

DSGT: Assignment 1

Q-1) A. Convert to predicate logic

$$1. \forall x (\text{snowFall}(x) \wedge \text{whiteWindsBlow}(x)) \rightarrow (\text{OneWolfDies}(x) \wedge \text{packSurvives}(x))$$

$$2. \sim \text{knows}(\text{Jon Snow, anything}) \wedge \sim \text{understand}(\text{You, Life Beyond Wall})$$

$$3. \sim \text{walksInto}(\text{One, Mordor})$$

$$4. \forall x (\text{choice}(\text{set})(x) \rightarrow (\text{hasPower}(x) \vee \sim \text{hasPower}(x)))$$

$$5. \forall x (\text{Tools}(x) \wedge \text{BelongsTo}(x, \text{IT}) \rightarrow (\text{userFriendly}(x) \wedge \text{idiotProof}(x)))$$

$$6. \forall x (\text{DarkrestTimes}(x) \wedge \text{TurnOn}(y, \text{Light})) \rightarrow \text{FoundHappiness}(x)$$

$$7. \exists x \exists y (\text{felt}(x) \wedge \text{known}(x) \wedge \sim \text{shinedThrough}(x, y) \wedge \text{shown}(y))$$

$$8. \forall x (\text{शिक्षा}(x) \rightarrow (\text{डिग्री}(x) \wedge \text{व्यापकवृष्टिकोण}(x, \text{जिवन}) \wedge \text{व्यापकवृष्टिकोण}(x, \text{समाज})))$$

9.1.B)

1. Everyone who is aware of Matrix is looking for Neo.
2. Everyone ^{one} ~~person~~ ^{wh} that sits on Iron Throne is both ambitious and ruthless.
3. All students who are admitted in Hogwarts and stay in Gryffindor are brave and chivalrous.
4. All wizards that are in Slytherin have a keen interest in dark magic and are likely death eaters.
5. There exists a gangster in the city who, when there exists a unique police officer who is brave, takes down that police officer.
6. There exists a magical creature such that every wizard can see it if and only if it resides in the forbidden forest.
7. There exists a comic store in Pasadena such that if the comic is a superhero comic then it sells that comic if and only if the name of the store is 'The Comic Center of Pasadena'.
8. There exists a unique individual ^{who} ~~that~~ is friendly and caring and this individual acts ^{caringly} towards any other person who ^{is} ~~also~~ a friend and caring.

Q-2. Propositions: $(P \wedge Q) \rightarrow S$ and $(Q \vee R) \rightarrow S$

a. Assume SI is available and Q is operational but UI can't access the data.

b. This means that either P or Q is false.

c. But P is true so Q must be false.

d. This means that database is not operational.

e. But the second proposition states that if either Q is operational or R is online then UI can access data.

f. This is a contradiction so the conclusion of argument must be true.

Q-3. $(P \wedge Q) \rightarrow R \wedge (\sim Q) \rightarrow (\sim R)$

If FI is corrupted (P) and BI is available (Q) then data is recoverable.

But if BI is not available then data is not recoverable since data accessibility depends on BI is available or not (Q), the given proposition is valid.

Q-4. a. $P \wedge (Q \vee R) \rightarrow S$

b. $S \wedge T \rightarrow U$

c. $\sim Q \vee \sim R \rightarrow \sim S$

d. $\sim P \rightarrow \sim U$

From proposition (a), we can infer that if Firewall is operational (P) and either server is running (Q

or VPN is connected (R) then system is secure (S).
From proposition (b), we can infer that if server is not running ($\sim Q$) or VPN is not connected ($\sim R$) then system is not secure ($\sim S$). From proposition (d), we can infer that if firewall is not operational ($\sim P$) then network is unsafe ($\sim U$).

So, if firewall is operational (P), server is running (Q) or the VPN is connected (R), and antivirus is updated (T) then the network is safe (U).

Also, if firewall is operational (P) but server is not running ($\sim Q$) or VPN is not connected ($\sim R$) so system is not secure ($\sim S$).

The system appears to be consistent.

- Q.5.
- a. $T \wedge W \rightarrow F$
 - b. $F \rightarrow E$
 - c. $\sim W \vee \sim T \rightarrow D$
 - d. $\sim D \rightarrow (T \vee W)$
 - e. $\sim E \rightarrow \sim F$
 - f. E (system's objective)

From proposition (a), if traffic lights are working (T) and weather is clear (W) then traffic flow is smooth (F).

From proposition (b), if traffic flow is smooth (F) then emergency response time is optimal (E).

From proposition (c), if weather is not clear ($\sim W$) or

Considering inference, the system's rules appear to be consistent.

System can achieve its objective by ensuring an optimal emergency response time (E) by maintaining smooth traffic flow (F) through proper functioning of traffic light (T) and clear weather (W) and deploying drones (D) when required.

Q.6. (i) Basis of Induction:-

For $n = 3$

$$\text{For } n = 3^3 = 27 > 3^2 \text{ i.e. } 9.$$

(ii) Inductive step:- Assume that $3^k > k^2$

For $n = k+1$

$$3^{k+1} > (k+1)^2$$

$$\therefore 3^k \cdot 3 > k^2 + 2k + 1$$

$$k^2 \cdot 3 > k^2 + 2k + 1$$

$$3k^2 > k^2 + 2k + 1$$

$$2k^2 > 2k + 1$$

$$k^2 > k + \frac{1}{2} \quad \left[\text{Dividing by 2} \right]$$

This is true for all positive integers $k \geq 3$,
so

$$3^{k+1} > (k+1)^2$$

$$3^{k+1} > k^2 + 2k + 1$$

By mathematical induction, we have shown that if the statement hold for k , it also holds for $k+1$.

Q.7. (i) Base case:

For $n = 1$

$F(1) = 1$ and $2 \times 1 = 2$, so base case holds

For $n = 2$

$F(2) = 1$ and $2 \times 2 = 4$, so base case holds

Q. (i) Inductive step: Assume that $F(k) < 2k$ holds for positive integers $k \geq 2$

So for $n = k+1$

$$F(k+1) < 2(k+1)$$

From definition of Fibonacci sequence,

$$F(k+1) = F(k) + F(k-1)$$

By inductive hypothesis, we know that

$$F(k) < 2k \text{ and } F(k-1) < 2(k-1)$$

Adding them,

$$F(k) + F(k-1) < 2k + 2(k-1)$$

$$F(k+1) < 2k + 2k - 2$$

$$F(k+1) < 4k - 2$$

$$\therefore k \geq 2$$

$$F(k+1) < 4k - 2 < 2k + 2$$

$$\therefore F(k+1) < 2(k+1)$$

Q. 8. $\boxed{(P \rightarrow Q) \wedge (R \rightarrow S) \rightarrow ((P \wedge R) \rightarrow (Q \wedge S))}$

$$\Rightarrow ((\sim P \vee Q) \wedge (\sim R \vee S)) \rightarrow (\sim(P \wedge R) \vee (Q \wedge S)) \dots \text{Implication law}$$

$$= \sim((\sim P \vee Q) \wedge (\sim R \vee S)) \vee (\sim(P \wedge R) \vee (Q \wedge S)) \dots \text{Implication law}$$

$$= (\sim(\sim P \vee Q) \vee \sim(\sim R \vee S)) \vee ((\sim P \vee \sim R) \vee (Q \wedge S)) \dots \text{De Morgan}$$

$$= (P \wedge \sim Q) \vee (R \wedge \sim S) \vee ((\sim P \vee \sim R) \vee (Q \wedge S)) \dots \text{De Morgan}$$

$$= (P \wedge \sim Q) \vee (R \wedge \sim S) \vee (P \wedge R) \vee (\sim Q \vee S) \dots \text{Distributive}$$

$$= (P \vee R) \wedge (P \vee \sim S) \wedge (R \vee \sim Q) \vee (R \vee S) \dots \text{Distributive}$$

$$= (P \vee R) \wedge (P \vee \sim S) \wedge R$$

Idempotent

$$= [P \wedge (P \vee \sim S)] \vee [R \wedge (P \vee \sim S)] \wedge R$$

Distributive

$$= P \vee (R \wedge (P \vee \sim S)) \wedge R$$

Absorptive

$$= (P \vee (R \wedge P)) \vee (P \vee (R \wedge \sim S)) \wedge R$$

Distributive

$$= P \vee R \vee (R \wedge \sim S) \wedge R$$

Idempotent

$$= (P \vee R \vee R) \wedge (P \vee R \vee \sim S) \wedge R$$

Distributive

$$= P \vee R \wedge (P \vee \sim S) \wedge R$$

Idempotent

$$= P \vee R \wedge R$$

Absorption

$$= P \vee R = T$$

\therefore It is a tautology

Q.9. $(P \wedge Q \wedge R) \wedge (\neg P \vee \neg Q) \wedge (Q \vee \neg R) \vee (\neg Q \vee R)$

Let $A = (P \wedge Q \wedge R) \wedge (\neg P \vee \neg Q) \wedge (Q \vee \neg R) \vee (\neg Q \vee R)$

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$P \wedge Q \wedge R$	$\neg P \vee \neg Q$	$Q \vee \neg R$	$\neg Q \vee R$	A
T	T	T	F	F	F	T	F	T	T	F
T	T	F	F	F	T	F	F	T	F	F
T	F	T	F	T	F	F	T	F	T	F
T	F	F	F	T	T	F	T	T	T	F
F	T	T	T	F	F	F	T	T	T	F
F	T	F	T	F	T	F	T	T	F	F
F	F	T	T	T	F	F	T	F	T	F
F	F	F	T	T	T	F	T	T	T	F

It is a contradiction as all values are false

Q.10.

(i) $\forall x [S(x) \rightarrow T(x) \vee I(x)]$

(ii) $\forall x [S(x) \wedge T(x) \rightarrow \neg I(x)]$

(iii) $\forall x [(K(x) \wedge S(x)) \rightarrow M(x)]$

(iv) $\forall x [P(x) \rightarrow I(x)]$

(v) $\exists x [K(x) \wedge P(x)]$

Let's consider the statement, "All keynote speakers are tech entrepreneurs who are speaking at multiple sessions". According to this statement, any keynote speaker $K(x)$ must also fulfil the conditions of being a tech entrepreneur ($S(x)$) and speaking at multiple sessions.

Vivian Ludrick

$(m(x))$. However, based on the statement: - "No tech entrepreneurs who is speaking at the conference is an investor", we know that tech entrepreneurs $(T(x))$ who are speaking $(S(x))$ cannot be investors $(I(x))$. These two statements contradict each other: - Therefore there is an inconsistency in provided conditions.

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick

Vivian Ludrick