The theoretical model of the proprioception sonification model proposed in this paper is clarified, and the symbol definitions are listed in Table S1.

The decibels to linear gain conversion is as follows:

 (1)

This formula converts a value from the logarithmic dB scale to a linear multiplication factor. For example, +6 dB corresponds to a linear gain of approximately 2.0 (doubling the amplitude).

The relative distance between the listener to the sound source is: (assuming the listener is at the origin [0, 0, 0])

 (2)

The base azimuth and elevation in world coordinates is calculated as:

 (3)

 (4)

 (5)

where  is the horizontal angle of the sound source, and is the vertical angle, which means the angle between the horizontal (X-Y) plane and the vector pointing to the source.

Distance-dependent beep repetition rate maps the source’s distance to the repetition rate of the beep sound.

 (6)

The clip function ensures the rate stays within the preset range [2,10] Hz to prevent the beep sound from too weak or too loud to hear.

The elevation-dependent piano note frequency is determined by the formula below:

 (7)

It maps the source’s vertical angle to the pitch (frequency) of a continuous piano-like tone from C4 to C6.

The piano sound (digital sine wave) is generated using this formula:

 (8)

This formula is for generating a pure tone digitally. The mod 2π means keep the phase  within the interval [0, 2π) by wrapping it around every time it exceeds 2π.

For the spatialization effect of volume attenuation, the Inverse-Square Law can be written as:

 (9)

This models a fundamental law of physics. In a free field, the intensity of sound decreases with the square of the distance from the source. This makes distant sounds quieter than rear sounds.

The distance-dependent air absorption (filter cutoff frequency) is represented by the low-pass filter’s cutoff frequency:

 (10)

It simulates the effect of air absorbing high-frequency sound content over distance. When the distance  is small, the cutoff frequency is close to the maximum, the sound is bright and clear. As the distance increases, the cutoff frequency approaches the minimum, therefore the sound becomes muffled, as if it has traveled a long way.

The balance between the direct sound and reverb sound is shown as the wet ratio formula:

 (11)

This controls how much environment reflection (reverb, or “wet” signal) is mixed with the direct sound (“dry” signal). As the distance increases, the reverb mix linearly increases. It mimics the real-world perception that most distant sounds have a higher proportion of reflected sound compared to direct sound.

For the most important part of spatialization: HRTF interpolation is carried out using barycentric Coordinates on a Sphere. The target point’s direct  is represented as follows:

 (12)

It is a combination of final head-relative azimuth and elevation. And the final HRTF is the weighted sum of the HRTFs at the three vertices:

 (13)

HRTF data is only measured at discrete points. To get the HRTF for any arbitrary direction, we must interpolate between the known points. Barycentric interpolation is a standard and accurate method for doing this on a triangular mesh.

These sonification strategies also contribute to sound externalization. To improve antero-posterior localization accuracy, the audio signal is constrained to the 8000–16000 Hz range [1]. Additionally, to enhance front–back differentiation, the sound source in the posterior field is replaced with a running water (stream) sound instead of pink noise.

TABLE S1

Symbol Definitions for Proprioceptive Sonification

|  |  |  |
| --- | --- | --- |
| Symbol | Value | Description |
| Input & State | | |
|  | - | World coordinates of the sound source |
|  | - | Calculated Euclidean distance to the source |
|  | - | Head yaw and pitch angles from tracking |
|  | - | Final head-relative azimuth and elevation |
|  | - | Nominal gain level in decibels |
|  | - | A generic frequency (e.g., ) |
|  | 48000 Hz | System sample rate |
|  | - | Amplitude of a generated sine wave |
|  | - | Signal value and phase at sample |
| Beep Parameters | | |
|  | 10.0 Hz | Maximum beep rate (at minimum distance) |
|  | 8.0 Hz | Range of beep rate change (from max to min) |
|  | 0.1 m | Minimum distance for beep rate scaling |
|  | 0.9 m | Distance range over which beep rate scales |
| Piano Parameters | | |
|  | 1046.5 Hz | Piano frequency at maximum positive elevation (C6) |
|  | 523.25 Hz | Piano frequency at zero elevation (C5) |
|  | 261.63 Hz | Piano frequency at maximum negative elevation (C4) |
|  | +90° | Maximum positive elevation angle for scaling |
|  | -90° | Maximum negative elevation angle for scaling |
| Effects Parameters | | |
|  | - | Intensity attenuation gain factor |
|  | 8000 Hz | Maximum cutoff frequency for air absorption filter |
|  | 1000 Hz | Minimum cutoff frequency for air absorption filter |
|  | 0.05 | Exponential decay constant for air absorption |
|  | 0.4 | Maximum wet (reverb) signal ratio (40%) |
|  | 0.2 m | Minimum distance for reverb calculation |
|  | 1.5 m | Distance range over which reverb ratio scales |
| HRTF Parameters | | |
|  | - | Unit vector representing the target sound direction |
|  | - | Unit vectors for the vertices of the enclosing simplex |
|  | - | Barycentric weights for interpolation |
|  | - | The Left/Right Head-Related Transfer Function |

References

[1] E. H. A. Langendijk and A. W. Bronkhorst, “Contribution of spectral cues to human sound localization,” *The Journal of the Acoustical Society of America*, vol. 112, no. 4, pp. 1583–1596, Oct. 2002, doi: 10.1121/1.1501901.