RYERSON UNIVERSITY

Department of Electrical, Computer, and Biomedical Engineering

Course Number	ELE829
Course Title	System Models & Identification
Semester/Year	Fall 2022
Instructor	Professor M.S. Zywno, Ph.D.

Final Project	Stochastic Models for Noise Filters

Section No.	**04**
Submission Date	**December 1, 2022**
Due Date	**December 1, 2022**

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(Note: remove the first 4 digits from your student ID: *xxxx12345*)

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www.ryerson.ca/senate/current/pol60.pdf

Final Project Grading Sheet

Assigned OE Process: 04 **Assigned PEM Process: 42**	
Any deductions will be recorded here	
ONE PAGE EXECUTIVE SUMMARY Anything that is important about this report should be included on this page - it is your "bottom line". If you don't know what to include, think about a busy CEO of your company who will not want to thumb through the whole report - he/she needs to know "the why, "the what" and the end result. The rest is for "the middle management" to pore over.	/10
Are appropriate diagnostics for OE Model included?	/5
Is OE Process Identified and Validated?	/5
Results for OE Process and Discussion:	/25
Are appropriate diagnostics for the deterministic part of PEM model included?	/5
Are appropriate diagnostics for the stochastic part of PEM model included?	/5
Is PEM Process Identified and Validated?	/5
Results for PEM Process and Discussion:	/30
General: Clarity, writing style, grammar, spelling, layout of the report	/30
TOTAL: /100	

Executive Summary

The purpose of this final project is to demonstrate knowledge gained over the period of the course regarding Modern Systems Identification. This includes collection of I/O data and using skills learned in ELE639 to diagnose characteristics of the system; system identification, which includes the use of one of Ljung's Models (OE, BJ, PEM ARMAX or ARX); and finally validating the model of the system using both visual (correlation) and numerical (Chi-Squared Test).

This project will be split into two processes.

The first is an OE process. In this process we will first take a sample of the flag and after looking at the bode plot using SPA (or ETFE), the hankel model, and determine the number of lags through CRA (or Correl). Since the number of lags is substantial we must use a second data set or else the model will not validate. Now we can determine the OE model of the system. The model will then be validated through the residual chi-square test as well as visually using the cross and autocorrelation functions. After passing the validation, if the model can be simplified it will be compared back to the original.

The second is a PEM process. Similarly to the OE process, we will use diagnostics to determine an OE model of the system and check its validation. We will see that the ACF and chi-square test fails, and the CCF passes. Now we know the process must have colored noise so we run the partial autocorrelation function to determine a noise filter and create a compensated model. After the new compensated model is created, the validation process can be run again with it passing all parameters. In this process there are no simplifications that can be made to the model so we can just leave it as is.

In the end both processes have been successfully modeled using the skills learned in both ELE639 and ELE829!

PROCESS OE

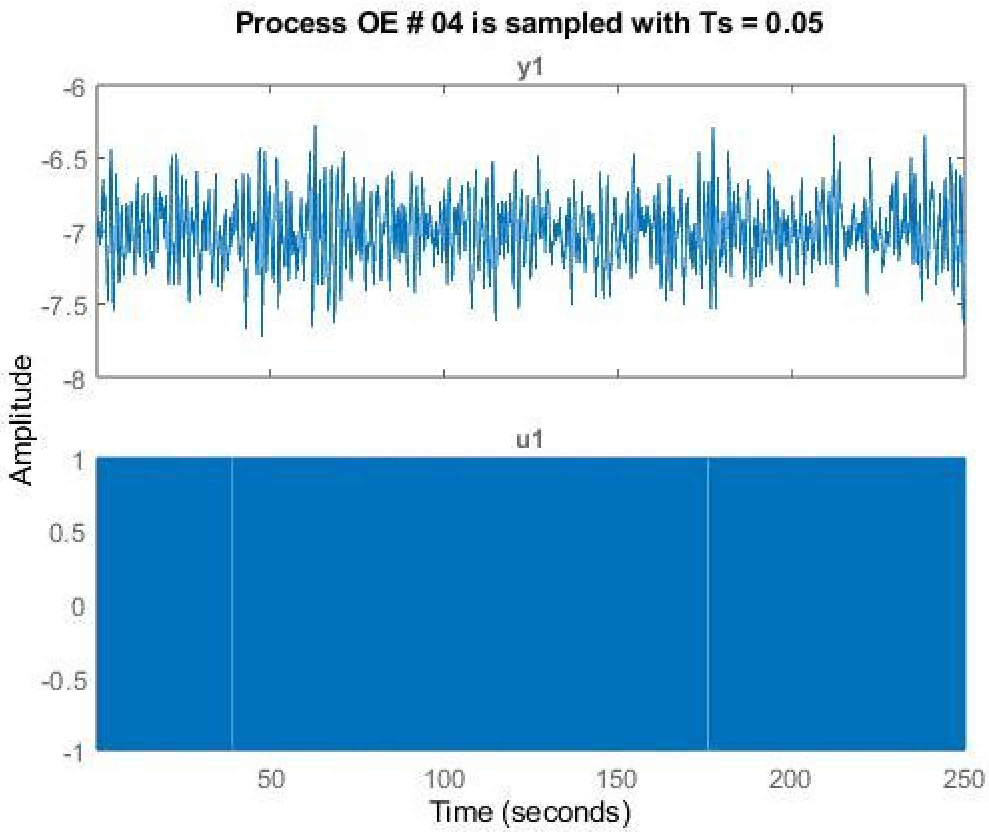

Figure 1: Fast switching Noise sample

```
>> mean(y1)

ans =

   -6.9993
```

We found the DC offset so that can be removed from the system as seen in line 12 of the OE code

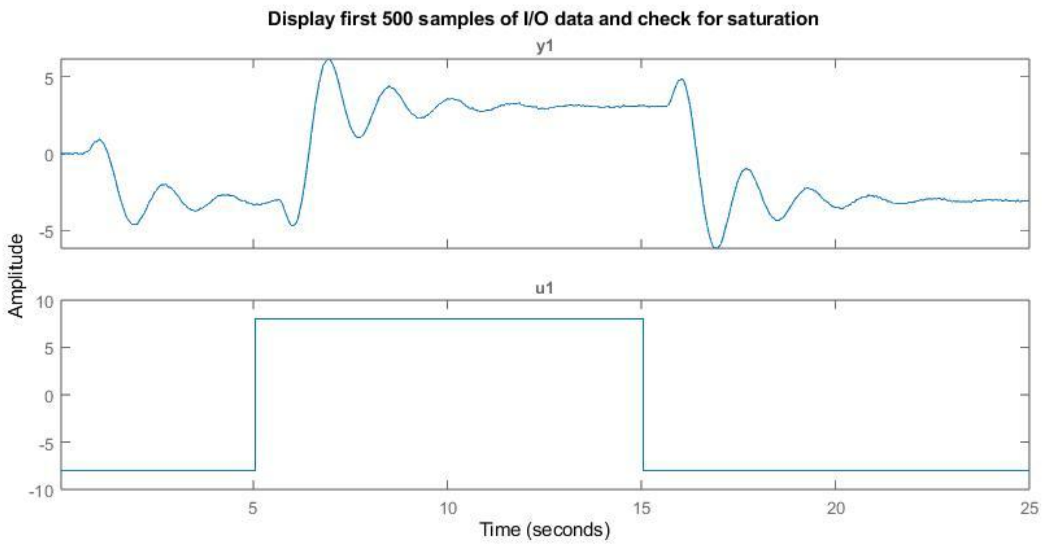

Figure 2: First 500 Samples of slow switching sample (K = 80)

From a slow switch rate and high signal to noise ratio we can see that that in the system there is a dip as the signal switches which is indicative of a right hand side zero, there are multiple oscillations which is indicative of a pair of complex conjugate poles and a slow settling time which shows a presence of a third pole. (three poles, one zero).

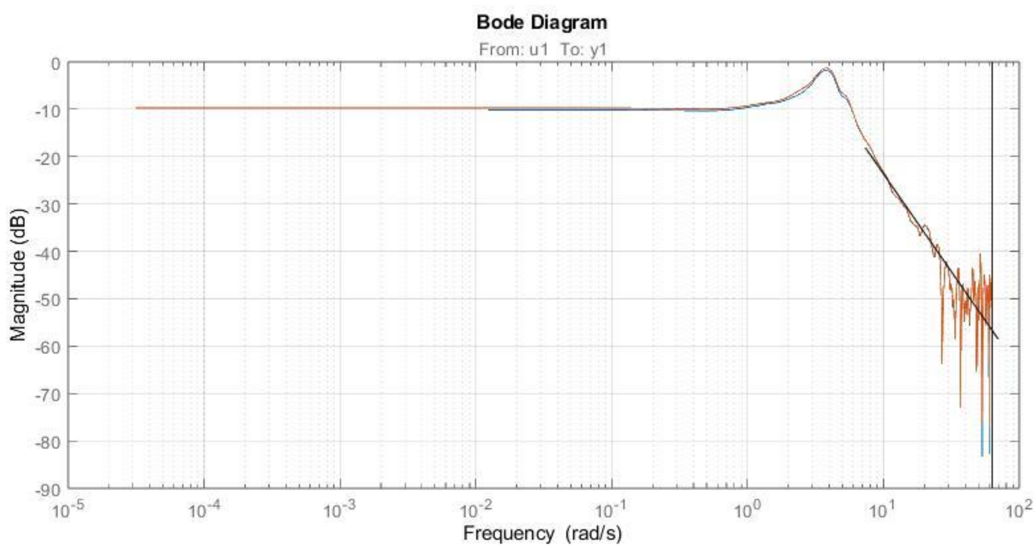

Figure 3: SPA and ETFE

The bode shows an initial upward slope - indicative of a zero
Then a -40dB/dec slope which following a zero would mean that there is a presence of 3 poles

From the bode plot we can conclude that this is a third order system

nb = 1, nf = 3

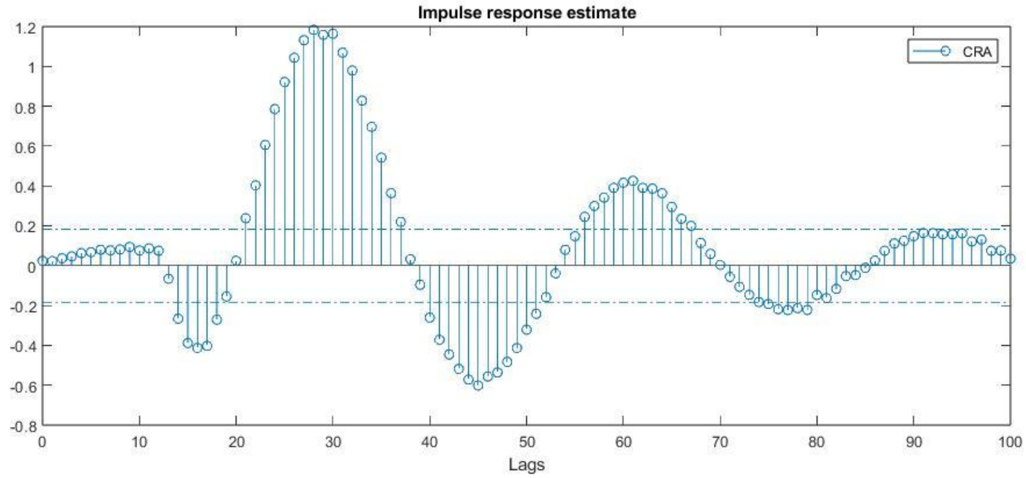

Figure 4: CRA

Looking at the CRA figure we can see there are 14 points within the confidence area but since one of the points seems to be part of the dip we can say there are 13 lags in this system. Since there are a significant amount of lags we must create a new data set to validate against.

nk = 13

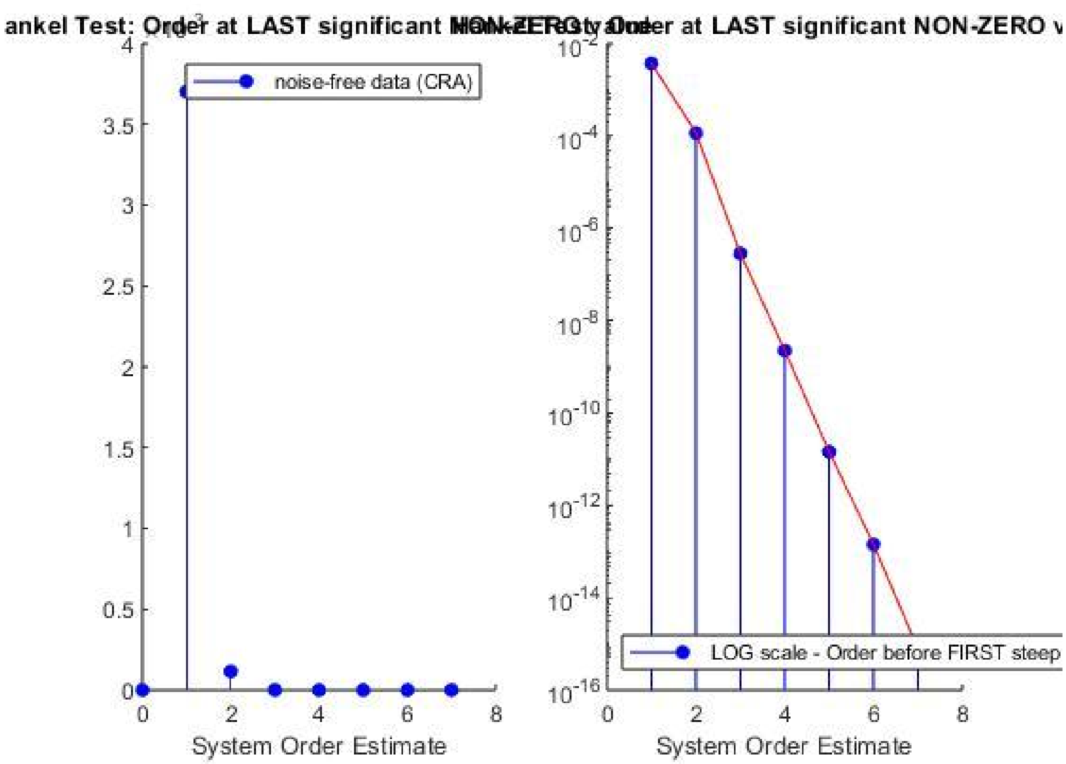

Figure 5: Hankel Test

Hankel test shows that this is a second order system. But since we've seen signs of a third order system we will work with that.
For the oe model we will use [3, 3, 13]

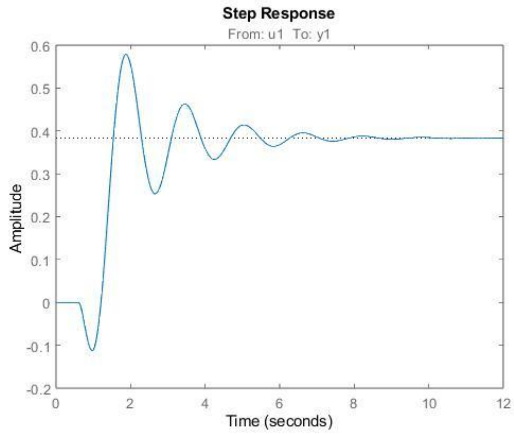

Figure 6a: OE Step Response

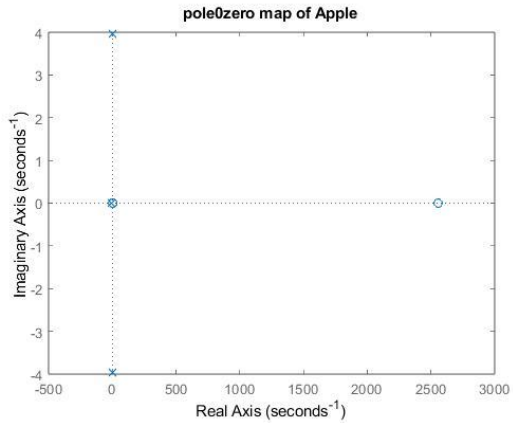

Figure 6b: OE Pole-Zero Map

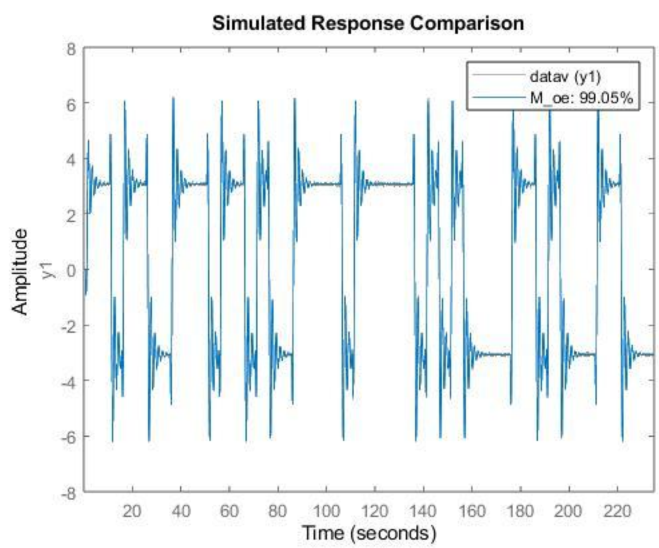

Figure 6c: OE Comparison

y(t) = [B(z) / F(z)] u(t) + e(t)
B(z) = -0.006674 z^-13 + 0.001647 z^-14 + 0.007097 z^-15
F(z) = 1 - 2.763 z^-1 + 2.579 z^-2 - 0.8102 z^-3

y(t) = [B(s) / F(s)] u(t) + e(t)
B(s) = 0.002403 s^2 - 6.149 s + 18.45
F(s) = s^3 + 4.209 s^2 + 19.61 s + 48.12

exp(-0.6*s) * $\frac{0.0024028\,(s-2556)(s-3.003)}{(s+3.008)(s^2 + 1.201s + 16)}$

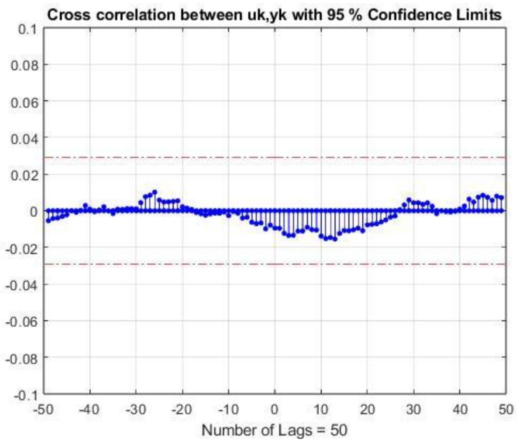

Figure 7a: Cross Correlation

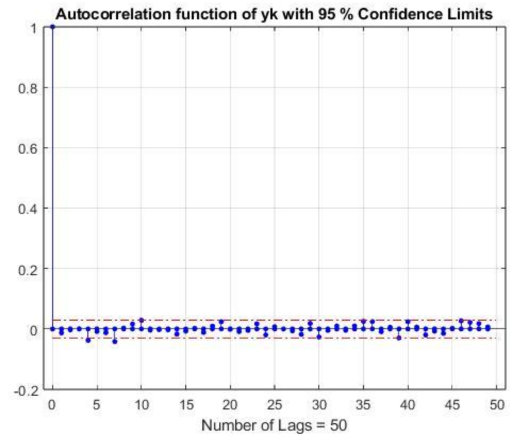

Figure 7b: Auto correlation

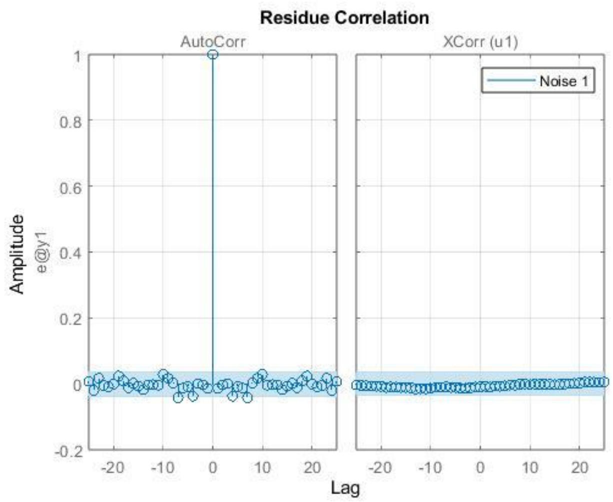

Figure 7c: Residue Correlation

S1 = 20.5218
X1 - 30.4355

From the validation data and graphs in figures 7a,b,c above we see that the model is accepted as it passes the validation tests.

From the function described earlier:

exp(-0.6*s) * $\frac{0.0024028\,(s-2556)(s-3.003)}{(s+3.008)(s^2+1.201s+16)}$

We can adjust it further by removing the zero situated at x = 2556 and also adjusting the dc gain to compensate, by doing so we end up with:

exp(-0.6*s) * $\frac{-6.1429(s-3.003)}{(s+3.008)(s^2+1.2s+16)}$

By running the step response of the new equation:

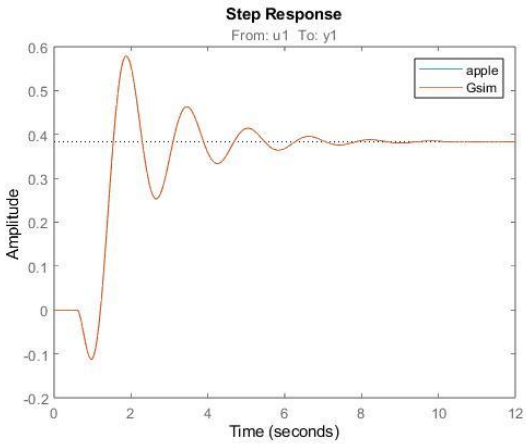

Figure 8a: new model step

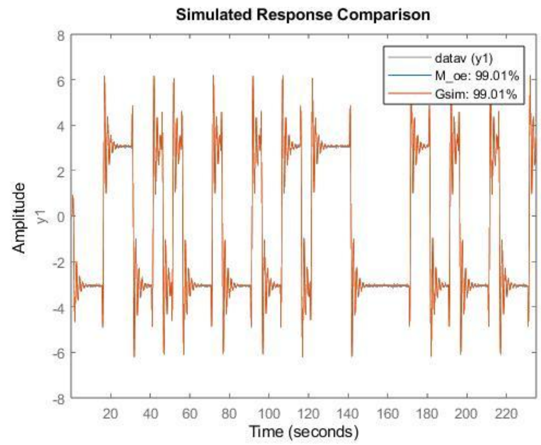

Figure 8b: new model comparison

The new adjusted model follows the OE model exactly as seen in figure 8a, and when compared to the validation data set in figure 8b they both show a 99.01% correlation.

PROCESS PEM

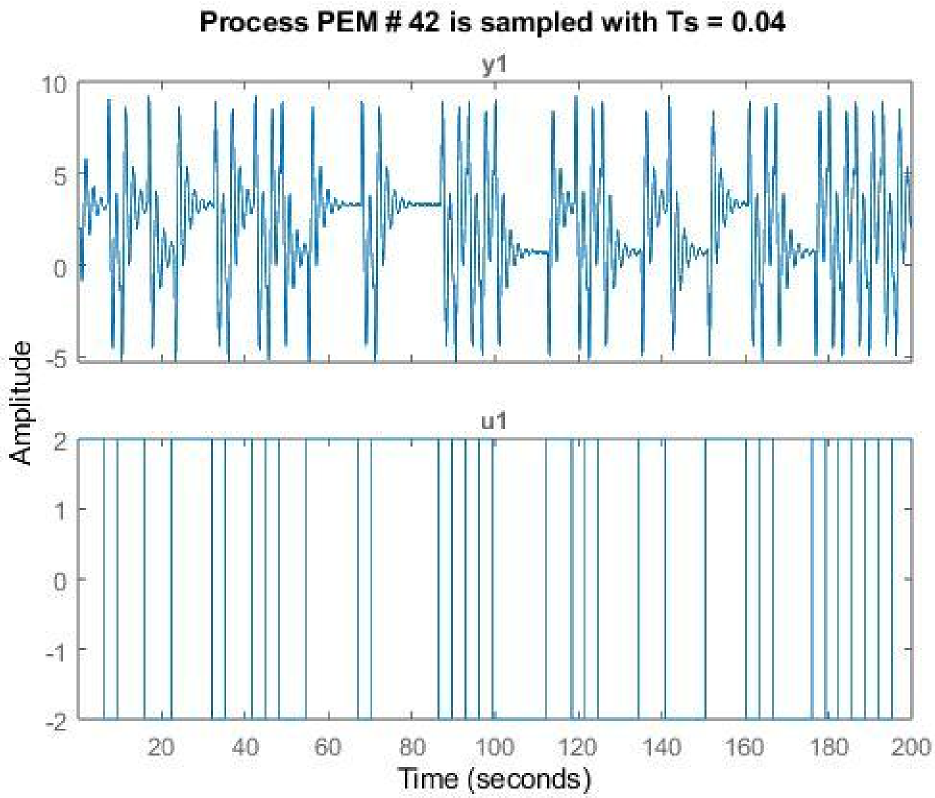

Figure 9: noise sample process 2

The DC offset is found and removed as can be seen in line 14 of the PEM code

```
ans =

    2.0506
```

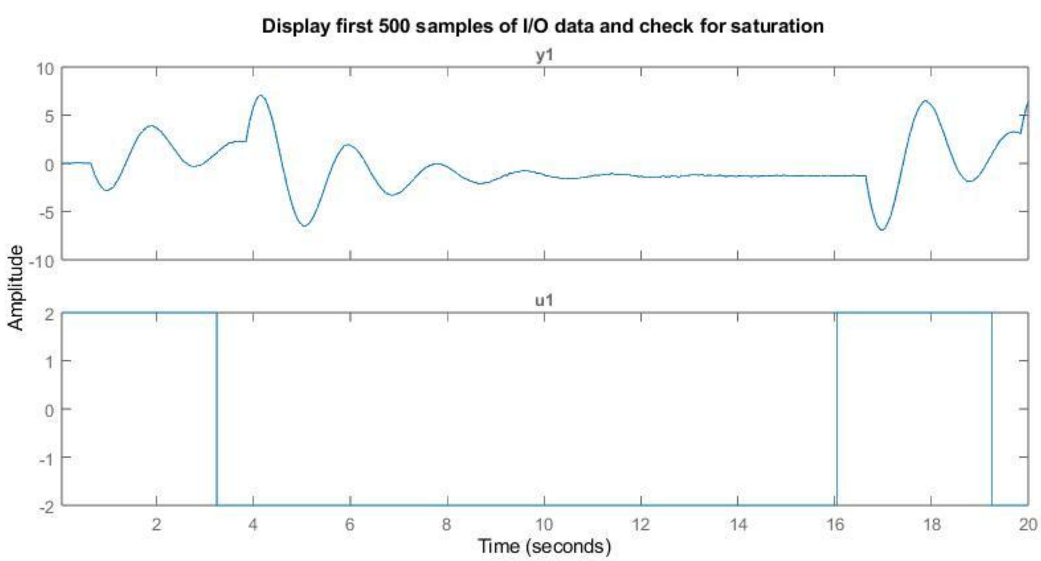

Figure 10: first 500 samples (K = 80)

From the slow switch rate we can see that as the signal switches there is a dip which is indicative of a right hand side zero. We can also see the signal oscillate so there is a presence of two complex conjugate poles. Maybe a presence of a third zero as it takes a bit of time for the signal to settle.

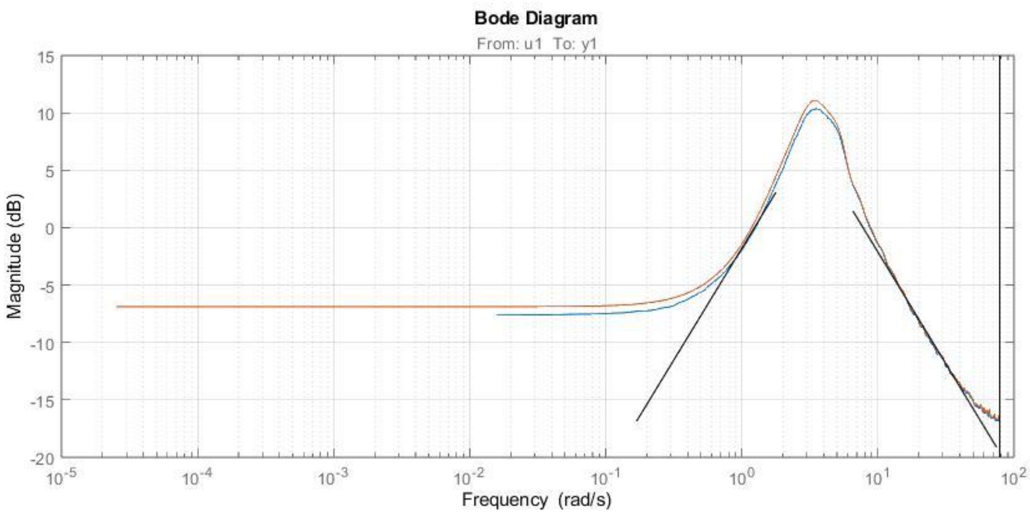

Figure 11: SPA and ETFE

From the bode plot we see a positive slope of +20dB/dec which is indicative of the presence of a zero. We then see a slope of -20dB/dec which normally is indicative of a single pole, but following a zero means that there are two poles.

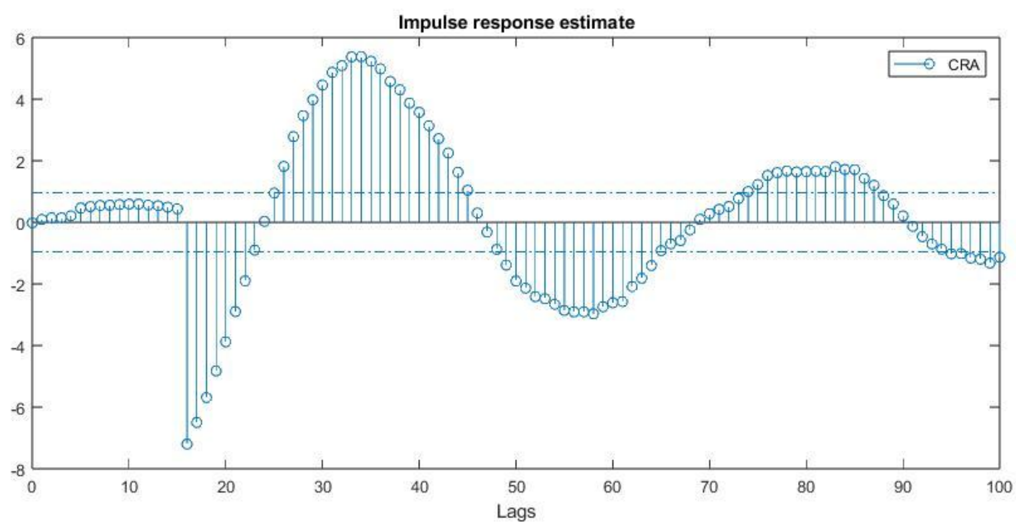

Figure 11: CRA

Looking at the cra model, we see that there are 16 lags so we know nk = 16

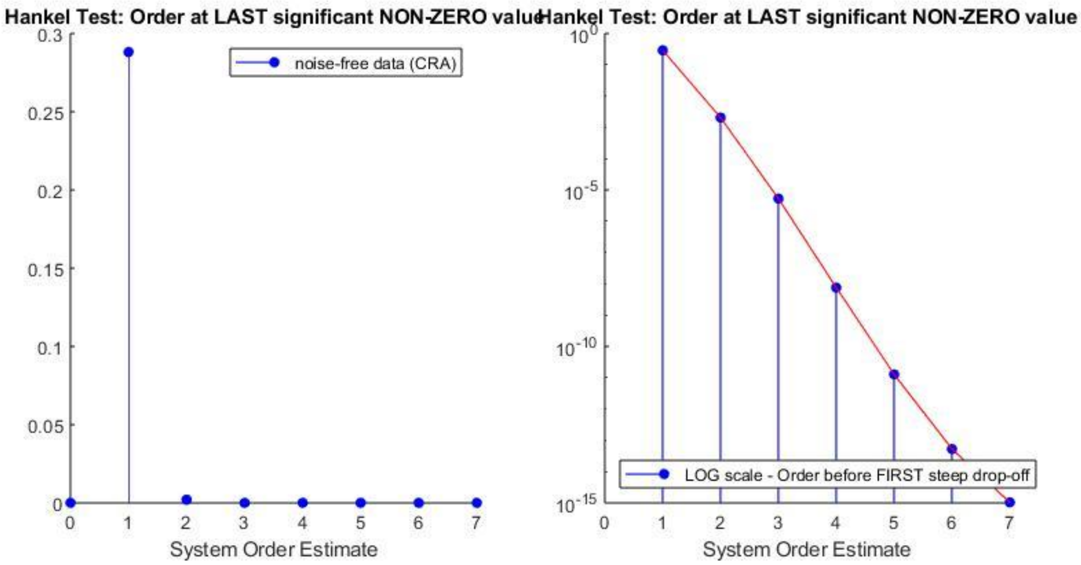

Figure 12: Hankel Test

From the hankel test we see that it is a second order system as the slope after 2 is the largest Therefore we can say the system is [2 2 16] so now we can run the OE model of the system

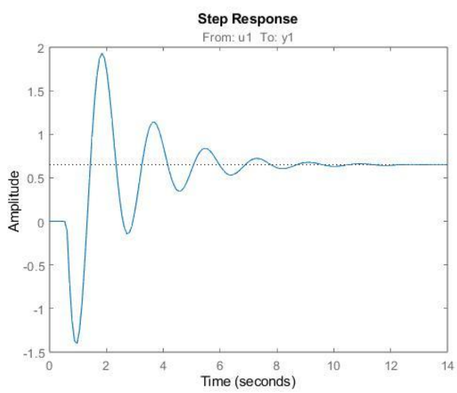

Figure 13a: OE Step Response

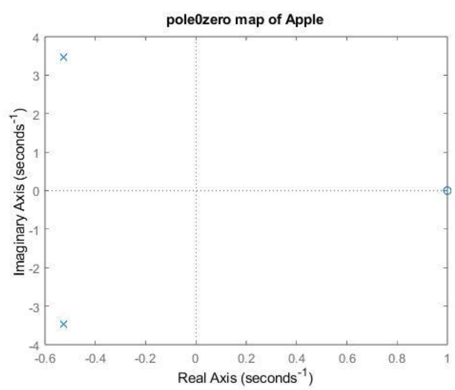

Figure 13b: OE Pole-Zero Map

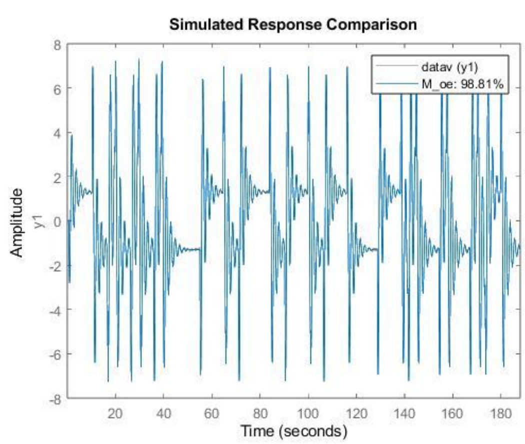

Figure 13c: OE Comparison

From the oe model, we can see that in figure 13c that the OE model follows the validation set very well, having a 98% correlation

y(t) = [B(z) / F(z)] u(t) + e(t)
B(z) = -0.3047 z^-16 + 0.3172 z^-17
F(z) = 1 - 1.94 z^-1 + 0.9589 z^-2
Fit to estimation Data: 98.83%
FPE: 0.001027, MSE: 0.001025

y(t) = [B(s) / F(s)] u(t) + e(t)
B(s) = -7.966s + 7.962
F(s) = s^2 + 1.05 s + 12.25

$$\exp(-0.6*s) * \frac{-7.9657\ (s-1)}{(s^2 + 1.05s + 12.25)}$$

Now we can run the validation of the oe model on the validation set
We get a failing chi-square test of S1 = 160.7904, X1 = 30.4355

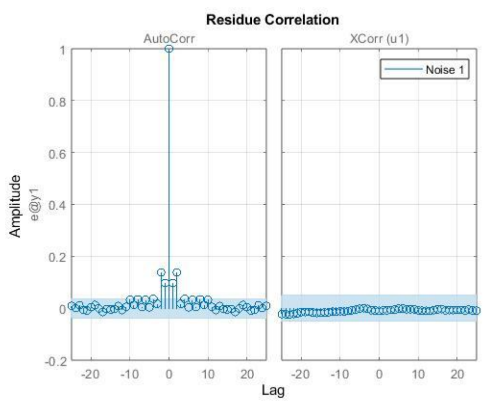

Figure 14a: OE Residue Correlation

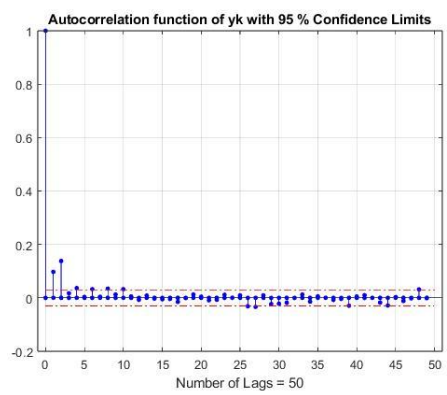

Figure 14b: OE Autocorrelation

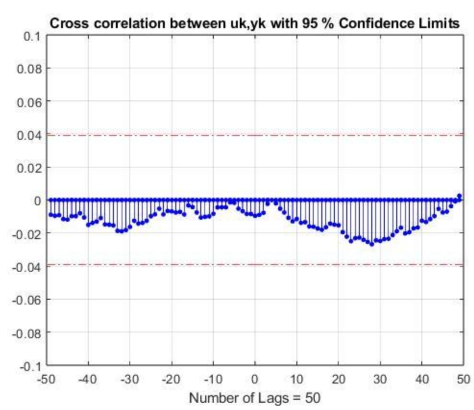

Figure 14c: OE Cross Correlation

Obviously the cross correlation will pass but as can be seen in figure 14b the autocorrelation function fails, so now we can check the partial autocorr function to determine what should be done next.

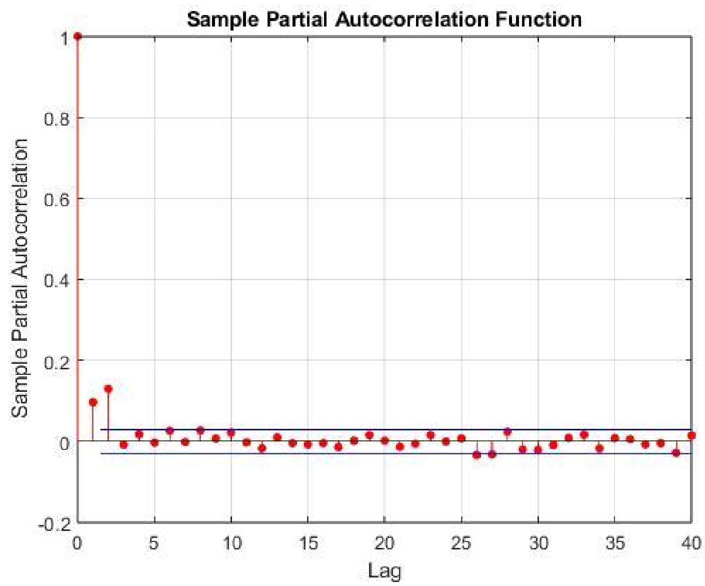

Figure 15: OE Partial Autocorrelation

From the partial autocorrelation function we can see that there are two positive points which leads me to believe that we should use a BJ model [nb, nc, nd, nf, nk]
We can use the values used in the OE model for nb, nf, and nk, which are 2, 2 and 16 respectively. For the values of nc and nd we know from the parcorr function that we have to use an AR filter, so nc = 0 and from figure 15, nd = 2

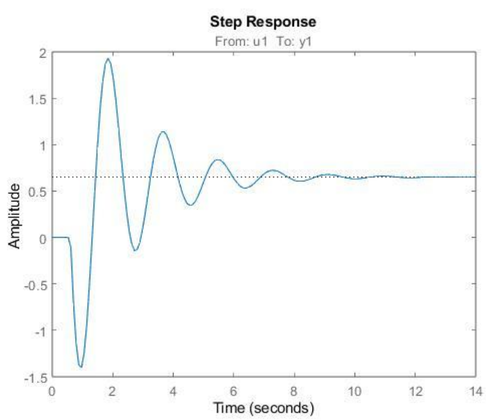

Figure 16a: BJ Step Response

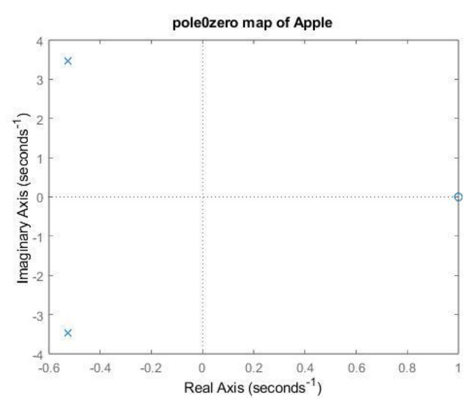

Figure 16b: BJ Pole-Zero Map

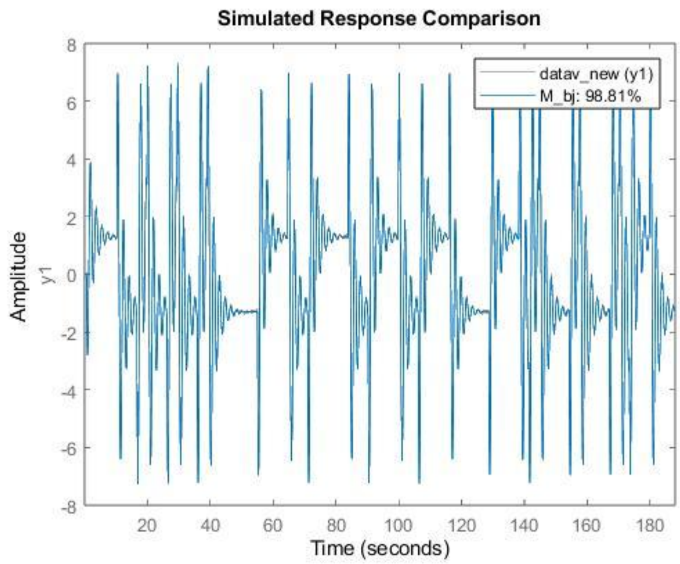

Figure 16c: BJ comparison

Discrete Model:
y(t) = [B(z) / F(z)] u(t) + [1 / D(z)] e(t)

B(z) = -0.3047 z^-16 + 0.3172 z^-17
D(z) = 1 - 0.0822 z^-1 - 0.1495 z^-2
F(z) = 1 - 1.94 z^-1 + 0.9589 z^-2

Continuous Model:
y(t) = [B(s)/F(s)]u(t) + [C(s)/D(s)]e(t)
B(s) = -7.966 s + 7.962
C(s) = s^3 + 79.67 s^2 + 7589 s + 1.886e05
D(s) = s^3 + 73.92 s^2 + 7980 s + 1.449e05
F(s) = s^2 + 1.05 s + 12.25

We can now run the validation on the new BJ model

S1 = 17.0149 X1 = 30.4355

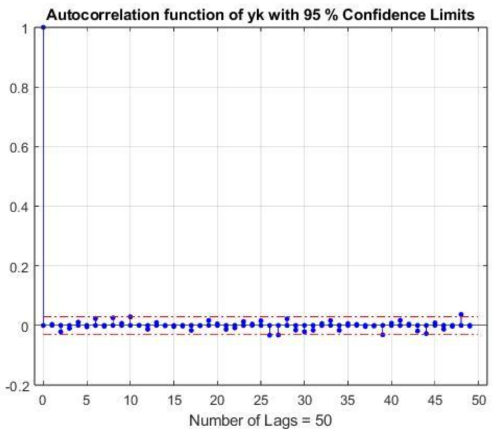

Figure 17a: BJ autocorrelation

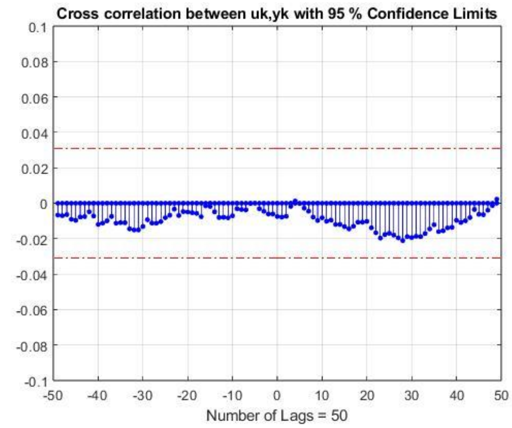

Figure 17b: BJ cross Correlation

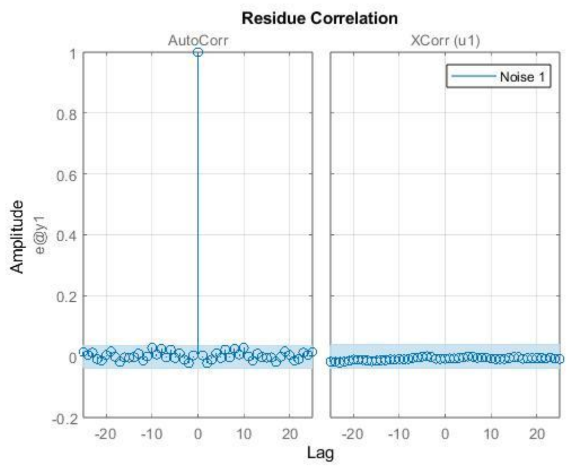

Figure 17c: BJ residue correlation

From the validation sequence we can see that everything passes which confirms that the model for the signal is correct!

From the continuous model we know the equation of the system is

exp(-0.6*s) * $\frac{-7.9657\,(s-1)}{(s^2+1.05s+12.25)}$

And the noise filter is

$\frac{1}{(z-0.4299)(z+0.3477)}$

Since there are no simplifications that can be made to the continuous model, we can just look at the comparisons in figures 13c and 16c.