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STAT 131 Final Project Report
Estimating Korean Stock Market Volatility

Project Overview

The goal of our project is to estimate Korean Stock Market Volatility (VKOSPI) using the Korean Blue Chip Index (KOSPI). We will be using the *Options in Korean Stock Market Dataset* found on Kaggle**.** We will be trying various models such as GARCH, EGARCH, and IGARCH in hopes of achieving our goal.

Description of Dataset

A description of the dataset is as follows:
- Every row in the data represents a trading day (excluding weekends and holidays).
- The column headers indicate whether the data pertains to Foreigners (abbreviated as "For") or Individuals (abbreviated as "Indiv").
- The column labeled "Amount" reflects the product of the price and quantity, whereas the "Quantity" column reflects only the quantity.
- The data for options trading is represented by "Call" for the Call option of KOSPI200, "Put" for the Put option of KOSPI200, and "Future" for the Future of KOSPI200.
- The "Day_till_expiration" column indicates the number of days left until the expiration date of the options.
- It is important to note that the expiration date for these options is every second Thursday.
- Time ranges from June 1, 2009 to November 9, 2019.
- The size of the sample is 2580 variables with 14 different columns.

The relevant columns of the dataset we will be using are:
- KOSPI200
- VKOSPI

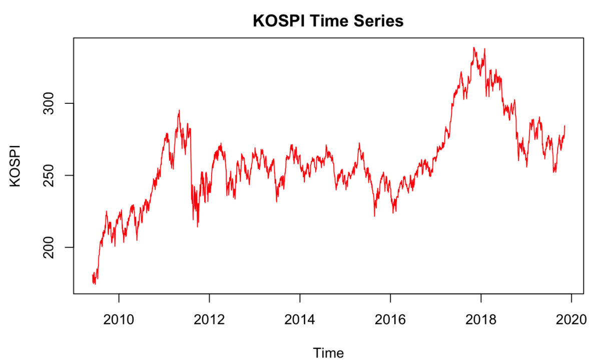
Motivation of the Study

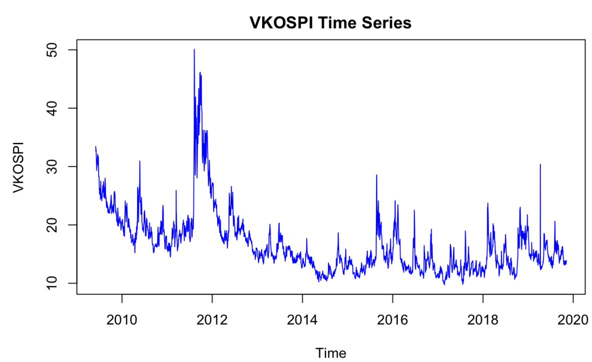
The Korean Composite Stock Price Index (KOSPI) displays the stock prices of companies listed on the Korea Stock Exchange, and serves as a benchmark to gauge the overall movement of the market. It is also utilized to assess investment performance, compare returns with other financial instruments, and forecast economic conditions. The index was established on January 4, 1980, with a market capitalization of 100 as the baseline, and subsequent market capitalization is indexed to this point for comparison. Additionally, the Korea Exchange calculates and announces the Volatility Index, VKOSPI, which predicts the expected future volatility of the KOSPI 200 index based on option prices. This is similar to the Chicago Board Options Exchange's (CBOE) volatility index (VIX) that is calculated based on S&P 500 index options. VKOSPI is expected to function as both an investment indicator for detecting market fluctuation risks and an investment tool for managing volatility risk.

During these times of uncertainty, market volatility has reached unprecedented levels. The VIX, also known as the "fear index," skyrocketed to its highest point amidst and even after the COVID-19 pandemic. Estimating volatility is essential in making informed investment decisions, thus our project aims to estimate the volatility of the Korean stock market using KOSPI200.

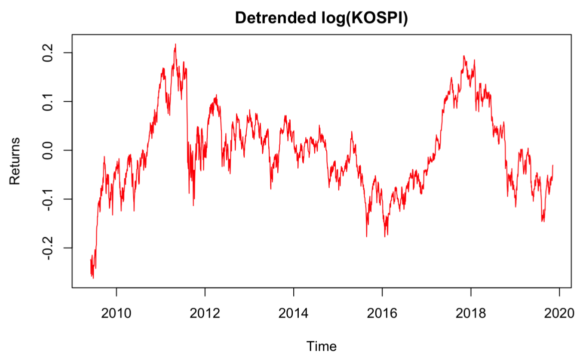
Exploratory Data Analysis

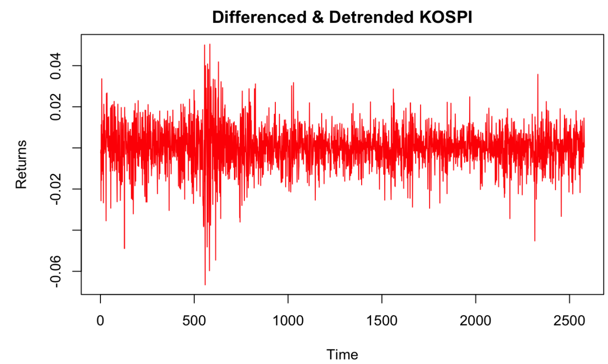
Before we start modeling, we first inspect the KOSPI series and VKOSPI series which we will be using. Trace plots of both series are given below.





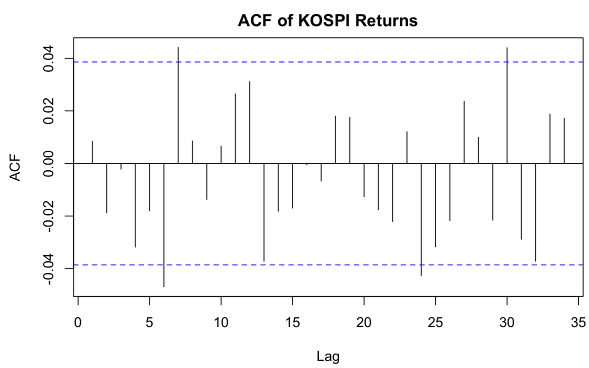
There is a clear increasing linear trend in the KOSPI series as well as a slight decreasing trend in the VKOSPI series. As we will be using the KOSPI series for our modeling, we need to pre-process the returns. First, we took the log of the KOSPI series in hopes to stabilize the variance. We then removed the linear trend by modeling with the residuals of a linear model fit on the logged KOSPI series. However, after detrending, we could still see an evident seasonal pattern. Thus, we decided to difference (lag 1) the series. The left plot below shows the detrended returns while the right plot shows the detrended returns after differencing.

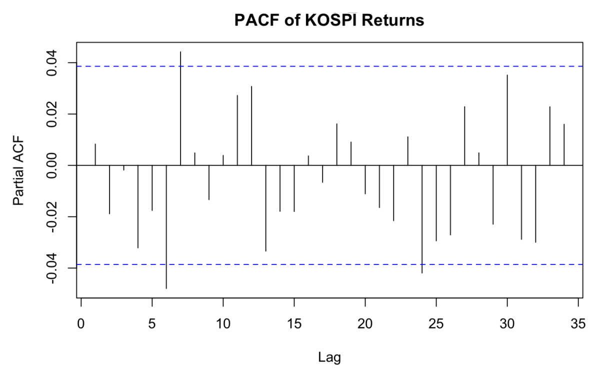




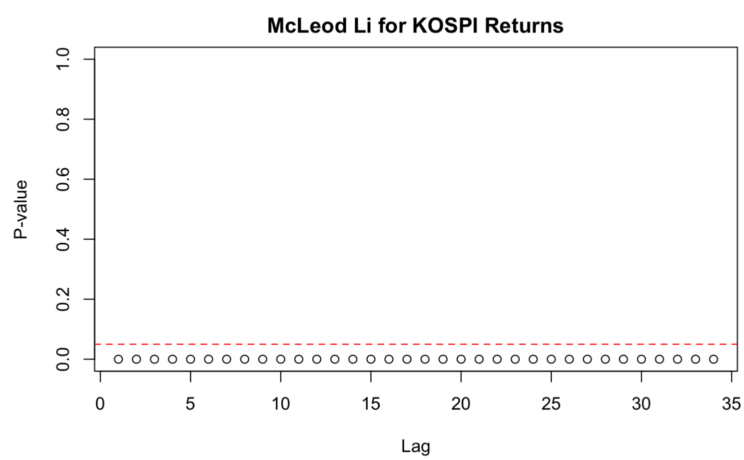
The mean of the differenced and detrended series is around $7.49e^{-5}$ which is not significantly different than 0.

For the remainder of this paper, we will be referring to the differenced and detrended log(KOSPI) as our KOSPI returns. Through the ACF and PACF plot of the returns shown below, we can confirm that the returns are somewhat uncorrelated over time.





Moreover, if we look at the results of the McLeod Li test, we detect significant ARCH effects (all lags had significant p-values). Therefore, we will try to fit some GARCH models. The McLeod Li test results are shown below.



Modeling & Diagnostics

Model 1: GARCH

Our first try to fit a GARCH model on our KOSPI returns. To choose the order of our model we looked at the ACF and PACF of the squared returns as well as the EACF table of the returns. As the ACF and PACF plots were unclear, we used the EACF table (shown below) to determine which order to use.

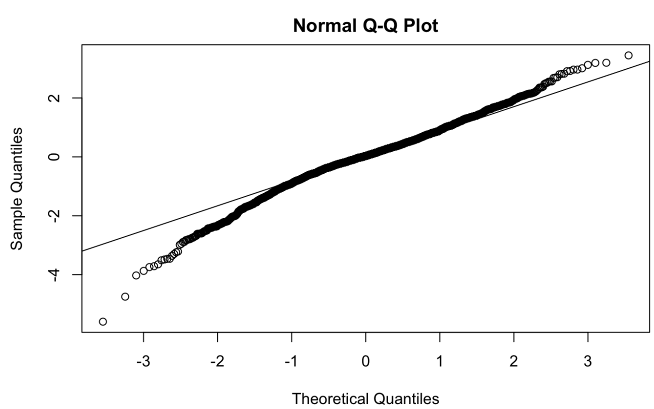
AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	o	o	o	o	o	x	x	o	o	o	o	o	o	o
1	x	o	o	o	o	o	x	o	o	o	o	o	x	o
2	x	x	o	o	o	x	o	o	o	o	o	o	x	o
3	x	x	x	o	o	x	x	o	o	o	o	o	x	o
4	x	x	x	x	o	x	o	o	o	o	o	o	x	o
5	x	x	x	x	x	o	o	o	o	o	o	o	o	o
6	x	x	x	x	x	x	o	o	o	o	o	o	o	o
7	x	x	x	x	o	x	x	o	o	o	o	o	o	o

Based on the above table, we chose a GARCH(1,1) model. The estimated GARCH(1,1) model was found to be:

$$x_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = 1.450e^{-6} + 4.535e^{-2} X_{t-1}^2 + 9.386e^{-1} \sigma_{t-1}^2$$

All 3 coefficients were found to be significant at the .05 significance level. Moreover, our model passed the Box-Ljung test at the .05 significance level (p-value = $.3216$), showing that our model residuals are not significantly correlated. However, our model did not pass the normality assumption of the residuals as it failed the Jarque Bera test at the .05 significance level (p-value $< 2.26e^{-16}$) . Below we show the QQ-plot of our model's residuals where we can see clear deviations from the normal distribution on both ends.



As a result our model may not be able to capture all the information related to the non-normality of the residuals.

Model 2: EGARCH

EGARCH (Exponential Generalized Autoregressive Conditional Heteroskedasticity) is an extension of the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model that allows for asymmetric volatility. The EGARCH model is specified in terms of the logarithm of the conditional variance, which allows for the conditional variance to be negative. The EGARCH model also includes an exponential term, which helps to capture the long-term effects of volatility shocks. The main difference between the EGARCH and GARCH models is that the EGARCH model allows for asymmetric volatility effects, while the GARCH model assumes that volatility responds symmetrically to positive and negative shocks.

We fitted an EGARCH(1, 1) model on the returns. The distribution of the standardized residuals is assumed to be normal. The fitted model is as follows:

$$\epsilon_t = \sigma_t z_t$$
$$ln(\sigma_t^2) = -0.073\frac{|-0.1699|+0.0796*(-0.1699)}{\sigma_{t-1}} + 0.98ln(\sigma_{t-1}^2)$$

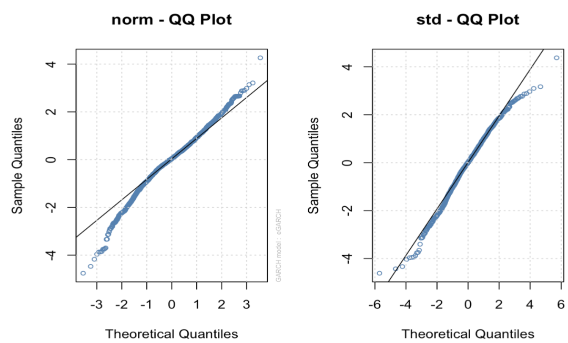
The α_1 coefficient is statistically significant as its p-value is 0.00000, telling that past volatility has an impact on current volatility. Also, weighted Ljung-Box test on standardized residuals says no serial correlation meaning that the test found no significant evidence of autocorrelation in the standardized residuals up to a certain lag. In addition, the adjusted Pearson Goodness-of-Fit test returns a low number, meaning that the model does not fit the data well. Therefore, we will attempt a new EGARCH model by alternating to t-distribution.

Next, we attempted on t-distribution of EGARCH(1,1) and the fitted model is as follows:

$$\epsilon_t = \sigma_t z_t$$
$$ln(\sigma_t^2) = 0.0002 + -0.087\frac{|-0.172|+0.088*(-0.172)}{\sigma_{t-1}} + 0.98ln(\sigma_{t-1}^2)$$

The diagnostic result is similar to the normal distribution of EGARCH(1,1). he alpha1 coefficient is also statistically significant as its p-value is 0.00000, telling that past volatility has an impact on current volatility. Weighted Ljung-Box Test on Standardized Residuals also says no serial correlation meaning that the test found no significant evidence of autocorrelation in the standardized residuals up to a certain lag. In addition, even though the Adjusted Pearson Goodness-of-Fit Test is higher than that of the normal distribution, the number is still low so we should conclude that the model does not fit the data well.

For both of the EGARCH models, we may notice that leverage effect presents because α_1 is smaller than 0.



Comparing between two different distributions of EGARCH, the EGARCH-t distribution QQ-plot seems to be more straight. However, we will try to estimate with IGARCH to check if it returns better results.

<u>Model 3: IGARCH</u>

The IGARCH (Integrated Generalized Autoregressive Conditional Heteroskedasticity) model is a special case of the GARCH model, where the sum of the AR and MA coefficients is constrained to equal one (thereby exhibiting persistence). This restriction implies that the shocks to the volatility have a permanent effect, and that the volatility process is non-stationary.

As a result, IGARCH is specifically designed to capture non-stationary processes, with the persistence of the volatility shocks lasting indefinitely; therefore, it might provide better forecasts for non-stationary volatilities.

Another advantage to using IGARCH is that they have fewer parameters due to the constraint, which can lead to simpler models and potentially more stable parameter estimates.

We first fit an IGARCH(1, 1) model on the returns. The distribution of the standardized residuals is assumed to be normal. The fitted model is as follows:

$$
\begin{aligned}
X_t &= 0.000134 + \epsilon_t \\
\epsilon_t &= \sigma_t z_t \\
\sigma_t^2 &= 0 + 0.046422 \cdot \epsilon_{t-1}^2 + 0.953578 \cdot \sigma_{t-1}^2
\end{aligned}
$$

On output interpretations, we observed that notably, the alpha1 coefficient is statistically significant with a p-value of essentially 0, indicating that past volatility has a significant impact on current volatility.

The Ljung-Box test results for the standardized residuals and squared residuals indicate no significant serial correlation, suggesting that the model has adequately captured the autocorrelation structure in the returns and volatility.

The adjusted Pearson goodness-of-fit test assesses the fit of the model's assumed distribution to the data. The p-values are very small, indicating that the normal distribution may not be an appropriate assumption for the standardized residuals. This might suggest considering alternative distribution assumptions, such as the Student's t-distribution or the Generalized Error Distribution.

Next, we attempt fitting the same model but with a t-distribution assumption. Similarly, here is the fitted model equation:

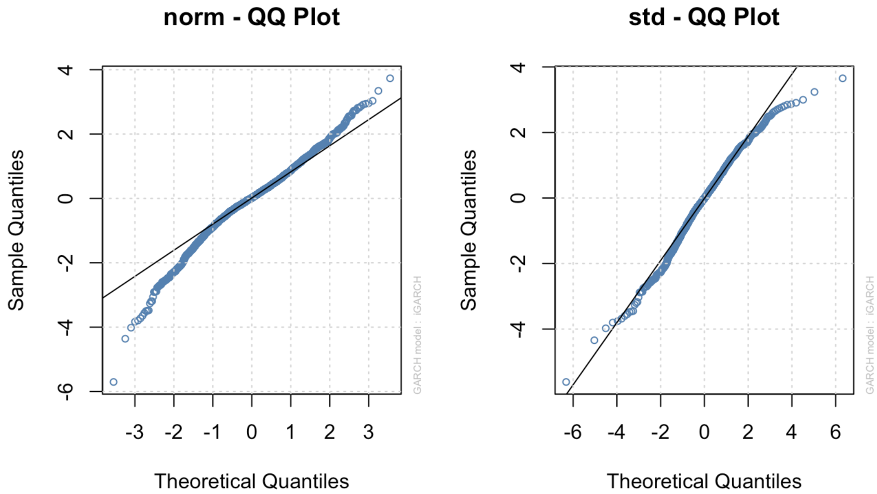
$$
\begin{aligned}
X_t &= 0.000356 + \epsilon_t \\
\epsilon_t &= \sigma_t z_t \\
\sigma_t^2 &= 0 + 0.046147 \cdot \epsilon_{t-1}^2 + 0.953853 \cdot \sigma_{t-1}^2 \\
z_t &\sim t(5.235451)
\end{aligned}
$$

We observed that parameters mu, alpha1, and shape are all statistically significant, suggesting that the t-distribution assumption improves the model fit.

The Ljung-Box test results for the standardized residuals and squared residuals indicate no significant serial correlation, suggesting that the model has adequately captured the autocorrelation structure in the returns and volatility.

The adjusted Pearson goodness-of-fit test p-values are smaller than those for the normal distribution assumption, but still significant, indicating that the t-distribution provides a better fit than the normal distribution, but there might still be room for improvement.

The following QQ-plot visualizes the distinctions between the two distribution assumptions (where the normal distribution is on the left and t-distribution is on the right).



Final Model

Previously, we fit a few models on predicting volatility. We have GARCH and its variants EGARCH and IGARCH. We are now interested in determining which is the best. To accomplish this, we can use metrics such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), as well as the log-likelihood of each model. The model with the lowest AIC and BIC values and the highest log-likelihood is generally considered the best fitting model. The following table summarizes these metrics.

Model	**AIC**	**BIC**	**Log Likelihood**
GARCH	-16817.98	-16800.41	8411.99
EGARCH	-6.58	-6.57	8494.37
IGARCH	-6.56	-6.55	8467.40

We note that the AIC and BIC of GARCH are much lower than those of EGARCH and IGARCH. This could possibly be explained by the model complexity. Notice that while EGARCH and IGARCH can capture certain features of the data that GARCH models cannot (such as asymmetric volatility and integrated behavior), they might introduce additional complexity that is not justified by the improvement in the likelihood. If the extra parameters in EGARCH and IGARCH models do not provide a substantial increase in the likelihood, the AIC and BIC values will penalize these models for the additional complexity, resulting in higher values compared to the GARCH(1,1) model.

Hence, even though EGARCH maximized log-likelihood, GARCH still beats it on both AIC and BIC. Thus, we conclude that GARCH(1, 1) is our best model.

Conclusion

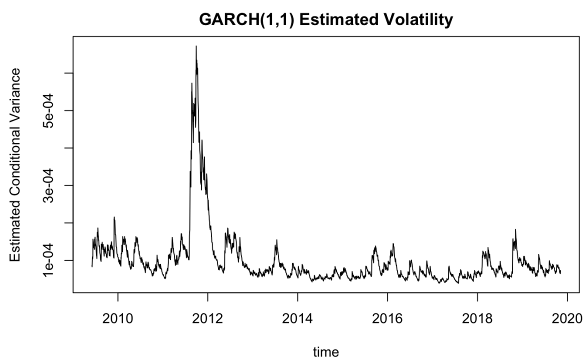
As mentioned above, out of the GARCH(1,1), EGARCH(1,1), and IGARCH(1,1) models, the GARCH(1,1) model best fits the data in terms of the AIC and BIC criteria. The estimated GARCH(1,1) model is defined as follows:

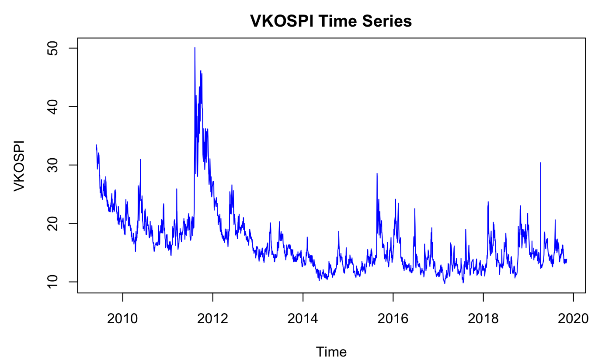
$$x_t = \sigma_t \epsilon_t$$

$$\sigma_t^{\ 2} = 1.450e^{-6} + 4.535e^{-2} X_{t-1}^{\ 2} + 9.386e^{-1} \sigma_{t-1}^{\ 2}$$

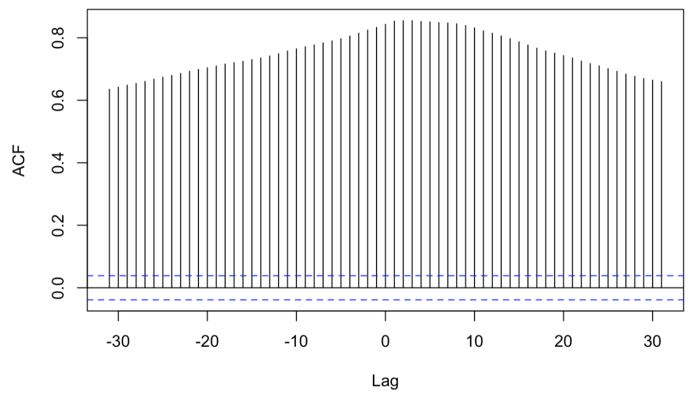
We can see that our intercept ($1.450e^{-6}$) is very low, showing that the baseline volatility is also low. Moreover the α_1 parameter is positive ($4.535e^{-2}$), indicating that past shocks have a positive, albeit small, impact on current volatility. The β_1 coefficient is very close to 1 ($9.386e^{-1}$) indicating that volatility is highly persistent: it takes a long time to revert back to the long-term average after a shock.

To understand our model's performance, it is important to analyze how the final model's estimated volatilities compare to the VKOSPI series. Below we plot the estimated conditional variances (estimated volatilities) from the GARCH(1,1) model alongside the VKOSPI series.





We can see that the estimated volatilities of our GARCH(1,1) model follow a similar trend as the VKOSPI series. To further inspect the similarities between our model's estimates and the VKOSPI series, we analyze the Cross Correlation Function (CFF) between the two.
The CCF plot is given below.



From the CCF plot, we can see that our model's estimated volatilities are highly correlated with the VKOSPI series. Moreover, the peak of the plot occurs near (but not exactly at) lag 0 showing that our model does pretty well in estimating a similar volatility trend as found in the VKOSPI series.

Although our GARCH(1,1) model does not pass the Jarque Bera test of the normality of the residuals, we can conclude that our final model does a decent job in estimating the volatility of the KOSPI series. Further improvements could be made by assuming a different distribution, other than normal, for our model's residuals such as the Student's t-distribution or a skewed distribution.

Resources

Staff, M. F. (2015, December 14). *How to calculate return on indices in a stock market*. The Motley Fool. Retrieved April 25, 2023, from https://www.fool.com/knowledge-center/how-to-calculate-return-on-indices-in-a-stock-mark.aspx

V-lab: EXPONENTIAL GARCH volatility documentation. Real-time Financial Volatility, Correlation, And Risk Measurement, Modeling, And Forecasting. (n.d.). Retrieved April 25, 2023, from https://vlab.stern.nyu.edu/docs/volatility/EGARCH