# Moment Swaps

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**Abstract**

In this paper we introduce moment swaps. These derivatives depend on the realized higher moments of the underlying. A special case is the nowadays popular Variance Swaps. After introducing moment swaps we discuss how to hedge these derivatives. Moreover, we show how the classical hedge of the Variance Swap in terms of a position in log-contracts and a dynamic trading strategy can be significantly enhanced by using third moment swaps.

# 1 Introduction

This paper deals with moment swaps. These derivatives depend on the realized higher moments of the underlying. More precisely, their payoff is a function of powers of the (daily) log-returns and give you protection to different kinds of market *shocks*. In case of squared log-returns we have the so-called Variance Swaps. Variance swaps are nowadays liquidly traded for several underlyers and were created to cover against changes in the volatility regime. Besides the latter, skewness, kurtosis, etc. also play an important role. To protect against a wrongly estimated skewness or kurtosis, moment derivatives of higher order can be useful. Moreover, we will show that the classical hedge of the Variance Swap using log-contracts can significantly improved using moment swaps.

Furthermore, recent studies [20], [21], [10] and [11] suggest that functionals of powers of returns seem the natural choice to complete the market. It was shown for example in [10] and [11] that under an incomplete Lévy market allowing trade in the power-assets of all orders leads to a complete market. Power assets are strongly related to the realized higher moments and in a discrete time framework, they mainly coincide (see [11]).

This paper is organized as follows. In the next section we introduce moment swaps on stocks and futures. After linking the prices between the two cases, we discuss the hedging of higher order moment swaps. Basically, the hedge consists out of a position in the so-called log-contract, dealt with in more detail in Section 4, a dynamic trading strategy and a static position in moment swaps of lower order. In Section 3, we introduce some stochastic volatility models (Heston and time-changed Lévy models). We give closed-formula expressions for the price of the log-contract and assess the hedging of the nowadays popular Variance Swap using a 3rd order moment swap. We will show that the classical hedge of a Variance Swap in terms of a static position in the log-contract and a dynamic trading strategy can be significantly enhanced by taking a position in third moment swaps by running a simulation study under the above mentioned stochastic volatility models.

# 2 Moment Swaps

## 2.1 Introducing Moment Swaps

Consider a stock (index), which is assumed to pay out a continuous dividend-yield $q \geq 0$. Denote by $S = \{S_t, t \geq 0\}$ the price process of the stock. Assume we have also available the classical bank-account/bond, with a compound interest rate $r$. The bond price process is hence given by $B = \{B_t = \exp(rt), t \geq 0\}$.

Consider a finite set of equally spaced discrete times $\{t_i = i\Delta t, i = 0, 1, \ldots, n\}$, with $\Delta t = T/n$ at which the path of the underlying is monitored. We denote the price of the underlying at these points, i.e. $S_{t_i}$, by $S_i$ for simplicity. Typically, $\{t_0 = 0, t_1 = \Delta t, t_2 = 2\Delta t \ldots, t_n = T\}$ corresponds to daily closing times and

$S_i$ is the closing price at day $i$. Note that then

$$\log(S_i) - \log(S_{i-1}), \quad i = 1, \ldots, n,$$

correspond to the daily log-returns.

We assume that a future is available on $S$ which expires at time $T$. By risk-neutral valuation, the price process of the future is given by $F = \{F_t = \exp((r-q)(T-t))S_t\}$. In line with the above notation, let us write $F_i = F_{t_i}$.

Next, we define the *moment swaps* on the stock. The $k$th-moment swap is a contract where the parties agree to exchange the amount

$$MOMS^{(k)}_{stock} = N \times \left( \sum_{i=1}^{n} (\log(S_i) - \log(S_{i-1}))^k \right) = N \times \left( \sum_{i=1}^{n} \left( \log\left( \frac{S_i}{S_{i-1}} \right) \right)^k \right),$$

at maturity, where $N$ is the nominal amount, which we take for simplicity from now on equal to $N = 1$ for all swaps considered.

A special case of these swaps is the 2nd-moment swap, better known as the Variance Swap. The payoff function in that case is given by

$$VS = \sum_{i=1}^{n} (\log(S_i) - \log(S_{i-1}))^2 .$$

Basically this contract swaps fixed (annualized) variance (second moment) by the realized variance (second moment) and as such provides protection against unexpected or unfavorable changes in volatility. Higher moment swaps provide the same kind of protection. The $MOMS^{(3)}$ is related to realized skewness and provides protection against changes in the symmetry of the underlying distribution. $MOMS^{(4)}$ derivatives are linked to realized kurtosis and provide protection against the unexpected occurences of very large jumps, or in other words changes in the tail behavior of the underlying distribution.

It will turn out to be quite convenient to first consider moment swaps with the future price as underlying. On can make easily the transformation to the case where the stock price is taken as the underlying for the moment derivative. The $k$th-moment swap on the future is a contract where the parties agree to exchange the amount

$$MOMS^{(k)}_{future} = \sum_{i=1}^{n} (\log(F_i) - \log(F_{i-1}))^k = \sum_{i=1}^{n} \left( \log\left( \frac{F_i}{F_{i-1}} \right) \right)^k,$$

at maturity.

The relation between the future and the stock moment swaps can be easily deduce using the identity $F_i = \exp((r-q)(T-t_i))S_i$. Indeed,

$$\begin{aligned} & MOMS^{(k)}_{future} \\ = \; & \sum_{i=1}^{n} \left( \log\left( \frac{F_i}{F_{i-1}} \right) \right)^k \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n}\left(-(r-q)\Delta t+\log\left(\frac{S_i}{S_{i-1}}\right)\right)^k \\
&= \sum_{i=1}^{n}\sum_{j=0}^{k}\binom{k}{j}(-(r-q)\Delta t)^j\left(\log\left(\frac{S_i}{S_{i-1}}\right)\right)^{k-j} \\
&= \sum_{j=0}^{k-2}\binom{k}{j}(-(r-q)\Delta t)^j MOMS_{stock}^{(k-j)}-(r-q)T(-(r-q)\Delta t)^{k-1} \\
&\qquad +k(-(r-q)\Delta t)^{k-1}LOG_{stock},
\end{aligned}
$$

where due to telescoping

$$
LOG_{stock}=\sum_{i=1}^{n}\log\left(\frac{S_i}{S_{i-1}}\right)=\log(S_T)-\log(S_0)
$$

is the so-called *log-contract* (on the stock). This log-contract will play an important role also later on in the hedging of moment-swaps. Ignoring higher order powers of $\Delta t$ - which is assumed to be small, the above calculation leads to the following approximation:

$$
MOMS_{future}^{(k)}\approx MOMS_{stock}^{(k)}-k((r-q)\Delta t)MOMS_{stock}^{(k-1)}. \tag{1}
$$

## 2.2 Hedging Moment Swaps

We focus on hedging the moment swaps which are written on the future price as underlying.

In line with the results obtained by [6], first consider the following (Taylor-like) expansion of the $k$th power of the logarithmic function

$$
\begin{aligned}
&(\log(x))^k = \qquad (2)\\
&\quad k!\left(x-1-\log(x)-\frac{(\log(x))^2}{2!}-\frac{(\log(x))^3}{3!}-\ldots-\frac{(\log(x))^{k-1}}{(k-1)!}+\mathcal{O}((x-1)^{k+1})\right).
\end{aligned}
$$

Substituting $x$ by $F_i/F_{i-1}$ leads to

$$
\begin{aligned}
&(\log(F_i/F_{i-1}))^k = \\
&\quad k!\left(\frac{\Delta F_i}{F_{i-1}}-\log(F_i/F_{i-1})-\sum_{j=2}^{k-1}\frac{(\log(F_i/F_{i-1}))^j}{j!}+\mathcal{O}((\Delta F_i/F_{i-1})^{k+1})\right),
\end{aligned}
$$

where $\Delta F_i = F_i - F_{i-1}$. Summing over $i$ gives a decomposition of the $MOMS_{future}^{(k)}$ payoff:

$$
MOMS_{future}^{(k)} \tag{3}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} (\log(F_i/F_{i-1}))^k \\
&= k! \sum_{i=1}^{n} \left( \frac{\Delta F_i}{F_{i-1}} - \log(F_i/F_{i-1}) - \sum_{j=2}^{k-1} \frac{(\log(F_i/F_{i-1}))^j}{j!} + \mathcal{O}((\Delta F_i/F_{i-1})^{k+1}) \right) \\
&= -k!(\log(F_T) - \log(F_0)) \\
&\quad +k! \sum_{i=1}^{n} \frac{\Delta F_i}{F_{i-1}} - \sum_{j=2}^{k-1} \frac{k!}{j!} MOMS_{future}^{(j)} + \mathcal{O}(\sum_{i=1}^{n} (\Delta F_i/F_{i-1})^{k+1}) \qquad (4)
\end{aligned}
$$

due to telescoping. Thus up to $(k+1)$th-order terms the sum of the $k$th powered log-returns decomposes into

- the payout from $-k!$ log-contracts on the future with payoff

$$
LOG_{future} = \log(F_T) - \log(F_0) = LOG_{stock} - (r-q)T;
$$

- a dynamic strategy $(k! \sum_{i=1}^{n} \frac{\Delta F_i}{F_{i-1}})$ in futures;
- a series of moment contracts of order strictly smaller than $k$.

The log-contract can be hedge by a dynamic trading strategy in futures in combination with a static position in bonds and vanilla options (see Carr and Lewis [6]) .

For $k = 2$, Equation (4) leads to the classical hedge of the variance swap in terms of log-contracts:

$$
VS_{future} = MOMS_{future}^{(2)} \approx 2 \left( \sum_{i=1}^{n} \frac{\Delta F_i}{F_{i-1}} - LOG_{future} \right). \qquad (5)
$$

Hence hedging the variance swap comes down into a short position of 2 logcontracts and following a dynamic strategy (which can be entered at zero costs) in terms of futures. The performance of this hedge will be discussed in the next section.

For $k = 3$, Equation (4) leads to

$$
MOMS_{future}^{(3)} \approx 6 \left( \sum_{i=1}^{n} \frac{\Delta F_i}{F_{i-1}} - LOG_{future} - \frac{1}{2} VS_{future} \right).
$$

Rearranging terms results in

$$
VS_{future} \approx 2 \left( \sum_{i=1}^{n} \frac{\Delta F_i}{F_{i-1}} - LOG_{future} \right) - \frac{1}{3} MOMS_{future}^{(3)}. \qquad (6)
$$

Additionally shorting 1/3 third moment swaps, $MOMS_{future}^{(3)}$, will thus lead to an improvement of the hedging strategy of the variance swap. In Section 5 we quantify this improvement.

# 3 Some Advanced Models

In this section, we consider some advanced models under which we will try to assess the hedging performance of the variance swap using moment derivatives. We completely, follow the notation of [24] (see also [25]). For more details we refer to [23]. Here, we will give the dynamics of the model in terms of the characteristic function $\phi(u,t)$ of the logarithm of the price process $\log S_t$, i.e.,

$$\phi(u,t) = E[\exp(\mathrm{i}u\log(S_t))].$$

## 3.1 The Heston Stochastic Volatility Model

The stock price process in the Heston Stochastic Volatility model (HEST) follows the Black-Scholes SDE in which the volatility is behaving stochastically over time:

$$\frac{\mathrm{d}S_t}{S_t} = (r-q)\mathrm{d}t + \sigma_t \mathrm{d}W_t, \quad S_0 \geq 0,$$

with the (squared) volatility following the classical Cox-Ingersoll-Ross (CIR) process:

$$\mathrm{d}\sigma_t^2 = \kappa(\eta - \sigma_t^2)\mathrm{d}t + \theta\sigma_t\mathrm{d}\tilde{W}_t, \quad \sigma_0 \geq 0,$$

where $W = \{W_t, t \geq 0\}$ and $\tilde{W} = \{\tilde{W}_t, t \geq 0\}$ are two correlated standard Brownian motions such that $\mathrm{Cov}[\mathrm{d}W_t \mathrm{d}\tilde{W}_t] = \rho \mathrm{d}t$.

The characteristic function $\phi(u,t)$ is in this case given by (see [15] or [2]):

$$\begin{aligned}
\phi(u,t) &= E[\exp(\mathrm{i}u\log(S_t))|S_0,\sigma_0^2] \\
&= \exp(\mathrm{i}u(\log S_0 + (r-q)t)) \\
&\quad \times \exp(\eta\kappa\theta^{-2}((\kappa - \rho\theta u\mathrm{i} - d)t - 2\log((1-g\mathrm{e}^{-dt})/(1-g)))) \\
&\quad \times \exp(\sigma_0^2\theta^{-2}(\kappa - \rho\theta\mathrm{i}u - d)(1-\mathrm{e}^{-dt})/(1-g\mathrm{e}^{-dt})),
\end{aligned}$$

where

$$\begin{aligned}
d &= ((\rho\theta u\mathrm{i} - \kappa)^2 - \theta^2(-\mathrm{i}u - u^2))^{1/2}, && (7)\\
g &= (\kappa - \rho\theta u\mathrm{i} - d)/(\kappa - \rho\theta u\mathrm{i} + d). && (8)
\end{aligned}$$

## 3.2 Lévy Models with Stochastic Time

The Lévy models with stochastic time considered in this paper are build out of two independent stochastic processes as in [8]. This technique dates back to [19]. The first process is a Lévy process. The behavior of the asset price will be modeled by the exponential of the Lévy process, $X = \{X_t, t \geq 0\}$ suitably time-changed, by a second process, a stochastic clock, that builds in a stochastic volatility effect by making time stochastic.

The economic time elapsed in $t$ units of calendar time will be given by the integrated process $Y = \{Y_t, t \geq 0\}$

$$Y_t = \int_0^t y_s \mathrm{d}s.$$

of a subordinator (a positive process), $y = \{y_t, t \geq 0\}$. Let us denote by $\varphi(u; t, y_0)$ the characteristic function of $Y_t$ given $y_0$. The (risk-neutral) price process $S = \{S_t, t \geq 0\}$ is now modeled as follows:

$$S_t = S_0 \frac{\exp((r-q)t)}{E[\exp(X_{Y_t})|y_0]} \exp(X_{Y_t}), \tag{9}$$

where $X = \{X_t, t \geq 0\}$ is a Lévy process. The factor $\exp((r-q)t)/E[\exp(X_{Y_t})|y_0]$ puts us immediately into the risk-neutral world by a mean-correcting argument. Basically, we model the stock price process as the ordinary exponential of a time-changed Lévy process. The process incorporates jumps (through the Lévy process $X_t$) and stochastic volatility (through the time change $Y_t$). The characteristic function $\phi(u,t)$ for the log of our stock price is given by:

$$\begin{aligned} \phi(u,t) &= E[\exp(\mathrm{i}u \log(S_t))|S_0, y_0] \\ &= \exp(\mathrm{i}u((r-q)t + \log S_0)) \frac{\varphi(-\mathrm{i}\psi_X(u); t, y_0)}{\varphi(-\mathrm{i}\psi_X(-\mathrm{i}); t, y_0)^{\mathrm{i}u}}, \end{aligned} \tag{10}$$

where

$$\psi_X(u) = \log E[\exp(\mathrm{i}uX_1)]; \tag{11}$$

$\psi_X(u)$ is called the characteristic exponent of the Lévy process.

Here we consider two possible processes for $y$: the CIR process (continuous) and the Gamma-OU process (jump process) and two possible Lévy processes: the NIG and VG process.

**NIG Lévy Process:** A NIG process is based on the Normal Inverse Gaussian (NIG) distribution, $\mathrm{NIG}(\alpha, \beta, \delta)$, with parameters $\alpha > 0$, $-\alpha < \beta < \alpha$ and $\delta > 0$. Its characteristic function is given by:

$$\phi_{NIG}(u; \alpha, \beta, \delta) = \exp\left(-\delta\left(\sqrt{\alpha^2 - (\beta + \mathrm{i}u)^2} - \sqrt{\alpha^2 - \beta^2}\right)\right).$$

Since this is an infinitely divisible characteristic function, one can define the NIG-process $X^{(NIG)} = \{X_t^{(NIG)}, t \geq 0\}$, with $X_0^{(NIG)} = 0$, as the process having stationary and independent NIG distributed increments. An increment over the time interval $[s, s+t]$ follows a $\mathrm{NIG}(\alpha, \beta, \delta t)$ law.

**VG Lévy Process:** The characteristic function of the $\mathrm{VG}(C, G, M)$ distribution, with parameters $C > 0$, $G > 0$ and $M > 0$ is given by:

$$\phi_{VG}(u; C, G, M) = \left(\frac{GM}{GM + (M-G)\mathrm{i}u + u^2}\right)^C.$$

This distribution is infinitely divisible and one can define the VG-process $X^{(VG)} = \{X_t^{(VG)}, t \geq 0\}$ as the process which starts at zero, has independent and stationary increments and where the increment $X_{s+t}^{(VG)} - X_s^{(VG)}$ over the time interval $[s, s+t]$ follows a VG$(Ct, G, M)$ law.

**CIR Stochastic Clock:** Carr, Geman, Madan and Yor [8] use as the rate of time change the CIR process that solves the SDE:

$$\mathrm{d}y_t = \kappa(\eta - y_t)\mathrm{d}t + \lambda y_t^{1/2}\mathrm{d}W_t,$$

where $W = \{W_t, t \geq 0\}$ is a standard Brownian motion. The characteristic function of $Y_t$ (given $y_0$) is explicitly known (see [12]):

$$\begin{aligned}\varphi_{CIR}(u, t; \kappa, \eta, \lambda, y_0) &= E[\exp(\mathrm{i}uY_t)|y_0] \\ &= \frac{\exp(\kappa^2\eta t/\lambda^2)\exp(2y_0\mathrm{i}u/(\kappa + \gamma\coth(\gamma t/2)))}{(\cosh(\gamma t/2) + \kappa\sinh(\gamma t/2)/\gamma)^{2\kappa\eta/\lambda^2}},\end{aligned}$$

where

$$\gamma = \sqrt{\kappa^2 - 2\lambda^2\mathrm{i}u}.$$

# 4 The Log-Contract

In Section 2, the log-contract played a very important role. We note here that, under a lot of models, closed expressions for the price of the log-contract is available. Indeed, e.g. we have under the Black-Scholes model with $\sigma$ as usual denoting the volatility parameter.

$$\begin{aligned}LOG_{stock} &= \exp(-rT)E_Q[\log(S_T) - \log(S_0)|\mathcal{F}_0] \\ &= \exp(-rT)E_Q[(r - q - \sigma^2/2)T + \sigma W_T|\mathcal{F}_0] \\ &= \exp(-rT)(r - q - \sigma^2/2)T.\end{aligned}$$

For the more advanced models, the price of the log-contract can be easily obtained from the (risk-neutral) characteristic function of $\log(S_T)$, i.e.

$$\phi(u, t) = E_Q[\exp(\mathrm{i}u\log(S_T)|\mathcal{F}_0].$$

Then,

$$-\mathrm{i}\left.\frac{\partial\phi(u, t)}{\partial u}\right|_{u=0} = E_Q[\log(S_T)|\mathcal{F}_0].$$

Hence

$$\begin{aligned}LOG_{stock} &= \exp(-rT)E_Q[\log(S_T) - \log(S_0)|\mathcal{F}_0] \\ &= \exp(-rT)\left(-\mathrm{i}\frac{\partial\phi(0, t)}{\partial u} - \log(S_0)\right)\end{aligned}$$

| model | $LOG_{stock} \times 10000$ |
|---|---|
| HEST | -32.64 |
| VG-CIR | -48.33 |
| NIG-CIR | -48.30 |

Table 1: $LOG_{stock}$ prices

Doing this exercise for the Heston model, we obtain:

$$LOG_{stock}^{(HEST)} = \frac{\exp(-rT)}{2\kappa} \left(2\kappa(r-q)T - \eta\kappa T - \eta \mathrm{e}^{-\kappa T} + \eta - \sigma_0^2 + \sigma_0^2 \mathrm{e}^{-\kappa T}\right).$$

For the case of the time-changed Lévy models, with $\phi(u,t)$ as in (10), this comes down to:

$$\begin{aligned} LOG_{stock} &= \exp(-rT)\left((r-q)T - \varphi'(0)\psi_X'(0) - \log(\varphi(-\mathrm{i}\psi_X(\mathrm{i}))\right). \\ &= \exp(-rT)\left((r-q)T + E[Y_T]E[X_1] - \log(E[\exp(X_{Y_T})])\right), \end{aligned}$$

where $\psi_X$ is as in (11).

Applying the above formulas for the models with parameters given in Table 2, we obtain the prices in Table 1. Note that the parameters in Table 2 were obtained, as described in [24] from a global calibration procedure on the same volatility surface (of the EuroStoxx50). All models fitted this surface quite well and hence vanilla prices under the different models were almost equal. However, as already reported in [24] (and [25]), exotic prices can differ quite significantly and one has a clear issue of model risk. This is again illustrated here, the log-contract prices differ almost 50 percent is some cases.

# 5 Variance Swap Hedging using Moment Swaps

In this section we compare the performance of the hedging of a Variance Swap. We compare the strategy without moment swaps described in (5) with the strategy including a 3rd moment swap as in (6) and this for the different models described in Section 3. The parameters are again taken from [24] and given in Table 2.

In Figures 1-3, one sees the Profit and Loss (P&L) distribution for the hedging of 10000 Variance Swap contracts. Each P&L distribution was obtained by running a million scenarios. Table 3 overviews the mean and the standard deviation of the P&L distribution.: $\mu_1$ and $\sigma_1$ corresponds to resp. the mean and standard deviation of the hedging without moment derivatives as in (5); $\mu_2$ and $\sigma_2$ refers to the case including a 3rd order moment swap as in (6).

As expected from a theoretical point of view, we here also empirically observe that the introduction of a 3rd moment swap in the hedge of a Variance Swap leads to a significant improvement of the hedge performance. Not only is the P&L-mean much closer to zero, also the standard deviation is much smaller. The tails of the original hedge are much fatter than that of the enhanced hedge.

| |
|---|
| HEST |
| $\sigma_0^2 = 0.0654, \kappa = 0.6067, \eta = 0.0707, \theta = 0.2928, \rho = -0.7571$ |
| VG-CIR |
| $C = 18.0968, G = 20.0276, M = 26.3971, \kappa = 1.2145, \eta = 0.5501,$ $\lambda = 1.7913, y_0 = 1$ |
| NIG-CIR |
| $\alpha = 16.1975, \beta = -3.1804, \delta = 1.0867, \kappa = 1.2101, \eta = 0.5507,$ $\lambda = 1.7864, y_0 = 1$ |

Table 2: Parameters

| Model | $\mu_1$ | $\sigma_1$ | $\mu_2$ | $\sigma_2$ |
|---|---|---|---|---|
| HEST | 0.065 | 2.298 | -0.0644 | 0.0057 |
| VG-CIR | 6.722 | 20.568 | -0.6671 | 1.647 |
| NIG-CIR | 6.986 | 27.137 | -0.732 | 3.150 |

Table 3: P&L Characteristics

We note that the standard deviation in the Heston case is already quite low using no 3rd moment swaps. This is due to the continuous paths of the model. In absence of jumps the Taylor-like approximation cutted off after the quadratic term in (2) is already very accurate. However real markets jump and it are precisely these jumps that can cause significant hedging errors.


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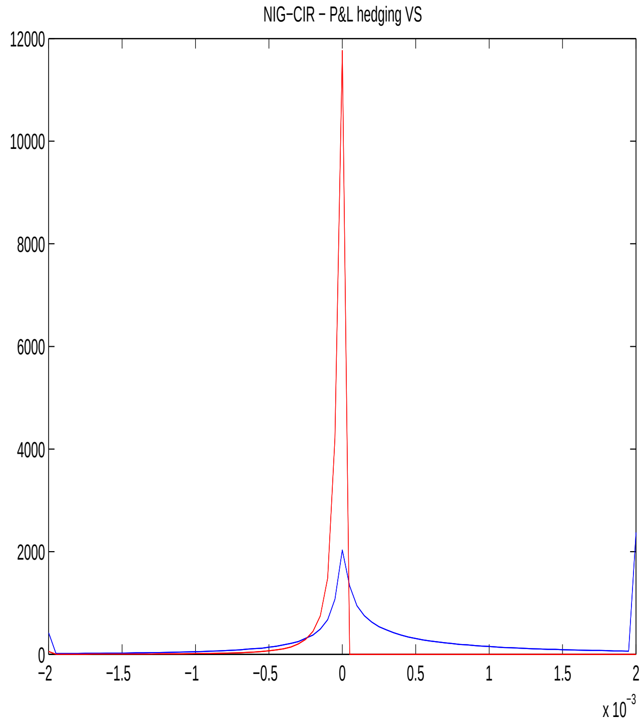


Figure 1: Variance Swap hedge P&L under NIG-CIR. Hedge (5) in blue; Hedge (6) in red

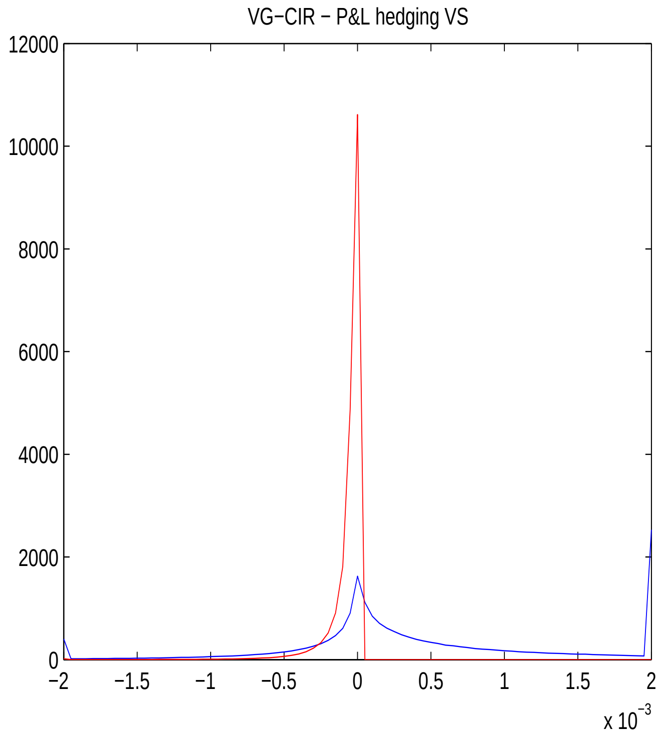


Figure 2: Variance Swap hedge P&L under VG-CIR. Hedge (5) in blue; Hedge (6) in red

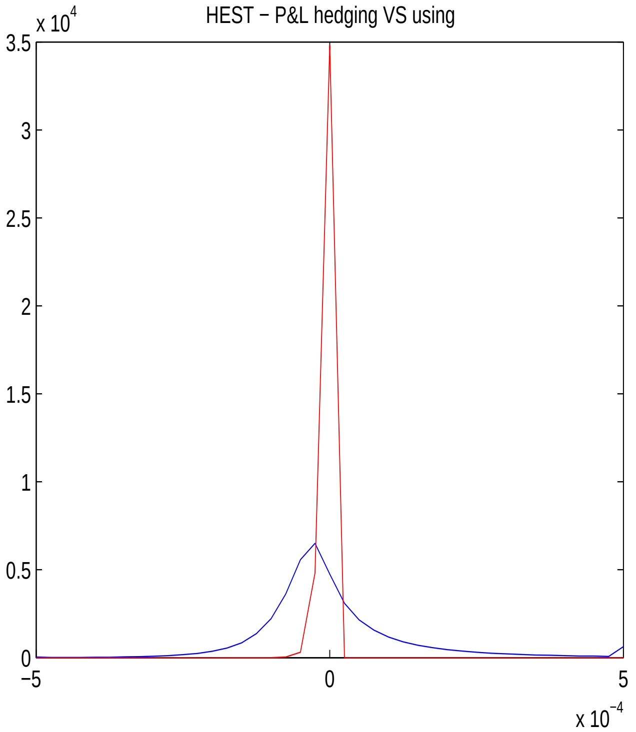


Figure 3: Variance Swap hedge P&L under HEST. Hedge (5) in blue; Hedge (6) in red