ایمان شرفی ۹۸۲۱۰۱۸٤ — تمرین سری دوم کنترل فازی

3.1.a) The equilibrium of a fuzzy Complement C is defined as $a \in [0,1]$ Such that $C(a) = a$.

$$C_w(a) = (1 - a^w)^{1/w}$$

Ans. $C(a) = a \Rightarrow (1-a^w)^{1/w} = a$

$$\rightarrow 1 - a^w = a^w \rightarrow 2a^w = 1 \longrightarrow \boxed{a^w = \tfrac{1}{2}}$$

if: $w = 1 \rightarrow \boxed{a = \tfrac{1}{2}}$

3.3.b)

$$\mu_F(x) = \frac{x}{x+2}, \quad \mu_G(x) = 2^{-x}$$

$$\mu_{\bar F}(x) = \frac{2}{x+2}, \quad \mu_{\bar G}(x) = 1-2^{-x} \rightarrow \text{from } 3.1.$$

$$\mu_{F\cap \bar G}(x) = t_{ab}\left(\mu_F(x), \mu_{\bar G}(x)\right) = \mu_F(x)\cdot\mu_{\bar G}(x) = (1-2^{-x})\frac{x}{x+2}$$

$$\mu_{\overline{\bar F\cap G}}(x) = 1-\mu_{\bar F\cap G}(x) = 1-t_{ab}\left(\mu_{\bar F}(x), \mu_G(x)\right) = 1-2^{-x}\cdot\frac{2}{x+2}$$

$$\mu_{\overline{F\cup G}}(x) = 1-\mu_{F\cup G}(x) = 1-S_{as}\left(\mu_F(x), \mu_G(x)\right)$$

$$= 1-\left(\mu_F(x) + \mu_G(x) - \mu_F(x)\cdot\mu_G(x)\right)$$

$$= 1-\left(\frac{x}{x+2} + 2^{-x} - \frac{x}{x+2}\cdot 2^{-x}\right)$$

$$= 1 - \frac{x}{x+2} - \frac{2}{x+2}\cdot 2^{-x}$$

3.5) a) $C_\lambda(a) = \dfrac{1-a}{1+\lambda a}, \lambda \in (-1, \infty) \rightarrow$ Sugeno Class

$C_w(a) = (1-a^w)^{1/w}, w \in (0, \infty) \rightarrow$ Yager Class

$$C_\lambda[C_\lambda(a)] = C_\lambda\left[\frac{1-a}{1+\lambda a}\right] = \frac{1 - \frac{1-a}{1+\lambda a}}{1 + \lambda \frac{1-a}{1+\lambda a}}$$

$$= \frac{\frac{a(\lambda+1)}{1+\lambda a}}{\frac{(1+\lambda)}{1+\lambda a}} = a \Rightarrow \boxed{C_\lambda[C_\lambda(a)] = a}$$ Sugeno is involutive.

$$C_w[C_w(a)] = C_w\left[(1-a^w)^{1/w}\right] = \left\{1 - \left(\underbrace{\left(1-a^w\right)^{1/w}}_{1-a^w}\right)^w\right\}^{1/w}$$

$$= \left(a^w\right)^{1/w} = a \Rightarrow \boxed{C_w[C_w(a)] = a}$$ Yager is involutive.

b) show $u(a,b) = c[t(c(a), c(b))]$

is an S-norm.

- برای اینکه u یک s نرم باشد باید چهار اصل زیر برقرار باشند.

Axiom 1. $u(1,1) = c[t(0,0)] = 1$

$u(0,a) = c[t(1, 1-a)] = a$

$u(a,0) = c[t(1-a, 1)] = a \rightarrow$ Boundary Conditions.

Axiom 2. $u(a,b) = c[t(1-a, 1-b)] = c[t[1-b, 1-a]]$

$= c[t(c(b), c(a))] = u(b,a) \rightarrow$ Commulative Condition

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Axiom 3. $a \le a',\ b \le b' \Rightarrow u(a,b) \overset{?}{\le} u(a',b')$

$$u(a,b) = c\left[t(1-a,1-b)\right] = 1 - t(1-a,1-b) = 1 - \left((1-a)(1-b)\right)$$

$$u(a',b') = c\left[t(1-a',1-b')\right] = 1 - t(1-a',1-b') = 1 - \left((1-a')(1-b')\right)$$

$$\Rightarrow \cancel{1} - \left((1-a')(1-b')\right) \ge \cancel{1} - (1-a)(1-b)$$

$$\Rightarrow (1-a)(1-b) \ge (1-a')(1-b') \xrightarrow[1-b \ge 1-b']{1-a \ge 1-a'}$$ بنابراین این حکم همواره برقرار است

$$\boxed{u(a,b) \le u(a',b')}$$ (Nondecreasing Condition)

Axiom 4. $u\left(u(a,b),c\right) \overset{?}{=} u\left(a,u(b,c)\right)$

$$u\left(c\left[t(1-a,1-b)\right],c\right) = c\left[t\left(1-\left(1-t(1-a,1-b)\right),1-c\right)\right]$$

$$= c\left[t\left(t(1-a,1-b)\right),1-c\right] = c\left[t\left(1-a,t(1-b,1-c)\right)\right]$$

$$= u\left(a,u(b,c)\right)$$ (Assosiative Cond.)

* بنابر چهار شرط فوق $u(a,b) = c\left[t(c(a),c(b))\right]$ یک s-norm است.

c)

راه حل دوم:

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \Rightarrow c\left[S(A,B)\right] = t\left(c(A),c(B)\right)$$

$$\Rightarrow S(A,B) = c\left[t(c(A),c(B))\right]$$

بنابراین $u(A,B)$ یک s-norm است و یک associated class هستند.

3.7.b) $\quad C[S_{ds}(a,b)] \overset{?}{=} t_{dp}[C(a), C(b)]$

$$S_{ds}^{(a,b)} = \begin{cases} a & \text{if } b=0 \\ b & \text{if } a=0 \\ 1 & \text{otherwise} \end{cases}$$

$$t_{dp}^{(a,b)} = \begin{cases} a & \text{if } b=1 \\ b & \text{if } a=1 \\ 0 & \text{otherwise} \end{cases}$$

if: $\begin{cases} a=0 \\ b\neq 0 \end{cases} \Rightarrow C[S_{ds}(a,b)] = C[b] = 1-b$

$t_{dp}[1, 1-b] = 1-b$

if: $\begin{cases} a\neq 0 \\ b=0 \end{cases} \Rightarrow C[S_{ds}(a,b)] = C[a] = 1-a$

$t_{dp}(1-a, 1) = 1-a$

if: $a,b\neq 0 \Rightarrow C[1] = 0$

$t_{dp}[1-a, 1-b] = 0$

$\xRightarrow{\text{بنابراین}} C[S_{ds}(a,b)] = t_{dp}[C(a), C(b)]$

بازای همه حالات ممکن برای a و b صادق است

بنابراین شرط associated class را دارا میباشند.

3.8) Generalized Means $\rightarrow v_\alpha(a,b) = \left(\frac{a^\alpha + b^\alpha}{2}\right)^{1/\alpha}$

① $\alpha \to -\infty \Rightarrow \lim\limits_{\alpha\to-\infty} v_\alpha(a,b) = \lim\limits_{\alpha\to-\infty}\left(\frac{a^\alpha+b^\alpha}{2}\right)^{1/\alpha} \xrightarrow[1>a>b>0]{\text{فرض}} = \lim\limits_{\alpha\to-\infty} b\left(\frac{\left(\frac{a}{b}\right)^\alpha+1}{2}\right)^{1/\alpha}$

$= b\left(\frac{1}{2}\right)^0 = b = \min(a,b)$

* این رابطه برای فرض $1>b>a>0$ نیز صادق است.

② $\alpha \to +\infty \Rightarrow \lim\limits_{\alpha\to+\infty} v_\alpha(a,b) = \lim\limits_{\alpha\to+\infty}\left(\frac{a^\alpha+b^\alpha}{2}\right)^{1/\alpha} \xrightarrow[1>a>b>0]{\text{فرض}} \lim\limits_{\alpha\to+\infty} a\left(\frac{1+\left(\frac{b}{a}\right)^\alpha}{2}\right)^{1/\alpha}$

$= a\left(\frac{1}{2}\right)^0 = a = \max(a,b)$

* این رابطه برای $1>b>a>0$ نیز صادق است.