۰۹۸۲۱۰۱۸۴۰ - ایمان سترکی

تمرین سری سوم کنترل فازی

Ex 4.2) ⓐ $U_1 = \{a, b, c\}$, $U_2 = \{s, t\}$, $U_3 = \{x, y\}$, $U_4 = \{i, j\}$

$$Q = 0.4/(b,t,y,i) + 0.6/(a,s,x,i) + 0.9/(b,s,y,i) + 1/(b,s,y,j) + 0.6/(a,t,y,i) + 0.2/(c,s,y,i)$$

Projection of Q on $U_1 \times U_2 \times U_4$:

$$Q_1 = \{0.4/(b,t,i) + 0.6/(a,s,i) + 0.9/(b,s,i) + 1/(b,s,j) + 0.6/(a,t,i) + 0.2/(c,s,i)\}$$

Projection of Q on $U_1 \times U_3$:

$$Q_2 = \{0.6/(a,y) + 0.6/(a,x) + 1/(b,y) + 0.2/(c,y)\}$$

Projection of Q on U_4 :

$$Q_3 = \{0.9/i + 1/j\}$$

ⓑ Extension of Q_1 : $\rightarrow$

$$Q_4 = \{0.4/(b,t,x,i) + 0.4/(b,t,y,i) + 0.6/(a,s,x,i) + 0.6/(a,s,y,i) + 0.9/(b,s,x,i) + 0.9/(b,s,y,i) + 1/(b,s,x,j) + 1/(b,s,y,j) + 0.6/(a,t,x,i) + 0.6/(a,t,y,i) + 0.2/(c,s,x,i) + 0.2/(c,s,y,i)\}$$

Extension of Q_2:
$$
\begin{aligned}
Q_5 = \Big\{ & \frac{0.6}{(a,s,y,i)} + \frac{0.6}{(a,s,y,j)} + \frac{0.6}{(a,t,y,i)} + \frac{0.6}{(a,t,y,j)} \\
& + \frac{0.6}{(a,s,x,i)} + \frac{0.6}{(a,s,x,j)} + \frac{0.6}{(a,t,x,i)} + \frac{0.6}{(a,t,x,j)} \\
& + \frac{1}{(b,s,y,i)} + \frac{1}{(b,s,y,j)} + \frac{1}{(b,t,y,i)} + \frac{1}{(b,t,y,j)} \\
& + \frac{0.2}{(c,s,y,i)} + \frac{0.2}{(c,s,y,j)} + \frac{0.2}{(c,t,y,i)} + \frac{0.2}{(c,t,y,j)} \Big\}
\end{aligned}
$$

Extension of Q_3:
$$
\begin{aligned}
Q_6 = \Big\{ & \frac{0.9}{(a,s,x,i)} + \frac{0.9}{(a,s,y,i)} + \frac{0.9}{(a,t,x,i)} + \frac{0.9}{(a,t,y,i)} \\
& + \frac{0.9}{(b,s,x,i)} + \frac{0.9}{(b,s,y,i)} + \frac{0.9}{(b,t,x,i)} + \frac{0.9}{(b,t,y,i)} \\
& + \frac{0.9}{(c,s,x,i)} + \frac{0.9}{(c,s,y,i)} + \frac{0.9}{(c,t,x,i)} + \frac{0.9}{(c,t,y,i)} \\
& + \frac{1}{(a,s,x,j)} + \frac{1}{(a,s,y,j)} + \frac{1}{(a,t,x,i)} + \frac{1}{(a,t,y,i)} \\
& + \frac{1}{(b,s,x,j)} + \frac{1}{(b,s,y,j)} + \frac{1}{(b,t,x,j)} + \frac{1}{(b,t,y,j)} \\
& + \frac{1}{(c,s,x,j)} + \frac{1}{(c,s,y,j)} + \frac{1}{(c,t,x,j)} + \frac{1}{(c,t,y,j)} \Big\}
\end{aligned}
$$

Ex 4.3) $Q_1 = \begin{pmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0 \\ 0 & 0.2 & 1 \end{pmatrix}$, $Q_2 = \begin{pmatrix} 0.6 & 0.6 & 0 \\ 0 & 0.6 & 0.1 \\ 0 & 0.1 & 0 \end{pmatrix}$, $Q_3 = \begin{pmatrix} 1 & 0 & 0.7 \\ 0 & 1 & 0 \\ 0.7 & 0 & 1 \end{pmatrix}$

max–min:

$$Q_1 \circ Q_2 = \max \begin{pmatrix} (0.6, 0, 0) & (0.6, 0, 0.1) & (0, 0, 0) \\ (0.3, 0, 0) & (0.3, 0.2, 0) & (0, 0.1, 0) \\ (0, 0, 0) & (0, 0.2, 0.1) & (0, 0.1, 0) \end{pmatrix} = \begin{pmatrix} 0.6 & 0.6 & 0 \\ 0.3 & 0.3 & 0.1 \\ 0 & 0.2 & 0.1 \end{pmatrix}$$

$$Q_1 \circ Q_3 = \max \begin{pmatrix} (1, 0, 0.7) & (0, 0, 0) & (0.7, 0, 0.7) \\ (0.3, 0, 0) & (0, 0.2, 0) & (0.3, 0, 0) \\ (0, 0, 0.7) & (0, 0.2, 0) & (0, 0, 1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0.3 \\ 0.7 & 0.2 & 1 \end{pmatrix}$$

$$Q_1 \circ Q_2 \circ Q_3 = \max \begin{pmatrix} (0.6, 0, 0) & (0, 0.6, 0) & (0.6, 0, 0) \\ (0.3, 0, 0.1) & (0, 0.3, 0) & (0.3, 0, 0.1) \\ (0, 0, 0.1) & (0, 0.2, 0) & (0, 0, 0.1) \end{pmatrix} = \begin{pmatrix} 0.6 & 0.6 & 0.6 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.2 & 0.1 \end{pmatrix}$$

max Product:

$$Q_1 \circ Q_2 = \max \begin{pmatrix} (0.6, 0, 0) & (0.6, 0, 0.07) & (0, 0, 0) \\ (0.18, 0, 0) & (0.18, 0.12, 0) & (0, 0.02, 0) \\ (0, 0, 0) & (0, 0.12, 0.1) & (0, 0.02, 0) \end{pmatrix} = \begin{pmatrix} 0.6 & 0.6 & 0 \\ 0.18 & 0.18 & 0.02 \\ 0 & 0.12 & 0.02 \end{pmatrix}$$

$$Q_1 \circ Q_3 = \max \begin{pmatrix} (1, 0, 0.49) & (0, 0, 0) & (0.7, 0, 0.7) \\ (0.3, 0, 0) & (0, 0.2, 0) & (0.21, 0, 0) \\ (0, 0, 0.7) & (0, 0.2, 0) & (0, 0, 1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0.7 \\ 0.3 & 0.2 & 0.21 \\ 0.7 & 0.2 & 1 \end{pmatrix}$$

$$Q_1 \circ Q_2 \circ Q_3 = \max \begin{pmatrix} (0.6, 0, 0) & (0, 0.6, 0) & (0.42, 0, 0) \\ (0.18, 0, 0.014) & (0, 0.18, 0) & (0.13, 0, 0) \\ (0, 0, 0.014) & (0, 0.12, 0) & (0, 0, 0.02) \end{pmatrix} = \begin{pmatrix} 0.6 & 0.6 & 0.42 \\ 0.18 & 0.18 & 0.13 \\ 0.014 & 0.12 & 0.02 \end{pmatrix}$$

4.4) $A = \left\{ 0.5/_{(-1)} + 0.8/_{(0)} + 1/_{(1)} + 0.4/_{(2)} \right\}$, $f(x) = x^2$

$$f(A) = \left\{ 0.8/_{(0)} + 1/_{(1)} + 0.4/_{(4)} \right\}$$

$\mu_B(y) = \max\limits_{x \in f^{-1}(y)} \mu_A(x), y \in V$

4.5) $\mu_{AE}(x,y) = e^{-(x-y)^2}$, $\mu_{ML}(y,z) = \dfrac{1}{1+e^{-(y-z)}}$

$$\mu_{AE \circ ML}(x,z) = \max_{y \in V} \left[t[\mu_{AE}(x,y), \mu_{ML}(y,z)] \right]$$

$$= \max_{y \in V} \left[\frac{e^{-(x-y)^2}}{1+e^{-(y-z)}} \right]$$

$$\Rightarrow \frac{\partial}{\partial y}\left[\mu_{AE \circ ML}(x,z) \right] = \frac{\left\{ 2(x-y)e^{-(x-y)^2}\left(1+e^{-(y-z)}\right) + e^{-(y-z)} \cdot e^{-(x-y)^2} \right\}}{\left\{1+e^{-(y-z)}\right\}^2} = 0$$

$$\Rightarrow e^{-(y-z)} + 2(x-y)\left(1+e^{-(y-z)}\right) = 0$$

if $(x,z) = (0,0) \Rightarrow e^{-y} - 2y(1+e^{-y}) = 0 \Rightarrow \dfrac{e^{-y}}{1+e^{-y}} = 2y \Rightarrow 2y(1+e^{y}) - 1 = 0$

$\boxed{y_1 = 0.222}$

if $(x,z) = (0,1) \Rightarrow e^{-y+1} + 2(-y)(1+e^{-y+1}) = 0$ $\rightarrow \boxed{y_2 = 0.331}$

if $(x,z) = (1,0) \Rightarrow e^{-y} + 2(1-y)(1+e^{-(y)}) = 0$ $\rightarrow \boxed{y_3 = 1,123}$

if $(x,z) = (1,1) \Rightarrow e^{-y+1} + 2(1-y)(1+e^{-y+1}) = 0$ $\rightarrow \boxed{y_4 = 1,222}$

$(x,y,z) = (0, 0.222, 0) \Rightarrow \mu_{AE \circ ML}(0,0) = 0.5286$

$(x,y,z) = (0, 0.331, 1) \Rightarrow \mu_{AE \circ ML}(0,1) = 0.3036$

$(x,y,z) = (1, 1{,}123, 0) \Rightarrow \mu_{AE \circ ML}(1,0) = 0.7432$

$(x,y,z) = (1, 1{,}222, 1) \Rightarrow \mu_{AE \circ ML}(1,1) = 0.5286$