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9914 SE Comps A.

Assignment: 2

Q.1. e(000) = 00000

e(001) = 00110

e(010) = 01001

e(011) = 01111

e(100) = 10011

e(101) = 10101

e(110) = 11010

e(111) = 11000

Decoding Table:

00000	00110	01001	01111	10011	10101	11010	11000
00001	00111	01000	01110	10010	10100	11011	11001
00010	00100	01011	01101	10001	10111	11000	11010
00100	00010	01101	01011	10111	10001	11110	11100
01000	01110	00001	00111	11011	11101	10010	10000
10000	10110	11001	11111	00011	00101	01010	01000
10001	10111	11000	11110	00010	00100	01011	01001

- ~~If~~ e(001) = 00110 → for 00111
- e(110) = 11010 → for 11001
- e(111) = 11000 → for 01010

Q.2. $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The encoding function is $B^3 \to B^6$

$B^3 = \{000, 001, 010, \cancel{00}011, 100, 101, 110, 111\}$

$\therefore e(000) = 000\, x_1 x_2 x_3$ [Here $b_1 = 0$, $b_2 = 0$, $b_3 = 0$]

$\therefore x_1 = b_1 h_{11} + b_2 h_{21} + b_3 h_{31}$

$x_1 = 0 \cdot 1 + 0 \cdot 1 + 0 \cdot 0$

$x_1 = 0$

$x_2 = b_1 h_{12} + b_2 h_{22} + b_3 h_{32}$

$= 0 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 = 0$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$x_3 = 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1$

$x_3 = 0$

$e(000) = 000000$

$\therefore e(001) = 001\, x_1 x_2 x_3$ [Here $b_1 = 0$, $b_2 = 0$, $b_3 = 1$]

$\therefore x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{12} + b_3 \cdot h_{13}$

$x_1 = 0 \cdot 1 + 0 \cdot 1 + 1 \cdot 0$

$x_1 = 0$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$x_2 = 0 \cdot 0 + 0 \cdot 1 + 1 \cdot 1$

$x_2 = 1$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$x_3 = 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1$

$x_3 = 1$

$\therefore e(001) = 001011$

$e(010) = 010\, x_1 x_2 x_3$ [Here $b_1 = 0$, $b_2 = 1$, $b_3 = 0$]

$\therefore x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$x_1 = 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1$

$x_1 = 1$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 1$

$= 1$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$= 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 = 1$

$\therefore e(001) = 001011$

$e(010) = 010\, x_1 x_2 x_3$ [Here $b_1 = 0, b_2 = 1, b_3 = 0$]

$\therefore x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$x_1 = 0 \cdot 1 + 1 \cdot 1 + 0 \cdot 1$

$x_1 = 1$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$x_2 = 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 = 1$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$= 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$

$e(010) = 010110$

$e(011) = 011\, x_1 x_2 x_3$ [Here $b_1 = 0, b_2 = 1, b_3 = 1$]

$\therefore x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$x_1 = 0 \cdot 1 + 1 \cdot 1 + 1 \cdot 0$

$x_1 = 1$

$x_2 = b_1 \cdot h_{21} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 0 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 = 0$

$x_3 = b_1 \cdot h_{13} + b_2 h_{23} + b_3 h_{33}$

$= 0 \cdot 0 + 1 \cdot 0 + 1 \cdot 1 = 1$

$e(011) = 011101$

$\therefore e(100) = 100\, x_1 x_2 x_3$ [Here $b_1 = 1, b_2 = 0, b_3 = 0$]

$x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$= 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 = 1$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 = 0$

$\therefore e(100) = 100100$

$\therefore e(101) = 101\, x_1 x_2 x_3$ [Here $b_1 = 1, b_2 = 0, b_3 = 1$]

$x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$x_1 = 1 \cdot 1 + 0 \cdot 1 + 1 \cdot 0$

$x_1 = 1$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 = 1$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$= 1 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 = 1$

$\therefore e(101) = 101111$

$\therefore e(110) = 110\, x_1 x_2 x_3$ [Here $b_1 = 1, b_2 = 1, b_3 = 0$]

$x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$= 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 0$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 = 1$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$= 1 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$

$\therefore e(110) = 110010$

$\therefore e(111) = 111\, x_1 x_2 x_3$ [Here $b_1 = 1, b_2 = 1, b_3 = 1$]

$x_1 = b_1 \cdot h_{11} + b_2 \cdot h_{21} + b_3 \cdot h_{31}$

$= 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 0 = 0$

$x_2 = b_1 \cdot h_{12} + b_2 \cdot h_{22} + b_3 \cdot h_{32}$

$= 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 = 0$

$x_3 = b_1 \cdot h_{13} + b_2 \cdot h_{23} + b_3 \cdot h_{33}$

$= 1 \cdot 0 + 1 \cdot 0 + 1 \cdot 1 = 1$

$e(111) = 111001$

Decoding table:

000000	001011	010110	011101	100100	101111	110010	111001
000001	001010	010111	011100	100101	101110	110011	111000
000010	001001	010100	011111	100110	101101	110000	111011
000100	001111	010010	011001	100000	101000	110110	111101
001000	000011	011110	010101	101100	100111	111010	110001
010000	011011	000110	001101	100100	111111	100010	101001
100000	101011	110110	111101	000100	001111	010010	011001

i. The word 011001 is located in 8^{th} column. The word at the top is 111001.

$\therefore e(111) = 111001$

$\therefore$ 011001 can be decoded as 111.

ii. The word 101001 is located in 8^{th} column. The word at the top is 110010

$\because e(110) = 110010$

$\therefore$ 101001 can be decoded as 110.

iii. The word 111010 is located in 7^{th} column. The word at the top is 110010.

$\because e(110) = 110010$

$\therefore$ 111010 can be decoded as 110.

iv. The word 101011 is located in 2^{nd} column. The word at ~~to~~ the top is 001011.

$\therefore$ 101011 can be decodes as 001.

v. The word 110110 is located in 3^{rd} column. The word at the top is 010110.

$\because e(010) = 010110$ $\therefore$ 110110 can be decoded as 010

Q.3. a] The given set of positive rational number closed under the multiplication operation. ∴ closure property is satisfied.

b] Multiplication operation is always associative. Hence associative property is satisfied.

c] Here identity element $e = 1$ and $1 \in Q$. Hence, identity property is satisfied.

d] $a \cdot a^{-1} = 1$ $\therefore a^{-1} = \frac{1}{a} \in Q$.

∴ Inverse property is satisfied.

For every element there is an inverse present.

∴ Q is a Group.

Q.4. $Z_n = \{0, 1, 2, \dots, n-1\}$

$Z_4 = \{0, 1, 2, 3\}$

$a \circ b$ = remainder of ab divided by 4 Here $n=4$

a.

0	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

b. All entries in the table belong to set Z.

∴ Closure property is satisfied.

For Associativity

$1 \circ (2 \circ 3) = (1 \circ 2) \circ 3$

$1 \circ 2 = 2 \circ 3$

$2 = 2$

∴ Associativity is verified

∴ It is a semigroup.

Q-5.a]

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

b] $3^0 = 1$

$3^1 = 3$

$3^2 = 3 \times_7 3 = 2$

$3^3 = 3 \times_7 3 = 2 \times_7 3 = 6$

$3^4 = 3^3 \times_7 3 = 6 \times_7 3 = 4$

$3^5 = 3^4 \times_7 3 = 4 \times_7 3 = 5$

$3^6 = 3^5 \times_7 3 = 5 \times_7 3 = 1$

$\therefore$ Hence $|3| = 6$

$\therefore$ 3 is generator of this group and this group is cyclic.