# **Word Representations**

by

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# GloVe

Global Vectors for Word Representation Jeffrey Pennington, Richard Socher, Christopher D. Manning

Presented by Guangyu Zhou

### Outline

Papers: Word embedding -- GloVe: Global Vectors for Word Representation

- 1. Motivation
- 2. Notations
- 3. Model Construction
- 4. Complexity
- 5. Experiments

# Preliminary: Word embedding

Word embedding refers to a kind of methods that learn a distributed dense vector for each word in a vocabulary.

- Traditional word embedding methods first obtain the co-occurrence matrix then perform dimension reduction with PCA. (Global matrix factorization)
- Recent methods use neural language models that directly learn word vectors by predicting the **context** words of the target word. (Local context window)

#### 1. Motivation

Two main model families for learning word vectors are:

- (1) Global matrix factorization -- Latent Semantic Analysis
- (2) Local context window -- Skip-gram model (Word2Vec)

(1) Global matrix factorization utilizes low rank approximations to decompose large matrix that capture statistical information about a corpus.

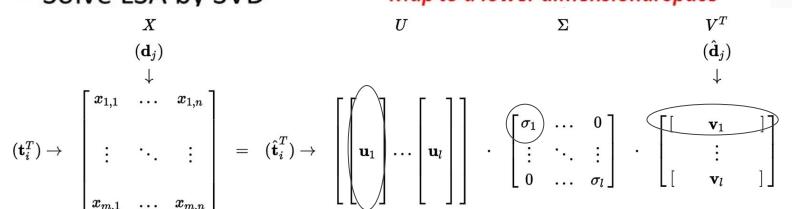
Latent Semantic Analysis -- term-document matrix

Latent semantic analysis (LSA) is a technique in natural language processing, in particular distributional semantics, of **analyzing relationships between a set of documents and the terms** they contain by producing a set of concepts related to the documents and terms.

# Latent Semantic Analysis

Solve LSA by SVD

#### Map to a lower dimensional space



- Perform SVD on document-term adjacency matrix
- 2. Construct  $C_{M\times N}^k$  by only keeping the largest k singular values in  $\Sigma$  non-zero

# The two main model families for learning word vectors Pros & Cons

### (1) Global matrix factorization (LSA)

- + Efficiently leverage statistical information
- Do relatively poorly on the word analogy task, indicating a sub-optimal vector space structure.

### (2) Local context window (Skip-gram)

- + Do better on the analogy task
- Poorly utilize statistics of the corpus since they train on separate local context windows instead of on global co-occurrence counts.

#### 1. Motivation

GloVe combines the advantages of two major model families by producing a new global **log-bilinear regression model** 

It efficiently leverage statistical information by training only on the nonzero elements in a word-word co-occurrence matrix, rather than entire sparse matrix or on individual context windows in a large corpus

### **Log-bilinear Model**

Given context  $\mathbf{w}_{1:n-1}$ , model predicts the next word  $\mathbf{W}_n$  by linearly combining the representations of context.

$$\hat{r} = \sum_{i=1}^{n-1} C_i r_{w_i}$$
  $r_{w_i}$  is the real-valued word vector representing word  $w_i$ 

Then the description for the next word is computed based on the similarity between the predicted distribution and representations of all words in the vocabulary.

$$P(w_n = w | w_{1:n-1}) = \frac{\exp(\hat{r}^T r_w)}{\sum_{i} \exp(\hat{r}^T r_i)}$$

#### 2. Notations

X : matrix of word-word co-occurrence counts

 $X_{ij}$ : the number of times word i occurs in the context of word j

 $W_i$ : vector form for word i

 $X_i = \sum_k X_{ik}$ : the number of times any word appears in the context of word i

 $P_{ij} = P(j|i) = X_{ij}/X_i$ : probability that word j appears in the context of word i

#### 3. Model Construction

**Step 1:** Co-occurrence probabilities for target words *ice* and *steam* with selected context words from a **6 billion token corpus**. Only in the ratio does noise from non-discriminative words like *water* and *fashion* cancel out.

**Argument**: the appropriate **starting point for word vector learning** should be with **ratios of co-occurrence probabilities** rather than **the probabilities themselves**.

Probability and Ratio	k = solid	k = gas	k = water	k = fashion
P(k ice)	$1.9 \times 10^{-4}$	$6.6 \times 10^{-5}$	$3.0\times10^{-3}$	$1.7 \times 10^{-5}$
P(k steam)	$2.2 \times 10^{-5}$	$7.8 \times 10^{-4}$	$2.2\times10^{-3}$	$1.8\times10^{-5}$
P(k ice)/P(k steam)	8.9	$8.5 \times 10^{-2}$	1.36	0.96

## Step 2:

**Noting**: the ratio  $P_{ik}/P_{jk}$  depends on three words i, j, k. w and  $\tilde{w}$ , are learned independently.

 $\Rightarrow$  General model form:  $F(w_i, w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{ik}}$ 

## Step 3:

**Intention**: encode information of ratio  $P_{ik}/P_{jk}$  into function F

**⇒** Most natural way to do - vector difference:

$$F(w_i - w_j, \tilde{w}_k) = \frac{P_{ik}}{P_{jk}}$$

## Step 4:

**Imbalance**: the right-hand side of **F** is a **scalar**, while parameters in left-hand side are **vectors**.

⇒ To avoid this issue, take the dot product

$$F\left((w_i - w_j)^T \tilde{w}_k\right) = \frac{P_{ik}}{P_{jk}}$$

## Step 5:

**Note**: for word-word co-occurrence matrices  $X_{ij}$ , the distinction between a word and a context word is arbitrary and that we are **free to exchange the two roles**.

To do so consistently, we must not only exchange  $w \leftrightarrow w^{\tilde{}}$  but also  $X \leftrightarrow X^T$ 

⇒Our final model should be **invariant under this relabeling** ⇒ **store symmetry** 

## Step 6:

$$F\left(\left(w_i-w_j\right)^T\tilde{w}_k\right)=\frac{P_{ik}}{P_{jk}}\,,$$

$$F\left((w_i-w_j)^T\tilde{w}_k\right) = \frac{F(w_i^T\tilde{w}_k)}{F(w_j^T\tilde{w}_k)} \qquad F(w_i^T\tilde{w}_k) = P_{ik} = \frac{X_{ik}}{X_i}$$

$$F(w_i^T \tilde{w}_k) = P_{ik} = \frac{X_{ik}}{X_i}$$



$$F = \exp$$



$$w_i^T \tilde{w}_k = \log(P_{ik}) = \log(X_{ik}) - \log(X_i)$$

## Step 7:

⇒ continue to store symmetry

$$w_i^T \tilde{w}_k = \log(P_{ik}) = \log(X_{ik}) - \log(X_i)$$

$$w_i^T \tilde{w}_k + b_i + \tilde{b}_k = \log(X_{ik})$$

### Step 8:

Problem: 
$$w_i^T \tilde{w}_k + b_i + \tilde{b}_k = \log(X_{ik})$$
  $\log(X_{ik}) \rightarrow \log(1 + X_{ik})$ 

- 1. Actually ill-defined since the **logarithm diverges** whenever its argument is 0.
- 2. Additive shifting? => model weights all co-occurrences equally

$$J = \sum_{i,j=1}^{r} f(X_{ij}) \left( w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2$$

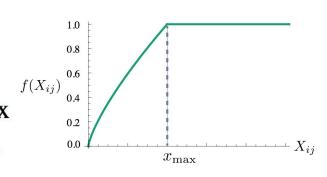
## Step 9:

Constraints: f(0) = 0; non-decreasing; relatively small for large values of x

⇒ One class of functions that work well

$$f(x) = \begin{cases} (x/x_{\text{max}})^{\alpha} & \text{if } x < x_{\text{max}} \end{cases}$$

$$1 & \text{otherwise}$$



#### **Model Construction summary**

Procedures : generate inspirations from example ⇒ maths representation

- ⇒ continuously adding constraints to polish up the cost function
  - 1. Vector difference
  - 2. Inner product
  - 3. Keep symmetry
  - 4. Avoid logarithmic divergence
  - 5. Adjust weights

Cost Function: 
$$J = \sum_{i,j=1}^{V} f(X_{ij}) (w_i^T \tilde{w}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$
,

The computational complexity of the model depends on the number of nonzero elements in the matrix X.

$$O(|C|) = ?O(|V|^2)$$

$$|C| \sim \sum_{i,j} X_{i,j} = \sum_{r=1}^{|X|} \frac{k}{r^{\alpha}} = kH_{|X|,\alpha}$$

$$|C| \sim |X|^{\alpha} H_{|X|,\alpha}$$

$$|C| \sim |X|^{\alpha} H_{|X|,\alpha}$$

$$|C| \sim |X|^{\alpha} H_{|X|,\alpha}$$

$$|X| = \begin{cases} O(|C|) & \text{if } \alpha < 1 \\ O(|C|^{1/\alpha}) & \text{if } \alpha > 1 \end{cases}$$

### Tasks:

Word analogies
Word similarity
Named entity recognition

### Corpus:

2010 Wikipedia dump with 1 billion tokens; 2014 Wikipedia dump with 1.6 billion tokens; Gigaword 5 which has 4.3 billion tokens; Combination Gigaword5 and Wikipedia2014; 42 billion tokens of web data, from Common Crawl.

frog
frogs
toad
litoria
leptodactylidae
rana
lizard
eleutherodactylus







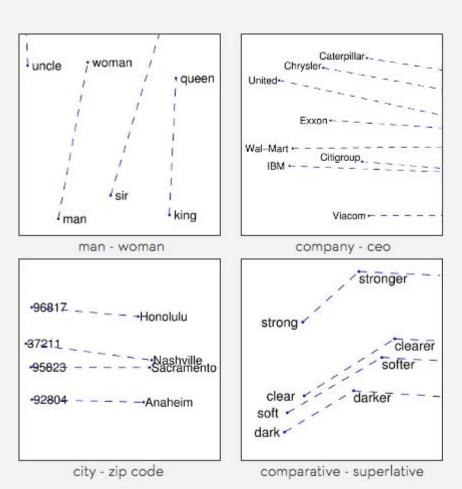
4. leptodactylidae



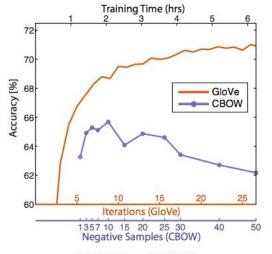
5. rana

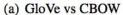


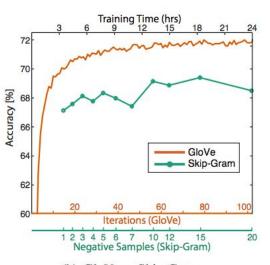
7. eleutherodactylus



Model	Size	WS353	MC	RG	<b>SCWS</b>	RW
SVD	6B	35.3	35.1	42.5	38.3	25.6
SVD-S	6B	56.5	71.5	71.0	53.6	34.7
SVD-L	6B	65.7	72.7	75.1	56.5	37.0
CBOW <sup>†</sup>	6B	57.2	65.6	68.2	57.0	32.5
SG <sup>†</sup>	6B	62.8	65.2	69.7	58.1	37.2
GloVe	6B	65.8	72.7	77.8	53.9	38.1
SVD-L	42B	74.0	76.4	74.1	58.3	39.9
GloVe	42B	75.9	83.6	82.9	<b>59.6</b>	47.8
CBOW*	100B	68.4	79.6	75.4	59.4	45.5







(b) GloVe vs Skip-Gram

## Conclusion:

- GloVe combines advantages from global matrix factorization & local context window.
- 2. This well-written paper gives us a complete view of model intuition and construction.
- 3. GloVe outperforms Word2Vec.

Improving Distributional Similarity with Lessons Learned from Word Embeddings

# The Contributions of Word Embeddings

#### **Novel Algorithms**

(objective + training method)

- Skip Grams + Negative Sampling(SGNS)
- CBOW + Hierarchical Softmax
- Noise Contrastive Estimation
- GloVe
- . . .

#### **New Hyperparameters**

(preprocessing, smoothing, etc.)

- Subsampling
- Dynamic Context Windows
- Context Distribution Smoothing
- Adding Context Vectors
- •

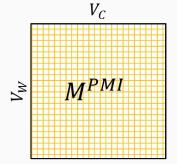
What's really improving performance?

# Count-based word embedding models

PPMI Matrix (positive pointwise mutual information)

- Each row is a word w in vocabulary V<sub>w</sub>
- Each column is a context c in V<sub>c</sub>

$$PMI(w,c) = \log \frac{\hat{P}(w,c)}{\hat{P}(w)\hat{P}(c)} = \log \frac{\#(w,c)\cdot|D|}{\#(w)\cdot\#(c)}$$
$$PPMI(w,c) = \max \left(PMI(w,c),0\right)$$



Singular Value Decomposition(SVD)

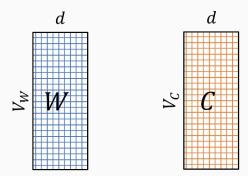
Factorize PPMI into  $U \cdot \Sigma \cdot V^{\top}$  (UU<sup>T</sup>=I,  $\Sigma$  diagonal eigenvalue matrix,  $VV^{\top}$ =I)

Keeping only top d element of  $\Sigma$ :

$$U_d \cdot \Sigma_d \cdot V_d^{\top}$$

$$W^{\text{SVD}} = U_d \cdot \Sigma_d \qquad C^{\text{SVD}} = V_d$$

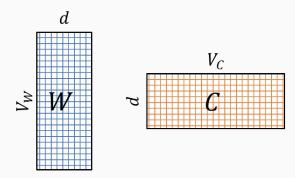
Take SGNS's embedding matrices(W and C)



Take SGNS's embedding matrices(W and C)

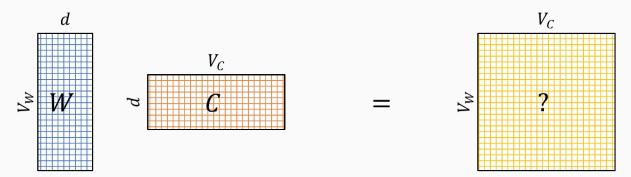
Multiply them

What do you get?



 $A V_w \times V_c$  matrix

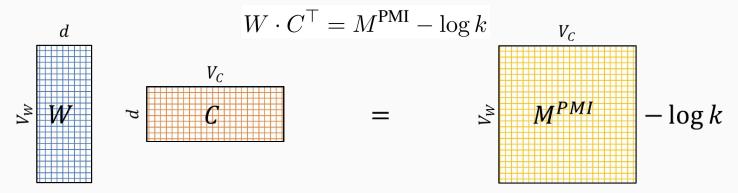
Each cell describes the relation between a specific word-context pair



"Neural Word Embeddings as Implicit Matrix Factorization" Levy & Goldberg, NIPS 2014

The proof shows for large enough d and enough iterations

SGNS factorizes word-context PMI matrix, shifted by a global constant.



"Neural Word Embeddings as Implicit Matrix Factorization" Levy & Goldberg, NIPS 2014

# The Contributions of Word Embeddings

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- GloVe
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#### **New Hyperparameters**

(preprocessing, smoothing, etc.)

- Subsampling
- Dynamic Context Windows
- Context Distribution Smoothing
- Adding Context Vectors
- •

What's really improving performance?

# New Hyperparameters

Preprocessing

- (word2vec)
- Dynamic Context Windows
- Subsampling
- Deleting Rare Words
- Postprocessing

(GloVe)

- Adding Context Vectors
- Association Metric

(SGNS)

- Shifted PMI
- Context Distribution Smoothing

## **Dynamic Context Windows**

### Marco saw a furry little wampimuk hiding in the tree

word2vec:	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$		$\frac{4}{4}$	$\frac{3}{4}$	$\frac{2}{4}$	$\frac{1}{4}$
GloVe:	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{1}$		<u>1</u>	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
Aggressive:	1 8	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{1}$		<u>1</u>	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

The Word-Space Model (Sahlgren, 2006)

### Adding Context Vectors

- SGNS creates word vectors w
- SGNS creates auxiliary context vectors c
  - So do GloVe and SVD
- Instead of just w
- Represent a word as: w + c
- Adding second order similarity  $(\mathbf{w}_{x} \cdot \mathbf{w}_{y}, \mathbf{c}_{x} \cdot \mathbf{c}_{y})$

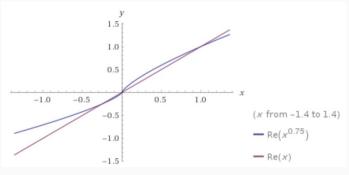
### **Context Distribution Smoothing**

- SGNS samples c' $\sim$ P to form negative (w,c') examples
- In practice, it's a smoothed unigram distribution

$$P^{0.75}(c) = \frac{(\#c)^{0.75}}{\sum_{c' \in V_C} (\#c')^{0.75}}$$

- Adding bias towards rare words
- Applicable to PMI:

$$PMI_{\alpha}(w,c) = \log \frac{\hat{P}(w,c)}{\hat{P}(w)\hat{P}_{\alpha}(c)}$$
$$\hat{P}_{\alpha}(c) = \frac{\#(c)^{\alpha}}{\sum_{c} \#(c)^{\alpha}}$$



### Recap: All Hyperparameters

- Preprocessing
  - Dynamic Context Windows (dyn)
  - Subsampling (sub)
  - Deleting Rare Words (del)
- Postprocessing
  - Adding Context Vectors (w+c)
- Association Metric
  - Shifted PMI (neg)
  - Context Distribution Smoothing (cds)

- Others
  - Context window size (win)
  - Eigenvalue Weighting (eig)
  - Vector Normalization (nrm)

### Systematic Experiments Setup

### 9 Hyperparameters

Hyper-	Explored	Applicable		
parameter	Values	Methods		
win	2, 5, 10	All		
dyn	none, with	All		
sub	none, dirty, clean <sup>†</sup>	All		
del	none, with <sup>†</sup>	All		
neg	1, 5, 15	PPMI, SVD, SGNS		
cds	1,0.75	PPMI, SVD, SGNS		
w+c	only $w, w + c$	SVD, SGNS, GloVe		
eig	0, 0.5, 1	SVD		
nrm	none <sup>†</sup> , row, col <sup>†</sup> , both <sup>†</sup>	All		

# **4** Word Representation Algorithms

- 1. PPMI (Sparse)
- 2. SVD
- 3. SGNS
- 4. GloVe

### 8 Benchmarks

- 6 Word Similarity
   Task Datasets
- 2 Analogy Task
   Dataset (Google,
   MSR)

Lots of experiments...

### Results (in detail)

Method	WordSim	WordSim	Bruni et al.	Radinsky et al.	Luong et al.	Hill et al.	Google	MSR
	Similarity	Relatedness	MEN	M. Turk	Rare Words	SimLex	Add / Mul	Add / Mul
PPMI	.709	.540	.688	.648	.393	.338	.491 / <b>.650</b>	.246 / .439
SVD	.776	.658	.752	.557	.506	.422	.452 / .498	.357 / .412
<b>SGNS</b>	.724	.587	.686	.678	.434	.401	.530 / .552	.578 / <b>.592</b>
GloVe	.666	.467	.659	.599	.403	.398	.442 / .465	.529 / .576

Table 2: Performance of each method across different tasks in the "vanilla" scenario (all hyperparameters set to default): win = 2; dyn = none; sub = none; neg = 1; cds = 1; w+c = only w; eig = 0.0.

Method	WordSim	WordSim	Bruni et al.	Radinsky et al.	Luong et al.	Hill et al.	Google	MSR
	Similarity	Relatedness	MEN	M. Turk	Rare Words	SimLex	Add / Mul	Add / Mul
PPMI	.755	.688	.745	.686	.423	.354	.553 / <b>.629</b>	.289 / .413
SVD	.784	.672	.777	.625	.514	.402	.547 / .587	.402 / .457
<b>SGNS</b>	.773	.623	.723	.676	.431	.423	.599 / .625	.514 / .546
GloVe	.667	.506	.685	.599	.372	.389	.539 / .563	.503 / <b>.559</b>
CBOW	.766	.613	.757	.663	.480	.412	.547 / .591	.557 / <b>.598</b>

Table 3: Performance of each method across different tasks using word2vec's recommended configuration: win = 2; dyn = with; sub = dirty; neg = 5; cds = 0.75; w+c = only w; eig = 0.0. CBOW is presented for comparison.

Method	WordSim	WordSim	Bruni et al.	Radinsky et al.	Luong et al.	Hill et al.	Google	MSR
	Similarity	Relatedness	MEN	M. Turk	Rare Words	SimLex	Add / Mul	Add / Mul
PPMI	.755	.697	.745	.686	.462	.393	.553 / .679	.306 / .535
SVD	.793	.691	.778	.666	.514	.432	.554 / .591	.408 / .468
<b>SGNS</b>	.793	.685	.774	.693	.470	.438	.676 / <b>.688</b>	.618 / <b>.645</b>
GloVe	.725	.604	.729	.632	.403	.398	.569 / .596	.533 / .580

Table 4: Performance of each method across different tasks using the best configuration for that method and task combination, assuming win = 2.

### Results (in detail)

.764

.772

.745

.735

.766

.794

.679

.690

.617

.701

.681

.700

.776

.772

.746

.741

.770

.775

SVD

SGNS

GloVe

**PPMI** 

SVD

**SGNS** 

5

10

win	Method	WordSim	WordSim	Bruni et al.	Radinsky et al.	Luong et al.	Hill et al.	Google	MSR
		Similarity	Relatedness	MEN	M. Turk	Rare Words	SimLex	Add / Mul	Add / Mul
2	PPMI	.732	.699	.744	.654	.457	.382	.552 / .677	.306 / .535
	SVD	.772	.671	.777	.647	.508	.425	.554 / .591	.408 / .468
	SGNS	.789	.675	.773	.661	.449	.433	.676 / <b>.689</b>	.617 / <b>.644</b>
	GloVe	.720	.605	.728	.606	.389	.388	.649 / .666	.540 / .591
	PPMI	.732	.706	.738	.668	.442	.360	.518 / .649	.277 / .467

.639

.663

.631

.663

.628

.678

.499

.454

.416

.235

.312

.281

.416

.403

.389

.336

.419

.422

.532 / .569

.692 / .714

.700 / .712

.532 / .605

.526 / .562

.694 / .710

.369 / .424

.605 / .645

.541 / .599

.249 / .353

.356 / .406

.520 / .557

GloVe .746 .643 .754 .616 .266 .375 .702 / .712 .463 / .519 **SGNS-LS** .766 .681 .781 .689 .451 .414 .739 / .758 .690 / .729 10 GloVe-LS .678 .361 .371 .732 / .750.628 / .685 .624 .752 .639 Table 5: Performance of each method across different tasks using 2-fold cross-validation for hyperparameter tuning. Configurations on large-scale (LS) corpora are also presented for comparison.

### Hyperparameter Settings

### Classic Setting/Vanilla Recommended word2vec Setting

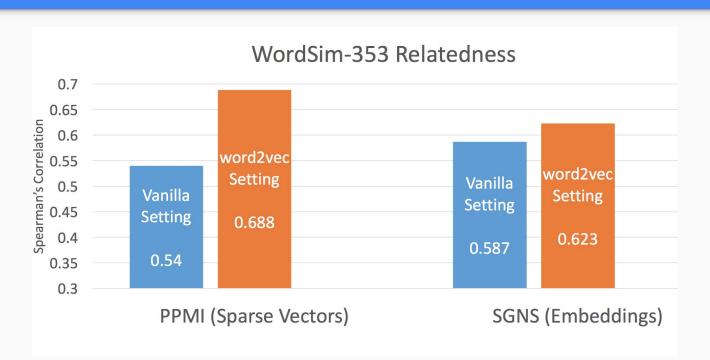
- → Preprocessing
  - <None>
- → Postprocessing
  - <None>
- → Association Metric
  - ◆ PMI/PPMI

- → Preprocessing
  - Dynamic Context Window
  - Subsampling
- → Postprocessing
  - <None>
- → Association Metric
  - Shifted PMI/PPMI
  - Context Distribution Smoothing

### **Optimal Setting**

→ Enable full range of hyper-parameters and choose the best performance

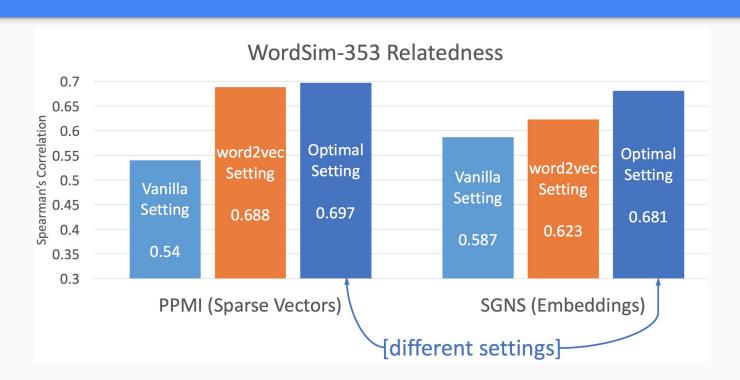
### **Experiments: Comparison**



#### Note on measurement

Spearman's Correlation:
Pearson correlation coefficient
between the ranked variables

### **Experiments: Hyperparameter Tuning**



### **Overall Results**

- → Hyperparameters vs Algorithms
  - Hyperparameters often have stronger effects than algorithms.
- → Hyperparameters vs Big Data
  - Hyperparameters often have stronger effects than more data.
- → Questions on prior superiority claims
- **→** ...

# Re-evaluating Prior Claims

### 1. Embedding vs count-based methods

--Don't Count, Predict! (Baroni et al., 2014)

Hyperparameter settings account for most of the reported gaps. Embeddings do not really outperform count-based methods (except MSR analogy task).

# Re-evaluating Prior Claims

- 1. Embedding vs count-based methods
- 2. GloVe vs SGNS?

--GloVe (Pennington et al., 2014)

Difference on hyperparameter settings account for most of the reported gaps. It is observed that SGNS outperformed GloVe almost on every task.

## Re-evaluating Prior Claims

- 1. Embedding vs count-based methods
- 2. GloVe vs SGNS
- 3. Linguistic Regularities in Word Representations

"PPMI vectors perform on par with SGNS on analogy tasks." (Levy and Goldberg, 2014)

It holds on semantic tasks but not on syntactic analogies. In the aspect of syntactic analogies, there is a gap in favor of SGNS.

**Examples: Syntactic**: "Good is to best as smart is to smartest" (MSR)

**Semantic**: "Paris is to France as Tokyo is to Japan." (Google)

### Conclusions

- Contributions of Word Embeddings: New hyperparameters more than novel algorithms, that is, hyperparameters mostly are improving performances.
- 2. Beneficial configurations and practical recommendations.
- 3. Primary suggestion: Look for new hyperparameters and adapt parameters across different models

### References

- 1. Pennington, Jeffrey, et al. "Glove: Global vectors for word representation." *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*. 2014.
- 2. Levy, Omer, et al. "Improving distributional similarity with lessons learned from word embeddings." *Transactions of the Association for Computational Linguistics* 3 (2015): 211-225.
- Levy, Omer, and Yoav Goldberg. "Neural word embedding as implicit matrix factorization."
   Advances in neural information processing systems. 2014.

# Thanks! Q&A

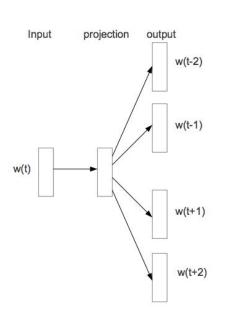


# Refresh: Skip-gram model

- Objective: Find word representations that are useful for predicting the surrounding words in a sentence or a document.
- Cost function:  $\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p(w_{t+j}|w_t)$

with probabilities defined as, 
$$p(o|c) = \frac{\exp\left(u_o^T v_c\right)}{\sum_{w=1}^W \exp\left(u_w^T v_c\right)}$$

which is impractical because the cost of computing gradient is proportional to W.



# Refresh: Skip-gram + negative sampling = Word2Vec

Overall objective function:  $J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J_t(\theta)$ 

$$J_t(\theta) = \log \sigma \left( u_o^T v_c \right) + \sum_{i=1}^k \mathbb{E}_{j \sim P(w)} \left[ \log \sigma \left( -u_j^T v_c \right) \right] = \log \sigma \left( u_o^T v_c \right) + \sum_{j \sim P(w)} \left[ \log \sigma \left( -u_j^T v_c \right) \right]$$

Distinguish the target word  $W_0$  from draws from the noise distribution  $P_n(w)$  using logistic regression

# Supplementary Topic: Link GloVe with word2vec



GloVe

Local cost function:

where 
$$f(x)$$
 is weighting function, shown

 $l_G(w_i, c_i) = f(\#(w_i, c_i)) (W_i \cdot C_i^T + b_{W_i} + b_{C_i} - \log \#(w_i, c_i))^2$ 

as:

$$f(x) = \begin{cases} (x/x_{\text{max}})^{\alpha} & x < x_{\text{max}} \\ 1 & \text{otherwise} \end{cases}$$

Optimal solution:

$$W_i \cdot C_j^T = \log \# \left( w_i, c_j \right) - b_{W_i} - b_{C_j}$$



SGNS

Local objective:

 $W_i \cdot C_i^T = PMI(w_i, c_j) - \log k$ 

$$= \log \#(w_i, c_j) - \log \#(w_i) - \log \#(c_j) + \log \Sigma_w \#(w) - \log k.$$

 $l_{S}\left(w_{i}, c_{j}
ight) = \#\left(w_{i}, c_{j}
ight)\log\sigma\left(W_{i}\cdot C_{j}^{T}
ight) + k\cdot\#\left(w_{i}
ight)\cdotrac{\#\left(c_{j}
ight)}{\sum_{v,v}\#\left(w
ight)}\log\sigma\left(-W_{i}\cdot C_{j}^{T}
ight)$ 

 $\sigma(x) = \frac{1}{1+e^{-x}}$ 

Credit: Linking GloVe with word2vec, by Tianze Shi and Zhiyuan Liu (Tsinghua University)