Control Systems – Team Analysis Report

LTTS GLOBAL ENGINEERING ACADEMY

L&T Technology Services

Version Number: 1.4
Team Number: 2
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Module: Control Systems

Document History

Ver. Rel. No.	Release Date	Prepared. By	Reviewed By	Approved By	Remarks/Revision Details
1.0	14/04/2021	Pushkar Antony	Rama Subba Reddy	Dr.Prithvi Sekhar	Addition of System without controller and lag lead controller
1.1	14/04/2021	Rama Subba Reddy	Pushkar Antony	Dr.Prithvi Sekhar	Addition of Stanley Controller
1.2	14/04/2021	Amit Kumar	Pushkar Antony	Dr.Prithvi Sekhar	Addition of On-Off Controller
1.3	14/04/2021	Amiya Panda	Amiya Panda	Dr.Prithvi Sekhar	Addition of PID Controller
1.4	15/04/2021	Pushkar Antony	Amit Kumar	Dr.Prithvi Sekhar	Final checks and Alignments

Contents

1) Lateral Dynamics Control- Open Loop without controller

Plant Description

Vertical dynamics, or ride dynamics, basically refers to the vertical response of the vehicle to road disturbances. Longitudinal dynamics involves the straight-line acceleration and braking of the vehicle. Lateral dynamics is concerned with the vehicle's turning behavior.

```
%-------------------------------------------------------------------------
% Equation1: Ybeta*Beta + Yr*r + Ydelta*delta + Fya + m*g*theta =
%               m*V*d(Beta)/dt + m*V*r
% Equation2: Nbeta*Beta + Nr*r + Ndelta*delta - (c-a)*Fya = Izz*dr/dt
%-------------------------------------------------------------------------
% Variables:
%   Ybeta = Cf+Cr --> Damping - in - Sideslip
%   Yr = (aCf - bCr)/V --> Lateral Force/Yaw Coupling
%   Ydelta = ~Cf --> Control Force
%   Nbeta = a*Cf - b*Cr --> Directional Stability
%   Nr = (a^2)Cf + (b^2)Cr --> Yaw Damping
%   Ndelta = -aCf --> Control Moment
%-------------------------------------------------------------------------
% Values: Ydelta = 2461; m = 1775; V = 100; Nr = -67.06; Ndelta = 2803.079;
%         Yr = 0.46384; Izz = 1960; Ybeta = -4772; Nbeta = 46.38; V1=50;
%         Yr1 = 0.92768; Nr1 = -134.12;
```

Open Loop Control

System with Velocity of 100km/hr:-

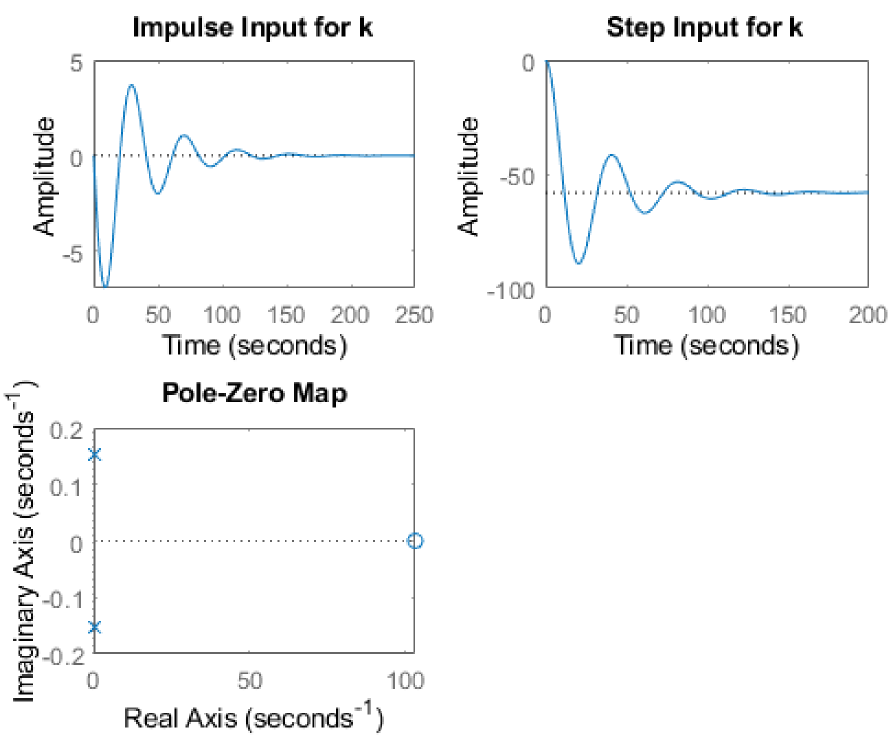
```
clc;
Ydelta = 2461;
m = 1775;
V = 100;
Nr = -67.06;
Ndelta = 2803.079;
Yr = 0.46384;
Izz = 1960;
Ybeta = -4772;
Nbeta = 46.38;
sys = tf([((Ydelta)/(m*V)),-((Nr*Ydelta)+(Ndelta*((m*V)-Yr)))/...
    (Izz*m*V)],[1,(-(Nr/Izz)-(Ybeta/(m*V))),((Nr*Ybeta)+...
    (Nbeta*((m*V)-Yr)))/(Izz*m*V)])
figure(1);
subplot(2,2,1);
impulse(sys);
title('Impulse Input for k');
subplot(2,2,2);
step(sys);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([((Ydelta)/(m*V)),-((Nr*Ydelta)+(Ndelta*((m*V)-Yr)))/...
```

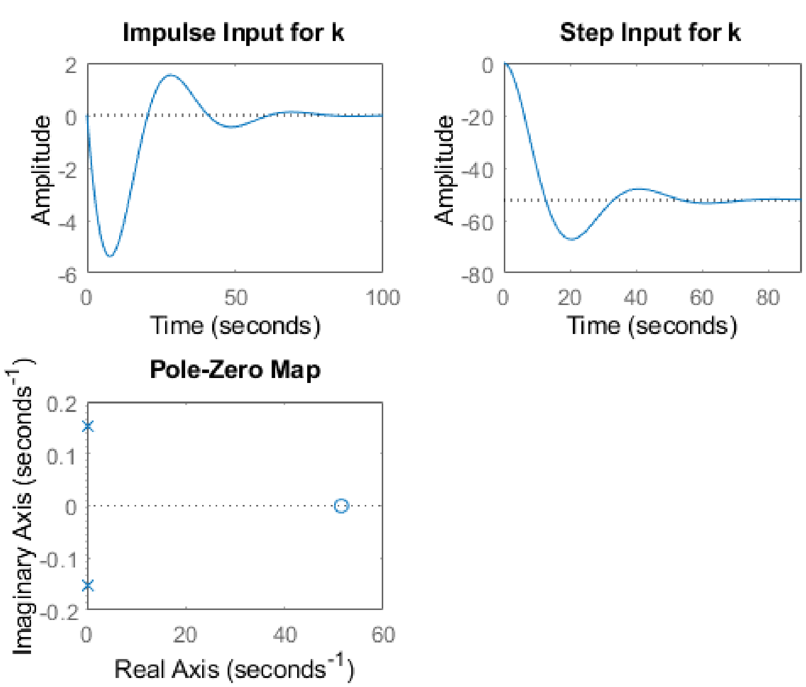
```
    (Izz*m*V)],[1,(-(Nr/Izz)-(Ybeta/(m*V))),((Nr*Ybeta)+...
    (Nbeta*((m*V)-Yr)))/(Izz*m*V)])
pzmap(sys)
hold on;
S = stepinfo(sys)
%-------------------------------------------------------------------------
% System with velocity of 50km/hr:-
V1 = 50;
Yr1 = 0.92768;
Nr1 = -134.12;
sys = tf([((Ydelta)/(m*V1)),-((Nr1*Ydelta)+(Ndelta*((m*V1)-Yr1)))/...
    (Izz*m*V1)],[1,(-(Nr1/Izz)-(Ybeta/(m*V1))),((Nr1*Ybeta)+...
    (Nbeta*((m*V1)-Yr1)))/(Izz*m*V1)])
figure(2);
subplot(2,2,1);
impulse(sys);
title('Impulse Input for k');
subplot(2,2,2);
step(sys);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([((Ydelta)/(m*V1)),-((Nr*Ydelta)+(Ndelta*((m*V1)-Yr)))/...
    (Izz*m*V1)],[1,(-(Nr/Izz)-(Ybeta/(m*V1))),((Nr*Ybeta)+...
    (Nbeta*((m*V1)-Yr)))/(Izz*m*V1)])
pzmap(sys)
hold on;
S = stepinfo(sys)
```

```
sys =

      0.01386 s - 1.43
  ------------------------
  s^2 + 0.0611 s + 0.02458

Continuous-time transfer function.


z =

  103.1148


p =

  -0.0305 + 0.1538i
  -0.0305 - 0.1538i


k =
```

```
    0.0139


S =

  struct with fields:

        RiseTime: 7.6539
    SettlingTime: 126.2645
     SettlingMin: -89.1467
     SettlingMax: -41.4693
       Overshoot: 53.2876
      Undershoot: 0
            Peak: 89.1467
        PeakTime: 21.1043


sys =

     0.02773 s - 1.428
  ------------------------
  s^2 + 0.1222 s + 0.02734

Continuous-time transfer function.


z =

   51.5401


p =

  -0.0440 + 0.1535i
  -0.0440 - 0.1535i


k =

    0.0277


S =

  struct with fields:

        RiseTime: 8.5851
    SettlingTime: 64.9768
     SettlingMin: -67.2096
     SettlingMax: -47.9433
       Overshoot: 28.6676
```

```
Undershoot: 0
      Peak: 67.2096
  PeakTime: 20.3506
```





Math Analysis:

Independent: Time(t) Dependent: Vehicle side slip angle (Beta), Front Steer angle (Delta) Constant: Damping in Sideslip (Ybeta), Lateral Force/Yaw Coupling(Yr), Control Force (Ydelta), Directional Stability (Nbeta), Yaw Damping (Nr), Control Moment(Ndelta).

```
%--------------------------------------------------------------------------
% Roots:
%   System with V = 100km/hr:
%       1) Zero = 103.1148
%       2) Pole = -0.0305+0.1538i, -0.0305-0.1538i
%   System with V = 50m/s:
%       1) Zero = 51.5401
%       2) Pole = -0.0440+0.1535i, -0.0440-0.1535i
%--------------------------------------------------------------------------
% Time Response Analysis:
%   1)System with V = 100km/hr:
%        RiseTime: 7.6539
%        SettlingTime: 126.2645
%        SettlingMin: -89.1467
%        SettlingMax: -41.4693
%        Overshoot: 53.2876
%        Undershoot: 0
%        Peak: 89.1467
%        PeakTime: 21.1043
%   2)System with V = 50m/s:
%        RiseTime: 8.5851
%        SettlingTime: 64.9768
%        SettlingMin: -67.2096
%        SettlingMax: -47.9433
%        Overshoot: 28.6676
%        Undershoot: 0
%        Peak: 67.2096
%        PeakTime: 20.3506
```

Comparison Analysis:

1) The order of the system is 2nd order. When the Velocity of the system is changed, we can get to see some changes.
2) The system with higher velocity has the highest rise time and reaches the peak the fastest.
3) That is, the system with the higher velocity is the fastest.
4) The system with lesser velocity has the lower settling time, denoting that it is more accurate.
5) The system with lesser velocity has lower overshoot percentages.
6) The system with higher velocity has the highest peak.

2) Lateral Dynamics Control- Closed Loop with Lag-Lead Controller

Plant Description

Introduction: Vertical dynamics, or ride dynamics, basically refers to the vertical response of the vehicle to road disturbances. Longitudinal dynamics involves the straight-line acceleration and braking of the vehicle. Lateral dynamics is concerned with the vehicle's turning behavior.

```
%-------------------------------------------------------------------------
% Equation1: Ybeta*Beta + Yr*r + Ydelta*delta + Fya + m*g*theta =
%              m*V*d(Beta)/dt + m*V*r
% Equation2: Nbeta*Beta + Nr*r + Ndelta*delta - (c-a)*Fya = Izz*dr/dt
%-------------------------------------------------------------------------
% Variables:
%   Ybeta = Cf+Cr --> Damping - in - Sideslip
%   Yr = (aCf - bCr)/V --> Lateral Force/Yaw Coupling
%   Ydelta = ~Cf --> Control Force
%   Nbeta = a*Cf - b*Cr --> Directional Stability
%   Nr = (a^2)Cf + (b^2)Cr --> Yaw Damping
%   Ndelta = -aCf --> Control Moment
%-------------------------------------------------------------------------
% Values: Ydelta = 2461; m = 1775; V = 100; Nr = -67.06; Ndelta = 2803.079;
%         Yr = 0.46384; Izz = 1960; Ybeta = -4772; Nbeta = 46.38; V1=50;
%         Yr1 = 0.92768; Nr1 = -134.12;
```

Closed Loop (negative feedback) without Controller :-

```
clc;
transfn = tf([0.01386,-1.43],[1,0.07496,-1.40542])
figure(1);
subplot(2,2,1);
impulse(transfn);
title('Impulse Input without controller');
subplot(2,2,2);
step(transfn);
title('Step Input without controller');
subplot(2,2,3);
[z,p,k]= tf2zp([0.01386,-1.43],[1,0.07496,-1.40542])
pzmap(transfn)
title('pz map without controller');
subplot(2,2,4);
bode(transfn)
[Gm,Pm,Wcg,Wcp] = margin(lag)
title('Bode plot without controller');
hold on;
S = stepinfo(transfn)
```

```
transfn =
```

```
     0.01386 s - 1.43
  -----------------------
  s^2 + 0.07496 s - 1.405

Continuous-time transfer function.


z =

  103.1746


p =

   -1.2236
    1.1486


k =

    0.0139


Unrecognized function or variable 'lag'.

Error in ClosedLoop_LagLead_Controller (line 46)
[Gm,Pm,Wcg,Wcp] = margin(lag)
```

Closed Loop (negative feedback) with Controller:-

System with lag-controller/compensator:-

```
sys = tf([0.01386,-1.43],[1,0.0611,0.02458])
sys2 = tf([1,0.05],[1,0.95])
lag = sys*sys2
figure(2);
subplot(2,2,1);
impulse(lag);
title('Impulse Input for k');
subplot(2,2,2);
step(lag);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([0.01386,-1.42,-0.0715],[1,1.011,0.08263,0.02335])
pzmap(lag)
subplot(2,2,4);
bode(lag)
[Gm,Pm,Wcg,Wcp] = margin(lag)
hold on;
S = stepinfo(lag)
%---------------------------------------------------------------------------
% System with lead-controller/compensator:-
```

```
sys = tf([0.01386,-1.43],[1,0.0611,0.02458])
sys2 = tf([1,0.95],[1,0.05])
lag = sys*sys2
figure(3);
subplot(2,2,1);
impulse(lag);
title('Impulse Input for k');
subplot(2,2,2);
step(lag);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([0.01386,-1.417,-1.358],[1,0.1111,0.02763,0.001229])
pzmap(lag)
subplot(2,2,4);
bode(lag)
[Gm,Pm,Wcg,Wcp] = margin(lag)
hold on;
S = stepinfo(lag)
%-------------------------------------------------------------------------
% System with lag-lead controller/compensator:-
sys = tf([0.01386,-1.43],[1,0.0611,0.02458])
sys2 = tf([1,0.95],[1,0.05])
sys3 = tf([1,0.25],[1,0.85])
lag = sys*sys2*sys3
figure(4);
subplot(2,2,1);
impulse(lag);
title('Impulse Input for k');
subplot(2,2,2);
step(lag);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([0.01386,-1.413,-1.713,-0.3396],[1,0.9611,0.1221,0.02472,0.001045])
pzmap(lag)
subplot(2,2,4);
bode(lag)
[Gm,Pm,Wcg,Wcp] = margin(lag)
hold on;
S = stepinfo(lag)
```

Math Analysis:

Independent: Time(t) Dependent: Vehicle side slip angle (Beta), Front Steer angle (Delta) Constant: Damping in Sideslip (Ybeta), Lateral Force/Yaw Coupling(Yr), Control Force (Ydelta), Directional Stability (Nbeta), Yaw Damping (Nr), Control Moment(Ndelta).

```
%-------------------------------------------------------------------------
% Roots:
%   1) Closed Loop without controller:
%       1) Zero = 103.1148
```

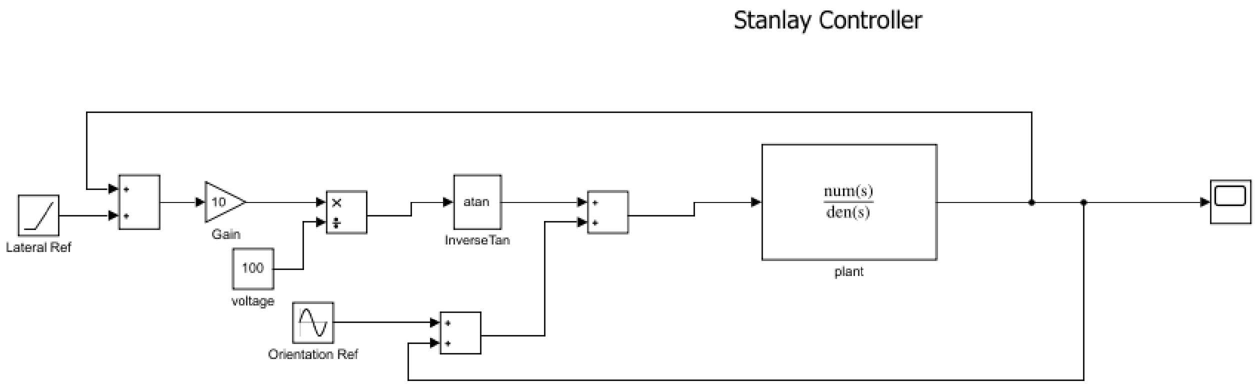
```
%       2) Poles = -1.2236, 1.1486
%   2) Closed Loop with Lag Controller:
%       1) Zeros = 102.5034, -0.0503
%       2) Poles = -0.9499 + 0.0000i, -0.0306 + 0.1538i, -0.0306 - 0.1538i
%   3) Closed Loop with Lead Controller:
%       1) Zeros = 103.186, -0.9495
%       2) Poles = -0.0305 + 0.1538i, -0.0305 - 0.1538i, -0.0500 + 0.0000i
%   4) Closed Loop with Lag-lead Controller:
%       1) Zeros =  103.1486, -0.9506, -0.2499
%       2) Poles =  {-0.8500 + 0.0000i, -0.0306 + 0.1538i,
%                    -0.0306 - 0.1538i, -0.0500 + 0.0000i}
%---------------------------------------------------------------------------
% Time Response Analysis:
%   1) Closed Loop without Controller:
%        RiseTime: NaN
%        SettlingTime: Nan
%        SettlingMin: NaN
%        SettlingMax: NaN
%        Overshoot: NaN
%        Undershoot: Nan
%        Peak: Inf
%        PeakTime: Inf
%   2) Closed Loop with Lag Controller:
%        RiseTime: 2.1472
%        SettlingTime: 138.4252
%        SettlingMin: -9.7437
%        SettlingMax: 0.5176
%        Overshoot: 218.2161
%        Undershoot: 16.9030
%        Peak: 9.7437
%        PeakTime: 12.1189
%   3) Closed Loop with Lead Controller:
%        RiseTime: 20.8356
%        SettlingTime: 96.3507
%        SettlingMin: -1.1153e+03
%        SettlingMax: -929.8799
%        Overshoot: 0.8998
%        Undershoot: 0
%        Peak: 1.1153e+03
%        PeakTime: 70.8488
%   4) Closed Loop with Lag-Lead Controller:
%        RiseTime: 17.9517
%        SettlingTime: 94.2687
%        SettlingMin: -330.5149
%        SettlingMax: -270.3005
%        Overshoot: 1.6628
%        Undershoot: 0
%        Peak: 330.5149
%        PeakTime: 67.8339
%---------------------------------------------------------------------------
% Frequency Response Analysis:
```

```
%   1) Closed Loop without controller:
%        Gm = 0.0031
%        Pm = 170.0613
%   2) Closed Loop with Lag Controller:
%        Gm = 0.3266
%        Pm = -137.446
%   3) Closed Loop with lead Controller:
%        Gm = 0.00090467
%        Pm = 148.5963
%   4) Closed Loop with Lag-Lead Controller:
%        Gm = 0.0031
%        Pm = 170.0613
```

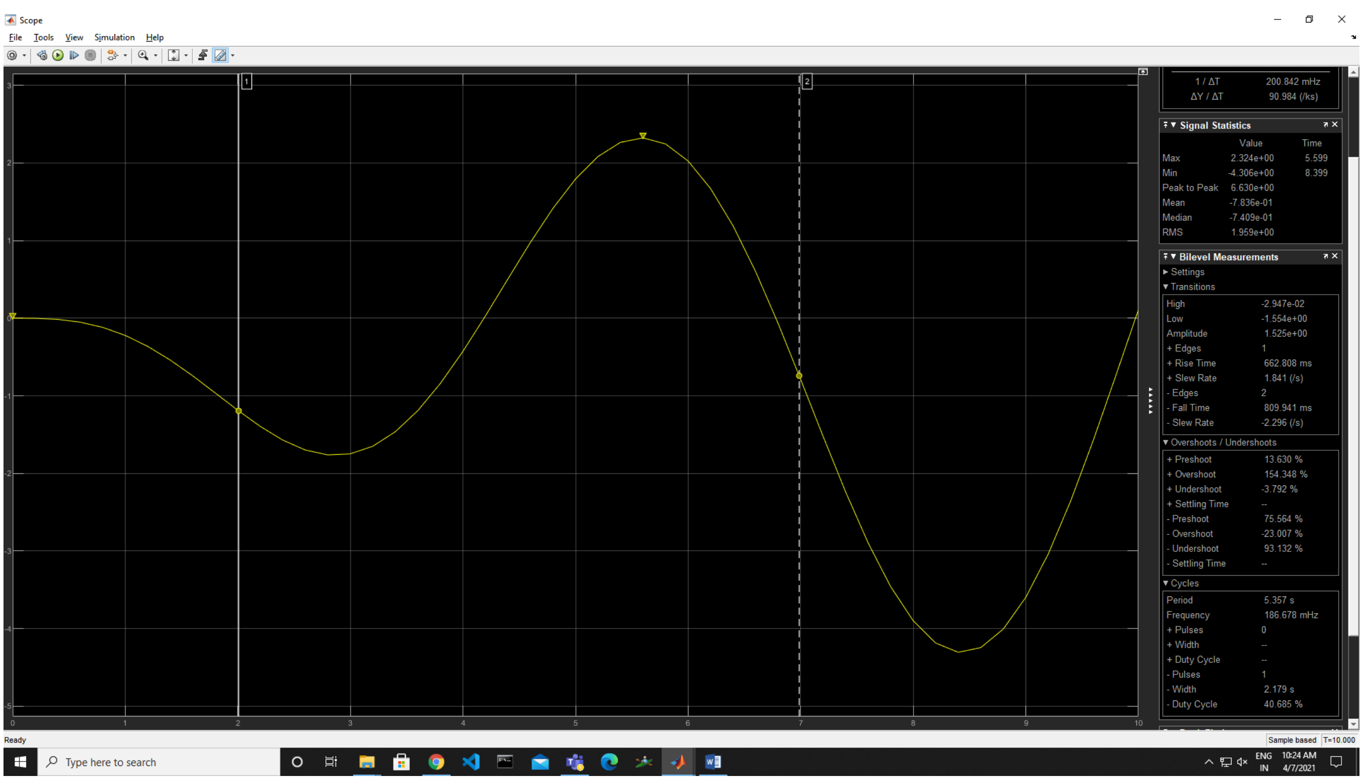
Comparison Analysis:

1) On giving -ve feedback without any controller, the system goes unstable.
2) On adding a controller/compensator the system goes from unstable to stable.
3) This happens because the controller adds poles/zeros making the system reach stability.
4) On adding a Lag Compensator, the rise time is least and settling time is highest comparing to the same system in other 2 controllers.
5) The Lag Controller, adds a zero and a pole to the system.
6) The lag Controller, has a +ve Gm proving its stability.
7) The lag controller also has the highest overshoot percentage.
8) On adding a Lead Compensator, the rise time and settling time is in-between comparing to the same system with other 2 controllers.
9) The Lead Controller, adds a zero and a pole to the system.
10) The lead Controller, has a +ve Gm proving its stability.
11) The lead controller provides the minimum overshoot percentage compared to the other 2 controllers.
12) On adding a Lag-Lead Compensator, the rise time is in-between and the settling time is lowest comparing to the same system in other controllers.
13) The Lag-Lead Controller, adds a zero and a pole to the system.
14) The Lag-Lead Controller, has a +ve Gm indicating its stability.
15) The Lag-Lead Controller provides the second best overshoot percentage compared to the other 2 controllers.
16) Overall, the lag-lead controller provides the best of all worlds and is the best controller to use.

3) Lateral Dynamics Control- Closed Loop with Stanley Controller



Anylasis
By using this controller the system response is fast because rise time so less



4) Lateral Dynamics Control- Closed Loop with On-Off Controller

```
%%Indiviual work
%Title:Control System -Lateral control -Second Order System
%Author:Amit Kumar
%Ps No: 99003775
%Date:14/04/2021
%Version 1.0

%%Transfer function for On_off or Bang-bang,or Two-Step Controller=H(s)=(K/T.s+1)*e^-r.s
%Variable description for Control Transfer Function
%where r for tau value k is gain and T is time constant
%The time constant is a measure of the capacitance of the system. The higher the time constant,
%the longer it takes for the system to react to changing inputs or disturbances.
%In most of the control systems with feedback loop, the system can not respond instantly
%to any disturbance and it takes time (delay) until the controller output has any effect on
%the measured (plant) output. This time delay is know as dead time.


%Implementations
%Ideal Equation
% Equation1 = m*V*(d(beta)/dt) + m*V_r = Y_beta * beta + Y_r * r + Y_delta * delta + F_ya + m*g*theta
% Equation2 = I_zz * ((dr)/(dt)) = N_beta *beta + N_r * r + N_delta *Delta - (c-a) * F_ya

%Roots:
%Roots for On-Off Controller is
%pole=-1/T;
%Zero=-1/r;

%   System with V = 100km/hr:
%       1) Zero = 103.1148
%       2) Pole = -0.0305+0.1538i, -0.0305-0.1538i

% System with V = 50km/hr:
%       1) Zero = 51.5401
%       2) Pole = -0.0440+0.1535i, -0.0440-0.1535i

%%Variable Description
%Variable Description
%m - Total Vehicle mass(kg)
%V - Magnitude of vehicle velocity (v)
%Y_delta - Control force derivative (Newton/rad)
%N_r - Yaw damping derivative (Newton-metre-s/rad)
%N_delta - Control Moment Derivative (Newton-metre/rad)
%r - Yaw velocity (rad/sec)
%Vr - Velocity of the rear tire (metre/sec)
%theta - Road side Slope (rad)
%g - Acceleration due to gravity (metre/s^2)
%c - Distance from front axle to aerodynamics side force (metre)
%a - Distance from mass center to front axle (metre)
%I_zz - Total vehicle yaw mass moment of inertia (kg-metre^2)
```

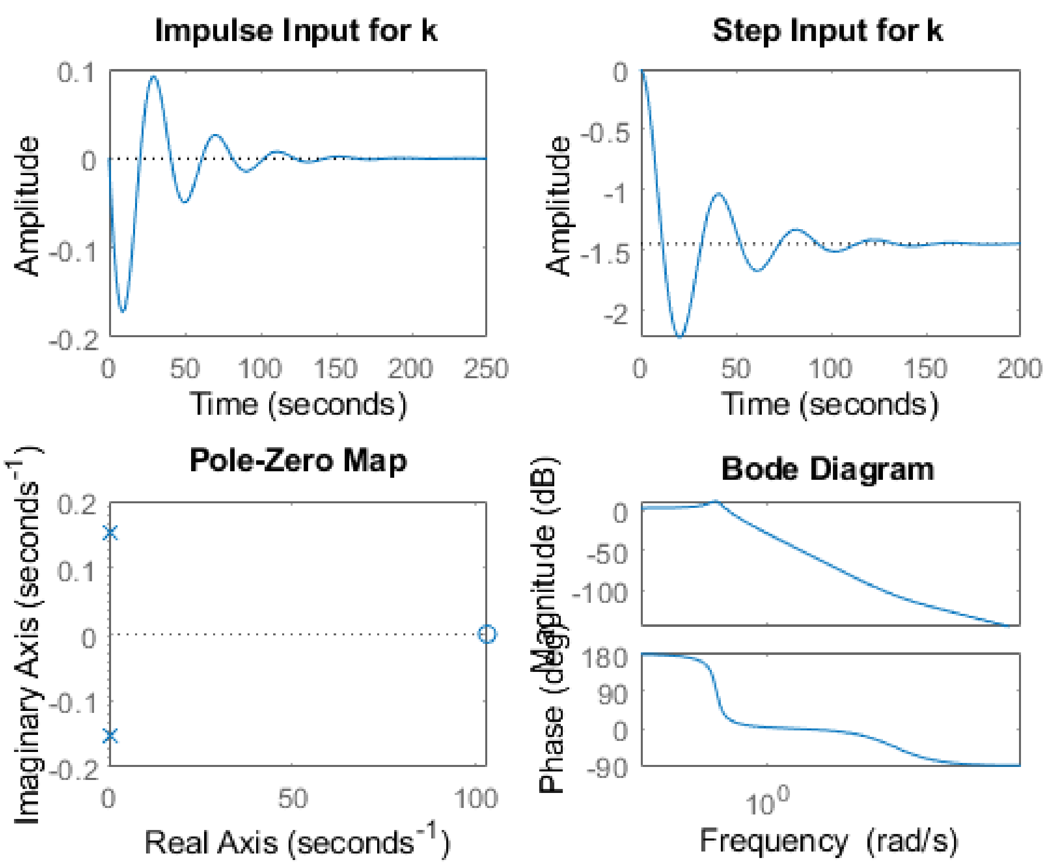
```
%Y-r - Lateral  force yaw coupling derivative (Newton-sec/rad)
%Y-beta - Damping in sideslip derivative (Newton/rad)
%beta - Vehicle side slip angle (rad)
%F_ya - Aerodynamics side force disturbance (Newton)
%N_beta - Directional stability derivative (Newton-metre/rad)

%%Math Analysis
%Independend: Time(t)
%Dependend:Vehicle side slip angle (Beta), Front Steer angle (Delta)
%Constant: Damping in Sideslip (Ybeta), Lateral Force/Yaw Coupling(Yr)
% Control Force (Ydelta), Directional Stability (Nbeta),
% Yaw Damping (Nr), Control Moment(Ndelta).

%%Plant Description
%Implementaions1
%This Plant is valid for any type of Vehicle
%The on-off control is the simplest form of a controller,
%which switches ON when the error is positive and switches OFF
%when the error is zero or negative. An on-off controller doesn't
%have intermediate states but only fully ON or fully OFF states.
%Due to the switching logic, an on-off controller is often called a bang-bang controller or a two-step
controller.
clc;
K=0.8;
T=0.60;
r=3;
M=K*exp(-r);
D=T+1;
Ydelta = 2461;
m = 1775;
V = 100;
Nr = -67.06;
Ndelta = 2803.079;
Yr = 0.46384;
Izz = 1960;
Ybeta = -4772;
Nbeta = 46.38;
sys1 = tf([((M*(Ydelta)/(m*V))),M*(-((Nr*Ydelta)+(Ndelta*((m*V)-Yr))))/...
    (Izz*m*V)],[1*D,(-(Nr/Izz)-(Ybeta/(m*V)))*D,((Nr*Ybeta)+...
    (Nbeta*((m*V)-Yr)))*D/(Izz*m*V)])
figure(1);
subplot(2,2,1);
impulse(sys1);
title('Impulse Input for k');
subplot(2,2,2);
step(sys1);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]=tf2zp([((M*(Ydelta)/(m*V))),M*(-((Nr*Ydelta)+(Ndelta*((m*V)-Yr))))/...
    (Izz*m*V)],[1*D,(-(Nr/Izz)-(Ybeta/(m*V)))*D,((Nr*Ybeta)+...
    (Nbeta*((m*V)-Yr)))*D/(Izz*m*V)])
```

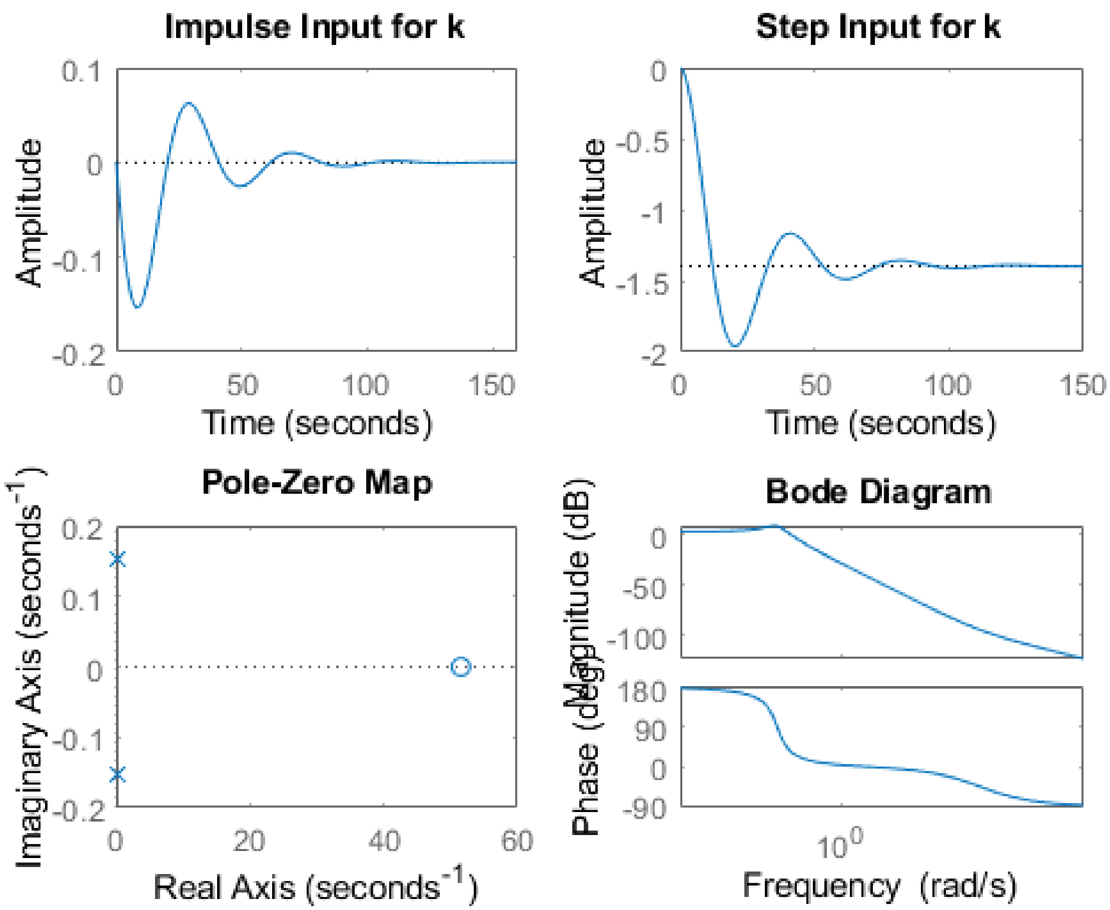
```
pzmap(sys1)
subplot(2,2,4);
bode(sys1)
hold on;
S = stepinfo(sys1)


V1 = 50;

sys2 = tf([((M*(Ydelta)/(m*V1))),M*(-((Nr*Ydelta)+(Ndelta*((m*V1)-Yr))))/...
    (Izz*m*V1)],[1*D,(-(Nr/Izz)-(Ybeta/(m*V1)))*D,((Nr*Ybeta)+...
    (Nbeta*((m*V1)-Yr)))*D/(Izz*m*V1)])

figure(2);
subplot(2,2,1);
impulse(sys2);
title('Impulse Input for k');
subplot(2,2,2);
step(sys2);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]=tf2zp([((M*(Ydelta)/(m*V1))),M*(-((Nr*Ydelta)+(Ndelta*((m*V1)-Yr))))/...
    (Izz*m*V1)],[1*D,(-(Nr/Izz)-(Ybeta/(m*V1)))*D,((Nr*Ybeta)+...
    (Nbeta*((m*V1)-Yr)))*D/(Izz*m*V1)])
pzmap(sys2)
subplot(2,2,4);
bode(sys2)
hold on;
S = stepinfo(sys2)

%%Analysis
%If we take Gain  value and Time constant value as zero we  get values of
%Rise time,Overshoot and peak time zero
%Not changes in UnderShoot
%If Velocity decrease Frequency Increases
%poles are complex  cojugate  at origin due to that system is
%marginal stable
%When velocity is more RiseTime is less and vise versa
%when Velocity of vehicle  is less Settling time is less
%Undershoot  is independent to vehicle speed
%If velocity is larger peaktime is more
```

```
sys1 =

      0.0005522 s - 0.05694
  -----------------------------
  1.6 s^2 + 0.09776 s + 0.03933

Continuous-time transfer function.
```

```
z =

  103.1148


p =

  -0.0305 + 0.1538i
  -0.0305 - 0.1538i


k =

   3.4514e-04


S =

  struct with fields:

        RiseTime: 7.6539
    SettlingTime: 126.2645
     SettlingMin: -2.2192
     SettlingMax: -1.0323
       Overshoot: 53.2876
      Undershoot: 0
            Peak: 2.2192
        PeakTime: 21.1043


sys2 =

     0.001104 s - 0.05692
  ---------------------------
  1.6 s^2 + 0.1408 s + 0.0408

Continuous-time transfer function.


z =

   51.5401


p =

  -0.0440 + 0.1535i
  -0.0440 - 0.1535i
```

```
k =

   6.9029e-04


S = 

  struct with fields:

        RiseTime: 8.1126
    SettlingTime: 86.9226
     SettlingMin: -1.9606
     SettlingMax: -1.1646
       Overshoot: 40.5421
      Undershoot: 0
            Peak: 1.9606
        PeakTime: 20.9366
```





5) Lateral Dynamics Control- Closed Loop with PID Controller

Control System - Lateral Control - Second Order System

```
%Name - Amiya Kumar Panda
%PS No - 99003783
%Date - 14/04/2021
%Version - 1.1.1
```

Plant Description

```
%This plant has been modeled for lateral contol of any vehicle.
%In this model two different values of velocity has been taken to get the better result analysis.

% Implementation
% Ideal Equation-
% Equation1 = m*V*(d(beta)/dt) + m*V_r = Y_beta * beta + Y_r * r + Y_delta * delta + F_ya +
```

```
m*g*theta
% Equation2 = I_zz * ((dr)/(dt)) = N_beta *beta + N_r * r + N_delta *Delta - (c-a) * F_ya

%Variable Description
%m - Total Vehicle mass(kg)
%V - Magnitude of vehicle velocity (v)
%Y_delta - Control force derivative (Newton/rad)
%N_r - Yaw damping derivative (Newton-metre-s/rad)
%N_delta - Control Moment Derivative (Newton-metre/rad)
%r - Yaw velocity (rad/sec)
%Vr - Velocity of the rear tire (metre/sec)
%theta - Road side Slope (rad)
%g - Acceleration due to gravity (metre/s^2)
%c - Distance from front axle to aerodynamics side force (metre)
%a - Distance from mass center to front axle (metre)
%I_zz - Total vehicle yaw mass moment of inertia (kg-metre^2)
%Y-r - Lateral  force yaw coupling derivative (Newton-sec/rad)
%Y-beta - Damping in sideslip derivative (Newton/rad)
%beta - Vehicle side slip angle (rad)
%F_ya - Aerodynamics side force disturbance (Newton)
%N_beta - Directional stability derivative (Newton-metre/rad)
```

Math analysis

```
% Independent: Time(t)
% Dependent: Vehicle side slip angle (Beta), Front Steer angle (Delta)
% Constant: Damping in Sideslip (Ybeta), Lateral Force/Yaw Coupling(Yr),Control Force (Ydelta),
Directional Stability (Nbeta),Yaw Damping (Nr), Control Moment(Ndelta).
```

Tool Analysis

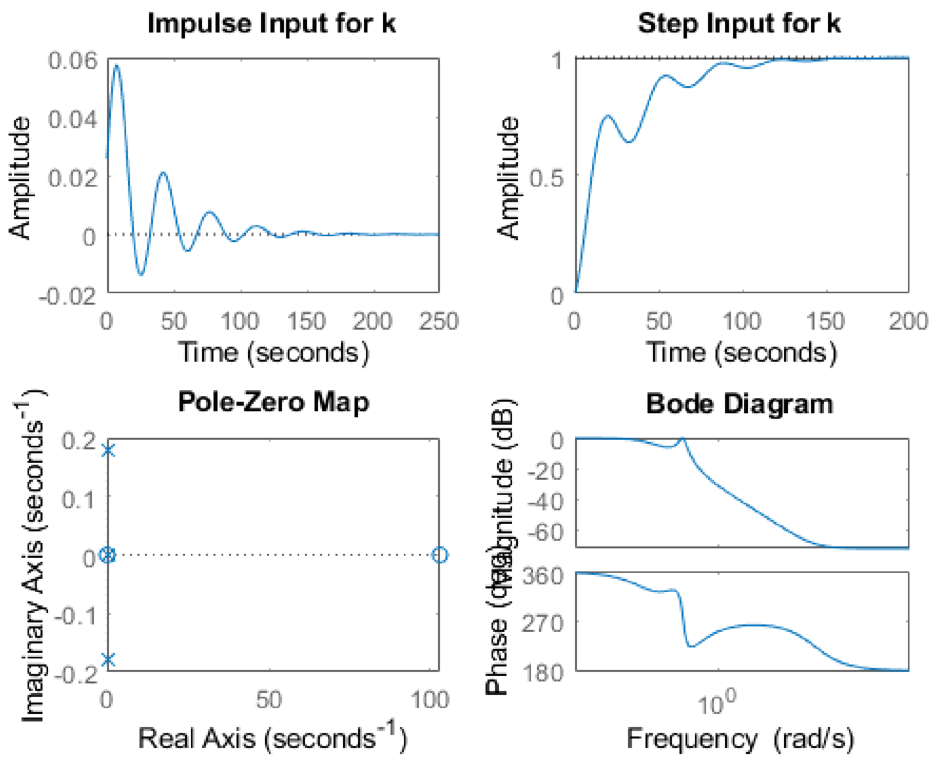
```
clear all;
close all;
clc;
Ydelta = 2461;
m = 1775;
V = 100;
Nr = -67.06;
Ndelta = 2803.079;
Yr = 0.46384;
Izz = 1960;
Ybeta = -4772;
Nbeta = 46.38;

sys = tf([((Ydelta)/(m*V)),-((Nr*Ydelta)+(Ndelta*((m*V)-Yr)))/(Izz*m*V)],[1,(-(Nr/Izz)-
(Ybeta/(m*V))),((Nr*Ybeta) + (Nbeta*((m*V)-Yr)))/(Izz*m*V)])
sys_N1=feedback(sys,1)
    [GC_PID,info_PI] = pidtune(sys,'PID');
     sys_N1_PID = feedback(sys * GC_PID,1);
```

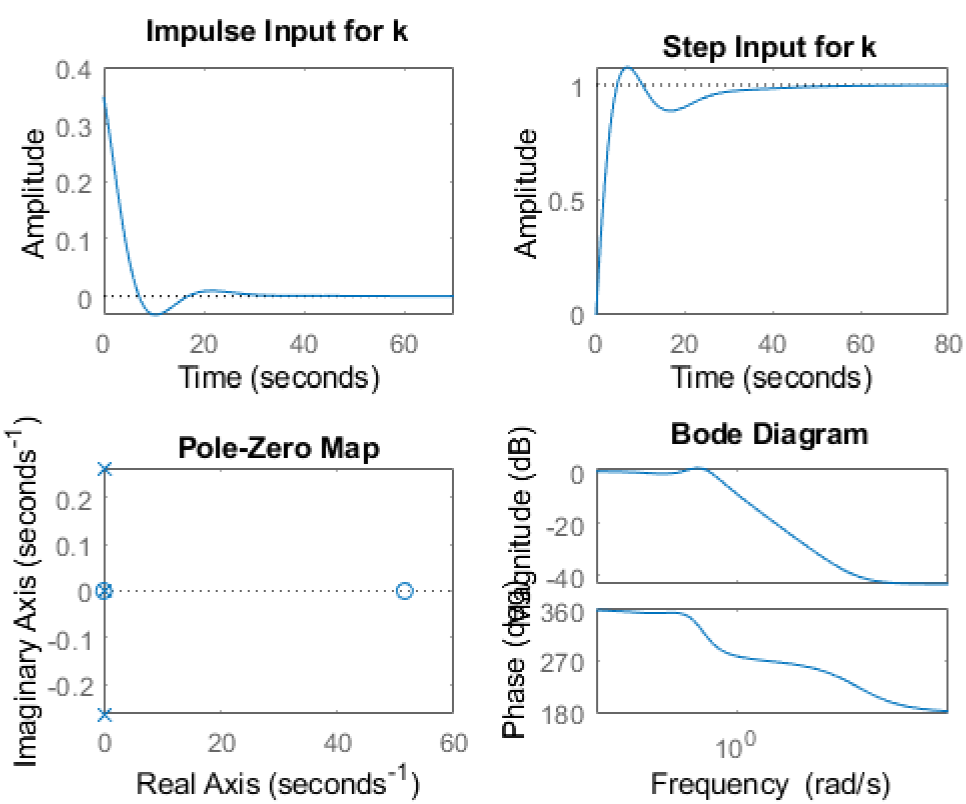
```
figure(1);
subplot(2,2,1);
impulse(sys_N1_PID);
title('Impulse Input for k');
subplot(2,2,2);
step(sys_N1_PID);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([((Ydelta)/(m*V)),-((Nr*Ydelta)+(Ndelta*((m*V)-Yr)))/(Izz*m*V)],[1,(-(Nr/Izz)-
(Ybeta/(m*V))),((Nr*Ybeta) + (Nbeta*((m*V)-Yr)))/(Izz*m*V)])
pzmap(sys_N1_PID)
subplot(2,2,4);
bode(sys_N1_PID)
hold on;
S = stepinfo(sys_N1_PID)


V1 = 50;
sys1 = tf([((Ydelta)/(m*V1)),-((Nr*Ydelta)+(Ndelta*((m*V1)-Yr)))/(Izz*m*V1)],[1,(-(Nr/Izz)-
(Ybeta/(m*V1))),((Nr*Ybeta) + (Nbeta*((m*V1)-Yr)))/(Izz*m*V1)])
sys_N2=feedback(sys1,1)
[GC_PID,info_PI] = pidtune(sys1,'PID');
     sys_N2_PID = feedback(sys1 * GC_PID,1);
figure(2);
subplot(2,2,1);
impulse(sys_N2_PID);
title('Impulse Input for k');
subplot(2,2,2);
step(sys_N2_PID);
title('Step Input for k');
subplot(2,2,3);
[z,p,k]= tf2zp([((Ydelta)/(m*V1)),-((Nr*Ydelta)+(Ndelta*((m*V1)-Yr)))/(Izz*m*V1)],[1,(-(Nr/Izz)-
(Ybeta/(m*V1))),((Nr*Ybeta) + (Nbeta*((m*V1)-Yr)))/(Izz*m*V1)])
pzmap(sys_N2_PID)
subplot(2,2,4);
bode(sys_N2_PID)
hold on;
S = stepinfo(sys_N2_PID)
```

```
sys =

      0.01386 s - 1.43
  ------------------------
  s^2 + 0.0611 s + 0.02458

Continuous-time transfer function.


sys_N1 =
```

```
     0.01386 s - 1.43
  -----------------------
  s^2 + 0.07496 s - 1.405

Continuous-time transfer function.


z =

  103.1148


p =

  -0.0305 + 0.1538i
  -0.0305 - 0.1538i


k =

    0.0139


S =

  struct with fields:

        RiseTime: 46.8233
    SettlingTime: 114.3984
     SettlingMin: 0.8746
     SettlingMax: 0.9999
       Overshoot: 0
      Undershoot: 0.0250
            Peak: 0.9999
        PeakTime: 228.4047


sys1 =

     0.02773 s - 1.429
  ------------------------
  s^2 + 0.08798 s + 0.0255

Continuous-time transfer function.


sys_N2 =

    0.02773 s - 1.429
  ----------------------
  s^2 + 0.1157 s - 1.404
```

```
Continuous-time transfer function.


z =

   51.5401


p =

  -0.0440 + 0.1535i
  -0.0440 - 0.1535i


k =

    0.0277


S =

  struct with fields:

        RiseTime: 3.5091
    SettlingTime: 35.6672
     SettlingMin: 0.8867
     SettlingMax: 1.0787
       Overshoot: 7.8736
      Undershoot: 0.6738
            Peak: 1.0787
        PeakTime: 7.1842
```


Impulse Input for k
Amplitude
Time (seconds)
Step Input for k
Amplitude
Time (seconds)
Pole-Zero Map
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)


Impulse Input for k
Amplitude
Time (seconds)
Step Input for k
Amplitude
Time (seconds)
Pole-Zero Map
Imaginary Axis (seconds-1)
Real Axis (seconds-1)
Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

Comparison Analysis

```
%Speed
% As the rising time is less in the system 2 (V=50). So, we can conclude
% that less the velocity of the system response will be fast.

%Accuracy
% As the settlimng time is less for the system 2 as compare to the system1.
% Here, we can concluded that less the velocity the response settles very fast.

% Stablity
%As the poles are complex conjugate for both the system and the poles are
%left side of the s-plane. The system is stable.
% As the no. of zeros are also less than the no. of poles the system gets stable.
```