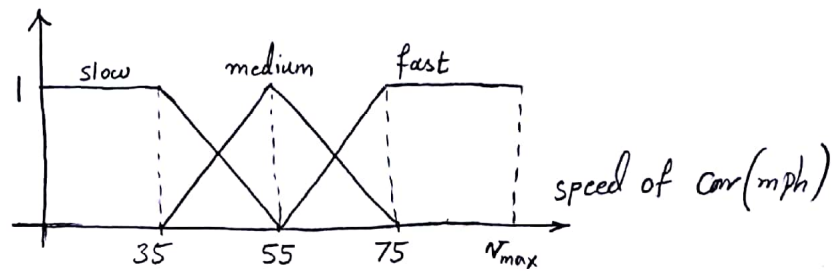
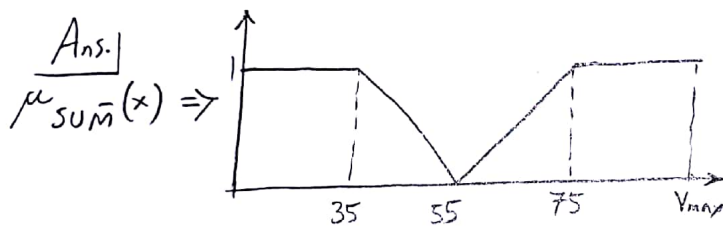
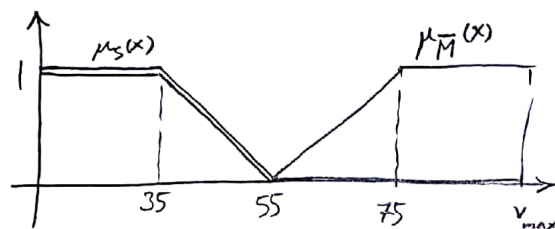


5.4) (5.12) x is S or x is not M

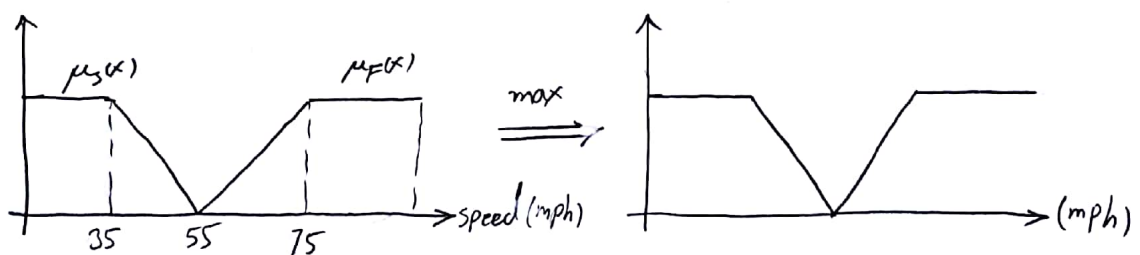


$$\mu_A(x) = \mu_{SUM}(x) = \max[\mu_S(x), 1 - \mu_M(x)]$$



(5.13) x is not S and x is not F

$$\begin{aligned} \Rightarrow \mu_B(x) &= \mu_{\bar{S} \cap \bar{F}}(x) = 1 - \mu_{S \cup F}(x) \\ &= 1 - \max[\mu_S(x), \mu_F(x)] \end{aligned}$$

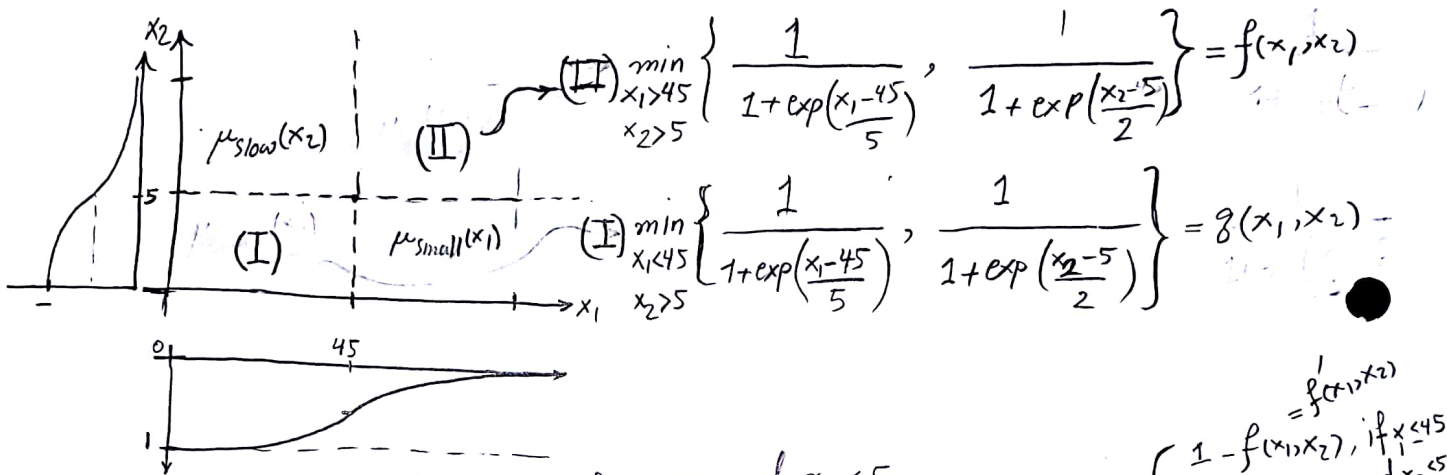


Ans.

5.5) IF x_1 is slow and x_2 is small, then y is large.

$$\mu_{\text{slow}}(x_1) = \frac{1}{1 + \exp\left(\frac{x_1 - 45}{5}\right)}, \quad \mu_{\text{small}}(x_2) = \frac{1}{1 + \exp\left(\frac{x_2 - 5}{2}\right)}, \quad \mu_{\text{large}}(y) = \frac{1}{1 + \exp(2.5 - 2y)}$$

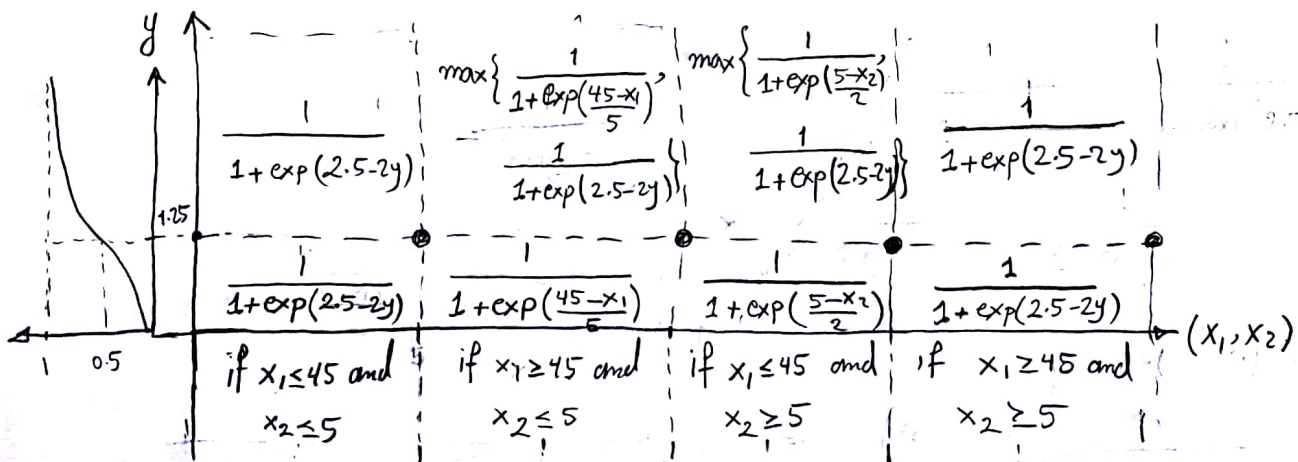
$$\mu_p(x) = \min \left[\mu_{\text{slow}}(x_1), \mu_{\text{small}}(x_2) \right]$$



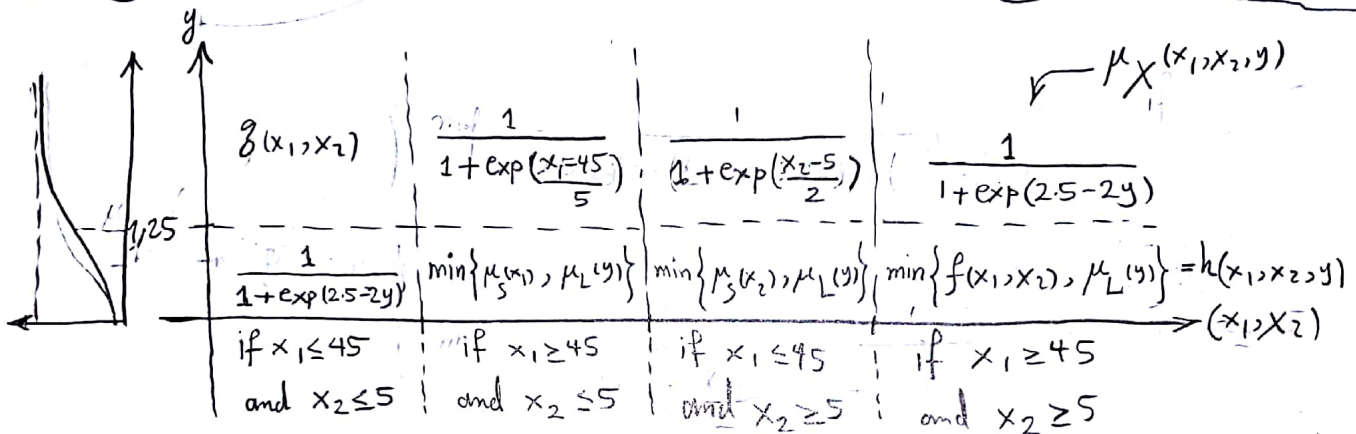
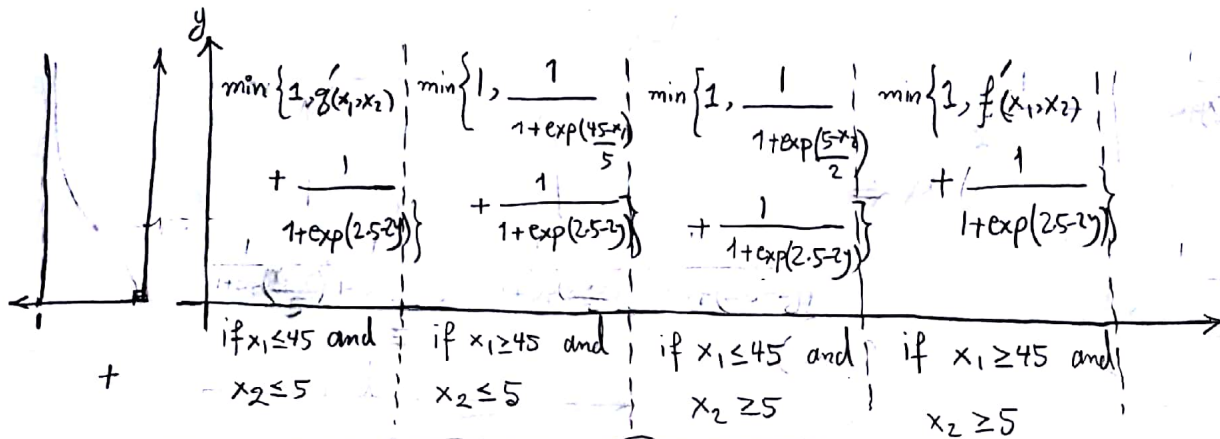
$$\Rightarrow \mu_{\text{PI}}(x_1, x_2) = \begin{cases} f(x_1, x_2), & \text{if } x_1 \leq 45 \text{ and } x_2 \leq 5 \\ \frac{1}{1 + \exp\left(\frac{x_1 - 45}{5}\right)}, & \text{if } x_2 \leq 5 \text{ and } x_1 \geq 45 \\ \frac{1}{1 + \exp\left(\frac{x_2 - 5}{2}\right)}, & \text{if } x_2 > 5 \text{ and } x_1 < 45 \\ g(x_1, x_2), & \text{if } x_1 > 45 \text{ and } x_2 > 5 \end{cases}$$

$$\Rightarrow \mu_{\text{PI}}(x_1, x_2) = \begin{cases} 1 - f(x_1, x_2), & \text{if } x_1 \leq 45 \text{ and } x_2 \leq 5 \\ \frac{1}{1 + \exp\left(\frac{45 - x_1}{5}\right)}, & \text{if } x_2 \leq 5 \text{ and } x_1 \geq 45 \\ \frac{1}{1 + \exp\left(\frac{5 - x_2}{2}\right)}, & \text{if } x_2 > 5 \text{ and } x_1 < 45 \\ 1 - g(x_1, x_2), & \text{if } x_1 > 45 \text{ and } x_2 > 5 \end{cases}$$

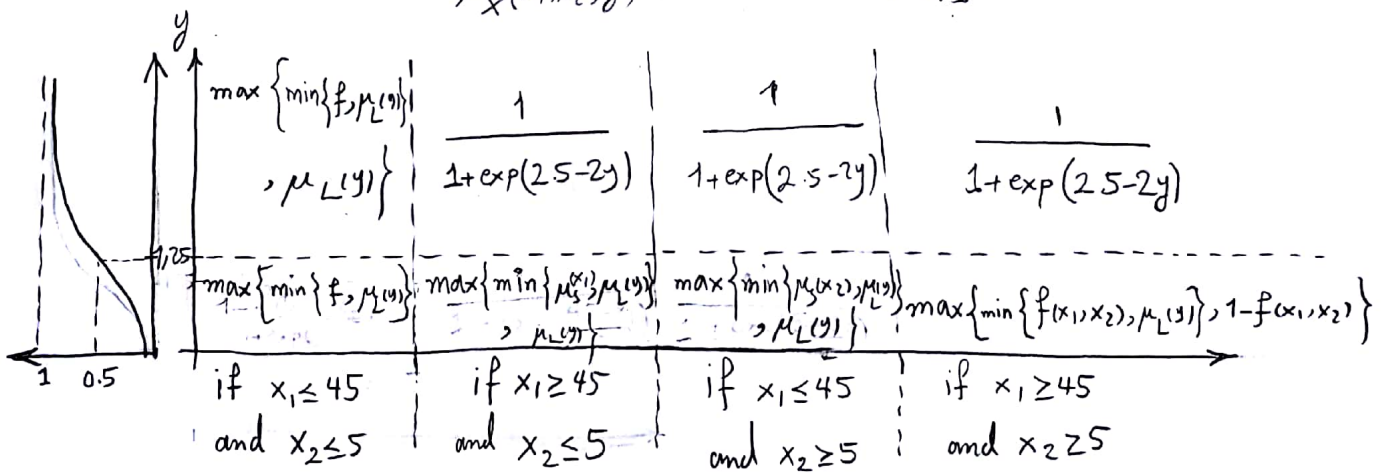
$$\mu_{Q_D}(x_1, x_2, y) = \mu_{P \vee Q}(x_1, x_2, y) = \max \left[\mu_{\text{PI}}(x_1, x_2), \mu_{\text{PI}_2}(y) \right]$$



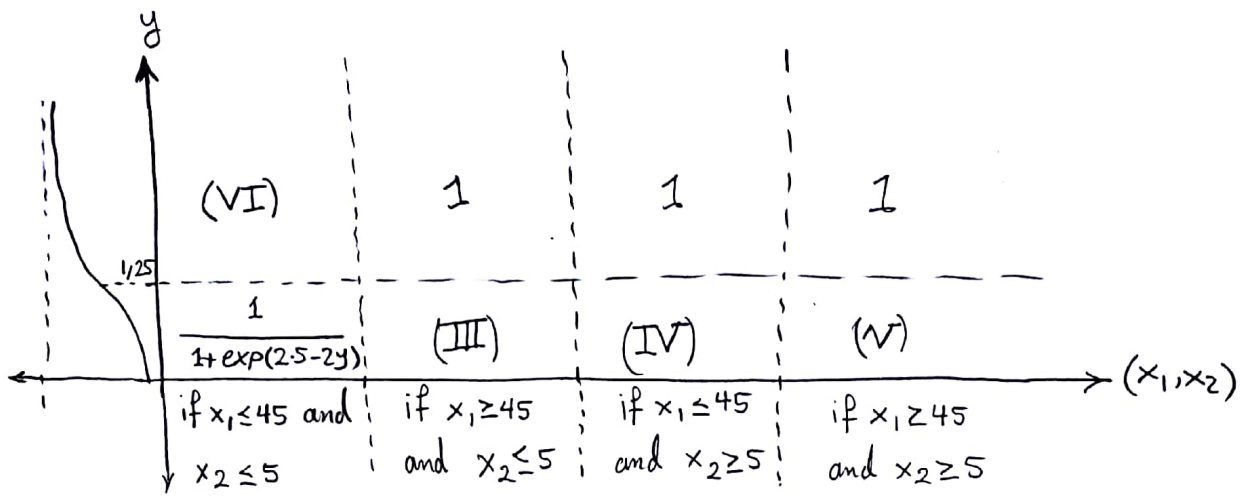
$$\mu_{Q_L}(x_1, x_2, y) = \min [1, 1 - \mu_{FP1}^{(x_1, x_2)} + \mu_{FP2}(y)]$$



$$\mu_{Q_Z}(x_1, x_2, y) = \max \left[\underbrace{\min \left(\mu_{FP1}(x_1, x_2), \mu_{FP2}(y) \right)}_{\mu_X(x_1, x_2, y) \uparrow}, \underbrace{1 - \mu_{FP1}(x_1, x_2)}_{\mu_{FP1}(x_1, x_2)} \right]$$



$$\mu_{Q_G}(x_1, x_2, y) = \begin{cases} 1 & \text{if } \mu_{FP1}(x_1, x_2) \leq \mu_{FP2}(y) \\ \mu_{FP2}(y) & \text{otherwise.} \end{cases}$$



$$(III) \mu_{Q_G}(x_1, x_2, y) = \begin{cases} 1, & \text{if } \mu_{\text{small}}(x_1) \leq \mu_L(y) \\ \mu_L(y), & \text{otherwise.} \end{cases}$$

$$\{x_1 \leq 45, x_2 \leq 5, y \leq 1/25\}$$

$$(IV) \mu_{Q_G}(x_1, x_2, y) = \begin{cases} 1, & \text{if } \mu_{\text{slow}}(x_2) \leq \mu_{\text{Large}}(y) \\ \mu_{\text{Large}}(y), & \text{otherwise.} \end{cases}$$

$$\{x_1 \leq 45, x_2 \geq 5, y \leq 1/25\}$$

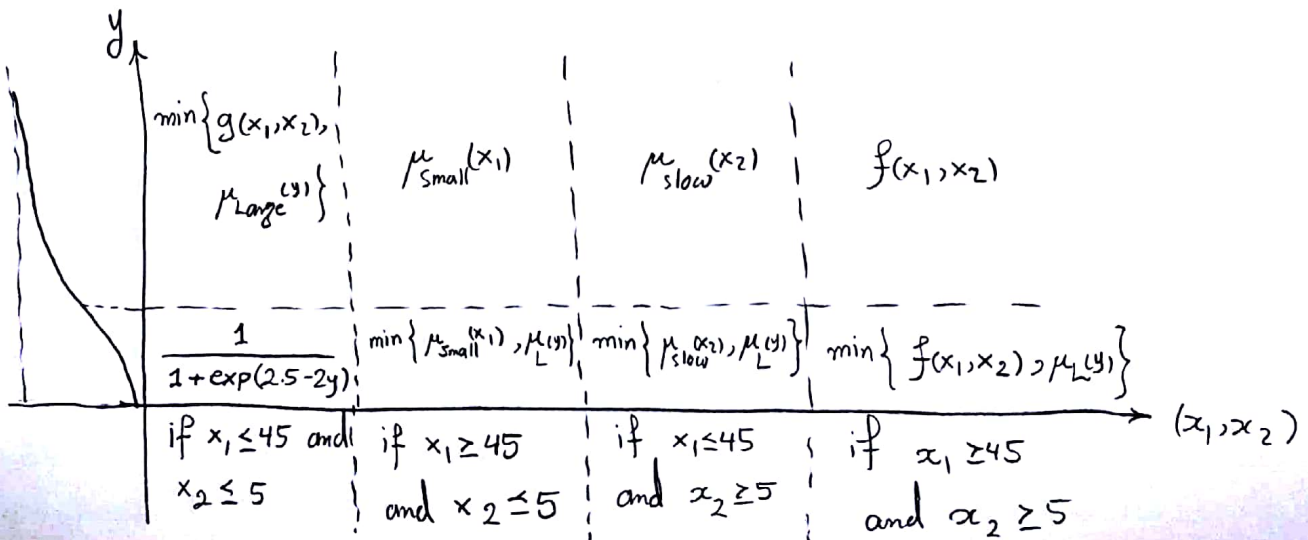
$$(V) \mu_{Q_G}(x_1, x_2, y) = \begin{cases} 1, & \text{if } f(x_1, x_2) \leq \mu_{\text{Large}}(y) \\ \mu_{\text{Large}}(y), & \text{otherwise.} \end{cases}$$

$$\{x_1 \geq 45, x_2 \geq 5, y \leq 1/25\}$$

$$(VI) \mu_{Q_G}(x_1, x_2, y) = \begin{cases} 1, & \text{if } g(x_1, x_2) \leq \mu_{\text{Large}}(y) \\ \mu_{\text{Large}}(y), & \text{otherwise.} \end{cases}$$

$$\{x_1 \leq 45, x_2 \leq 5, y \geq 1/25\}$$

$$\mu_{Q_{MM}}(x_1, x_2, y) = \min [\mu_{FP1}(x_1, x_2), \mu_{FP2}(y)]$$



5.6) Q is called reflexive if $\mu_Q(u, u) = 1$ for all $u \in U$.

$$\begin{aligned}\mu_{Q \circ Q}(x, z) &= \max_{y \in U} \min [\mu_Q(x, y), \mu_Q(y, z)] \\ &= \max_{y \in U} \min [1, 1] = 1\end{aligned}$$

So $Q \circ Q$ is reflexive.

$$\mu_Q(u, u) = \mu_{Q \circ Q}(u, u) = 1$$

$$\Rightarrow \mu_Q \leq \mu_{Q \circ Q} \Rightarrow \mu_Q \subseteq \mu_{Q \circ Q}$$

5.5 - a) $\mu_{Q_{MP}}(x_1, x_2, y) = \mu_{FP1}(x_1, x_2) \mu_{FP2}(y)$

