0.02	7%
Short ATM Put	71.3%	11.9%	29.0%	0.41	− 36%	− 3.1	11.2%	0.01	1%
Short 25D Put	16.4%	3.2%	23.8%	0.14	− 36%	− 3.2	1.8%	0.02	3%
Short 10D Put	− 3.2%	− 0.7%	20.6%	− 0.03	− 37%	− 4.2	− 2.8%	0.03	8%

Note: Notation: Total is the total performance over the period; P.a. is annualised return or CAGR; Vol is annualised volatility of daily log-returns; Sharpe is Sharpe ratio computed using 0.0% risk-free rate; Max DD is the maximum drawdown; Skew is the skeweness of daily log-returns; α_{AN}, β and R^2 is the intercept, the slope, and R^2 of the linear regression of weekly returns of the strategy vs weekly returns on the underlying coin. The first line highlighted by the blue colour corresponds to performances of BTC and ETH cryptocurrencies, respectively, which are used as benchmarks for regression analysis.

Table 2. Risk-adjusted USD performances of delta-hedged single-leg option strategies on BTC and ETH using USD-based accounting for option maturities of one week.

	Total	P.a.	Vol	Sharpe	MaxDD	Skew	α_{AN}	β	R^2
BTC	908.0%	62.0%	70.4%	0.88	− 77%	− 1.3	0.0%	1.00	100%
Long ATM Call	− 68.3%	− 21.3%	14.3%	− 1.49	− 69%	1.2	− 26.9%	0.05	11%
Long 25D Call	− 56.6%	− 16.0%	10.5%	− 1.52	− 57%	2.0	− 18.4%	0.02	2%
Long 10D Call	− 33.2%	− 8.1%	7.4%	− 1.10	− 34%	4.0	− 8.2%	− 0.00	0%
Short ATM Call	98.0%	15.3%	14.5%	1.06	− 18%	− 1.3	18.5%	− 0.05	10%
Short 25D Call	68.5%	11.5%	10.8%	1.07	− 12%	− 2.2	12.8%	− 0.02	3%
Short 10D Call	30.9%	5.8%	7.6%	0.76	− 8%	− 4.2	6.0%	− 0.00	0%
Long ATM Put	− 64.7%	− 19.5%	22.4%	− 0.87	− 65%	− 0.6	− 24.3%	0.05	12%
Long 25D Put	− 49.9%	− 13.5%	17.5%	− 0.77	− 51%	− 0.6	− 15.1%	0.01	2%
Long 10D Put	− 30.8%	− 7.4%	13.7%	− 0.54	− 33%	− 0.2	− 7.6%	0.00	0%
Short ATM Put	63.2%	10.8%	20.1%	0.53	− 27%	− 0.2	13.7%	− 0.04	11%
Short 25D Put	45.1%	8.1%	15.7%	0.52	− 24%	0.2	8.8%	− 0.01	1%
Short 10D Put	27.1%	5.1%	12.1%	0.42	− 17%	− 0.4	5.1%	− 0.00	0%
ETH	1652.5%	81.9%	89.3%	0.92	− 79%	− 1.3	0.0%	1.00	100%
Long ATM Call	− 63.1%	− 18.8%	19.4%	− 0.97	− 66%	0.5	− 27.0%	0.07	21%
Long 25D Call	− 50.2%	− 13.6%	14.0%	− 0.97	− 53%	2.5	− 17.5%	0.03	9%
Long 10D Call	− 34.4%	− 8.4%	8.9%	− 0.95	− 37%	4.5	− 9.7%	0.01	3%
Short ATM Call	54.8%	9.6%	20.3%	0.47	− 35%	− 0.7	17.2%	− 0.07	21%
Short 25D Call	38.6%	7.1%	14.9%	0.47	− 24%	− 2.9	11.0%	− 0.04	9%
Short 10D Call	26.9%	5.1%	9.5%	0.54	− 14%	− 5.3	6.7%	− 0.02	4%
Long ATM Put	− 61.1%	− 17.9%	27.4%	− 0.65	− 62%	− 0.7	− 23.4%	0.04	13%
Long 25D Put	− 34.6%	− 8.5%	20.3%	− 0.42	− 37%	0.1	− 9.1%	0.01	0%
Long 10D Put	− 10.7%	− 2.3%	16.3%	− 0.14	− 23%	1.3	− 1.0%	− 0.01	3%
Short ATM Put	36.3%	6.7%	24.0%	0.28	− 42%	0.2	11.2%	− 0.04	12%
Short 25D Put	3.1%	0.6%	18.2%	0.03	− 34%	− 0.5	1.4%	− 0.00	0%
Short 10D Put	− 4.8%	− 1.0%	14.6%	− 0.07	− 32%	− 1.9	− 2.1%	0.01	3%

Note: Notations are the same as in table 1.

Table 3. Risk-adjusted performances of delta-hedged multi-leg option strategies on BTC and ETH using Coin accounting for weekly rolls.

	Total	P.a.	Vol	Sharpe	MaxDD	Skew	α_{AN}	β	R^2
BTC	908.0%	62.0%	70.4%	0.88	− 77%	− 1.3	0.0%	1.00	100%
Long Straddle	− 92.8%	− 42.3%	32.0%	− 1.32	− 93%	1.3	− 53.9%	0.01	0%
Long 25D Strangle	− 81.2%	− 29.4%	27.3%	− 1.08	− 81%	0.3	− 33.8%	− 0.00	0%
Long 10D Strangle	− 56.3%	− 15.9%	22.6%	− 0.70	− 57%	− 0.2	− 16.3%	− 0.01	1%
Long Call Spread	− 53.1%	− 14.6%	10.2%	− 1.43	− 55%	2.0	− 15.8%	− 0.00	0%
Long Put Spread	− 44.7%	− 11.6%	23.0%	− 0.51	− 47%	− 1.1	− 10.6%	− 0.02	2%
Long 25D RR	− 27.8%	− 6.6%	21.4%	− 0.31	− 46%	0.3	− 7.2%	0.01	0%
Long 25D ButterFly	− 62.5%	− 18.5%	8.9%	− 2.08	− 62%	0.2	− 20.8%	0.01	1%
Short Straddle	307.5%	34.1%	27.8%	1.23	− 33%	− 1.9	31.3%	− 0.01	0%
Short 25D Strangle	168.4%	22.9%	23.2%	0.99	− 29%	− 0.9	21.0%	0.01	0%
Short 10D Strangle	75.7%	12.5%	18.6%	0.67	− 25%	− 0.4	11.5%	0.01	1%
Short Call spread	20.3%	3.9%	10.3%	0.38	− 20%	− 2.3	4.9%	− 0.00	0%
Short Put Spread	− 2.4%	− 0.5%	19.1%	− 0.03	− 33%	0.5	− 1.7%	0.02	3%
Short 25D RR	− 8.1%	− 1.8%	26.1%	− 0.07	− 35%	− 0.7	− 0.9%	− 0.01	0%
Short 25D ButterFly	28.8%	5.4%	8.9%	0.61	− 12%	− 0.7	6.4%	− 0.01	1%
ETH	1652.5%	81.9%	89.3%	0.92	− 79%	− 1.3	0.0%	1.00	100%
Long Straddle	− 91.9%	− 40.9%	37.5%	− 1.09	− 92%	2.9	− 51.3%	0.00	0%
Long 25D Strangle	− 74.1%	− 24.6%	31.0%	− 0.79	− 75%	2.8	− 26.5%	− 0.01	0%
Long 10D Strangle	− 42.8%	− 11.0%	25.9%	− 0.43	− 45%	3.3	− 8.9%	− 0.02	4%
Long Call Spread	− 47.2%	− 12.5%	12.3%	− 1.01	− 48%	2.1	− 13.8%	0.01	1%
Long Put Spread	− 14.7%	− 3.3%	27.0%	− 0.12	− 33%	2.5	0.1%	− 0.03	5%
Long 25D RR	− 37.7%	− 9.4%	25.6%	− 0.37	− 50%	− 1.6	− 12.3%	0.03	7%
Long 25D ButterFly	− 71.3%	− 23.0%	10.9%	− 2.10	− 72%	0.4	− 27.0%	0.01	2%
Short Straddle	220.8%	27.6%	33.6%	0.82	− 40%	− 3.0	26.6%	0.00	0%
Short 25D Strangle	74.8%	12.4%	27.5%	0.45	− 36%	− 3.1	12.0%	0.01	0%
Short 10D Strangle	25.3%	4.8%	22.1%	0.22	− 28%	− 4.0	3.2%	0.02	4%
Short Call spread	− 2.3%	− 0.5%	12.6%	− 0.04	− 15%	− 2.6	1.3%	− 0.01	1%
Short Put Spread	− 42.6%	− 11.0%	23.0%	− 0.48	− 51%	− 3.1	− 14.0%	0.03	6%
Short 25D RR	− 1.2%	− 0.2%	30.2%	− 0.01	− 32%	1.4	3.0%	− 0.03	6%
Short 25D ButterFly	54.8%	9.6%	10.8%	0.88	− 17%	− 0.7	10.7%	− 0.01	1%

Note: Notations are the same as in table 1.

are cheaper than at-the-money volatilities. Symmetrically, the long position in the bear spread is expected to be profitable if out-of-the-money put volatilities are cheaper than at-the-money volatilities. The risk reversal is close to being gamma neutral and it is profitable if OTM call volatilities are cheaper than OTM put volatilities. The butterfly is close to delta-neutral with negative theta at the inception and it provides the exposure to the risk-premia of the implied convexity (tails of returns distribution). For each strategy, we simulate both the long and short implementation.

For brevity, we skip results of strategies using USD-based accounting because USD risk-adjusted performances are close to Coin risk-adjusted performances of strategies under Coin-based accounting. Thus, we report the risk-adjusted coin performances using Coin-based accounting. To examine if the risk-premia depends on time-to-expiry, we simulate strategies using three rolls: weekly (Friday every week), monthly (last Friday of the month), quarterly (last Friday of the quarter). At every roll event, the strategy trades in options expiring at the roll maturity and holds them to the maturity. Delta hedging is implemented on hourly basis.

In tables 3–5, we show the risk-adjusted coin performances with weekly, monthly, and quarterly rolls, respectively. We see that the performance of all long strategies is negative. This indicates that the risk-premia in crypto options is negative for the level, the put and call skew, and the (tail) convexity. We observe that the most profitable strategies are short straddles, strangles and butterflies, which indicates that implied volatilities are higher that the realised volatilities on average. We see that the risk-premia is most negative for short-dated options.

For weekly rolls, call and put spread strategies with exposures to 25D call and 25D put volatilities relative to ATM volatilites are not profitable either on short or on long side after transaction costs. For monthly and quarterly rolls, short call spread is profitable, indicating that call volatilities for longer-dated options may be overvalues relative to put volatilities.

Risk-adjusted performances for short strategies with quarterly rolls are the poorest for BTC, and negative for ETH. For strategies on ETH with quarterly rolls the risk-adjusted performances of both long and short strategies are insignificant, so without accounting for trading costs the risk-premia may be zero.

We see that strategies with quarterly rolls have higher volatilities and betas to the performance of the underlying cryptocurrency because longer-dated options have higher vega exposures. We conclude that long strategies produce negative betas to the underlying performance because implied volatilities tend to increase when the underlying drops. As a the delta-hedged long strategies benefit from long vega exposure when the underlying drops.

In figures 8 and 9, we show the regime conditional performances of simulated strategies on BTC and ETH underlyings

Table 4. Risk-adjusted performances of delta-hedged option strategies on BTC and ETH using Coin accounting for monthly rolls.

	Total	P.a.	Vol	Sharpe	MaxDD	Skew	α_{AN}	β	R^2
BTC	908.0%	62.0%	70.4%	0.88	− 77%	− 1.3	0.0%	1.00	100%
Long Straddle	− 67.8%	− 21.1%	29.4%	− 0.72	− 69%	9.6	− 22.5%	0.00	0%
Long 25D Strangle	− 55.7%	− 15.6%	24.1%	− 0.65	− 58%	12.0	− 15.3%	− 0.01	0%
Long 10D Strangle	− 25.3%	− 5.9%	18.4%	− 0.32	− 38%	17.9	− 4.2%	− 0.02	2%
Long Call Spread	− 34.9%	− 8.6%	9.9%	− 0.87	− 39%	1.5	− 9.1%	0.00	0%
Long Put Spread	− 26.6%	− 6.2%	19.0%	− 0.33	− 36%	16.1	− 3.5%	− 0.03	5%
Long 25D RR	− 20.7%	− 4.7%	23.7%	− 0.20	− 37%	− 10.5	− 6.2%	0.02	4%
Long 25D ButterFly	− 28.0%	− 6.6%	8.8%	− 0.76	− 33%	0.4	− 7.8%	0.02	3%
Short Straddle	87.7%	14.0%	28.4%	0.49	− 36%	− 9.9	14.1%	0.00	0%
Short 25D Strangle	66.3%	11.2%	24.5%	0.46	− 34%	− 12.4	10.5%	0.01	0%
Short 10D Strangle	12.0%	2.4%	20.1%	0.12	− 33%	− 17.9	1.6%	0.02	2%
Short Call spread	22.2%	4.3%	10.0%	0.43	− 10%	− 1.6	4.9%	− 0.01	0%
Short Put Spread	3.2%	0.7%	20.4%	0.03	− 32%	− 16.9	− 1.4%	0.03	5%
Short 25D RR	6.3%	1.3%	24.7%	0.05	− 26%	9.6	3.2%	− 0.02	3%
Short 25D ButterFly	4.3%	0.9%	8.6%	0.10	− 14%	− 0.6	2.3%	− 0.02	3%
ETH	1652.5%	81.9%	89.3%	0.92	− 79%	− 1.3	0.0%	1.00	100%
Long Straddle	− 68.3%	− 21.3%	32.3%	− 0.66	− 70%	7.0	− 20.3%	− 0.02	1%
Long 25D Strangle	− 50.8%	− 13.8%	27.4%	− 0.50	− 52%	8.5	− 11.8%	− 0.02	1%
Long 10D Strangle	− 16.4%	− 3.7%	18.7%	− 0.20	− 25%	14.0	− 0.6%	− 0.03	4%
Long Call Spread	− 39.9%	− 10.1%	12.3%	− 0.82	− 40%	0.8	− 11.0%	0.01	1%
Long Put Spread	− 13.9%	− 3.1%	19.4%	− 0.16	− 26%	12.8	0.9%	− 0.04	6%
Long 25D RR	− 30.9%	− 7.4%	26.9%	− 0.28	− 46%	− 7.3	− 10.4%	0.04	9%
Long 25D ButterFly	− 30.2%	− 7.2%	11.2%	− 0.64	− 32%	0.5	− 7.5%	0.00	0%
Short Straddle	60.9%	10.4%	32.9%	0.32	− 41%	− 6.5	10.2%	0.02	1%
Short 25D Strangle	35.8%	6.6%	27.9%	0.24	− 37%	− 7.7	5.9%	0.02	1%
Short 10D Strangle	− 1.3%	− 0.3%	21.4%	− 0.01	− 36%	− 13.0	− 2.0%	0.03	4%
Short Call spread	23.9%	4.6%	12.6%	0.36	− 17%	− 1.0	5.9%	− 0.01	1%
Short Put Spread	− 19.0%	− 4.3%	21.5%	− 0.20	− 37%	− 12.5	− 6.9%	0.04	6%
Short 25D RR	15.3%	3.0%	27.1%	0.11	− 23%	7.3	6.7%	− 0.03	8%
Short 25D ButterFly	2.4%	0.5%	11.1%	0.04	− 12%	− 1.4	1.3%	− 0.00	0%

Note: Notations are the same as in table 1.

Table 5. Risk-adjusted performances of delta-hedged option strategies on BTC and ETH using Coin accounting for quarterly rolls.

	Total	P.a.	Vol	Sharpe	MaxDD	Skew	α_{AN}	β	R^2
BTC	908.0%	62.0%	70.4%	0.88	− 77%	− 1.3	0.0%	1.00	100%
Long Straddle	− 53.2%	− 14.6%	23.8%	− 0.62	− 57%	8.4	− 12.1%	− 0.02	1%
Long 25D Strangle	− 42.8%	− 11.0%	19.5%	− 0.57	− 47%	10.4	− 9.2%	− 0.02	1%
Long 10D Strangle	− 18.8%	− 4.3%	13.1%	− 0.33	− 28%	13.3	− 2.2%	− 0.02	2%
Long Call Spread	− 31.0%	− 7.5%	9.2%	− 0.81	− 33%	0.2	− 8.5%	0.01	2%
Long Put Spread	2.9%	0.6%	13.2%	0.04	− 25%	7.7	3.7%	− 0.03	3%
Long 25D RR	− 26.3%	− 6.2%	20.1%	− 0.31	− 42%	− 10.6	− 8.4%	0.04	6%
Long 25D ButterFly	− 23.3%	− 5.4%	10.9%	− 0.49	− 30%	2.3	− 5.3%	− 0.00	0%
Short Straddle	49.2%	8.7%	27.4%	0.32	− 39%	− 10.8	8.5%	0.02	1%
Short 25D Strangle	42.1%	7.6%	20.8%	0.37	− 31%	− 10.2	7.1%	0.01	1%
Short 10D Strangle	10.9%	2.2%	14.5%	0.15	− 25%	− 13.4	1.2%	0.02	2%
Short Call spread	32.4%	6.0%	9.3%	0.65	− 13%	− 0.3	7.2%	− 0.02	2%
Short Put Spread	− 20.6%	− 4.7%	16.1%	− 0.29	− 41%	− 9.0	− 6.2%	0.03	3%
Short 25D RR	21.6%	4.2%	18.4%	0.23	− 17%	9.6	7.1%	− 0.04	7%
Short 25D ButterFly	12.1%	2.4%	10.6%	0.23	− 9%	− 2.5	2.5%	0.00	0%
ETH	1652.5%	81.9%	89.3%	0.92	− 79%	− 1.3	0.0%	1.00	100%
Long Straddle	− 35.7%	− 8.8%	23.9%	− 0.37	− 49%	2.7	− 3.8%	− 0.03	1%
Long 25D Strangle	− 28.8%	− 6.8%	19.9%	− 0.34	− 40%	2.7	− 3.3%	− 0.02	1%
Long 10D Strangle	− 22.3%	− 5.1%	12.7%	− 0.41	− 28%	1.3	− 3.5%	− 0.01	1%
Long Call Spread	− 16.4%	− 3.7%	15.0%	− 0.25	− 24%	1.1	− 3.4%	0.00	0%
Long Put Spread	− 8.2%	− 1.8%	10.5%	− 0.17	− 18%	0.9	0.1%	− 0.02	3%
Long 25D RR	− 16.6%	− 3.7%	18.8%	− 0.20	− 31%	− 0.7	− 5.8%	0.03	6%
Long 25D ButterFly	− 15.9%	− 3.6%	10.5%	− 0.34	− 27%	3.6	− 2.4%	− 0.01	1%
Short Straddle	5.2%	1.1%	26.9%	0.04	− 42%	− 6.0	1.3%	0.03	1%
Short 25D Strangle	8.9%	1.8%	22.0%	0.08	− 38%	− 4.5	1.9%	0.02	1%
Short 10D Strangle	19.3%	3.7%	13.5%	0.28	− 18%	− 1.0	3.4%	0.01	0%
Short Call spread	0.4%	0.1%	15.7%	0.01	− 25%	− 0.9	1.3%	− 0.01	0%
Short Put Spread	− 9.3%	− 2.0%	11.0%	− 0.18	− 27%	− 1.1	− 3.2%	0.02	3%
Short 25D RR	9.0%	1.8%	18.8%	0.10	− 15%	0.6	4.7%	− 0.03	6%
Short 25D ButterFly	0.5%	0.1%	10.8%	0.01	− 16%	− 4.8	− 0.4%	0.01	1%

Note: Notations are the same as in table 1.

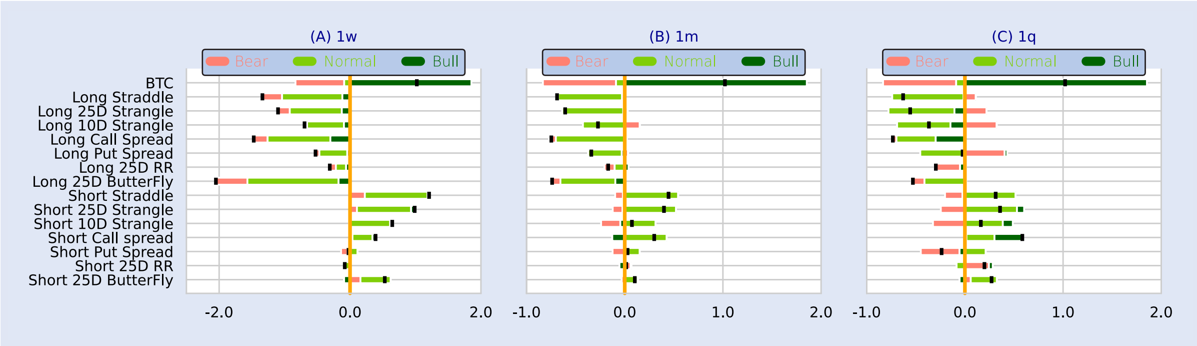

Figure 8. Regime conditional Sharpe ratios of delta-hedged option strategies on BTC with (A) weekly (1w), (B) monthly (1m), and (C) quarterly (1q) rolls. The black bar is the total Sharpe of the strategy.

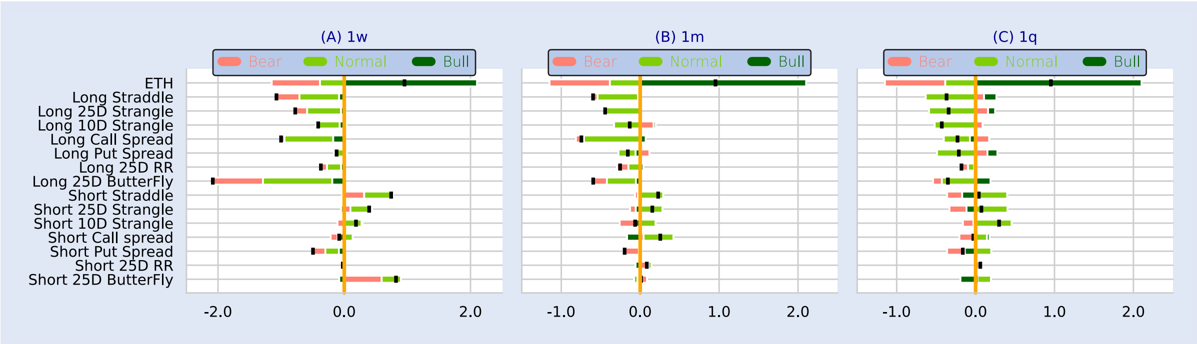

Figure 9. Regime conditional Sharpe ratios of delta-hedged option strategies on ETH with (A) weekly (1w), (B) monthly (1m), and (C) quarterly (1q) rolls. The black bar is the total Sharpe of the strategy.

for the three frequencies of the roll. The BTC and ETH regimes for the performance period are obtained by ordering the monthly returns from lowest to highest and splitting the 16% of worst returns into 'bear' regimes, 16% of best returns into 'bull' regimes, and remaining regimes into 'normal' regimes. The logic behind using these quantiles is that, under normal distribution, 'bear'/'normal'/'bull' regimes represent returns below/within/above one standard deviation. Using regime conditional performances, we compute the average performance multiplied by regime frequencies divided by the total volatility, so that we obtain the decomposition of the total Sharpe ratio by regimes.

The most interesting observation is that long ATM call on average is unprofitable in all regimes including the bull regime. The reason is that during bull periods implied call volatilies increase, so long positions rolls into options with higher implied volatilities than subsequent volatilities realised through delta-hedging. For ETH options, performances of long 25D and 10D puts are slightly positive in bear regimes. All short positions are most profitable in the normal regime (68% frequency).

It is remarkable that long volatility strategies with weekly rolls generate negative performances in all regimes. Long straddle strategies with monthly and quarterly rolls generate positive performance in bear regime. Long strangles with quarterly rolls generate positive performance in bear regimes too. Short volatility strategies with weekly rolls generate positive performance in all regimes. The reason is that any short-term losses cause with higher price volatility of the underlying coin tend to be compensated by higher risk-premiums for new option rolls.

6. Conclusions

We have considered the valuation and delta-hedging of inverse options. We have applied the uniqueness of fair price under different martingale measures and the numéraire invariance principle to derive formulas (9) and (12) for valuation and hedging of inverse options, respectively.

We have introduced USD- and Coin-based accounting rules for the measurement of realised performances of delta-hedged strategies in inverse options. We have simulated a number of popular option strategies on Bitcoin and Ether underlyings using data from Deribit exchange for past five years. We have shown that the risk-adjusted performance in coin terms of the account with Coin-based accounting is close to the risk-adjusted performance in USD terms of the account with USD-based accounting. This result validates empirically the numéraire invariance principle discussed in Appendix. An investor, who would like to have a long-term exposure to the underlying coin, should opt for Coin-based accounting, under which the strategy accrues the coin P&L (which is native to Deribit exchange). In opposite an investor, who would like to keep neutral exposure to coin performance, should choose the USD-based accounting, under which realised coin P&L is immediately swapped into USD. For an example, 15% per annum (p.a.) return for both USD and Coin performances implies that USD and Coin accounts increased by 15% being market neutral to BTC. However, the USD value of the Coin account and as well as the Coin value of USD account will be impacted by the change of BTC during the reporting period. These specifics are important for implementing and reporting of trading strategies in inverse options.

We have concluded that the risk-premia observed in crypto options is negative for both level, put and call skew, and convexity. Delta-hedging is necessary for harvesting the risk-premia because simple call overlay or put selling strategies bear directional market risk. As a result, short strategies with frequent rolls (such as weekly) appear to be best suited for harvesting the volatility risk-premia. Defensive long volatility strategies should be trading in options with longer maturities. This conclusion is also observed in traditional options markets on equity indices† .

In addition to systematic strategies in options on cryptocurrencies, our framework can also be applied to systematic strategies for hedging of impermanent loss using listed options, when providing liquidity to decentralised exchanges such as Uniswap †.

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References

Alexander, C., Ding, C. and Imeraj, A., Crypto quanto and inverse options. *Math. Finance*, 2023, **33**(4), 1005–1043.
Alexander, C. and Imeraj, A., Delta hedging bitcoin options with a smile. *Quant. Finance*, 2023, **23**(5), 799–817.
Black, F., The pricing of commodity contracts. *J. Financ. Econ.*, 1976, **3**(1-2), 167–179.
Black, F. and Scholes, M., The pricing of options and corporate liabilities. *J. Polit. Econ.*, 1973, **81**, 637–659.
Cao, M. and Celik, B., Valuation of bitcoin options. *J. Futures Markets*, 2020, **11**(85), 1007–1026.
Clark, I.J., *Foreign Exchange Option Pricing: A Practitioner's Guide*, 2011 (Wiley: Chichester).
Garman, M.B. and Kohlhagen, S.W., Foreign currency option values. *J. Int. Money. Finance.*, 1983, **2**(3), 231–237.
Geman, H., El Karoui, N. and Rochet, J-C., Changes of probability measure and option pricing. *J. Appl. Probab.*, 1995, **32**(2), 443–458.
Hilliard, J.E. and Ngo, J.T., Bitcoin: jumps, convenience yields, and option prices. *Quant. Finance*, 2022, **22**(11), 2079–2091.
Hou, A.J., Wang, W., Chen, C.Y.H. and Härdle, W.K., Pricing cryptocurrency options. *Financial Econom.*, 2020, **18**(2), 250–279.
Huang, J.Z., Ni, J. and Xu, L., Leverage effect in cryptocurrency markets. *Pacific-Basin Finance J.*, 2022, **73**, 101773.
Karandikar, R.L., On pathwise stochastic integration. *Stoch. Process. Their. Appl.*, 1995, **57**, 11–18.
Lewis, A.L., *Option Valuation Under Stochastic Volatility*, 2000 (Finance Press: Newport Beach).
Lipton, A., Lucic, V. and Sepp, A., Unified approach for hedging impermanent loss of liquidity provision. Preprint on SSRN: 4887298, 2024.
Lucic, V., Btc inverse call and the standard fx framework. Preprint on SSRN: 4113726, 2021.
Madan, D.B., Reyners, S. and Schoutens, W., Advanced model calibration on bitcoin options. *Digital Finance*, 2019, **1**, 117–137.
Matic, J., Packham, N. and Härdle, W.K., Hedging cryptocurrency options. *Rev. Derivatives Res.*, 2023, **26**, 91–133.
Reiswich, D. and Wystup, U., A guide to FX options quoting conventions. *J. Derivatives*, 2010, **18**(2), 58–68.
Sepp, A. and Rakhmonov, P., Log-normal stochastic volatility model with quadratic drift. *Int. J. Theoretical Appl. Finance*, 2023, **26**(8), 2450003.
Sondermann, D., *Introduction to Stochastic Calculus for Finance: A New Didactic Approach*, 2006 (Springer: Berlin).
Teng, H.-W. and Härdle, K.W., Financial analytics of inverse BTC options in a stochastic volatility world. Preprint on SSRN: 4238213, 2023.
Tosi, A. and Ziegler, A., The timing of option returns. Preprint on SSRN: 2909163, 2017.

Appendix. A note on the Numéraire invariance

We provide an alternative interpretation of equation (9) using the numéraire invariance principle for self-financing trading strategies. The numéraire invariance is a fundamental economic principle stating that a self-financing portfolio remains self-financing after the tradeable asset in whose units the portfolio holdings are expressed (most typically cash) is changed to another tradeable asset in the economy. Despite being very intuitive (in some sense obvious), the standard textbook treatment of numéraire invariance entails a full proof relying on manipulations using the Itô formula and properties of quadratic variation. We provide an alternative derivation which, in our mind, more closely reflects the underlying intuition.

We denote the price vector of tradeable assets by A_t and consider a trading strategy holding θ_t units of assets A_t. We assume that A_t is adapted to the underlying filtration $\{\mathcal{F}_t\}$, and recall that the strategy is called self-financing if for the resulting portfolio $V_T \equiv \theta_T \cdot A_T$ we have

$$V_T = \int_0^T \theta_t \cdot \mathrm{d}A_t. \tag{A1}$$

We assume that the paths of A_t and the portfolio holdings θ_t are corlol semimartingales, and that all the information in the underlying economy is encapsulated by A_t, that is, we assume that (modulo the null sets) $\{\mathcal{F}_t\}$ is generated by A_t. By the pathwise representation of the stochastic integral (e.g. Karandikar 1995, Sondermann 2006)

$$\int_0^T \theta_t \cdot \mathrm{d}A_t = \phi_T(\theta, A), \quad T \geq 0,$$

where for every $T > 0$, $\phi_T(\cdot,\cdot)$ is a measurable functional of the corlol paths. We put $\varphi_T(\cdot) \equiv \phi_T(\theta,\cdot)$, and from (A1) conclude that $\varphi_T(x) = \theta_T \cdot x_T$ holds $\mathbb{Q}_A$ a.s., where $\mathbb{Q}_A$ is the probability measure induced on the set of for corlol paths by A_t. Setting $x_t \equiv \frac{A_t}{M_t}$ for some strictly positive tradeable asset M_t yields

$$\theta_T \cdot \frac{A_T}{M_t} = \int_0^T \theta_t \cdot \mathrm{d}\left(\frac{A_t}{M_t}\right),$$

establishing the numéraire invariance.

† For an example see Tosi and Ziegler (2017).
† For an application see Lipton *et al.* (2024).