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Enhanced Whale Optimization Algorithm for Sizing Optimization of Skeletal Structures

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Abstract

The whale optimization algorithm (WOA) is a recently developed swarm-based optimization algorithm inspired by the hunting behavior of humpback whales. This study attempts to enhance the original formulation of the WOA in order to improve solution accuracy, reliability and convergence speed. The new method, called enhanced whale optimization algorithm (EWOA), is tested in sizing optimization problems of truss and frame structures. The EWOA is compared with WOA and other metaheuristic methods developed in literature in four optimization problems of skeletal structures. Numerical results demonstrate the efficiency of the EWOA and WOA with the former algorithm being more efficient than its standard version.

Keywords: enhanced whale optimization algorithm (EWOA), frame structures, structural optimization, truss, structures, whale optimization algorithm (WOA)

INTRODUCTION

Many nature-inspired population-based optimization algorithms such as genetic algorithms (GA) (Goldberg 1989), particle swarm optimization (PSO) (Eberhart and Kennedy 1995) and ant colony optimization (ACO) (Dorigo, Maniezzo, and Colorni 1996) have been introduced and have become more and more popular in recent years. Various methods of this category can be found in Kaveh (2014). These methods have been widely used in many engineering problems (Rao, Savasani, and Vakharia 2011; Figueredo and Sansen 2014; Gao et al. 2015; Tejani, Savsani, and Patel 2016). Other interesting applications of optimization methods can be found in Csébfalvi (2014), and Banichuk, Ragnedda, and Serra (2014).

Optimum design of structures can be classified as follows: (1) obtaining optimal size of structural members (sizing optimization); (2) finding the optimal form for the structure (shape optimization); (3) achieving optimal size and connectivity between structural members (topology optimization). Truss and frame optimization are very popular design problems and can be found frequently in papers. For example, CMLPSA (Corrected Multi-Level & Multi-Point Simulated Annealing) was utilized by Lamberti (2008); PSO (particle swarm optimization) was employed by Hasançebi et al. (2009); EHS (efficient harmony search algorithm) and SAHS (self adaptive harmony search algorithm) were utilized by Degertekin (2012); D-ICDE (improved constrained differential evolution using discrete variables) was used by Ho-Huu et al. (2015) and aeDE (adaptive elitist differential evolution) was employed by Ho-Huu et al. (2016) to optimize truss structures. For frame optimization, Camp, Bichon, and Stovall (2005) used ACO (ant colony optimization); Kaveh and Talatahari (2010b) employed ICA (imperialist competitive algorithm); Kaveh and Talatahari (2012) utilized CSS (charged system search) and Kazemzadeh Azad, Hasançebi, and Kazemzadeh Azad (2013) employed BB-BC (big bang–big crunch). The weight of these structures must be minimized subject to stress, stability and displacement constraints. This optimization task is in general difficult to solve because of non-linear constraints and non-convex feasible region.

In this study, a new nature-inspired meta-heuristic optimization algorithm, called whale optimization algorithm (WOA), is utilized in sizing optimization of skeletal structures. This method is introduced by Mirjalili and Lewis (2016) and it is inspired by the bubble-net hunting strategy of humpback whales. WOA simulates hunting behavior with random or the best search agent to chase the prey and the use of a spiral to simulate bubble-net attacking mechanism of humpback whales. Here, the original formulation of WOA is modified in order to improve its convergence behavior. The new algorithm, named enhanced whale optimization algorithm (EWOA), is tested in four structural optimization problems: two truss optimization problems (spatial 72-bar truss and spatial 582-bar tower) and two frame optimization problems (3-bay 15-story frame and 3-bay 24-story frame). The four test problems are solved with both EWOA and WOA, and optimization results are compared with the literature.

The remainder of the paper is organized as follows. The mathematical model of structural optimization is presented in Section 2. Section 3 describes the EWOA algorithm besides a brief of basic WOA. In order to show the capability of the proposed algorithms, four numerical examples are studied in Section 4. Finally, some conclusions are derived in Section 5.

STATEMENT OF THE OPTIMIZATION PROBLEM

Sizing optimization of skeletal structures can be stated as follows:

$$\begin{aligned}
&\text{Find} \quad \{X\} = [x_1, x_2, \ldots, x_{ng}] \\
&\text{to minimize} \quad W(\{X\}) = \sum_{i=1}^{nm} \rho_i A_i L_i \qquad (1) \\
&\text{subjected to:} \quad \begin{cases} g_j(\{X\}) \le 0, \; j = 1, 2, \ldots, nc \\ \mathrm{x}_{i\,\min} \le x_{\mathrm{i}} \le x_{i\max} \end{cases}
\end{aligned}$$

where $\{X\}$ is the vector containing the design variables; ng is the number of design variables; $W(\{X\})$ is the weight of the structure; nm is the number of elements of the structure; ρ_i, A_i and L_i denote the material density, cross-sectional area, and the length of the ith member, respectively. $x_{i\min}$ and $x_{i\max}$ are the lower and upper bounds of the design variable x_i, respectively. $g_j(\{X\})$ denotes design constraints; nc is the number of constraints.

To handle the constraints, the well-known penalty approach is employed. Thus, the objective function is redefined as follows:

$$f(\{X\}) = (1 + \varepsilon_1.\upsilon)^{\varepsilon_2} \times W(\{X\}), \qquad \upsilon = \sum_{j=1}^{nc} \max[0, g_j(\{X\})] \qquad (2)$$

where υ denotes the sum of the violations of the design constraints. The constant ε_1 is set equal to 1 while ε_2 starts from 1.5 and then linearly increases to 3.

OPTIMIZATION ALGORITHMS

Whale Optimization Algorithm

A recent addition to meta-heuristic algorithms is the whale optimization algorithm (WOA), that was introduced by Mirjalili and Lewis (2016). The WOA is inspired by the humpback whales hunting method that is called bubble-net hunting strategy. They prefer to hunt school of krill or small fishes close to the surface. Therefore, humpback whales swim around the prey within a shrinking circle and along a spiral-shaped path simultaneously to create distinctive bubbles along a circle or '9'-shaped path. To simulate this behavior in WOA, there is a probability of 50% to choose between the shrinking encircling mechanism and the spiral model to update the position of whales during optimization. Their formulations are designed as follows:

1. Shrinking encircling preys: In WOA, the currently best candidate solution is assumed as the target prey and the other search agents try to update their positions towards it. This behavior is represented by the following formula:

$$\vec{X}(t+1) = \vec{X}^*(t) - A.\vec{D} \qquad (3)$$

$$\vec{D} = | C.\vec{X}^*(t) - \vec{X}(t) | \qquad (4)$$

$$A = 2.a.r - a \qquad (5)$$

$$C = 2.r \qquad (6)$$

where $\vec{X}^*$ is the historically best position, $\vec{X}$ is a whale position and t indicates the current iteration. a is linearly decreased from 2 to 0 over the course of iterations and r is a random number uniformly distributed in the range of [0,1]. The sign "||" denotes the absolute value.

2. Spiral bubble-net feeding maneuver: A spiral equation is used between the position of whale and prey to mimic the helix-shaped movement of humpback whales as follows:

$$\vec{X}(t+1) = e^{bk}.\cos(2\pi k).\vec{D}' + \vec{X}^*(t) \qquad (7)$$

$$\vec{D}' = | \vec{X}^*(t) - \vec{X}(t) | \qquad (8)$$

where b is a constant for defining the shape of the logarithmic spiral, and k is a random number uniformly distributed in the range of [−1,1].

In order to have a global optimizer, when A is greater than 1 or less than −1, the search agent is updated according to a randomly chosen search agent instead of the best search agent:

$$\vec{X}(t+1) = \vec{X}_{\text{rand}} - A.\vec{D}'' \qquad (9)$$

$$\vec{D}'' = | C.\vec{X}_{\text{rand}} - \vec{X}(t) | \qquad (10)$$

where $\vec{X}_{\text{rand}}$ is selected randomly from whales in the current iteration. For further details, the reader may refer to Mirjalili and Lewis (2016).

Enhanced Whale Optimization Algorithm

The WOA is simple in concept and effective to explore global solutions. In order to improve the solution accuracy, reliability of search and convergence speed of WOA, a new algorithm is introduced in this study, the enhanced whale optimization algorithm (EWOA). A key point in improving an algorithm is to preserve the simplicity of the original method.

A random number in the [0, 1] range is extracted for each whale in each iteration. If it is greater than 0.5, Eq. (7) is selected; otherwise, Eq. (12) is chosen for updating whale's position.

In exploration phase of EWOA, one component of each whale is changed with the random value in the search space with a probability like p instead of Eq. (9).

$$p = 0.3(1 - \text{iter} / \text{iter}_{\max}) \qquad (11)$$

where iter and $\text{iter}_{\max}$ are current iteration number and the total number of the iteration for optimization process, respectively.

For a selected whale, an integer random number is extracted in the interval [1, ng] to choose which design variable should be randomly changed. At this point, another random number q is extracted in the interval [0, 1] and compared with the probability threshold p. The selected variable x_j is changed if $q < p$, according to $x_j = x_{j\min} + \text{random}.(x_{j\max} - x_{j\min})$, where *random* is a random number uniformly distributed in the interval [0, 1].

The modified algorithm should be capable of maintaining proper balance between the diversification and the intensification inclinations. According to this point and the above change, Eq. (3) is redefined as follows:

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \circ \vec{D}''' \quad (12)$$

$$\vec{D}''' = \vec{r} \circ |\vec{X}(t)| \quad (13)$$

$$\vec{A} = 2.\vec{a} \circ \vec{r} - \vec{a} \quad (14)$$

where $\vec{r}$ is a random vector that has each component uniformly distributed in the range of [0,1] and $\vec{a}$ is a vector that has each component equal to a. The sign "∘" denotes an element-by-element multiplication.

The flowchart of EWOA is shown in **Figure 1**.

TEST PROBLEMS AND OPTIMIZATION RESULTS

In this section, four benchmark examples are provided to demonstrate the effectiveness, robustness and efficiency of the WOA and EWOA. In order to reduce statistical errors, each test is repeated 20 times independently. In all problems, agents are allowed to select discrete values from the permissible list of cross sections (real numbers are rounded to the nearest integer in the each iteration). The algorithms are coded in MATLAB and the structures are analyzed using the direct stiffness method by our own codes.

Spatial 72-Bar Truss Problem

Figure 2 shows the schematic of a spatial 72-bar truss structure. The material density is 0.1 lb/in^3 (2,767.990 kg/m^3) and the modulus of elasticity is 10^7 psi (68.95 GPa). The elements are divided into sixteen groups, because of structural symmetry: (1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30}, (7) A_{31}–A_{34}, (8) A_{35}–A_{36}, (9) A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}–A_{58}, (14) A_{59}–A_{66} (15), A_{67}–A_{70}, and (16) A_{71}–A_{72}. The structure is subject to the two independent loading conditions listed in **Table 1**. The maximum stress developed in the elements must be less than ±25 ksi (±172.375 MPa). Maximum displacement of the uppermost nodes cannot exceed ±0.25 in (±6.35 mm), for each node, in all directions. In this case, the discrete sizing variables can be selected from a list of 64 discrete sections from 0.111 to 33.5 in^2 (71.613 to 21,612.860 mm^2) (Kaveh and Talatahari 2010b).

This example is also used for adjusting *b* [a constant for defining the shape of the logarithmic spiral in Eq. (7)], number of whales and $iter_{max}$ (total number of iterations). In order to adjust the value of b, number of whales and $iter_{max}$ are, respectively, set to 20 and 1,000, and different amounts of b are considered as 0.5, 1, 1.5 and 2. The results shown in **Table 2** demonstrate that the algorithm is not very sensitive to the values of b; however, statistical results indicate that 0.5 is the most efficient value. In order to adjust the number of whales, the value of $iter_{max}$ is set to 1,000 and various number of whales are selected as 10, 20, 30 and 40. Comparison of the results are shown in **Table 3**, and it can be seen that 20 is a quite suitable number. Different $iter_{max}$ are tested (500, 750, 1,000, 1,250 and 1,500) to adjust this variable. **Table 4** summarizes the results and can be concluded 1,000 is the most suitable value for $iter_{max}$.

Table 5 represents the results obtained by different optimization algorithms. The best designs of IRO (improved ray optimization) (Kaveh, Ilchi Ghazaan, and Bakhshpoori 2013), aeDE (adaptive elitist differential evolution) (Ho-Huu et al. 2016), WOA and EWOA are identical (i.e., 389.33 lb). The lightest designs obtained by DHPSACO (discrete heuristic particle swarm ant colony optimization) (Kaveh and Talatahari 2009a), ICA (imperialist competitive algorithm) (Kaveh and Talatahari 2010b) and CBO (colliding bodies optimization) (Kaveh and Ilchi Ghazaan 2015) are 393.380, 392.84 and 391.23 lb, respectively. EWOA was the most robust optimizer, achieving the lowest average weight over the independent optimization runs. **Figure 3** shows the convergence curves of the best and average results obtained by WOA and EWOA. The best designs have been located at 6,960 and 10,460 analyses for WOA and EWOA, respectively.

Spatial 582-Bar Tower Problem

The spatial 582-bar tower truss shown in **Figure 4** is optimized for minimum volume with the cross-sectional areas of the members being the design variables. The 582-members are divided into 32 groups, because of structural symmetry. Cross-sectional areas of elements (sizing variables) are selected from a discrete list of W-shaped standard steel sections based on area and radii of gyration properties. Cross-sectional areas of elements can vary between 6.16 and 215 in^2 (i.e. between 39.74 and 1,387.09 cm^2). A single load case is considered: lateral loads of 1.12 kips (5.0 kN) applied in both *x*- and *y*-directions and a vertical load of –6.74 kips (–30 kN) applied in the *z*-direction at all nodes of the tower. Limitation on stress and stability of truss elements are imposed according to the provisions of American Institute of Steel Construction (AISC) (1989) as follows.

The allowable tensile stresses for tension members are calculated as

$$\sigma_i^{+} = 0.6F_y \qquad (15)$$

where F_y is the yield strength.

The allowable stress limits for compression members are calculated depending on two possible failure modes of the members known as elastic and inelastic buckling. Therefore

$$\sigma_i^{-} = \begin{cases} \left[\left(1 - \dfrac{\lambda_i^2}{2C_c^2}\right)F_y\right] \Big/ \left[\dfrac{5}{3} + \dfrac{3\lambda_i}{8C_c} - \dfrac{\lambda_i^3}{8C_c^3}\right] & \text{for } \lambda_i < C \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \qquad (16)$$

where E is the modulus of elasticity; λ_i is the slenderness ratio $\left(\lambda_i = kl_i / r_i\right)$; C_c denotes the slenderness ratio dividing the elastic and inelastic buckling regions $C_c = \sqrt{2\pi^2 E / F_y}$; k is the effective length factor (k is set equal to 1 for all truss members); L_i is the member length; and r_i is the minimum radius of gyration.

The maximum slenderness ratio is limited to 300 for tension members, and it is recommended to be 200 for compression members. Moreover, nodal displacements in all coordinate directions must be less than ±3.15 in (i.e. ±8 cm) for this example.

Table 6 represents the results obtained by different optimization algorithms. The best design obtained by EWOA is better than other methods (1,294,929 in^3). The best volume found by PSO (particle swarm optimization) (Hasançebi et al. 2009), DHPSACO (Kaveh and Talatahari 2009a), HBB-BC (hybrid Big Bang–Big Crunch optimization) (Kaveh and Talatahari 2010a), CBO (Kaveh and Ilchi Ghazaan 2014) and WOA are 1,366,674, 1,346,227, 1,365,143, 1,334,994, and 1,302,038 in^3, respectively. EWOA was again the most robust optimizer, achieving the lowest average volume over the independent optimization runs. The stress ratio evaluated at the best design optimized by WOA and EWOA are shown in **Figure 5**. The maximum stress ratio and the maximum nodal displacements obtained by WOA are 99.87% and 3.1499 in, respectively. 99.90% and 3.1497 in are found by EWOA for maximum stress ratio and the maximum nodal displacements. **Figure 6** illustrates the convergence curves found by the proposed methods. The best designs are achieved after 18,840 and 19,300 analyses in WOA and EWOA, respectively. However, EWOA required only about 14,000 analyses to find better intermediate designs than WOA and 17,100 analyses to find an intermediate design with volume 1,302,000 in^3, better than the WOA optimized volume (1,302,038 in^3). Furthermore, EWOA required only 11,740 analyses to find a volume of 1,330,000 in^3, better than the design optimized by CBO (1,334,994 in^3 within 17,700 analyses).

3-Bay 15-Story Frame Problem

Figure 7 represents the schematic of a 3-bay 15-story frame. The applied loads and the numbering of member groups are also shown in this figure. The modulus of elasticity is 29 Msi (200 GPa) and the yield stress is 36 ksi (248.2 MPa). The effective length factors of the members are calculated as $k_x \geq 0$ for a sway-permitted frame and the out-of-plane effective length factor is specified as $k_y = 1.0$. Each column is considered as non-braced along its length, and the non-braced length for each beam member is specified as one-fifth of the span length. Limitation on displacement and strength are imposed according to the provisions of American Institute of Steel Construction (AISC) (2001) as follows:

(a) Maximum lateral displacement

$$\frac{\Delta_T}{H} - R \leq 0 \qquad (17)$$

where Δ_T is the maximum lateral displacement; H is the height of the frame structure; R is the maximum drift index which is equal to 1/300.

(b) The inter-story displacements

$$\frac{d_i}{h_i} - R_I \leq 0, \quad i = 1, 2, \ldots, ns \qquad (18)$$

where d_i is the inter-story drift; h_i is the story height of the ith floor; ns is the total number of stories; R_I is the inter-story drift index (1/300).

(c) Strength constraints

$$\begin{cases} \dfrac{P_u}{2\varphi_c P_n} + \dfrac{M_u}{\varphi_b M_n} - 1 \leq 0, & for \dfrac{P_u}{\varphi_c P_n} < 0.2 \\ \dfrac{P_u}{\varphi_c P_n} + \dfrac{8M_u}{9\varphi_b M_n} - 1 \leq 0, & for \dfrac{P_u}{\varphi_c P_n} \geq 0.2 \end{cases} \qquad (19)$$

where P_u is the required strength (tension or compression); P_n is the nominal axial strength (tension or compression); ϕ_c is the resistance factor (ϕ_c = 0.9 for tension, ϕ_c = 0.85 for compression); M_u is the required flexural strengths; M_n is the nominal flexural strengths; ϕ_b denotes the flexural resistance reduction factor (ϕ_b = 0.90). The nominal tensile strength for yielding in the gross section is calculated by

$$P_n = A_g . F_y \qquad (20)$$

The nominal compressive strength of a member is computed as

$$P_n = A_g . F_{cr} \qquad (21)$$

where

$$\begin{cases} F_{cr} = (0.658^{\lambda_c^2})F_y, & \text{for } \lambda_c \le 1.5 \\ F_{cr} = \left(\dfrac{0.877}{\lambda_c^2}\right)F_y, & \text{for } \lambda_c > 1.5 \end{cases} \quad (22)$$

$$\lambda_c = \frac{kl}{r\pi}\sqrt{\frac{F_y}{E}} \quad (23)$$

where A_g is the cross-sectional area of a member and k is the effective length factor that is calculated by (Dumonteil 1992):

$$k = \sqrt{\frac{1.6G_A G_B + 4.0(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \quad (24)$$

where G_A and G_B are stiffness ratios of columns and girders at two end joints, A and B, of the column section being considered, respectively.

Also, the sway of the top story is limited to 9.25 in (23.5 cm) in this example.

The designs optimized by HPSACO (heuristic particle swarm ant colony optimization) (Kaveh and Talatahari 2009b), HBB-BC (Kaveh and Talatahari 2010a), ICA (Kaveh and Talatahari 2010b), CSS (charged system search) (Kaveh and Talatahari 2012), CBO (Kaveh and Ilchi Ghazaan 2015), WOA and EWOA are compared in **Table 7**. The EWOA algorithm obtained the lowest weight overall, 88,090 lb. EWOA was the most robust optimizer also in this test problem, obtaining the lowest average weight over the independent optimization runs. Stress ratios and inter-story drifts evaluated for the best designs of WOA and EWOA are shown in **Figures 8** and **9**. **Figure 10** compares the best and average convergence histories of EWOA and WOA. The best designs are achieved after 19,060 and 19,940 analyses in WOA and EWOA, respectively.

3-Bay 24-Story Frame Problem

Figure 11 shows the schematic of a 3-bay 24-story frame. Frame members are collected in 20 groups (16 column groups and 4 beam groups). Each of the four beam element groups is chosen from all 267W-shapes, while the 16 column element groups are limited to W14 sections. The material has a modulus of elasticity equal to E = 29.732 Msi (205 GPa) and a yield stress of f_y = 33.4 ksi (230.3 MPa). The effective length factors of the members are calculated as $k_x \geq 0$ for a sway-permitted frame and the out-of-plane effective length factor is specified as $k_y = 1.0$. All columns and beams are considered as non-braced along their lengths. Similar to the previous example, the frame is designed following the LRFD-AISC specification and uses an inter-story drift displacement constraint (AISC 2001).

The optimized designs found by the different algorithms are compared in **Table 8**. The lightest design (i.e., 203,490 lb) is again obtained by EWOA. The best weights found by ACO (ant colony optimization) (Camp, Bichon, and Stovall 2005), HS (harmony search) (Degertekin 2008), ICA (Kaveh and Talatahari 2010b), CSS (Kaveh and Talatahari 2012), CBO (Kaveh and Ilchi Ghazaan 2015) and WOA are, respectively, 220,465, 214,860, 212,640, 212,364, 215,874 and 206,520 lb. The average optimized weight achieved by EWOA is better than those obtained by the other metaheuristic algorithms considered in this study. **Figure 12** compares the convergence curves obtained by EWOA and WOA, that found the optimum weight after 18,820 and 19,640 structural analyses, respectively. It should be noted that EWOA required only 10,500 analyses to find an intermediate design weighing 210,000 lb, better than the designs optimized by ICA and CSS (212,640 and 212,364 lb, respectively), and only 13,500 analyses to find an intermediate design weighing 206,000 lighter than the WOA optimized design (206,520 lb).

CONCLUDING REMARKS

This study presented an improved formulation of the whale optimization algorithm which tries to maintain a proper balance between the diversification and the intensification inclinations. The EWOA algorithm (enhanced whale optimization algorithm) was applied to weight minimization problems of skeletal structures. The simplicity of WOA is preserved in EWOA since no internal parameter is added. The suitability and efficiency of EWOA is illustrated through two truss and two frame optimization problems. EWOA converged to better designs in all the test problems. Also, the average weight/volume found by EWOA in the independent optimization runs is lower in all benchmark examples indicating that the search reliability of proposed method is superior. Besides, it can be seen from convergence history diagrams that the convergence rate of the EWOA algorithm is higher than that of the WOA.

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Table 1. Loading conditions for the spatial 72-bar truss problem

Node	Condition 1			Condition 2		
	F_x kips (kN)	F_y kips (kN)	F_z kips (kN)	F_x kips (kN)	F_y kips (kN)	F_z kips (kN)
17	0.0	0.0	–5.0 (–22.25)	–5.0 (–22.25)	5.0 (–22.25)	–5.0 (–22.25)
18	0.0	0.0	–5.0 (–22.25)	0.0	0.0	0.0
19	0.0	0.0	–5.0 (–22.25)	0.0	0.0	0.0
20	0.0	0.0	–5.0 (–22.25)	0.0	0.0	0.0

Table 2. Sensitivity of EWOA to the b parameter studied for the 72-bar truss problem

b	Results		
	Weight (lb)	Average optimized weight (lb)	Standard deviation on average weight (lb)
0.5	389.33	389.64	0.74
1	389.33	389.98	1.58
1.5	389.33	389.89	1.29
2	389.33	389.81	0.78

Table 3. Sensitivity of EWOA to the number of whales studied for the 72-bar truss problem

Number of whales	Results		
	Weight (lb)	Average optimized weight (lb)	Standard deviation on average weight (lb)
10	389.33	390.03	1.36
20	389.33	389.64	0.74
30	389.33	389.73	0.71
40	389.33	389.86	0.97

Table 4. Sensitivity of EWOA to the $iter_{max}$ parameter studied for the 72-bar truss problem

$iter_{max}$	Results		
	Weight (lb)	Average optimized weight (lb)	Standard deviation on average weight (lb)
500	389.33	390.28	1.89
750	389.33	390.49	1.53
1,000	389.33	389.64	0.74
1,250	389.33	389.90	0.95
1,500	389.33	389.93	1.33

Table 5. Optimized designs found by different algorithms in the 72-bar truss problem

Element group	Members	Cross-sectional areas (in^2)						
		DHPSACO (Kaveh and Talatahari 2009a)	ICA (Kaveh and Talatahari 2010b)	IRO (Kaveh, Ilchi Ghazaan, and Bakhshpoori 2013)	CBO (Kaveh and Ilchi Ghazaan 2015)	aeDE (Ho-Huu et al. 2016)	Present work	
							WOA	EWOA
1	A1–A4	1.800	1.99	1.99	2.13	1.99	1.99	1.99
2	A5–A12	0.442	0.442	0.563	0.563	0.563	0.563	0.563
3	A13–A16	0.141	0.111	0.111	0.111	0.111	0.111	0.111
4	A17–A18	0.111	0.141	0.111	0.111	0.111	0.111	0.111
5	A19–A22	1.228	1.228	1.228	1.228	1.228	1.228	1.228
6	A23–A30	0.563	0.602	0.563	0.442	0.442	0.442	0.442
7	A31–	0.111	0.111	0.111	0.141	0.111	0.111	0.111

	A34							
8	A35–A36	0.111	0.141	0.111	0.111	0.111	0.111	0.111
9	A37–A40	0.563	0.563	0.563	0.442	0.563	0.563	0.563
10	A41–A48	0.563	0.563	0.442	0.563	0.563	0.563	0.563
11	A49–A52	0.111	0.111	0.111	0.111	0.111	0.111	0.111
12	A53–A54	0.250	0.111	0.111	0.111	0.111	0.111	0.111
13	A55–A58	0.196	0.196	0.196	0.196	0.196	0.196	0.196
14	A59–A66	0.563	0.563	0.563	0.563	0.563	0.563	0.563
15	A67–A70	0.442	0.307	0.391	0.391	0.391	0.391	0.391
16	A71–A72	0.563	0.602	0.563	0.563	0.563	0.563	0.563
Weight		393.380	392.84	389.33	391.23	389.33	389.3	389.33

(lb)							3	
Average optimized weight (lb)		N/A	N/A	408.17	456.69	390.913	392.52	389.64
Worst optimized weight (lb)		N/A	N/A	N/A	N/A	393.325	399.65	391.83
Number of structural analyses		5,330	4,500	17,925	4,620	4,160	6,960	10,460
Constraint tolerance (%)		None	None	None	None	None	None	None

Table 6. Optimized designs found by different algorithms in the 582-bar tower problem

Element group	Optimal W-shaped sections					
	PSO (Hasançebi et al. 2009)	DHPSACO (Kaveh and Talatahari 2009a)	HBB-BC (Kaveh and Talatahari 2010a)	CBO (Kaveh and Ilchi Ghazaan 2014)	Present work	
					WOA	EWOA
1	W8 × 21	W8 × 24	W8 × 24	W8 × 21	W8 × 21	W8 × 21
2	W12 × 79	W12 × 72	W24 × 68	W14 × 82	W14 × 90	W14 × 90
3	W8 × 24	W8 × 28	W8 × 28	W8 × 28	W8 × 24	W8 × 24
4	W10 × 60	W12 × 58	W18 × 60	W12 × 50	W14 × 61	W10 × 60
5	W8 × 24	W8 × 24	W8 × 24	W8 × 24	W8 × 24	W8 × 24
6	W8 × 21	W8 × 24	W8 × 24	W8 × 21	W8 × 21	W8 × 21
7	W8 × 48	W10 × 49	W21 × 48	W12 × 53	W10 × 49	W14 × 48
8	W8 × 24	W8 × 24	W8 × 24	W12 × 26	W8 × 24	W8 × 24
9	W8 × 21	W8 × 24	W10 × 26	W8 × 21	W8 × 21	W8 × 21
10	W10 × 45	W12 × 40	W14 × 38	W14 × 48	W10 × 39	W10 × 49
11	W8 × 24	W12 × 30	W12 × 30	W8 × 24	W8 × 24	W8 × 24
12	W10 × 68	W12 × 72	W12 × 72	W14 × 61	W12 × 72	W16 × 67

13	W14 × 74	W18 × 76	W21 × 73	W14 × 82	W14 × 74	W18 × 76
14	W8 × 48	W10 × 49	W14 × 53	W12 × 50	W12 × 50	W10 × 49
15	W18 × 76	W14 × 82	W18 × 86	W14 × 74	W10 × 77	W18 × 76
16	W8 × 31	W8 × 31	W8 × 31	W8 × 40	W8 × 31	W8 × 31
17	W8 × 21	W14 × 61	W18 × 60	W12 × 53	W10 × 49	W14 × 61
18	W16 × 67	W8 × 24	W8 × 24	W6 × 25	W8 × 24	W8 × 24
19	W8 × 24	W8 × 21	W16 × 36	W8 × 21	W8 × 21	W8 × 21
20	W8 × 21	W12 × 40	W10 × 39	W8 × 40	W14 × 48	W14 × 34
21	W8 × 40	W8 × 24	W8 × 24	W8 × 24	W6 × 25	W8 × 24
22	W8 × 24	W14 × 22	W8 × 24	W8 × 21	W10 × 22	W8 × 21
23	W8 × × 21	W8 × 31	W8 × 31	W12 × 26	W8 × 21	W8 × 21
24	W10 × 22	W8 × 28	W8 × 28	W12 × 26	W8 × 24	W8 × 24
25	W8 × 24	W8 × 21	W8 × 21	W10 × 22	W8 × 21	W8 × 21
26	W8 × 21	W8 × 21	W8 × 24	W10 × 22	W10 × 22	W10 × 22
27	W8 × 21	W8 × 24	W8 × 28	W6 × 25	W8 × 24	W8 × 24
28	W8 × 24	W8 × 28	W14 × 22	W8 × 21	W8 × 21	W8 × 21
29	W16 × 21	W16 × 36	W8 × 24	W8 × 21	W8 × 21	W8 × 21

30	W8 × 21	W8 × 24	W8 × 24	W8 × 24	W8 × 24	W8 × 24
31	W8 × 24	W8 × 21	W14 × 22	W8 × 21	W8 × 21	W8 × 21
32	W8 × 24	W8 × 24	W8 × 24	W6 × 25	W8 × 28	W8 × 24
Volume (in^3)	1,366,674	1,346,227	1,365,143	1,334,994	1,302,038	1,295,738
Average optimized volume (in^3)	1,371,667	N/A	N/A	1,345,429	1,349,290	1,310,836
Worst optimized volume (in^3)	N/A	N/A	N/A	N/A	1,714,082	1,344,450
Number of structural analyses	N/A	8,500	12,500	17,700	18,840	19,300
Constraint tolerance (%)	None	None	None	None	None	None

Table 7. Optimized designs found by different algorithms in the 3-bay 15-story frame problem

Element group	Optimal W-shaped sections						
	HPSACO (Kaveh and Talatahari 2009b)	HBB-BC (Kaveh and Talatahari 2010a)	ICA (Kaveh and Talatahari 2010b)	CSS (Kaveh and Talatahari 2012)	CBO (Kaveh and Ilchi Ghazaan 2015)	Present work	
						WOA	EWOA
1	W21 × 111	W24 × 117	W24 × 117	W21 × 147	W24 × 104	W14 × 90	W14 × 99
2	W18 × 158	W21 × 132	W21 × 147	W18 × 143	W40 × 167	W30 × 173	W27 × 161
3	W10 × 88	W12 × 95	W27 × 84	W12 × 87	W27 × 84	W12 × 79	W27 × 84
4	W30 × 116	W18 × 119	W27 × 114	W30 × 108	W27 × 114	W27 × 114	W24 × 104
5	W21 × 83	W21 × 93	W14 × 74	W18 × 76	W21 × 68	W14 × 68	W21 × 68
6	W24 × 103	W18 × 97	W18 × 86	W24 × 103	W30 × 90	W30 × 90	W18 × 86
7	W21 × 55	W18 × 76	W12 × 96	W21 × 68	W8 × 48	W21 × 48	W21 × 48
8	W27 × 114	W18 × 65	W24 × 68	W14 × 61	W21 × 68	W14 × 68	W14 × 68

9	W10 × 33	W18 × 60	W10 × 39	W18 × 35	W14 × 34	W8 × 24	W8 × 31
10	W18 × 46	W10 × 39	W12 × 40	W10 × 33	W8 × 35	W14 × 48	W10 × 45
11	W21 × 44	W21 × 48	W21 × 44	W21 × 44	W21 × 50	W21 × 44	W21 × 44
Weight (lb)	95,850	97,689	93,846	92,723	93,795	88,651	88,090
Average optimized weight (lb)	N/A	N/A	N/A	N/A	98,738	92,903	90,784
Worst optimized weight (lb)	N/A	N/A	N/A	N/A	N/A	99,806	94,931
Number of structural analyses	6,800	9,900	6,000	5,000	9,520	19,060	19,940
Constraint tolerance	None	None	None	None	None	None	None

(%)							

Table 8. Optimized designs found by different algorithms in the 3-bay 24-story frame problem

Element group	Optimal W-shaped sections						
	ACO (Camp, Bichon, and Stovall 2005)	HS (Degertekin 2008)	ICA (Kaveh and Talatahari 2010b)	CSS (Kaveh and Talatahari 2012)	CBO (Kaveh and Ilchi Ghazaan 2015)	Present work	
						WOA	EWOA
1	W30 × 90	W30 × 90	W30 × 90	W30 × 90	W27 × 102	W30 × 90	W30 × 90
2	W8 × 18	W10 × 22	W21 × 50	W21 × 50	W8 × 18	W10 × 17	W10 × 30
3	W24 × 55	W18 × 40	W24 × 55	W21 × 48	W24 × 55	W21 × 62	W24 × 55
4	W8 × 21	W12 × 16	W8 × 28	W12 × 19	W6 × 8.5	W14 × 26	W6 × 8.5
5	W14 × 145	W14 × 176	W14 × 109	W14 × 176	W14 × 132	W14 × 109	W14 × 159
6	W14 × 132	W14 × 176	W14 × 159	W14 × 145	W14 × 120	W14 × 145	W14 × 99
7	W14 × 132	W14 × 132	W14 × 120	W14 × 109	W14 × 145	W14 × 109	W14 × 120
8	W14 × 13	W14 × 10	W14 × 90	W14 × 90	W14 × 82	W14 × 99	W14 × 74

	2	9					
9	W14 × 68	W14 × 82	W14 × 74	W14 × 74	W14 × 61	W14 × 53	W14 × 74
10	W14 × 53	W14 × 74	W14 × 68	W14 × 61	W14 × 43	W14 × 43	W14 × 43
11	W14 × 43	W14 × 34	W14 × 30	W14 × 34	W14 × 38	W14 × 34	W14 × 30
12	W14 × 43	W14 × 22	W14 × 38	W14 × 34	W14 × 22	W14 × 22	W14 × 22
13	W14 × 14 5	W14 × 14 5	W14 × 15 9	W14 × 14 5	W14 × 99	W14 × 12 0	W14 × 90
14	W14 × 14 5	W14 × 13 2	W14 × 13 2	W14 × 13 2	W14 × 10 9	W14 × 99	W14 × 12 0
15	W14 × 12 0	W14 × 10 9	W14 × 99	W14 × 10 9	W14 × 82	W14 × 10 9	W14 × 90
16	W14 × 90	W14 × 82	W14 × 82	W14 × 82	W14 × 90	W14 × 82	W14 × 99
17	W14 × 90	W14 × 61	W14 × 68	W14 × 68	W14 × 74	W14 × 90	W14 × 68
18	W14 × 61	W14 × 48	W14 × 48	W14 × 43	W14 × 61	W14 × 61	W14 × 61
19	W14 × 30	W14 × 30	W14 × 34	W14 × 34	W14 × 30	W14 × 38	W14 × 43
20	W14 × 26	W14 × 22	W14 × 22	W14 × 22	W14 × 22	W14 × 22	W14 × 22
Weight (lb)	220,465	214,860	212,640	212,364	215,874	206,520	203,490

Average optimized weight (lb)	229,555	222,620	N/A	215,226	225,071	216,475	208,648
Worst optimized weight (lb)	N/A	N/A	N/A	N/A	N/A	243,143	226,019
Number of structural analyses	15,500	13,924	7,500	5,500	8,280	19,640	18,820
Constraint tolerance (%)	None	None	None	None	None	None	None

Figure 1. Flowchart of the EWOA algorithm.

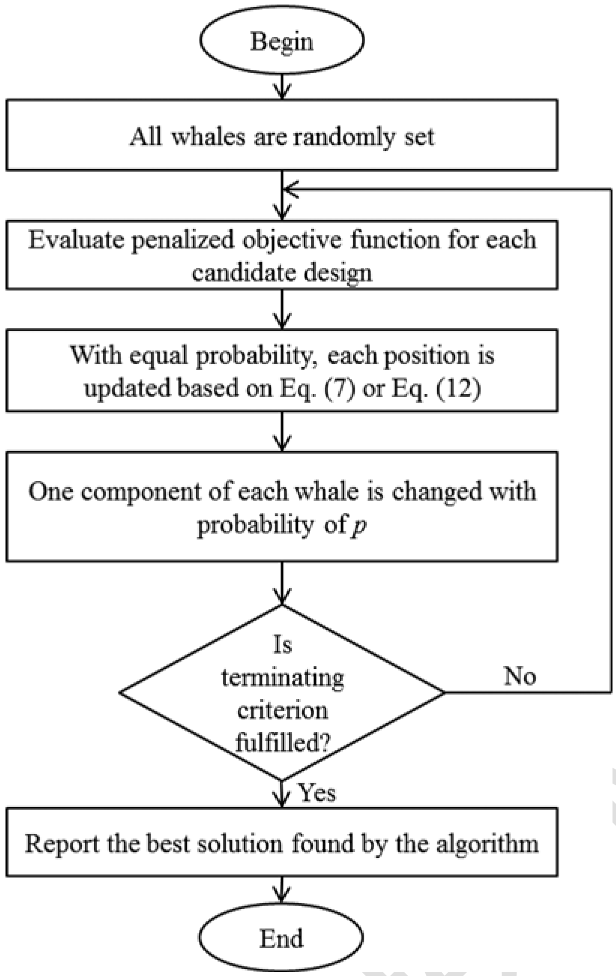

Figure 2. Schematic of the spatial 72-bar truss structure.

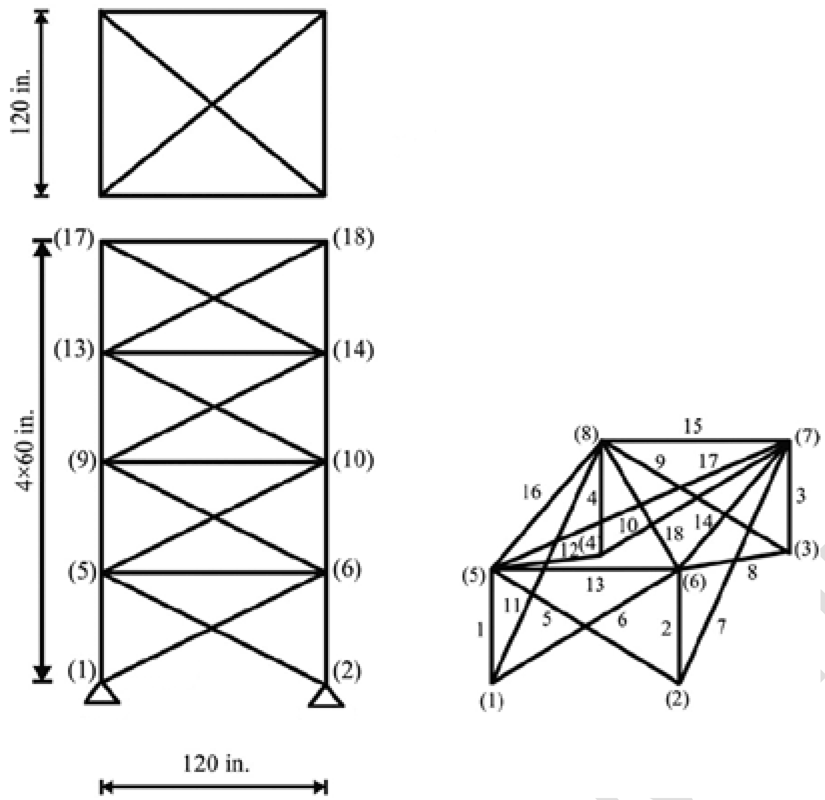

Figure 3. Convergence curves obtained by EWOA and WOA in the 72-bar truss problem.

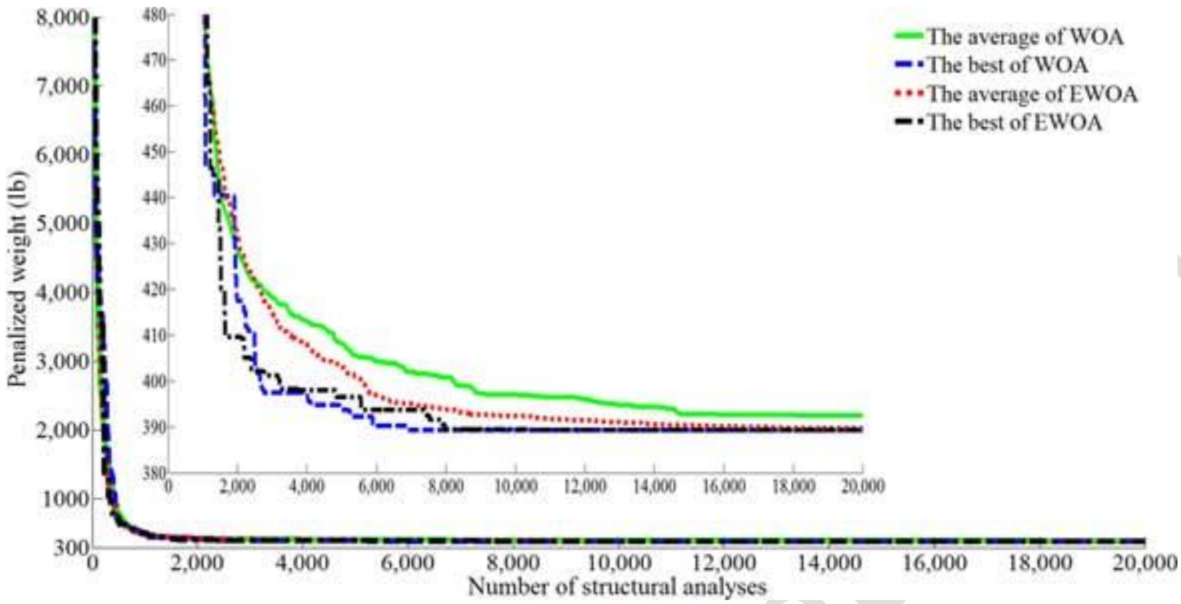

Figure 4. Schematic of the spatial 582-bar tower.

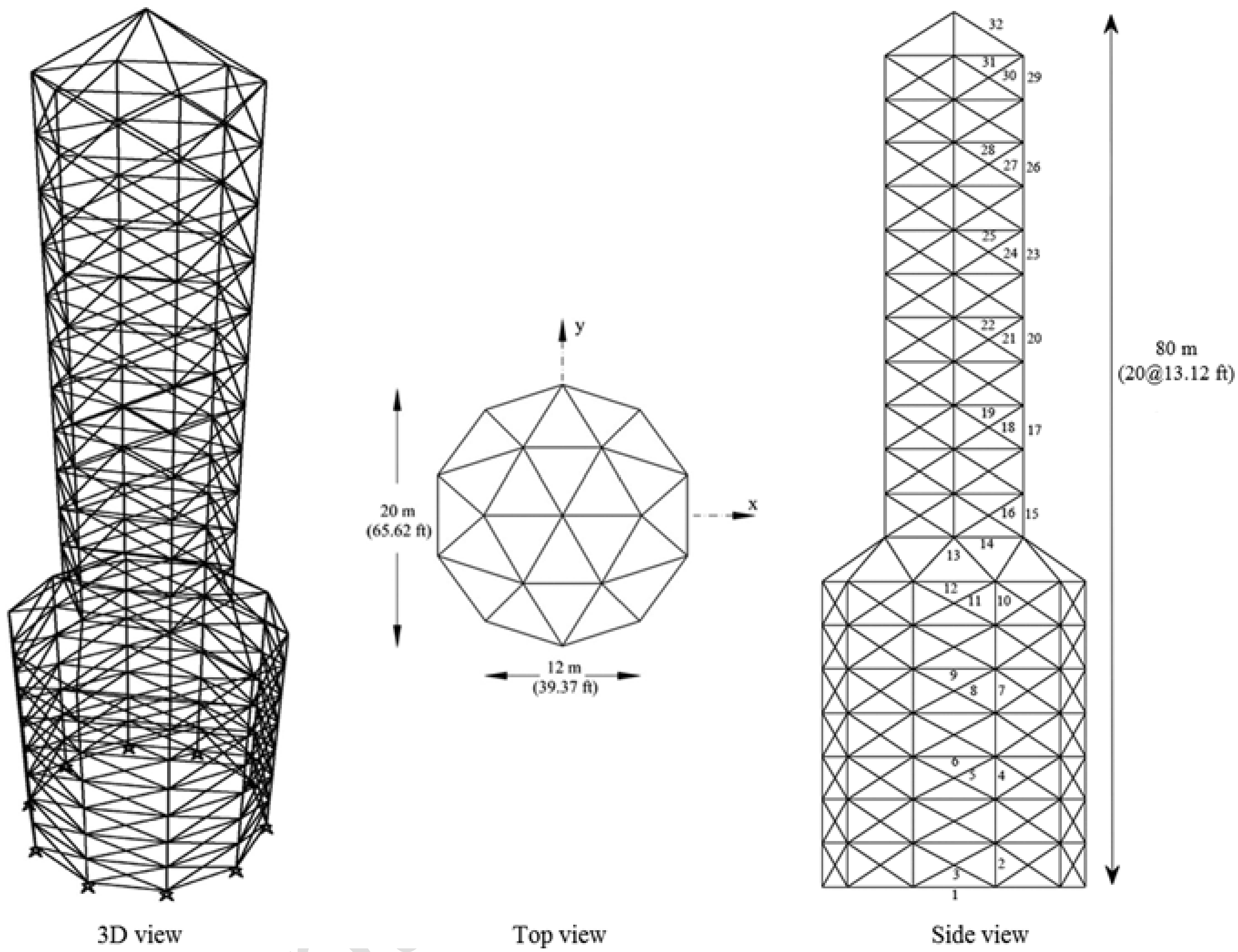

Figure 5. Stress ratios evaluated at the optimized designs found by EWOA and WOA in the 582-bar tower problem.

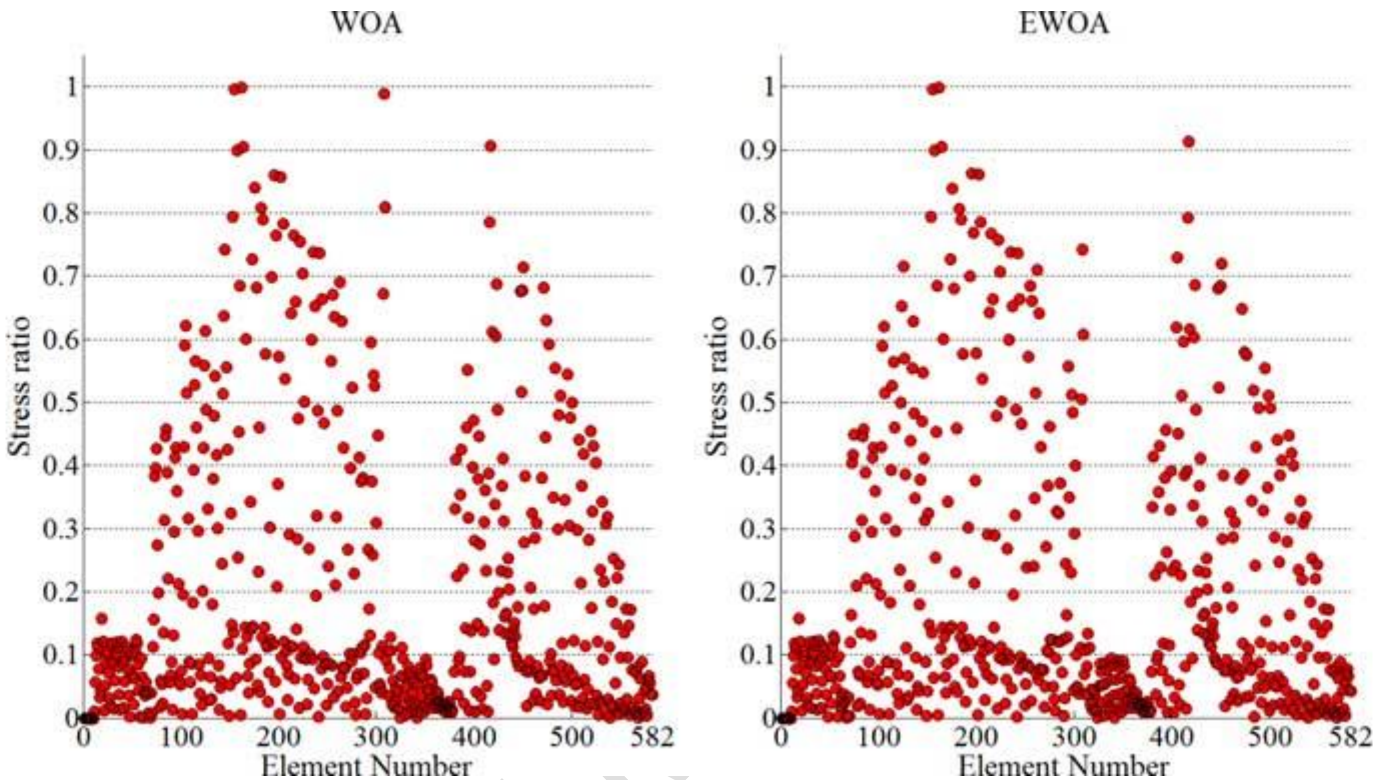

Figure 6. Convergence curves obtained by EWOA and WOA in the 582-bar tower problem.

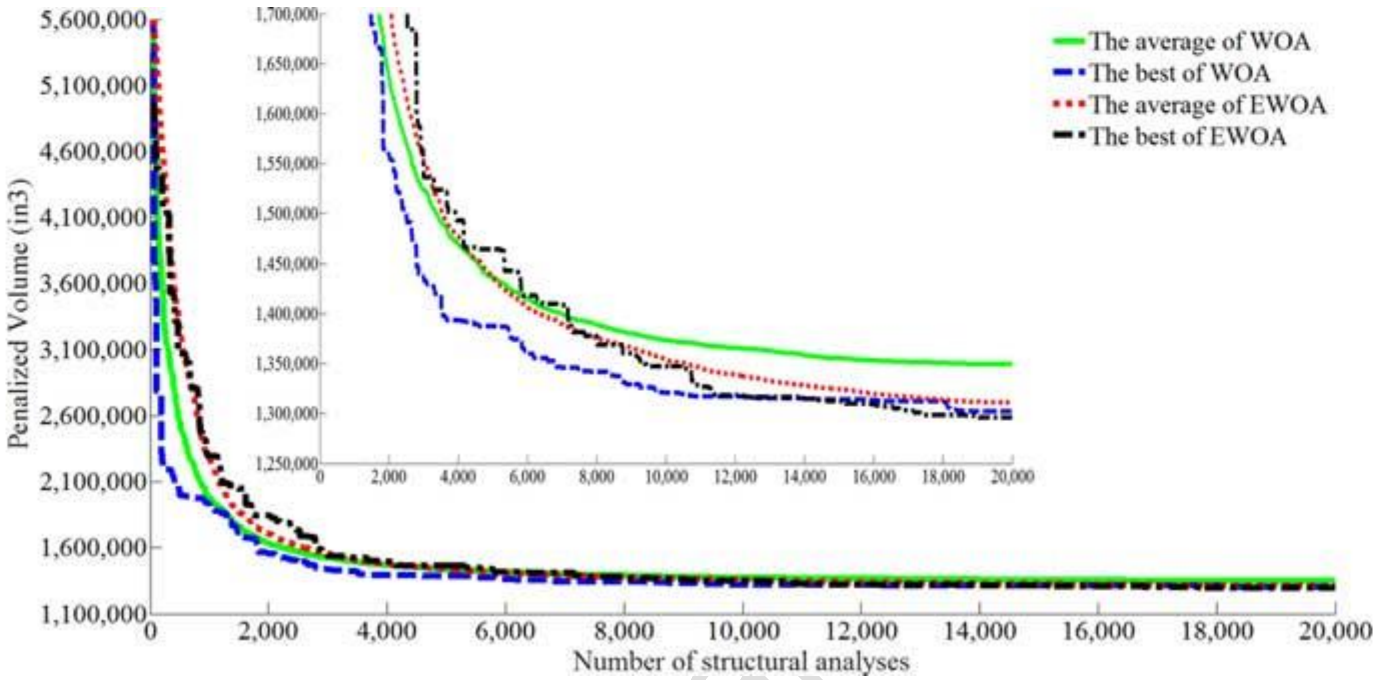

Figure 7. Schematic of the 3-bay 15-story frame.

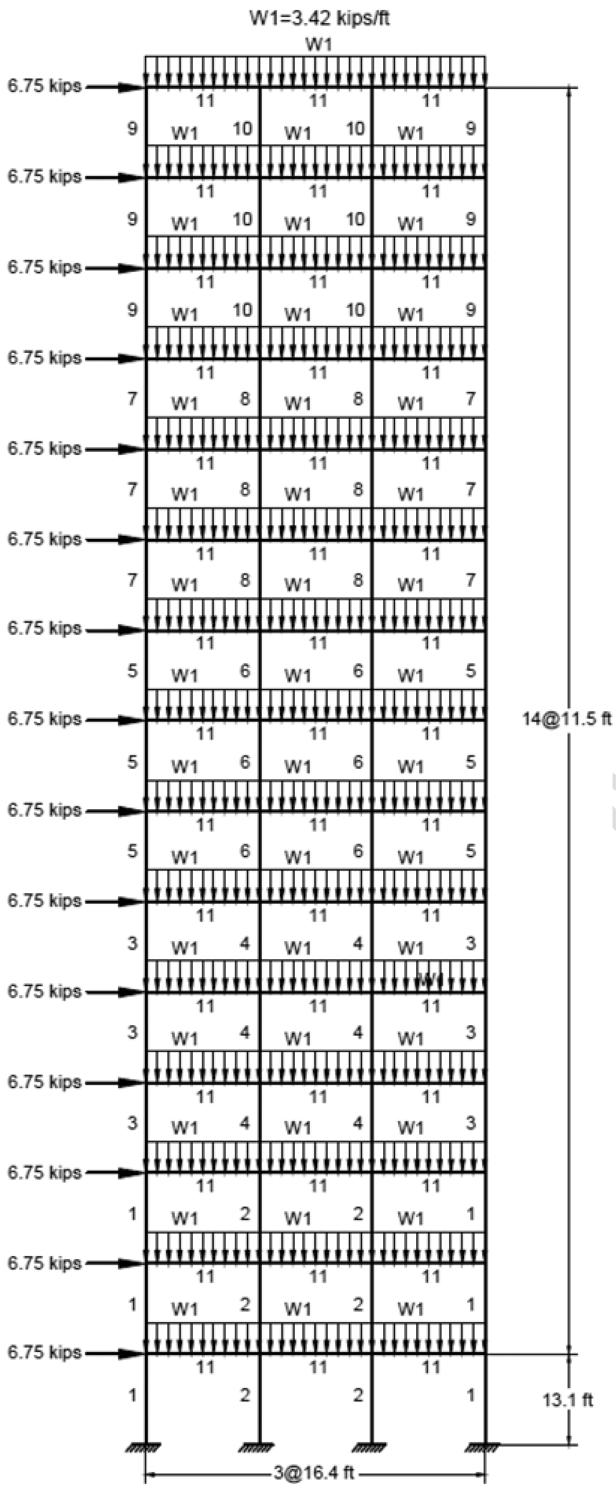

Figure 8. Stress ratios evaluated at the optimized designs found by EWOA and WOA in the 3-bay 15-story frame problem.

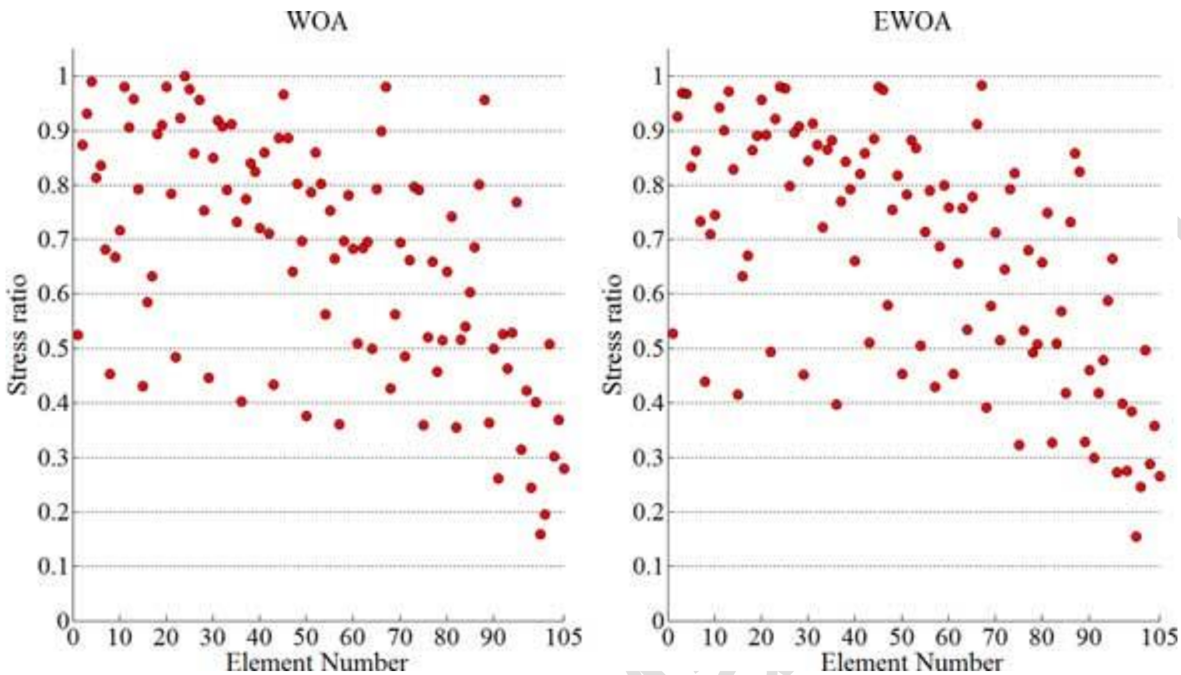

Figure 9. Inter-story drifts evaluated at the optimized designs found by EWOA and WOA in the 3-bay 15-story frame problem.

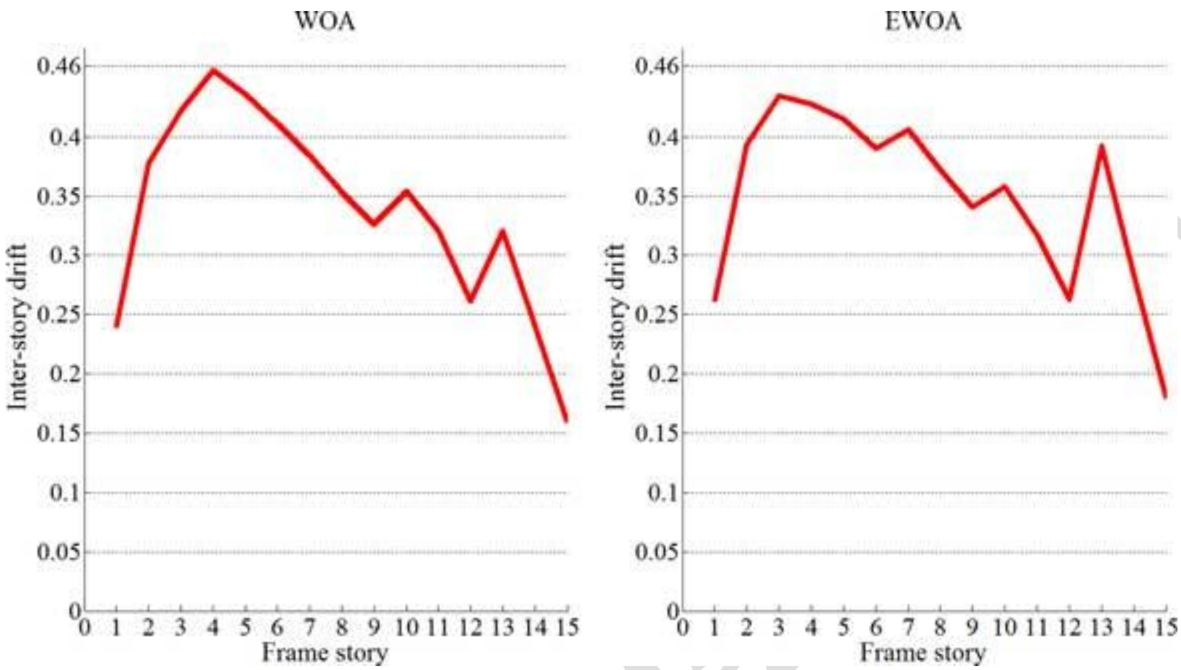

Figure 10. Convergence curves obtained by EWOA and WOA in the 3-bay 15-story frame problem.

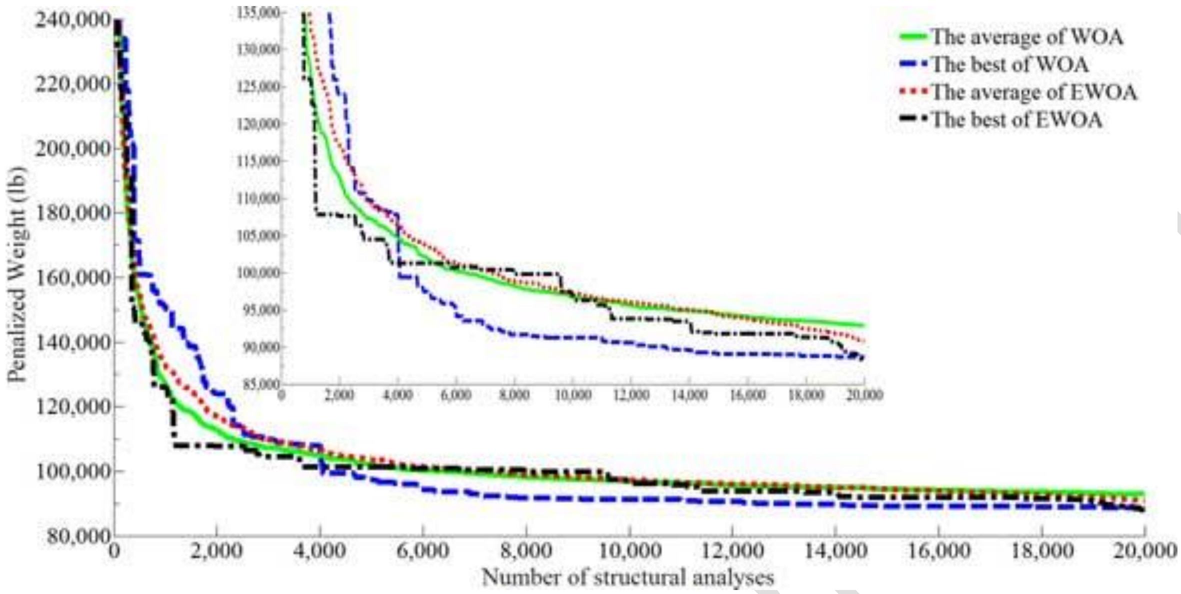

Figure 11. Schematic of the 3-bay 24-story frame.

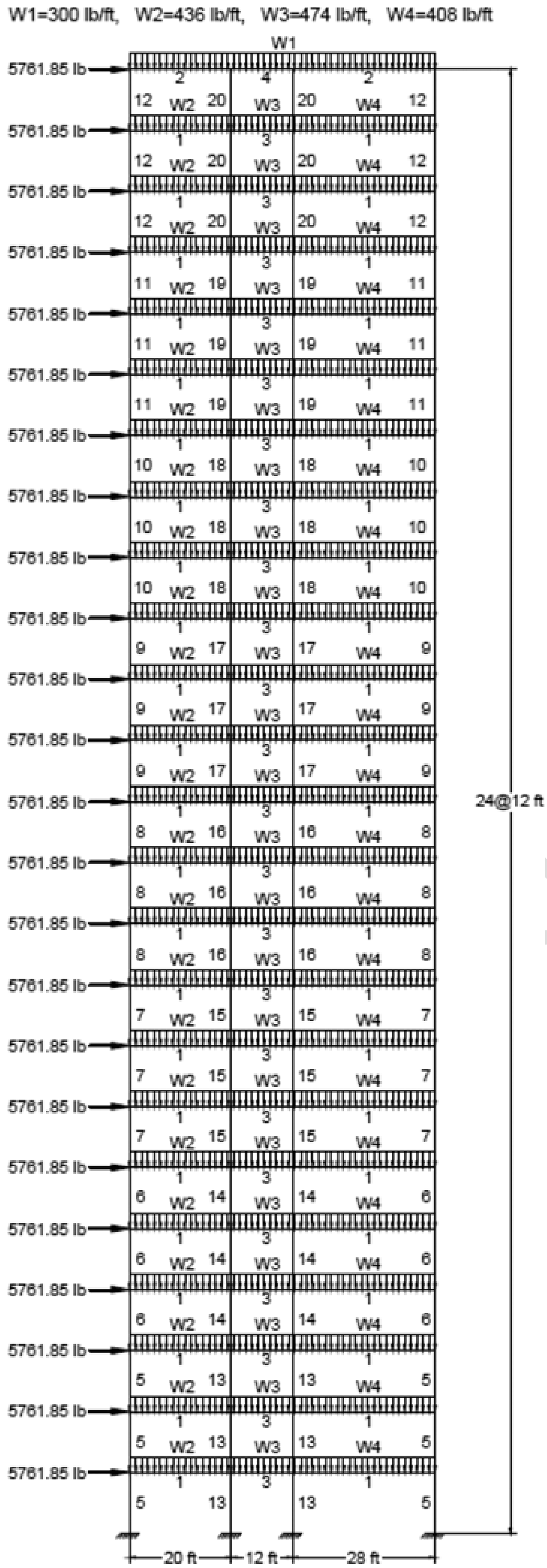

Figure 12. Convergence curves obtained by EWOA and WOA in the 3-bay 24-story frame problem.