

DeepDCF

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1 Discriminative Correlation Filters

1.1 DSST

(PCA-HOG+Gray)+DCF+Scale Estimation

key word: circular correlation,Parseval's identity, dense feature

1.1.1 Single Feature(gray)

patch: x_1, \dots, x_t

label: y_1, \dots, y_t

filter: w_t

test patch: z

$$\epsilon = \sum_{j=1}^t \|w_t \star x_j - y_j\|^2 = \sum_{j=1}^t \|\overline{W}_t \odot X_j - Y_j\|^2$$

$$W_t = \frac{\sum_{j=1}^t \overline{Y}_j \odot X_j}{\sum_{j=1}^t X_j \odot \overline{X}_j}$$

$$y = \mathfrak{F}^{-1}(\overline{W}_t \odot Z) = \mathfrak{F}^{-1}\left(\frac{\sum_{j=1}^t \overline{X}_j \odot Y_j \odot Z}{\sum_{j=1}^t \overline{X}_j \odot X_j}\right)$$

This is a little different between MOSSE[1] and this Derivation. No regularization term.

1.1.2 Multidimensional Features

patch: $f^l, l \in \{1, \dots, d\}$ label: g filter: h^l test patch: z^l

$$\epsilon = \sum_{l=1}^d \left\| h^l \star f^l - g \right\|^2 + \lambda \sum_{l=1}^d \left\| h^l \right\|^2$$

$$H^l = \frac{\overline{G} F^l}{\sum_{k=1}^d \overline{F^k} F^k + \lambda}$$

To obtain a robust approximation, here we update the numerator A_t^l and denominator B_t

$$A_t^l = (1 - \eta) A_{t-1}^l + \eta \overline{G}_t F_t^l$$

$$B_t^l = (1 - \eta) B_{t-1}^l + \eta \sum_{k=1}^d \overline{F}_t F_t^k$$

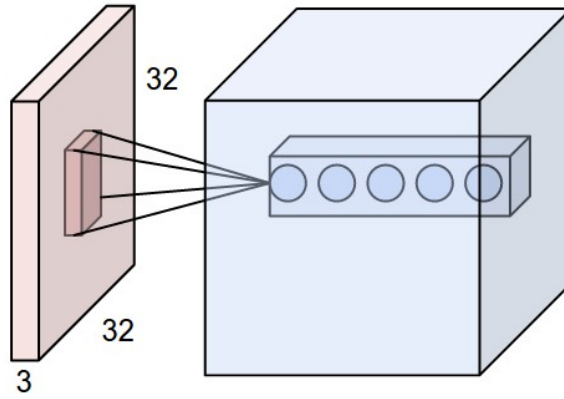
$$y = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^d \overline{A^l} Z^l}{B + \lambda} \right\}$$

1.2 CN

1.3 SRDCF

2 CNN

CNN can be treated as a encode function $\Phi(x)$ of receptive field.



$$\Phi(x) = f^l$$

$$x \in \mathbb{R}^{m \times n} \longrightarrow f \in \mathbb{R}^d$$

It is sparse (only a few input units contribute to a given output unit) and reuses parameters (the same weights are applied to multiple locations in the input)

3 DeepDCF

target patch: $x^l, l \in \{1, \dots, d\}$ idea label: y filter: w^l test patch: z^l test output: g

For the learning part.

$$\epsilon = \sum_{l=1}^d \|w^l \star x^l - y\|^2 + \lambda \sum_{l=1}^d \|w^l\|^2$$

$$W^l = \frac{\bar{Y} \odot X^l}{\sum_{k=1}^d \bar{X}^k \odot X^k + \lambda}$$

$$g = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^d \bar{A}^l \odot Z^l}{B + \lambda} \right\}$$

$$g = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^d \bar{A}^l \odot Z^l}{B + \lambda} \right\} = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^d \bar{\bar{Y}} \odot \bar{X}^l \odot Z^l}{\sum_{k=1}^d \bar{X}^k \odot X^k + \lambda} \right\} = \mathfrak{F}^{-1} \left\{ \frac{\sum_{l=1}^d (Z^l \odot \bar{X}^l) \odot Y}{\sum_{k=1}^d \bar{X}^k \odot X^k + \lambda} \right\}$$

$$g = \text{ifft2} \left(\left(\text{sum}(\text{fft2}(z)) .* \text{conj}(\text{fft2}(x)), 3 \right) .* \text{fft2}(y) \right) ./ \dots \\ \left(\text{sum}(\text{fft2}(x)) .* \text{conj}(\text{fft2}(x)), 3 \right) + \text{lambda} \right);$$

The forward pass derivation should be familiar to us by now. There need some patience for the backward pass.

First, let's begining with some fundamental theorems of DFT and Complex-Valued Derivatives.

Because the DFT is a linear operator, its gradient is simply the transformation matrix it self. During the back-progation, then, the gradient is conjugated.[3]

$$Y = \mathfrak{F} \{y\}, \frac{\partial l}{\partial Y} = \mathfrak{F} \left\{ \frac{\partial l}{\partial y} \right\}$$

The second theorem we will use is Complex-Valued Derivatives. [2] Because we need pass δ throught the frequency domain. We will be more familiar with the derivatives operator about complex .

$$\frac{\partial f(x, x^*)}{\partial x} = \overline{\frac{\partial f(x, x^*)}{\partial x^*}}$$

Now, Let's derive the formulas about the backward. For simplify, We start with $\frac{\partial l}{\partial g}$ and what we need is to results of $\frac{\partial l}{\partial x}$ and $\frac{\partial l}{\partial z}$.

$$G_{uv} = \frac{\sum_{l=1}^d (Z_{uv}^l \overline{X_{uv}^l}) Y_{uv}}{\sum_{k=1}^d X_{uv}^k \overline{X_{uv}^k} + \lambda}$$

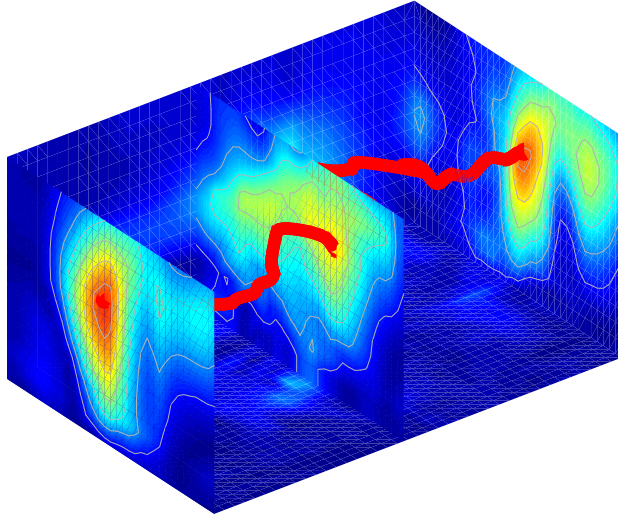
$$\frac{\partial l}{\partial Z_{uv}^l} = \frac{\partial l}{\partial G_{u,v}} \frac{\partial G_{u,v}}{\partial Z_{uv}^l} = \mathfrak{F} \left\{ \frac{\partial l}{\partial g} \right\}_{uv} \frac{\overline{X_{uv}^l} Y_{uv}}{\sum_{k=1}^d X_{uv}^k \overline{X_{uv}^k} + \lambda}$$

$$\frac{\partial l}{\partial X_{uv}^l} = \frac{\partial l}{\partial G_{u,v}} \frac{\partial G_{u,v}}{\partial X_{uv}^l} = \mathfrak{F} \left\{ \frac{\partial l}{\partial g} \right\}_{uv} \frac{\overline{Z_{uv}^l} Y_{uv} (\sum_{k=1}^d \overline{X_{uv}^k} X_{uv}^k + \lambda) - \overline{X_{uv}^l} (\sum_{l=1}^d (Z_{uv}^l \overline{X_{uv}^l}) Y_{uv})}{(\sum_{k=1}^d \overline{X_{uv}^k} X_{uv}^k + \lambda)^2}$$

$$\frac{\partial l}{\partial x_{uv}^l} = \mathfrak{F}^{-1} \left\{ \frac{\partial l}{\partial X} \right\}_{uv}^l$$

3.1 Training

I think that is a very perfect visualization of the training process.



References

- [1] David S Bolme, J Ross Beveridge, Bruce A Draper, and Yui Man Lui. Visual object tracking using adaptive correlation filters. In *Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on*, pages 2544–2550. IEEE, 2010.
- [2] Are Hjørungnes. *Complex-valued matrix derivatives: with applications in signal processing and communications*. Cambridge University Press, 2011.
- [3] Oren Rippel, Jasper Snoek, and Ryan P Adams. Spectral representations for convolutional neural networks. In *Advances in Neural Information Processing Systems*, pages 2449–2457, 2015.