ACKNOWLEDGEMENT

It is immense pleasure that we place on record our deep sense of gratitude to **Dr. NIJO VARGHESE,** Head of the Department of Physics, Sacred Heart College,for being our guide in this project work and also for giving us proper encouragement throughout the work.

We also express our sincere gratitude to **Dr. Sr. IRENE** , The Principal of our college, for her valuable support imparted on us.

Hearty thanks to our parents for all the support and encouragement they have forwarded us. Last but not the least, our sincere thanks to all our beloved friends who have directly and indirectly helped us through their valuable suggestions and selfless criticisms, which ensured that this project becomes a success.

We express our wholehearted gratitude for her guidance support in realizing our project. I wish to thank all the faculty members of the department for their wholehearted support.

ABSTRACT

Perturbations of stars and black holes have been one of the main topics of relativistic astrophysics for the last few decades.They are of particular importance today, because of their relevance to gravitational wave astronomy.In this review we present the theory of quasi-normal modes of compact objects from both the mathematical and astrophysical points of view .

The discussion includes black holes, perturbations of black holes(Schwarzschild), quasinormal modes of black holes and Pöschl-teller approximation method to find the frequency of the quasinormal modes.

We have employed the Python programming to obtain the plot of the potential and to calculate the frequency of quasinormal modes.

CONTENT

INTRODUCTION

One of the greatest creations of speculative science is the Einstein's General Theory of Relativity. Its first solution- a black hole- is happened to be more strange mysterious. Introducing the black holes through the words of Kip Thorne,

"…..black hole: a hole in space with a definite edge into which anything can fall and out of which nothing can escape, a hole with a gravitational force so strong that even light is caught and held in its grip, a hole that curves and warps time……. Well-tested laws of physics predict firmly that black holes exist. In our galaxy alone there may be millions, but their darkness hides them from view. Astronomers have great difficulty finding them."

-Kip S. Thorne

It is this weird nature made the black hole deeply entrenched in human imagination and one of the most studying objects in science.

From "dark stars" to "black holes"

The first scientific ideas on regions of gravity so strong that light cannot escape were kicked to the late 1780s. Combining Newton's theory of gravitation with his corpuscular theory of light, John Mitchell, a British natural philosopher, presented the idea of 'dark stars' at Royal society, London. He argued that if a star is compact enough, the escape velocity on its surface mainly greater the velocity of light corpuscles, and the star becomes invisible. Later in 1797, mathematician Laplace promoted the same idea and he wrote on his hook,

"It is therefore possible that greatest luminous bodies in the universe are on this very account invisible."

-Pierre-Simon Laplace

But the general acceptance of wave theory of light in the beginning of 19^{th} century forced him to drop out the notion of dark stars.

Findings about black holes:-

The modern understanding of black holes begins when Karl Schwarzschild derived an exact solution for Einstein's field equation in 1916, almost immediately after Einstein formulated his relativistic theory of gravity.

His solution showed that if the mass of the compact object confined within a critical circumference, the space will be strongly curved and the flow of time at the stars surface will be infinitely dilated. A star as small as a critical circumference, called Schwarzschild radius, must appear completely dark. This did not seem at all reasonable to physicists and astrophysicists of 1920s or even as late as the 1960s.Even Edenton, a relativity expert of his time and Einstein himself opposed the 'Schwarzschild singularity' theory.

But several discoveries in the 1930s made them recognized as realistic objects. Among them three major developments were,

- Chandrasekhar's 1931 proof that there is an upper limit on the mass of white dwarfs($M \leq 1.4 M_\odot$)
- Chadwick's 1932 discovery of the neutron and the subsequent idea – due to Baade and Zwicky – that entire stars made up of these particles may exist. Such neutron stars would be limited to a mass less than something like $3M_\odot$.
- The seminal work on gravitational collapse by Oppenheimer and Snyder from 1939, that provided the first demonstration of how the implosion of a star forms a black hole.

Eventhough these findings confirmed that the black holes are theoretically possible, it had to wait until 1970, for the first observation evidence for the actual existence of black holes in the Universe. It came from the detection of Cygnus X-1, one of the strongest X-ray sources we can detect from Earth. It is widely believed that the Cygnus X-1 is a binary black hole system with a comparison smaller than Earth but with a mass greater than that of a neutron star.

Presently we have many black hole candidates, with stronger evidences due to the advanced optical, X-ray and radio telescopes.It is now strongly suspected that almost all galaxies contain gigantic black holes in their centers, millions or even billions of times more massive than the sun. Our own galaxy,Milky Way is expected to harbor a supermassive black hole, known as Sagittarius A*, at the center. Several gravitational wave operators are now actively searching for the signal coming from black holes and may make it possible to directly observe them in the near future.

TYPES OF BLACK HOLES

There are four types of black holes: stellar, intermediate, supermassive, and miniature. The most commonly known way a black hole forms is by stellar death. As stars reach the ends of their lives, most will inflate, lose mass, and then cool to form white dwarfs. But the largest of these fiery bodies, those at least 10 to 20 times as massive as our own sun, are destined to become either super-dense neutron stars or so-called stellar-mass black holes.

It is expected that real black holes will have angular momentum, but may not be charged since atoms tend to be neutral. Because of this consider rotating and non-rotating black hole. The mass limits of the black holes of various classes are not precisely defined and several authors have proposed new classes. Here is a version of the scheme.

CLASS	MASS RANGE
Mini black holes	0 to 0.1 solar mass
Stellar mass black holes	0.1 to 10 solar masses
Intermediate mass black holes	100 to 10000 solar masses
Supermassive black holes	Greater than 10000 solar masses

In their final stages, enormous stars go out with a bang in massive explosions known as supernovae. Such a burst flings star matter out into space but leaves behind the stellar core. While the star was alive, nuclear fusion created a constant outward push that balanced the inward pull of gravity from the star's own mass. In the stellar remnants of a supernova, however, there are no longer forces to oppose that gravity, so the star core begins to collapse in on itself.

If its mass collapses into an infinitely small point, a black hole is born. Packing all of that bulk—many times the mass of our own sun—into such a tiny point gives black holes their powerful gravitational pull. Thousands of these stellar-mass black holes may lurk within our own Milky Way galaxy.

There are four basic kinds of black hole solutions Einstein's equations:

Schwarzschild: These are spherical and do not rotate. They are defined only by their total mass.

Reissner - Nordstrom: These possess mass and charge but do not rotate

Kerr: These rotate and are flattened at the poles, and only described by their mass and amount of spin (angular momentum).

Kerr - Nordstrom: These possess mass and charge, and they rotate.

Characteristics of black hole:-

Eventhough blackholes possess a complex mysterious nature, they have a perfect mathematical description. The mathematically defined black hole is the picture of simplicity as Chandrasekhar wrote in his monograph.

"The black holes of nature are the most perfect macroscopic objects there are in the Universe: the only elements in their construction are our concepts of space and time. And since the general theory of relativity provides only a single unique family of solutions for their descriptions, they are the simplest objects as well." - S. Chandrasekhar

As we mentioned already, the Schwarzschild solution is the first solution of Einstein's field equation, represents spacetime outside a spherically symmetric massive object. The line element, which describes such a geometry is given as,

$$ds^2=-(1 -\frac{2M}{r})dt^2 + (1 -\frac{2M}{r})^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \quad (1)$$

An important property of this solution is that it is independent of the temporal, t coordinate and depends only on r and that it is determined by only a single parameter,M , its mass. Far from the center of gravity ($r \to \infty$) the spacetime dissolve to the flat Minkowski spacetime,

$$ds^2= \eta_{\mu\nu} dx_\mu dx_\nu= - dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \quad (2)$$

One can notice from Eq.(1) that the gravity of the massive object not only curves the space around it but warp the time near its premises. One can visualize the Schwarzschild spacetime in the embedded diagram shown in figure 1.1 and observe the following points.

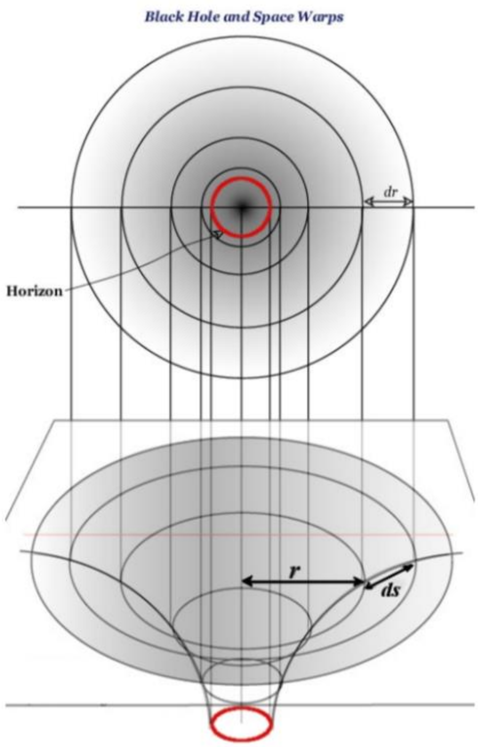


Figure 1.1: Embedded diagram of Schwarzschild black hole.

The solution is singular at two values of the radial coordinate, $r=r_e=2M$ and $r=0$. The first one is a coordinate singularity that can be transformed away by choosing suitable coordinate systems. The second one is the actual gravitational singularity marked by an infinite curvature. The radial distance, r_e is called the Schwarzschild radius and it marks a boundary called the event horizon of the black hole, which is a causality barrier through which anything can go inside but nothing can come out.

Schwarzchild black hole is the simplest case, with only one parameter determines its geometry, its mass. A theorem known as 'no-hair theorem' limits the other properties of a black hole reaching to an external observer. According to this theorem a black hole formed by the gravitational collapse of a charged rotating star, will rapidly relax to the stationary state, characterized by only three quantities, its mass, charge and angular momentum(MQJ). Any other hair, will disappear after the collapsing body settles down to its stationary configuration. Consequently, there are four verities of black holes in GTR,

Black hole	Signatures
Schwarzschild	M
Reissner-Nördstrom(RN)	M and Q
Kerr	M and J
Kerr-Newmann	M, Q and J

We summarise this section with the words of Wheeler,

"[The black hole] teaches us that space can be crumpled like a piece of paper into an infinitesimal dot, that time can be extinguished like a blown –out flame, and that the laws of physics that we regard as 'sacred', as immutable, are anything but."

-John Archibald Wheeler

GRAVITATIONAL PERTURBATIONS OF BLACK HOLES

The spherically symmetric uncharged stationary background spacetime is described by,

$ds^2 = \bar{g}_{\mu\nu}\, dx^\mu dx^\nu$

$= -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$ (3)

Consider small perturbations, $h_{\mu\nu}$ in background spacetime, $\bar{g}_{\mu\nu}$ with $|h\mu\nu|/|\bar{g}_{\mu\nu}| << 1$, such that the total metric can be taken as the sum of unperturbed background metric and the perturbation,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \qquad (4)$$

Defining $\psi(t,r) = \left(1 - \frac{2M}{r}\right)\frac{h1}{r}$, we get a second order equation,

$$\frac{d^2\psi_l}{dr_*^2} + (\omega^2 - V_{eff})\, \psi_l = 0 \qquad (5)$$

where we have employed the radial tortoise coordinate, defines as $r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$, such that it pushes the event horizon, 2M all the way to -∞. As,

$$r \to +\infty, r_* \to +\infty,$$

$$r \to 2M, r_* \to -\infty,$$

The effective potential V_{eff} ,usually called the Regge-Wheeler potential is given by,

$$V_{eff} = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right) \qquad (6)$$

The potential is positive real everywhere and has a maximum just outside the event horizon at $r \simeq 3.3M$. The asymptotic structure of the potential shows an inverse-square fall off as $r_* \to -\infty$. Modes with larger multipoles will experience a higher potential barrier.

Now the study of stability is reduced to finding the solution of differential equation, Eq. (5). With the time dependence of perturbation as $e^{-i\omega t}$, if the perturbation equation allow a solution with imaginaryfrequency, then the perturbations will grow exponentially in time and leads to an unstable configuration. A satisfactory explanation to this problem was first provided by Vishveshwara. By examining the asymptotic behavior of the solutions in Kruskal coordinates, he proved that the perturbations with imaginary frequency are physically unacceptable and hence that the Schwarzschild metric is stable.

Perturbations by classical wave fields

When a nearly spherical star collapses through its gravitational radius, non spherical perturbations can be expected in its electromagnetic and various other spin fields coupled to sources in the stars, along with the gravitational disturbances. One can also study the perturbation of these fields in curved spacetime, using the field theory.

If the fields are assumed to be week, the slight deformation in the background black hole spacetime, caused by the energy -momentum tensor of the field, can be neglected. The general approach to solving these problems is as follows. The field is expanded in spherical harmonics, scalar harmonics for scalar field(s=0), vector harmonics for electromagnetic field (s=1) and tensor harmonics for gravitational fields (s=2). Each spherical harmonics represented by the multipole number, l can evolve separately satisfying an equation of the form Eq. (1.16), with an effective potential characteristic of the field under consideration. For massless integer spin fields, in the Schwarzschild background spacetime, we can write[11] (A detailed derivation for various field perturbations are presented in the coming chapters),

$$V = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \sigma\frac{2M}{r^3}\right)$$

Where σ depends on the spin , s of the perturbing field as, $\sigma = 1 - s^2$ and is given by,

$$\sigma = \begin{cases} +1, \; scalar\ field \\ 0, electromagnetic\ field \\ -3, \; odd\ gravitational\ perturbation \end{cases}$$

DEFINITION OF QUASINORMAL MODES

Most of the problems concerning wave problems in black hole spacetimes can be reduced to a second order partial differential equation of the form,

$$\frac{\partial^2}{\partial x^2}\Psi - \frac{\partial^2}{\partial t^2}\Psi - V\Psi = 0 \qquad (7)$$

Here $x$is a spatial variable, usually but not always ranging(in a special coordinate system)from $-\infty$ to $+\infty$.when dealing with black hole spacetimes the horizon is usually at $-\infty$, andfor the rest of this discussion we shall assume so. Also,V is an x-dependent potential.To define in a phenomenological way what a QNM is, we shall proceed in the usual way by assuming a time dependence

$$\Psi(t,x) = e^{-i\omega t}\phi(x) \qquad (8)$$

Inserting this in Eq.(1)we get an ordinary differential equation in the spatial variable x,

$$\frac{d^2}{dx^2}\phi + (\omega^2 - V)\phi = 0 \qquad (9)$$

The form Eq.(8) is not restrictive, since once we have a solution for Eq.(9), a general time dependent solution can be given as a continuous Fourier transform of such solutions.The form (9)is ideal to study QNMs in a way that parallels a normal mode analysis.We shall now restrict ourselves to asymptotically flat spacetimes, and defer a discussion of asymptotically de Sitter (i.e., a spacetime with a positive cosmological constant) or anti-de Sitter (i.e., negative cosmological constant).In asymptotically flat spacetimes ,the potential V is positive and satisfies

$V \to 0,\ x \to -\infty$(10)

$$V \to 0,\ x \to +\infty \qquad (11)$$

Therefore, such potentials do not allow bound states,and this makes it impossible to do a normal mode expansion.The idea that the evolution of Ψ will generally involve a superposition of these QNMs can be shown to be correct by the use of Laplace transforms.Nevertheless, we saw that the signal is somehow dominated by characteristic oscillations,so we shall blindly continue with our analysis. Having in mind the form (10)-(11) of the potential we have that near the boundaries $-\infty$and $+\infty$ the solution behaves as plane waves,

$$\Phi \sim e^{\pm i\omega x} \ , \ x \ \to -\infty \qquad (12)$$

$$\Phi \sim e^{\pm i\omega x} \ , \ x \to +\infty \qquad (13)$$

The boundary conditions defining QNMs are that towards the boundaries the solutions should be purely outgoing at infinity (x=+∞)and ingoing at the horizon (x=−∞),

$$\Phi \sim e^{-i\omega x} \ , \ x \to -\infty \qquad (14)$$

$$\Phi \sim e^{+i\omega t} \ , \ x \to +\infty \qquad (15)$$

Here ingoing at the horizon means entering into the black hole, therefore leaving the domain we are studying.These are physically motivated boundary conditions. Only a discrete set of complex frequencies ω_{QN} satisfy these boundary conditions.These are the QN frequencies, and the associated wavefunctions ϕ, solutions of Eq.(9) are the QNMs.It has been proved by Vishveshwara that for the Schwarzschild geometry ω_{QN} must have a negative imaginary part; this has also been found for geometries other that Schwarzschild , for example Schwarzschild-anti-de Sitter,Schwarzschild-de Sitter, Kerr.This means on the one hand that QNMs decay exponentially in time,and the physical significance of this is that the black hole spacetime is losing energy in the form of gravitational waves.On the other hand,this also means that the spacetime is stable. In addition,the imaginary part being negative makes the numerical calculation of QN frequencies a non-trivial task: according to the conditions (14)-(15), and to the fact the QN frequencies have a negative imaginary part, one has that QNMs grow exponentially at the boundaries.Now in order to tell if a certain frequency is or not a QN frequency one must check that, for example,there is only an out-going $e^{i\omega x}$piece at infinity, or in other words, one must check that near infinity the $e^{\ -i\omega x}$is absent.

However,this last term is exponentially suppressed in relation to the other,so one must be able to distinguish numerically an exponentially small term from an exponentially large one.

$\omega_{QN}M$

n	*l=2:*	*l=3:*	*l=4:*
0	**0.59944-0.0927i**	**0.37367-0.08896i**	**0.80918-0.09416i**

1	**0.58264-0.28130i**	**0.34671-0.27391i**	**0.79669-0.28449i**
2	**0.55168-0.47909i**	**0.30105-0.47828i**	**0.77271-0.47991i**
3	**0.51196-0.69034i**	**0.25150-0.70514i**	**0.73984-0.68392i**

Table 1: The first four quasinormal frequencies for a Schwarzschild black hole, measured in units of the black hole mass M. To convert this to Hz one must multiply the numbers in the table by $32310\frac{M_{\odot}}{M}$. Thus , a one solar mass black hole has a typical ringing frequency of 10 kHz in the quadrupole mode, and a damping timescale, due to gravitational wave emission, of T=3.74x10^{-4}s.

Chandrasekhar and Detweiler have succeeded in finding some of the Schwarzschild QN frequencies this way,in 1975.Since then,numerous techniques have been developed.Some of them are analytical tools like the WKJB technique of Schutz and will,later refined to 3rd order,and recently extended to 6th order , or the “potential fit” one, in which one tries to fit the Schwarzschild potential to one which enables us to find exact results. However, the most successful attempt has been developed by Leaver,using a continued fraction form of the equations, which is rather easy to implement numerically. Exact results for QNMs of certain black hole space times only became available very recently.

In Table 1 the angular quantum number l gives the angular dependence of the gravitational wave. For example a wave with l=2, the lowest gravitational radiatable multipole, has a quadrupole distribution. Some comments are in order. First, QN frequencies come in pairs. That is, if $\omega = a + ib$ is a QN frequency, then so is $\omega = -a + ib$. We only show in Table 1 frequencies with a positive real part. There are countable infinity of QN frequencies. They are usually arranged by their imaginary part. Thus, the frequency with lowest imaginary part is called fundamental frequency and is labeled with the integer $n = 0$;the one with second lowest imaginary part is the first overtone $(n = 1)$ and so on. The real part can be shown to be, using a WKJB type of reasoning, of the order of the height of the potentialbarrier, i.e., $Re[\omega] \sim V_{max}$, whereV_{max} is the maximum value attained by the potential V.The real part is, in the Schwarzschild geometry, bounded from above, whereas the imaginary part seems to grow without bound for overtone number $n \to \infty$.

QUASINORMAL MODES OF BLACK HOLES

The asymptotic form of the effective potentials, V($r_* \to \pm\infty) \to 0$, suggests that the solution behaves as plane waves near the boundaries, $r_* \to \pm\infty$,

$$\varphi \sim \begin{cases} A_{in} e^{-i\omega r_*} - A_{out}\, e^{i\omega r_*} \text{as } r_* \to -\infty, \\ \quad B_{in} e^{-i\omega r_*} \text{as } r_* \to -\infty, \end{cases} \tag{16}$$

Where we have to impose purely ingoing boundary condition at the black hole event horizon $r_* \to -\infty$, means waves entering in to the black hole and obviously nothing can come out. QNMs are the manifestation of the resonance oscillations of the black hole spacetime itself and it dominates the soon after the initial transient stage. During this stage of evolution we require a purely outgoing boundary condition at , $r_* \to +\infty$. This means the incoming waves already passed and there is no other waves coming from ∞ to disturb the system and we set A_{in}=0 in Eq. (16). While applying this boundary condition in to Eq.(16), one can obtain a discrete set of solutions, the QNMs having the time dependence, $e^{i\omega_q t}$ with complex frequencies called the quasinormal (QN) frequencies.

$$\omega_q = \omega_R + i\omega_I \tag{17}$$

The real part of the QN frequency, ω_R corresponds to the actual oscillation frequency and the absolute value of the imaginary part, $|\omega_I|$ represents the rate at which each mode damps or grows. If the black hole is stable against the perturbations, it should have negative imaginary part for the QN frequencies, so that the perturbations will damp exponentially and the black hole can come to its equilibrium configuration. In this case part of the energy of the perturbations is carried away to infinity in the form gravitational radiations and another part goes in to the black hole through the event horizon. In this sense these modes are not normal but 'quasinormal'.

It was Chandrasekhar and Detweiler who first computed the quasinormal frequencies of the Schwarzschild black hole and explored their properties .

We would like to point out here that the attempts to calculate the QNM frequencies date back to the beginning of 70s.More specifically in their study of the black hole oscillations excited by an infalling particle Davis et al.[73] found that the peak of the spectrum is (for a Schwarzschild black hole) at around M_{ω}=0.32 (geometrical units).This number is very close to the one calculated by much more accurate methods later.In what will follow we will describe the various methods used to calculate the QNM frequencies.

PöSCHL-TELLER APPROXIMATION METHOD

There has been great interest in the long standing issue in classical relativistic theory of gravitation: small perturbations or Quasinormal modes associated with black holes. The question of stability of a black hole was first treated by Reggae and Wheeler who investigated linear perturbations of the exterior Schwarzschild space time. Further workon this problem led to the study of quasinormal modes which is believed as a characteristic sound of black holes. Quasinormal modes(QNMs) describe the damped oscillations under perturbations in the surrounding geometry of a black hole with frequencies and damping times of oscillations entirely fixed by the black hole parameters. The study of QNMs became an intriguing subject of discussion for last few decadesand references there in. QNMs carry unique finger prints of black holes and it is well known that they are crucial in studying the gravitational and electromagnetic perturbations around black hole space times. They are also seemed to have an observational significance as the gravitational waves produced by the perturbations, in principle, can be used for unambiguous detection of black holes. This motivates us to study the quasinormal mode spectra of black holes.

The motivation of the present work is to study the signature of cosmic strings on QNMs. It has been recognized that certain gauge theories allow the possibility of topological defects, such as strings, magnetic monopoles,…. and that these defects represent objects which might have been created in the very early universe. Cosmic strings are strand of matter which could be created in a cosmological phase transition. In 1976 Kibble suggested the possibility of strings in the early universe. These cosmic strings might be responsible for large-scale structure in the universe. Although little is known about these strings, it is clear that they raise a number of issues in fundamental physics and thus it seems to be of particular interest , both as a possible ”seed” for galaxy formation and as a possible gravitational lens. The QNMs of scalar perturbations around a Schwarzschild black hole pierced by a cosmic string was done earlier

Quasinormal mode frequencies

We shall evaluate the QNMs using Pöschl- Teller potential approximation proposed by Ferrari and Mashhoon. In this method the effective potential is approximated to the pöschl-Teller potential, such that we get a good fit to the original potential at its maximum and its maximum and it allows to solve the problem analytically, and is a good approximation for the low lying frequencies. Quasinormal mode frequencies obtained by the Pöschl-Teller approximation are found to be in good agreement with the results obtained by other methods. The Pöschl-Teller potential is defined as,

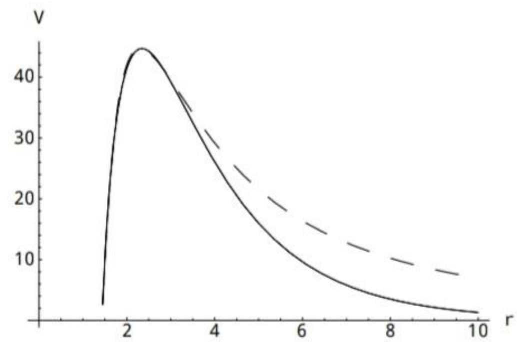
$$V_{PT} = \frac{V_0}{cosh^2\left(\frac{r_*}{b}\right)}$$

The quantity V_0 and b are given by the height and curvature of the potential at its maximum($r=r_{max}$).

$$V_0 = Vr_{max}\ , \frac{1}{b^2} = -\frac{1}{2V_0}\left[\frac{d^2V}{dr^{2}{}_*}\right]$$

In Fig.2 we compare the effective potential to the corresponding Pöschl-Teller potential.

In Fig.2 we compare the effective potential to the corresponding Pöschl-Teller potential.



The QNMs of the Pöschl-Teller potential can be evaluated analytically;

$$E=\frac{1}{b}\left[\sqrt{V_{0-}\frac{1}{4}}-\left(n+\frac{1}{2}\right)\right].$$

In this case the effective potential depends both on Q and E. So we calculate the Quasinormal modes. Here we first find QNMs for the case Q=0 in which the potential is independent of E, let it be E_0. We take this as the initial value E_0 for a fixed n,l(or k) and e and is used to evaluate the corresponding QNMs for Q≠0. i.e, we use E_0 as real to modify the potential and find E_1 and repeat the process successfully to get E_2,E_3,E_4,……..

TABULATION AND ERROR CALCULATION OF QNMS OF GRAVITATIONAL FIELD FOR DIFFERENT MULTIPOLES OF PERTURBATIONS OF BLACK HOLE

- The value of v, rmax, vd2, b, wr, wi

  Here, v is the effective potential,rmax is value of radial coordinate where the potential is maximum. vd2 = $\left[\frac{d^2V}{dr^2{}_*}\right]$ where r_* is the tortoise coordinate. b is the curvature of the potential at its maximum (r=rmax). wr and wi are the real and imaginary parts of the frequency.

  - For l=2

```
v= 0.15128669955195415
rmax= 3.280800000002703
-0.009919259599097724
5.523008910103807
0.378273648270257
0.09053036273131819

[Program finished]
```

- For l=3

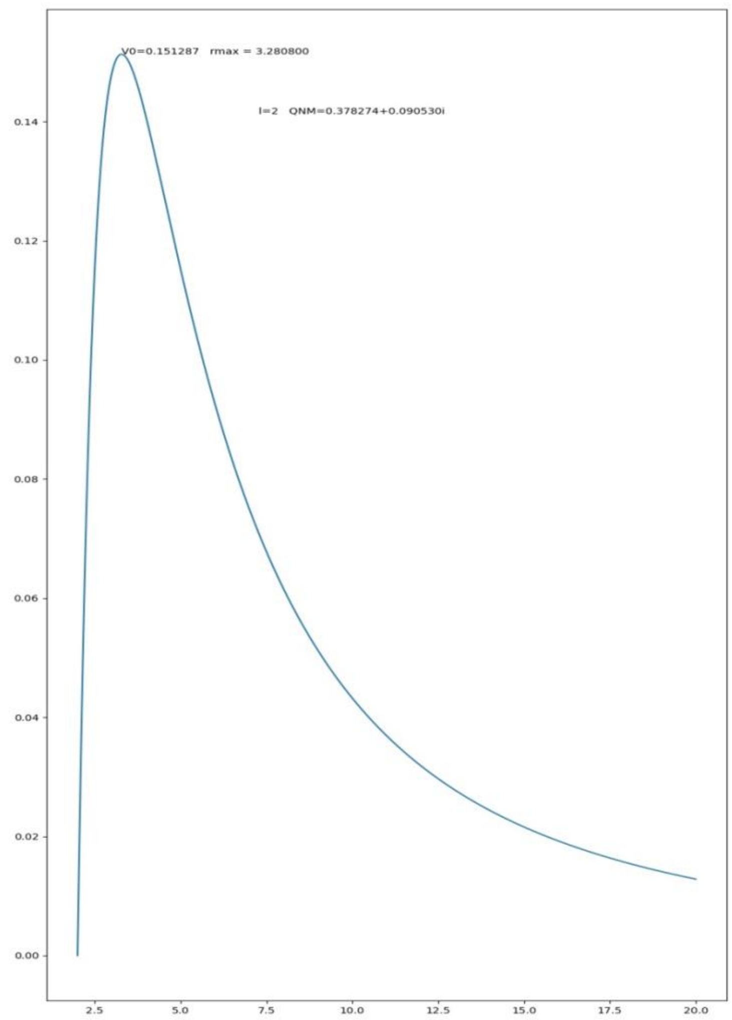
```
v= 0.37162504961055004
rmax= 3.106100000002334
-0.025919017958882243
5.3549844817613295
0.6024175585908376
0.09337095218538205

[Program finished]
```

- For l=4

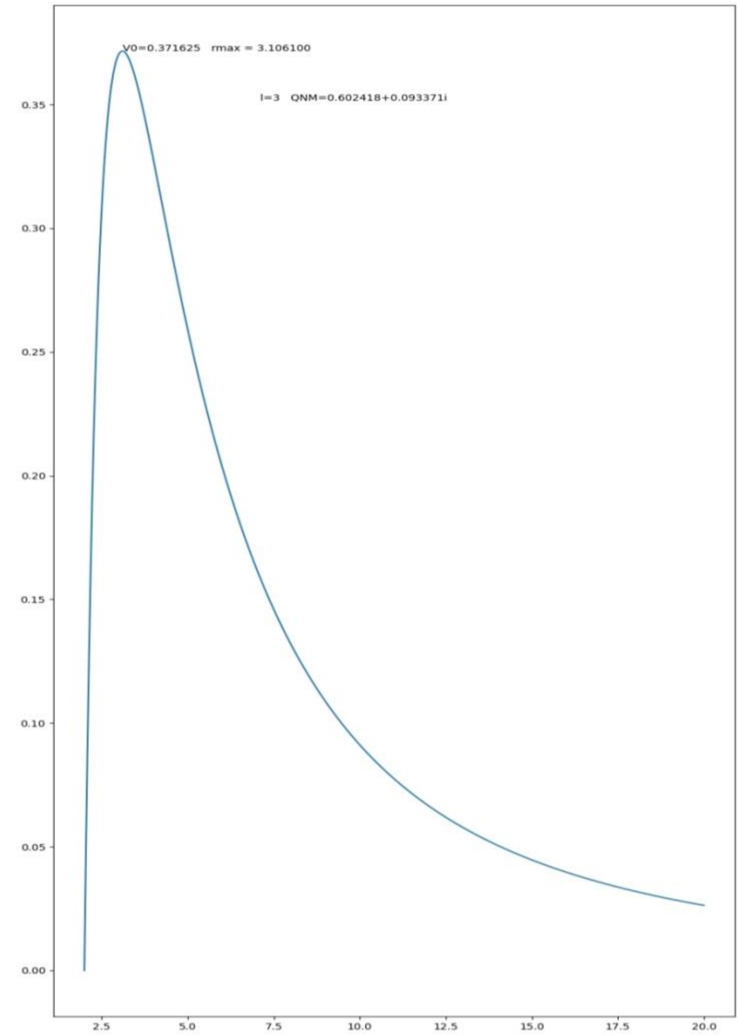
```
v= 0.667359781008887
rmax= 3.0575000000022317
-0.047726994676769816
5.2882620634021835
0.8114309974782294
0.09454902083243713

[Program finished]
```

The plots of the effective potential versus r are given below for the angular quantum number l = 2, 3 and 4. The plot also shows the quasinormal mode frequencies corresponding to the angular quantum number l.

❖ QNM for l=2

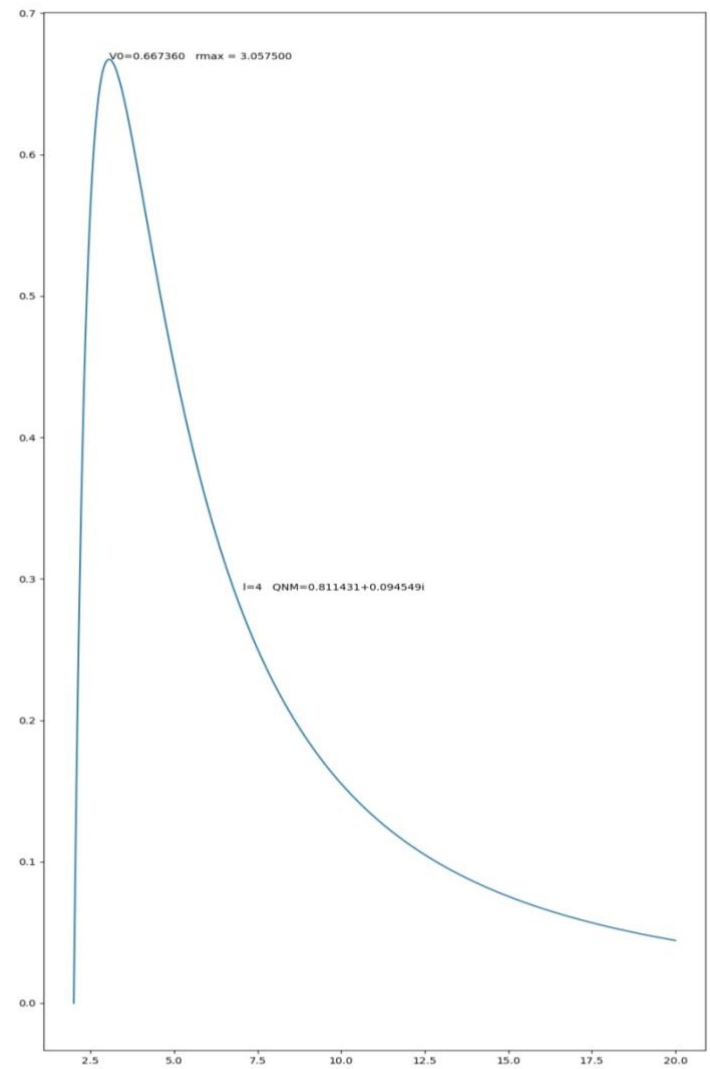

V0=0.151287 rmax = 3.280800
l=2 QNM=0.378274+0.090530i
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

❖ QNM for l=3

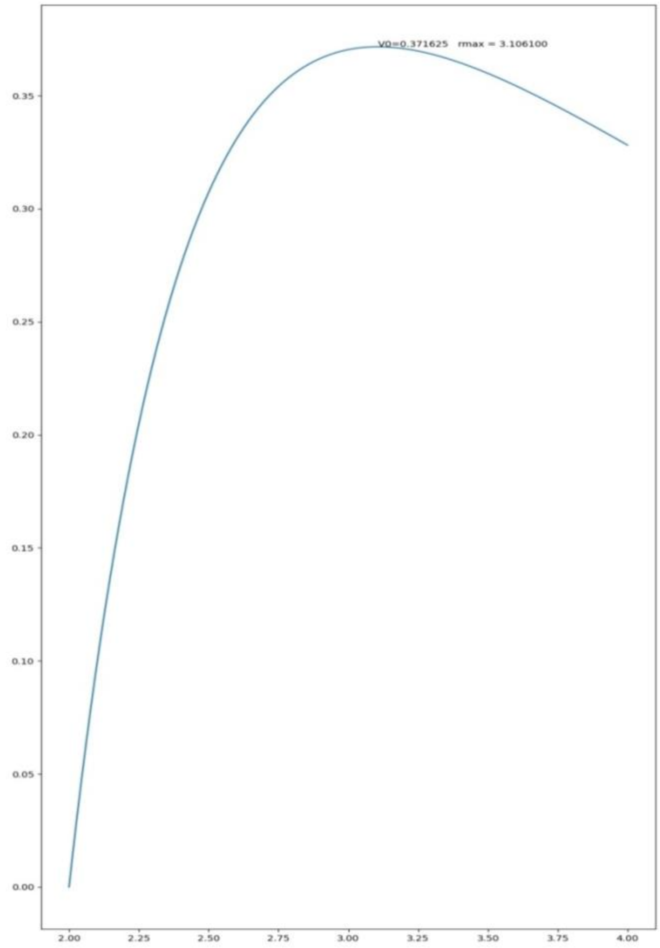

V0=0.371625 rmax = 3.106100
l=3 QNM=0.602418+0.093371i
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

❖ QNM for l=4

❖ QNM for small range (l=4)



❖ QNM for small range (l=4)
Here, the range taken for the potential is small (r=2 to r=4)



NORMAL MODE FREQUENCIES FOR SCHWARZSCHILD

Real frequency

l	Numerical	pöschl-teller	Error %
2	0.3737	0.378274	1.2
3	0.5994	0.602418	0.5
4	0.8092	0.811431	0.3

Imaginary

l	Numerical	pöschl-teller	Error %
2	0.0889	0.090530	1.8
3	0.0927	0.093371	0.7
4	0.0941	0.094549	0.4

*Numerical results from Chandrasekhar and Detweiler 1975.

CONCLUSION

The QNMs of Schwarzschild black holes for gravitational field of different modes have been calculated using pöschl-Teller method and calculated using python & this method help us to study about the parameters of black holes',ie this can be used for the qualitative study of black holes.

The modes are determined by the black hole mass M and the quantum number l.We have obtained the perturbation equation in Schwarzschild black hole spacetime and its deduction into a set of second order differential equations. We have evaluated the quasinormal mode frequencies for Schwarzschild black hole spacetimes having gravitational perturbation. The frequencies all have a negative imaginary part,which means that the blackhole is stable against these perturbations & it leads to decrease of the real oscillation frequency and to a slower decay.

PYTHON CODE

- Python code to plot the different angular quantum number
  - l=2

```python
import numpy as np
import math as m
l = 2.0
r = np.arange(2,20,.0001)
def v(r,l):
    v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
    return v

v2 = v(r,l)
v0 = max(v2)
c= np.where(v2 == v0)
r0= r[c[0][0]]
print ('v=',v0)
print('rmax=',r0)
vd2=(2.0*(-2+r0)*(-288+30* (7+l+l**2)*r0-4* (9+5*l+5*l**2)*r0**2+3*l*(l+1)*r0**3))/r0**8
b=m.sqrt((-2*v0/vd2))
print (vd2)
print(b)
wr=m.sqrt(v0-1/(4*b**2))
wi= (1/b)*(0+1.0/2)
print(wr)
print(wi)
```

- l=3

```python
import numpy as np
import math as m
l = 3.0
r = np.arange(2,20,.0001)
def v(r,l):
    v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
    return v

v2 = v(r,l)
v0 = max(v2)
c= np.where(v2 == v0)
r0= r[c[0][0]]
print ('v=',v0)
print('rmax=',r0)
vd2=(2.0*(-2+r0)*(-288+30* (7+l+l**2)*r0-4* (9+5*l+5*l**2)*r0**2+3*l*(l+1)*r0**3))/r0**8
b=m.sqrt((-2*v0/vd2))
print (vd2)
print(b)
wr=m.sqrt(v0-1/(4*b**2))
wi= (1/b)*(0+1.0/2)
print(wr)
print(wi)
```

- l=4

```
import numpy as np
import math as m
l = 4.0
r = np.arange(2,20,.0001)
def v(r,l):
   v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
   return v

v2 = v(r,l)
v0 = max(v2)
c= np.where(v2 == v0)
r0= r[c[0][0]]
print ('v=',v0)
print('rmax=',r0)
vd2=(2.0*(-2+r0)*(-288+30* (7+l+l**2)*r0-
4* (9+5*l+5*l**2)*r0**2+3*l*(l+1)*r0**3))/
r0**8
b=m.sqrt((-2*v0/vd2))
print (vd2)
print(b)
wr=m.sqrt(v0-1/(4*b**2))
wi= (1/b)*(0+1.0/2)
print(wr)
print(wi)
```

❖ python code to find QNM for different angular quantum number

- l=2

```
import numpy as np
import math as m
import matplotlib.pyplot as plt
l = 2.0
r = np.arange(2,20,.0001)
def v(r,l):
   v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
   return v
v2 = v(r,l)
v0 = max(v2)
maxint = np.where(v2 == v0)
rmax  = r[maxint[0][0]]
print ('Vo =%6.6f   rmax = %6.6f'%(v0,
rmax))
vd2 = (2.0*(-2+rmax)*(-288+30*(7+l+l**2)*
rmax-4*(9+5*l+5*l**2)*rmax**2+3*l*(1+l)*
rmax**3))/rmax**8
b = m.sqrt((-2*v0/vd2))
print ('vd2 =%6.6f   b = %6.6f'%(vd2,b))
wr = m.sqrt(v0-(1/(4*b**2)))
wi = (1/b)*(0+1.0/2)
print ('l=',int(l),' QNM= ',wr,'+',wi,'i')

plt.plot(r,v2)
plt.text(rmax, v0, 'V0=%f   rmax = %f'%(v0,
rmax))
plt.text(rmax+4, v0-0.02/l, 'l=%d
QNM=%f+%fi'%(int(l),wr,wi))
plt.show()
```

- l=3

```
import numpy as np
import math as m
import matplotlib.pyplot as plt
l = 3.0
r = np.arange(2,20,.0001)
def v(r,l):
    v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
    return v
v2 = v(r,l)
v0 = max(v2)
maxint = np.where(v2 == v0)
rmax  = r[maxint[0][0]]
print ('Vo =%6.6f   rmax = %6.6f'%(v0,
rmax))
vd2 = (2.0*(-2+rmax)*(-288+30*(7+l+l**2)*
rmax-4*(9+5*l+5*l**2)*rmax**2+3*l*(1+l)*
rmax**3))/rmax**8
b = m.sqrt((-2*v0/vd2))
print ('vd2 =%6.6f   b = %6.6f'%(vd2,b))
wr = m.sqrt(v0-(1/(4*b**2)))
wi = (1/b)*(0+1.0/2)
print ('l=',int(l),' QNM= ',wr,'+',wi,'i')

plt.plot(r,v2)
plt.text(rmax, v0, 'V0=%f   rmax = %f'%(v0,
rmax))
plt.text(rmax+4, v0-0.06/l, 'l=%d
QNM=%f+%fi'%(int(l),wr,wi))
plt.show()
```

- l=4

```
import numpy as np
import math as m
import matplotlib.pyplot as plt
l = 4.0
r = np.arange(2,20,.0001)
def v(r,l):
    v = (1-2/r)*((l*(l+1))/r**2-6/r**3)
    return v
v2 = v(r,l)
v0 = max(v2)
maxint = np.where(v2 == v0)
rmax  = r[maxint[0][0]]
print ('Vo =%6.6f   rmax = %6.6f'%(v0,
rmax))
vd2 = (2.0*(-2+rmax)*(-288+30*(7+l+l**2)*
rmax-4*(9+5*l+5*l**2)*rmax**2+3*l*(1+l)*
rmax**3))/rmax**8
b = m.sqrt((-2*v0/vd2))
print ('vd2 =%6.6f   b = %6.6f'%(vd2,b))
wr = m.sqrt(v0-(1/(4*b**2)))
wi = (1/b)*(0+1.0/2)
print ('l=',int(l),' QNM= ',wr,'+',wi,'i')

plt.plot(r,v2)
plt.text(rmax, v0, 'V0=%f   rmax = %f'%(v0,
rmax))
plt.text(rmax+4, v0-1.5/l, 'l=%d   QNM=%f+
%fi'%(int(l),wr,wi))
plt.show()
```

REFERENCES

- Studies on quasi normal modes and late–time tails in black hole spacetimes: nijo Varghese: October 2012
- The quasi-normal modes of the Schwarzschild black hole BY S. Chandrasekhar,F.R.S& S. Detwetler, Chicago, Illinois,60637(6 December 1974)
- Quasi normal modes of RN black hole space time with cosmic string in a Dirac field sini,V C kuriakose:December 10,2008.
- Quasi-normal modes of Schwarzschild-de sitter black holes A.Zhidenko St.Naukova 13,Dniepropetrovsk 49050,Ukraine Z A V@ff.dsu.dp.ua ;3 dec 2003
- Quasinormal Modes and Gravitational Radiation in Black Hole Spacetimes,Vitor Cardoso:published in 21 April 2004
- Black Holes explained by National geographic society:maya wei-haas;published September 21,2018