تمرین سری پنجم کنترل فازی

آیناز شریفی ۹۸۲۱۰۱۸۴

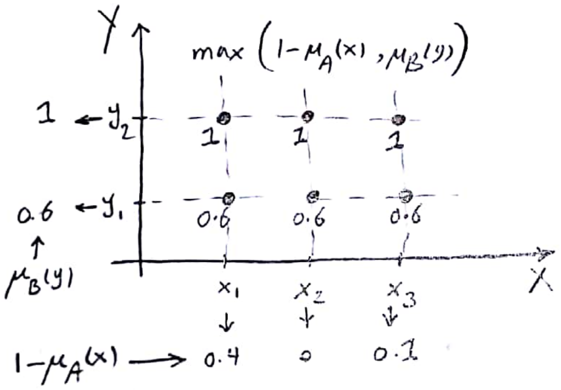
6.3) $U=\{x_1, x_2, x_3\}$, $V=\{y_1, y_2\}$

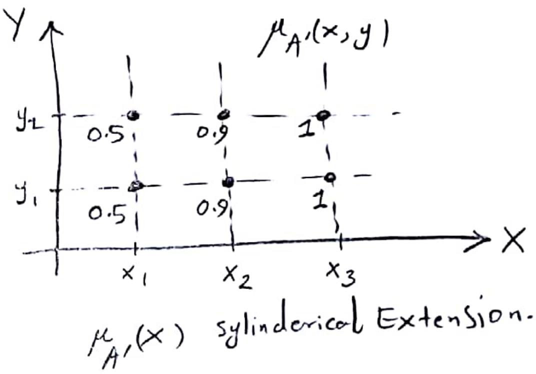
$A = 0.6/x_1 + 1/x_2 + 0.9/x_3$, $B = 0.6/y_1 + 1/y_2$

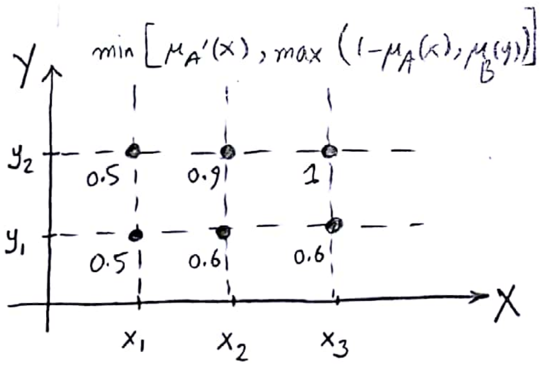
If x is A, Then y is B $\rightarrow A' = 0.5/x_1 + 0.9/x_2 + 1/x_3$

if x is A', then y is $B'(?)$

a) $\mu_{B'}(y) = \sup\limits_{x \in U} \min\Big[\mu_{A'}(x), \underbrace{\max\big(1-\mu_A(x), \mu_B(y)\big)}_{\text{Dienes-Rescher}}\Big]$



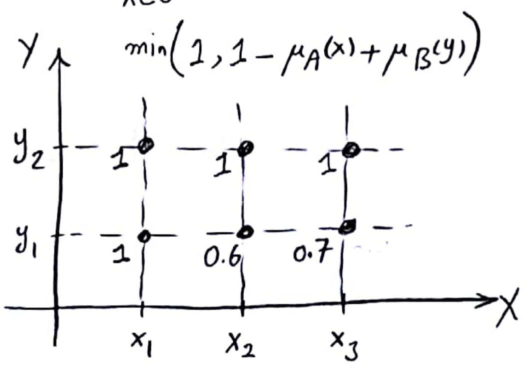


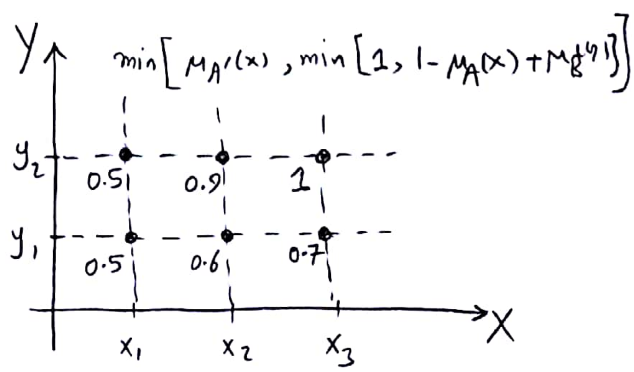


$\Rightarrow \mu_{B'}(y) = \{0.6/y_1, 1/y_2\}$

$= \mu_B(y)$

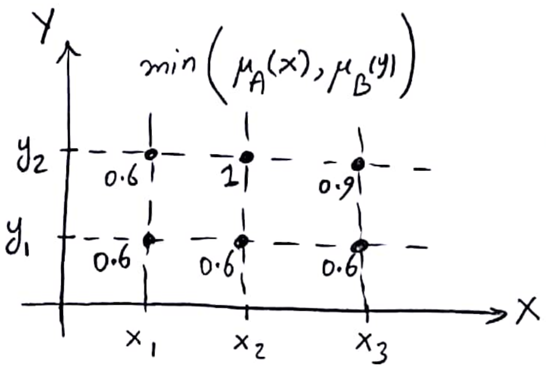
b) $\mu_{B'}(y) = \sup\limits_{x \in U} \min\Big[\mu_{A'}(x), \min\big[1, 1-\mu_A(x)+\mu_B(y)\big]\Big]$

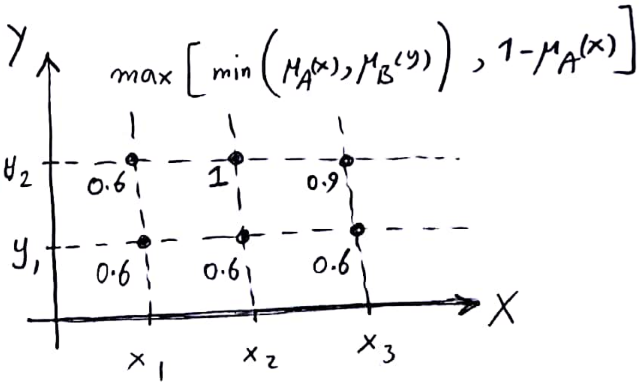


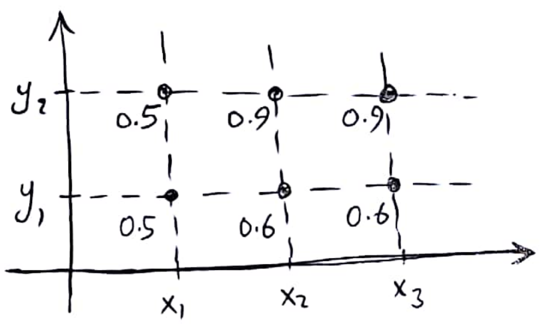


$$\Rightarrow \mu_{B'}(y) = \left\{ 0.7/y_1 \,,\, 1/y_2 \right\}$$

c) $$\mu_{B'}(y) = \sup_{x \in U} \min\left[\mu_{A'}(x) \,,\, \max\left[\min\left(\mu_A(x), \mu_B(y)\right), 1-\mu_A(x) \right] \right]$$

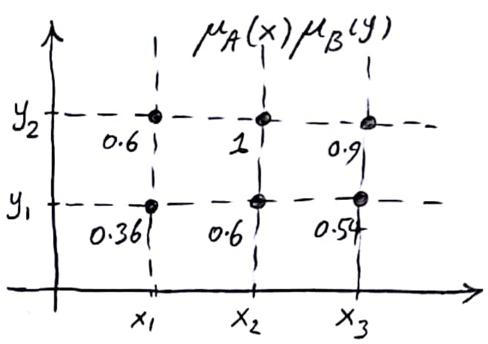


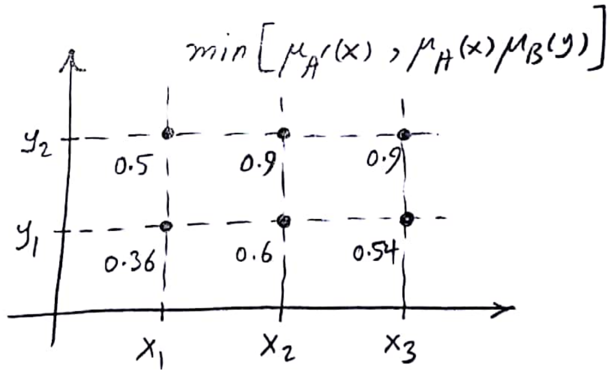




$$\Rightarrow \mu_{B'}(y) = \left\{ 0.6/y_1 \,,\, 0.9/y_2 \right\}$$

d) $$\mu_{B'}(y) = \sup_{x \in U} \min\left[\mu_{A'}(x) \,,\, \mu_A(x)\mu_B(y) \right]$$





$$\Rightarrow \mu_{B'}(y) = \left\{ 0.6/y_1 + 0.9/y_2 \right\}$$

6.5) $$\mu_{B'}(y) = \sup_{x \in U} t\left[\mu_{A'}(x), \mu_{A \to B}(x,y)\right]$$

$$= \sup_{x \in U} \min\left[\mu_{A'}(x), \min\left[1, 1 - \mu_A(x) + \mu_B(y)\right]\right]$$

b) $A' = \text{very } A$

$$\mu_{B'}(y) = \sup_{x \in U} \min\left[\left(\mu_A(x)\right)^2, \min\left[1, 1 - \mu_A(x) + \mu_B(y)\right]\right]$$

if $\mu_B(y) > \mu_A(x) \Rightarrow \mu_{B'}(y) = \sup_{x \in U} \min\left[\mu_A^2(x), 1\right] = \sup_{x \in U}\left[\mu_A^2(x)\right] = 1$

if $\mu_B(y) < \mu_A(x) \Rightarrow \mu_{B'}(y) = \sup_{x \in U} \min\left[\mu_A^2(x), 1 - \mu_A(x) + \mu_B(y)\right]$

با فرض اینکه $\mu_A^2(x)$ و $1-\mu_A(x)+\mu_B(y)$ در نقطه‌ای متقاطع باشند در نقطه‌ای تابع min ماکزیمم می‌گردد که
با هم برابر باشند:

$$\mu_A^2(x) = 1 - \mu_A(x) + \mu_B(y)$$

$$\Rightarrow \mu_A^2(x) + \mu_A(x) - (\mu_B(y) + 1) = 0 \rightarrow \mu_A(x) = \frac{-1 \pm \sqrt{5 + 4\mu_B(y)}}{2} \begin{cases} + & \text{ق‌ق} \\ - & \text{غ‌ق‌ق} \end{cases}$$

$$\Rightarrow \mu_A(x) = \frac{-1 + \sqrt{5 + 4\mu_B(y)}}{2}$$

$$\mu_{B'}(y) = \mu_A^2(x) = 1 - \left(\frac{-1 + \sqrt{5 + 4\mu_B(y)}}{2}\right) + \mu_B(y)$$

$$= \frac{2\mu_B(y) + 3 - \sqrt{5 + 4\mu_B(y)}}{2}$$

$$\Rightarrow \mu_{B'}(y) = \begin{cases} 1 & , \text{ if } \mu_B(y) > \mu_A(x) \\ \dfrac{2\mu_B(y) + 3 - \sqrt{5 + 4\mu_B(y)}}{2} & , \text{ if } \mu_B(y) < \mu_A(x) \end{cases}$$

d) $A' = \bar{A} \Rightarrow \mu_{B'}(y) = \sup\limits_{x \in U} \min[1 - \mu_A(x), \min[1, 1 - \mu_A(x) + \mu_B(y)]]$

if $\mu_B(y) > \mu_A(x) \Rightarrow \mu_{B'}(y) = \sup\limits_{x \in U} \min[1 - \mu_A(x), 1] = \sup\limits_{x \in U}[1 - \mu_A(x)] = 1$

if $\mu_B(y) < \mu_A(x) \Rightarrow \mu_{B'}(y) = \sup\limits_{x \in U} \min[1 - \mu_A(x), 1 - \mu_A(x) + \mu_B(y)]$

$$= \sup_{x \in U}(1 - \mu_A(x)) = 1$$

$\Rightarrow \boxed{\mu_{B'}(y) = 1}$

That's Impossible.

بنابراین جواب مستقل از $\mu_B(y)$ است یعنی استدلال براین اساس دارای نارسایی‌هایی است.

6.7) $\mu_{A'}(x) = \sup\limits_{y \in V} t[\mu_{B'}(y), \mu_{A \to B}(x, y)]$

$= \sup\limits_{y \in V} \min[\mu_{B'}(y), \min[\mu_A(x), \mu_B(y)]]$

a) $B' = \bar{B} \Rightarrow \mu_{A'}(x) = \sup\limits_{y \in V} \min[1 - \mu_B(y), \min(\mu_A(x), \mu_B(y))]$

if $\mu_A(x) \le \mu_B(y) \Rightarrow \mu_{A'}(x) = \sup\limits_{y \in V} \min[1 - \mu_B(y), \mu_A(x)]$

$\Rightarrow 1 - \mu_B(y) = \mu_A(x) \Rightarrow \mu_A(x) = \mu_B(y) = 0.5$ در نقطه‌ای ماکزیمم می‌گردد که با هم برابر باشند

$\Rightarrow \mu_{A'}(x) = 1 - \mu_B(y) = \mu_A(x) = 0.5$

if $\mu_A(x) > \mu_B(y) \Rightarrow \mu_{A'}(x) = \sup\limits_{y \in V} \min[1 - \mu_B(y), \mu_B(y)]$

$= 0.5$

$\Rightarrow \mu_{A'}(x) = 0.5$ جواب مستقل از $\mu_A(x)$ است.

C) $B' =$ more or less B

$$\mu_{A'}(x) = \sup_{y\in V} \min\left[\mu_B^{\frac{1}{2}}(y), \min\left(\mu_A(x), \mu_B(y)\right)\right]$$

$$\text{if } \mu_A(x) > \mu_B(y) \Rightarrow \mu_{A'}(x) = \sup_{y\in V} \min\left[\mu_B^{1/2}(y), \mu_B(y)\right] = \sup_{y\in V}\left[\mu_B(y)\right] = 1$$

$$\text{if } \mu_A(x) < \mu_B(y) \Rightarrow \mu_{A'}(x) = \sup_{y\in V} \min\left[\mu_B^{1/2}(y), \mu_A(x)\right] = \mu_A(x)$$

$$\mu_{A'}(x) = \begin{cases} 1, & \text{if } \mu_A(x) > \mu_B(y) \\ \mu_A(x), & \text{if } \mu_A(x) < \mu_B(y) \end{cases}$$

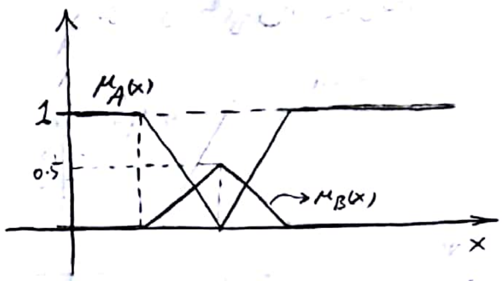
6.8) $\overline{A\wedge\bar{B}} = B\vee(\bar{A}\wedge\bar{B})$

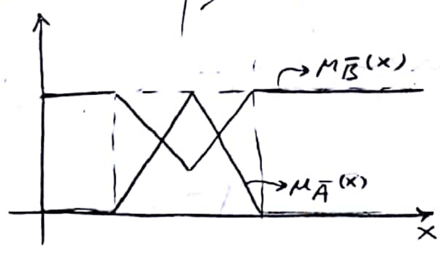
A	B	$\bar{B}$	$A\wedge\bar{B}$	$\overline{A\wedge\bar{B}}$	$\bar{A}$	$\bar{A}\wedge\bar{B}$	$B\vee(\bar{A}\wedge\bar{B})$
T	T	F	F	T	F	F	T
T	F	T	T	F	F	F	F
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	T

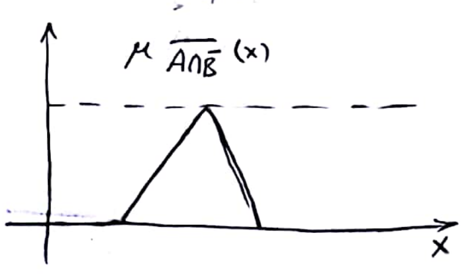
$$\Rightarrow \underbrace{\overline{A\wedge\bar{B}}}_{FP_1} \equiv \underbrace{B\vee(\bar{A}\wedge\bar{B})}_{FP_2}$$

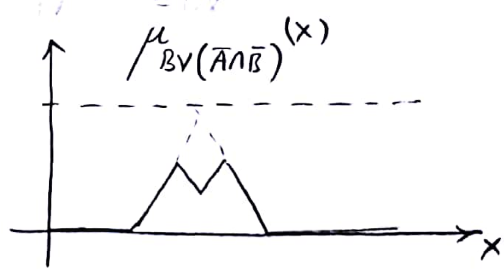
$FP_1 \rightarrow FP_2$

* حال باید ببینیم آیا مجموعه‌های فازی نیز این رابطه درست است یا خیر؟
برای این کار از مثال نقض استفاده می‌کنیم.









* بنابراین مثال نقض برای مجموعه‌های فازی از این رابطه نمی‌توان استفاده نمود