Computing the Discretised Schrödinger Equation
Part 2

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1 Derivative approximations

1.1 Some preliminaries

Let us define the vector $\vec{S}_i = \begin{pmatrix} a_i \\ b_i \end{pmatrix}$
Therefore the overall matrix of points S will be:

$$S = \begin{pmatrix} a_0 & \dots & a_n \\ b_0 & \dots & b_n \end{pmatrix}$$

Define: $\vec{F}(\vec{S}) = \frac{\partial \vec{S}}{\partial \tau}$
We can see that:

$$\frac{\partial}{\partial t}\begin{pmatrix} a_i \\ b_i \end{pmatrix} \approx \underbrace{\begin{pmatrix} b_i - (b_{i+1} + b_{i-1}) \\ a_{i+1} + a_{i-1} - a_i \end{pmatrix}}_{\vec{F}}$$

This is all the definitions we need for the derivative approximations.

1.2 Method 1: 2nd Order Approximation

This method produces errors on the order of $\mathcal{O}(d\tau^3)$
We begin by starting with an initial point on the line, $\vec{S}_0$. We must first approximate a half step by the following method: