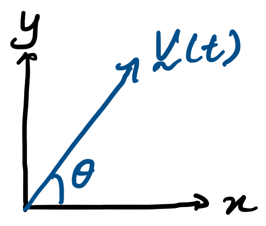
Rocket calculations

Sunday, May 24, 2020 4:40 PM

① From Tsiolkovsky's rocket equation, taking gravity into account, ignoring air resistance

$V(t) = V_{ex} \ln \frac{M_{initial}}{M(t)} - \int_0^t g(t)\,dt$



$\underset{\sim}{V}(t) = V(t)\cos\theta\, \underset{\sim}{i} + V(t)\sin\theta\, \underset{\sim}{j}$

$M(t)$ = rocket mass at time t

θ is a function of time

let $\dot{m}_{fuel}$ = mass flow rate

$\dot{m}_{fuel} = \frac{M_{initial} - M(t)}{t} = \frac{M_{initial} - M_{final}}{T}$, T = burn out time

$\Rightarrow M(t) = M_{initial} - \dot{m}_{fuel}\, t$

$= M_{initial} - (M_{initial} - M_{final})\frac{t}{T}$

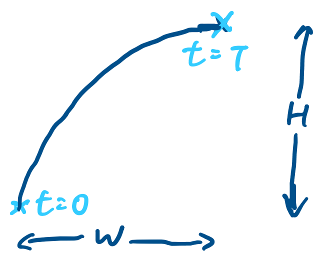
let $M_{final}/M_{initial} = 0.2$, 80% of rocket is fuel

$\frac{M_{initial}}{M(t)} = \frac{1}{1 - (1 - \frac{M_{final}}{M_{initial}})(\frac{t}{T})} = \frac{1}{1 - (1-0.2)\frac{t}{T}} = \frac{1}{1 - 0.8\frac{t}{T}}$

$g(t) = g_0 \sin\theta$, $g_0 = 9.81$

$x(t) = \int_0^t v(t)\cos\theta\, dt$

$y(t) = \int_0^t v(t)\sin\theta\, dt$



we can set a reasonable function of $\theta(t)$ to tilt the rocket

let the rocket tilt at constant rate

$\theta(t) = \frac{\pi}{2} - \frac{\pi}{2}(\frac{t}{T})$

$\Rightarrow \cos\theta = \cos(\frac{\pi}{2} - \frac{\pi}{2}(\frac{t}{T}))$

$= \sin \frac{\pi}{2T} t$

$\sin\theta = \sin(\frac{\pi}{2} - \frac{\pi}{2}(\frac{t}{T}))$

$= \cos\frac{\pi}{2T} t$

$\therefore\ \theta(t) = \frac{\pi}{2} - \frac{\pi}{2}(\frac{t}{T})$

$x(t) = \int_0^t \sin\frac{\pi}{2T} t\, V(t)\, dt$

$y(t) = \int_0^t \cos\frac{\pi}{2T} t\, v(t)\, dt$

$\Rightarrow v(t) = V_{ex} \ln(\frac{1}{1 - 0.8t/T}) - g_0 \int_0^t \cos\frac{\pi}{2T} t\, dt$

let $T = 5s$, $V_{ex} = 3000 m/s = 3 px/s$, $g_0 = 9.81 m/s^2 = 0.00981 px/s^2$

$v(t) = 3 \ln(\frac{1}{1 - 0.8t/5}) - 0.00981 \int_0^t \cos\frac{\pi}{10} t\, dt$

$= 3\ln(\frac{1}{1 - 0.16t}) - 0.00981 \cdot \frac{10}{\pi}(\sin\frac{\pi}{10} t)\Big|_0^t$

$= 3\ln(\frac{1}{1-0.16t}) - \frac{0.0981}{\pi} \sin\frac{\pi}{10} t$

$\therefore\ x(t) = \int_0^t (\sin\frac{\pi}{10} t)\left[3\ln(\frac{1}{1-0.16t}) - \frac{0.0981}{\pi}\sin\frac{\pi}{10}t\right] dt$

$y(t) = \int_0^t (\cos\frac{\pi}{10} t)\left[3\ln(\frac{1}{1-0.16t}) - \frac{0.0981}{\pi}\sin\frac{\pi}{10}t\right] dt$

---

② approximate behaviour as sine function Implemented in rocket animation!

as rocket travels further along x-direction, velocity increases exponentially

$\frac{dx(t)}{dt} \approx V_f(e^t - 1)$ as $\frac{dx(t)}{dt}\Big|_{t=0} = 0$, $t \in [0, 1]$

$x(t) = V_f(e^t - t) + C$

$x(t=0) = 0 \Rightarrow C = -V_f$, $x(t=1) = W \Rightarrow V_f(e-1) - V_f = W$

$V_f(e-2) = W$

$V_f = \frac{W}{e-2}$

$\therefore\ x(t) = V_f(e^t - t) - V_f$

$= \frac{W}{e-2}(e^t - t - 1)$

approximate trajectory to look like first quarter of sine curve

$y(x) = H \sin\frac{\pi x}{2W}$

$y(t) = H \sin\left[\frac{\pi}{2(e-2)}(e^t - t - 1)\right]$

adjustments for $\theta(t)$

$\frac{d\theta(t)}{dt} \approx \dot{\theta}_f(e^t - 1)$, $t \in [0,1]$ as $\frac{d\theta(t)}{dt}\Big|_{t=0} = 0$

$\theta(t) = \dot{\theta}_f(e^t - t) + C$

$\theta(t=0) = 0 \Rightarrow C = -\dot{\theta}_f$, $\theta(t=1) = \frac{\pi}{2} \Rightarrow \dot{\theta}_f(e-1) - \dot{\theta}_f = b$

$\dot{\theta}_f(e-2) = b$

$\dot{\theta}_f = \frac{b}{e-2}$

$\Rightarrow \theta(t) = \dot{\theta}_f(e^t - t) - \dot{\theta}_f$

$= \frac{b}{e-2}(e^t - t - 1)$