DeepDCF

首先还是先回顾DCF，对后面所需要用到的公式先进行理论上的推倒。并且同时梳理一下相关的论文。
MOSSE,CSK,KCF,CN(color names),DSST,SRDCF,DeepSRDCF,CF2.

Discriminative Correlation Filters

DSST

(PCA-HOG+Gray)+DCF+Scale Esimation

key word: **circular correlation**,**Parserval's identity**, **dense feature**

patch:

$$f_1, \ldots, f_t$$

label:

$$g_1, \ldots, g_t$$

filter:

$$h_t$$

test patch:

$$z$$

$$\epsilon = \sum_{j=1}^{t} \|h_t \star f_j - g_j\|^2 = \sum_{j=1}^{t} \left\| \overline{H}_t F_j - G_j \right\|^2$$

$$H_t = \frac{\sum_{j=1}^{t} \overline{G}_j F_j}{\sum_{j=1}^{t} \overline{F}_j F_j}$$

$$y = \mathfrak{F}^{-1} \overline{H}_t Z$$

那么这是最简单的单特征的情况。
对于Multidimensional Features情况稍微复杂一点。尤其是文中指出要是像上面那样优化一系列时间1,...,t的话，需要dxd个线性方程组，复杂度过高，所以只能对每一个时刻单独求。然后做线性加权的近似。

patch:

$$f^l, l \in \{1, \ldots, d\}$$

label:

$$g$$

filter:

$$h^l$$

test patch:

$$z^l$$

$$\epsilon = \sum_{l=1}^{d} \left\| h^l \star f^l - g \right\|^2 + \lambda \sum_{l=1}^{d} \left\| h^l \right\|^2$$

$$H^l = \frac{\overline{G} F^l}{\sum_{k=1}^{d} \overline{F^k} F^k + \lambda}$$

T obtain a robust approximation, here we update the numerator $$ A_ {t}^{l} $$ and denominator $$ B_{t} $$

$$A_t^l = (1-\eta) A_{t-1}^l + \eta \overline{G}_t F_t^l$$

$$B_t^l = (1-\eta) B_{t-1}^l + \eta \sum_{k=1}^{d} \overline{F}_t F_t^l$$

响应计算的结果

$$y = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^{d} \overline{A^l} Z^l}{B + \lambda} \right\}$$

CN

SRDCF

DeepDCF

将**CNN**网络看成是一种非线性映射。将原始rgb（或灰度扩展）图像进行映射。

$$\Phi(x) = f^l$$

$$\Phi(z) = z^l$$

patch:

$$x^l, l \in \{1, \dots, d\}$$

label:

$$g$$

filter:

$$h^l$$

test patch:

$$z^l$$

$$\epsilon = \sum_{l=1}^{d} \left\| h^l \star x^l - g \right\|^2 + \lambda \sum_{l=1}^{d} \left\| h^l \right\|^2$$

$$H^l = \frac{\overline{G} X^l}{\sum_{k=1}^{d} \overline{X^k} X^k + \lambda}$$

响应计算的结果

$$y = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^{d} \overline{A^l} Z^l}{B + \lambda} \right\}$$

其实到这了这，和上面的DSST实际上是一样的。因为我本身就是利用的DCF，公式没有任何变化，只是利用闭式解反传误差。下面为了更方便写代码(forward和backward)：
进行整合的推倒：

$$y = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^{d} \overline{A^l} Z^l}{B + \lambda} \right\} = \mathfrak{F}^{-1} \left\{ \frac{\sum_{k=1}^{d} \overline{\overline{G} X^l} Z^l}{\sum_{k=1}^{d} \overline{X^k} X^k + \lambda} \right\} = \mathfrak{F}^{-1} \left\{ \frac{\sum_{k=1}^{d} (Z^l \overline{X^l}) G}{\sum_{k=1}^{d} \overline{X^k} X^k + \lambda} \right\}$$

```
y=ifft2((sum3(fft2(z).*conj(fft2(X))).*fft2(g))./(sum3(fft2(X).*conj(fft2(X))
)+lambda)));
```

前项过程相对来说较为容易，反向就有点恶心了。
已知

$$\frac{\partial l}{\partial y}$$

求

$$\frac{\partial l}{\partial z}, \frac{\partial l}{\partial x}$$

那么还是先用公式证明：
公理1，

$$Y = \mathfrak{F}\{y\}, \frac{\partial l}{\partial Y} = \mathfrak{F}\left\{\frac{\partial l}{\partial y}\right\}$$

公理2,

$$\frac{\partial f(x, x^*)}{\partial x} = \overline{\frac{\partial f(x, x^*)}{\partial x^*}}$$

且易知

$$Y_{uv} = \frac{\sum_{k=1}^{d}(Z_{uv}^{l}\overline{X_{uv}^{l}})G_{uv}}{\sum_{k=1}^{d} X_{uv}^{k}\overline{X_{uv}^{k}} + \lambda}$$

那么

$$\frac{\partial l}{\partial Z_{uv}^{l}} = \frac{\partial l}{\partial Y_{u,v}}\frac{\partial Y_{u,v}}{\partial Z_{uv}} = \mathfrak{F}\left\{\frac{\partial l}{\partial y}\right\}_{uv}\frac{\overline{X_{uv}^{l}}G_{uv}}{\sum_{k=1}^{d} X_{uv}^{k}\overline{X_{uv}^{k}} + \lambda}$$

$$\frac{\partial l}{\partial X_{uv}^{l}} = \frac{\partial l}{\partial Y_{u,v}}\frac{\partial Y_{u,v}}{\partial X_{uv}} = \mathfrak{F}\left\{\frac{\partial l}{\partial y}\right\}_{uv}\frac{\overline{Z_{uv}^{l}G_{uv}}(\sum_{k=1}^{d}\overline{X_{uv}^{k}}X_{uv}^{k} + \lambda) - \overline{X_{uv}^{k}}(\sum_{l=1}^{d}(Z_{uv}^{l}\overline{X_{uv}^{l}})G_{uv})}{(\sum_{k=1}^{d}\overline{X_{uv}^{k}}X_{uv}^{k} + \lambda)^{2}}$$