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Q.1. $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad a, b \geq 1$

∴ master method is applicable

$f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

∴ $f(n) > n^{\log_b a}$ ⟶ Case 1 is applicable

$T(n) = \Omega(n^{\log_b a + \epsilon})$

here $(\epsilon = 0.42)$

$\boxed{T(n) = \theta(n^2)}$

Q.2. $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad a, b \geq 1$

∴ master method is applicable

$f(n) = n^2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

∴ $f(n) = n^{\log_b a}$ ⟶ Case 3

$T(n) = O(f(n) \cdot \log_b n)$

$\boxed{T(n) = O(n^2 \cdot \log_2 n)}$

Q.3. $T(n) = T(n/2) + 2^n$

As $a = 1 \geq 1$ and $b = 2 > 1$

∴ $f(n) = 2^n$

∴ $n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

∴ $f(n) > n^{\log_b a}$ ⟶ Case 1 is applicable

$f(n) = \Omega(n^{\log_b a + \epsilon})$

$\boxed{T(n) = O(f(n) = O(2^n))}$

Q.4. $T(n) = 2^n T(n/2) + n^n$

a is not constant. Therefore cannot apply master method here.

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Q.5. $T(n) = 16T(n/4) + n$

$a = 16 \geq 1 \qquad b = 4 > 1$

Master method is applicable.

$f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$f(n) < n^{\log_b a} \rightarrow$ case 2

$f(n) = O(n^{\log_b a - \epsilon})$ (here $\epsilon = 1$)

$\boxed{T(n) = O(n^{\log_b a}) = O(n)}$

Q.6. $T(n) = 2T(n/2) + n\log n$

$a = 2 \geq 1 \qquad b = 2 > 1$

Master method is applicable

$f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) > n^{\log_b a} \rightarrow$ case 1 is applicable

$\boxed{T(n) = O(f(n)) = O(n\log n)}$

Q.7. $T(n) = 2T(n/2) + n/\log n$

$a = 2 \geq 1 \qquad b = 2 > 1$

Master method is applicable

$f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

As there is non-polynomial comparison master method is not applicable.

Q.8. $T(n) = 2T(n/4) + n^{0.51}$

As $a = 2 \geq 1 \qquad b = 4 > 1$

Master method is applicable.

$f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$\therefore f(n) > n^{\log_b a} \rightarrow$ Case 1 is applicable

$\therefore f(n) > n^{\log_b a}$

$f(n) = n^{\log_b a + \epsilon}$

$\therefore T(n) = O(f(n)) = O(n^{0.51})$

Q.9 $T(n) = 0.5T(n/2) + 1/n$

As $a = 0.5 \not\geq 1$

$\therefore$ Master method is not applicable

Q.10. $T(n) = 16T(n/4) + n!$

$a = 16 \geq 1 \qquad b = 4 > 1$

master method is applicable

$f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$f(n) > n^{\log_b a}$

$\therefore T(n) = O(f(n)) = O(n!)$

Q.11. $T(n) = \sqrt{2}\,T(n/2) + \log n$

$a = \sqrt{2} \geq 1 \qquad b = 2 > 1$

Master method is applicable.

$f(n) = \log n$

$n^{\log_b a} = n^{\log_2 \sqrt{2}} = n^{1/2}$

$f(n) < n^{\log_b a}$

$\therefore T(n) = O(n^{\log_b a}) = O(\sqrt{n})$

Q.12. $T(n) = 3T(n/2) + n$

$a = 3 \geq 1 \qquad b = 2 > 1$

Master method is applicable.

$f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$f(n) < n^{\log_b a}$

$\therefore T(n) = O(n^{\log_b a}) = O(n^{1.58})$

13. $T(n) = 3T(n/3) + \sqrt{n}$

As $a = 3 \geq 1$ and $b = 3 > 1$

$\therefore f(n) = \sqrt{n}$

$\therefore n^{\log_b a} = n^{\log_3 3} = n$

$f(n) < n^{\log_b a}$

$f(n) = n^{\log_b a - \epsilon}$

$\therefore T(n) = O(n^{\log_b a}) = O(n)$

14. $T(n) = 4T(n/2) + cn$

$a = 4 \geq 1$ $b = 2 > 1$

Master method is applicable

$f(n) = cn$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) < n^{\log_b a}$

$\therefore T(n) = O(n^{\log_b a}) = O(n^2)$

15. $T(n) = 3T(n/4) + n\log n$

$a = 3 \geq 1$ $b = 4 > 1$

Master method is applicable

$f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$f(n) > n^{\log_b a}$

$\therefore T(n) = O(f(n)) = O(n\log n)$

16. $T(n) = 3T(n/3) + n/2$

$a = 3 \geq 1$ $b = 3 > 1$

Master method is applicable.

$f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$f(n) = n^{\log_b a}$

$T(n) = O(n\log_3 n)$

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Q.17. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 \geq 1 \quad b = 3 > 1$

Master method is applicable.

$f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$f(n) > n^{\log_b a}$

$\therefore T(n) = O(f(n)) = O(n^2 \log n)$

Q.18. $T(n) = 4T(n/2) + n/\log n$

$a = 4 \geq 1 \quad b = 2 > 1$

Master method is applicable.

$f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) < n^{\log_b a}$

$\therefore T(n) = O(n^{\log_b a}) = O(n^2)$

Q.19. $T(n) = 64T(n/8) - n^2 \log n$

$f(n) = -n^2 \log n \quad < 0$

$\therefore$ Master method is not applicable.

Q.20. $T(n) = 7T(n/8) + n^2$

$a = 7 \geq 1 \quad b = 3 > 1$

Master method is applicable.

$f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.77}$

$f(n) > n^{\log_b a}$

$T(n) = O(n^2)$