Homework 3 Monte Carlo VaR and Expected Shortfall
Due Friday, 19 February 2016 at 5:00 p.m.
Total 10 points

Instructions. This is a group assignment. Groups may include up to 4 people. Please submit a Word or .pdf document with your solutions via the Compass site prior to 5:00 p.m. on Friday, 19 February. Please also submit the R, Matlab, or Python scripts/computer programs and data files that you used. (You must use R, Matlab, Python or some other similar package; you may not use Excel.)

Assignment. You trade options. The current date is just after the close on trading on Friday, 12 February 2016.

Earlier this February, you decided that the implied volatilities of the S&P 500 (SPX) and Dow Jones (DJX) options traded on the CBOE were not consistent with each other; in particular, you thought the implied volatilities of the SPX options were too high relative to the implied volatilities of the DJX options. In an attempt to profit from this belief, you sold a March 18, 2016 straddle on the SPX and bought a March 18, 2016 straddle on the DJX. Specifically, you entered into the following trades:

(i) sold 50 March 18 SPX calls with strike price of 1,865;
(ii) sold 50 March 18 SPX puts with strike price of 1,865;
(iii) bought 550 March 18 DJX calls with strike of 160; and
(iv) bought 550 March 18 DJX puts with strike price of 160.

As of the close of trading on Friday, February 12, the March 18, 2016 SPX calls with a strike of 1,865 were bid at 49.50 and offered at 50.10; the March 18, 2016 SPX puts with a strike of 1,865 were bid at 55.50 and offered at 56.10. The March 18, 2016 DJX calls with strike of 160 were bid at 3.85 and offered at 4.05; the March 18, 2016 SPX puts with strike of 160 were bid at 4.70 and offered at 4.90. The S&P 500 was at 1,864.78 and the Dow Jones index was at 15,973.84.

The SPX index options are based on the S&P 500 index, and have a multiplier of 100. They are described at http://www.cboe.com/Products/indexopts/spx_spec.aspx. The DJX index options are based on 1/100th (one-one-hundredth) of the current value of the Dow Jones Industrial Average, and also have a multiplier of 100. They are described at http://www.cboe.com/products/indexopts/djx_spec.aspx.

1. Estimates of variances and covariances. (total 2 points) Work with continuously compounded (log) returns, and estimate the variances and covariances of the log returns using the exponentially weighted estimator that Christoffersen calls the "RiskMetrics model" and describes on pp. 69-70 of the textbook.

(a) (1 point) Collect historical data on the S&P 500 Index, the Dow Jones Industrial Average, the CBOE Volatility Index for the S&P 500 (VIX), and the CBOE's Dow Jones Volatility Index (VXD). From the two stock indexes and two volatility indexes, compute daily log relatives or

"returns" $\ln(x_t/x_{t-1})$. From the log relatives, estimate the covariance matrix of daily log relatives. (Use the exponentially-weighted "RiskMetrics" estimator.)

(b) (1 point) In the RiskMetrics model, the variance over K trading days is estimated as K times the daily variance. (See, for example, Christoffersen, p. 72.) Thus, you can annualize your variance and standard deviation estimates from part (a) by multiplying by 252 and $\sqrt{252}$, respectively. What are the annualized standard deviations of the log relatives?

2. Option implied volatilities. (1 point) Use the version of the Black-Scholes formula with a continuous dividend yield and the data (closing index values) from February 12, 2016 to compute the implied volatilities of the four options. Assume that the continuously compounded interest rate is 25 b.p. per year and that the (continuous) dividend yields on the SPX and DJX are 0.0222 (2.22%) and 0.0248 (2.48%), respectively. What are the implied volatilities of the four options?

3. Monte Carlo Value-at-Risk. (3 points) Assume that the log relatives are normally distributed and use the Monte Carlo method to estimate the VaR of your portfolio. Use a holding period of one day and a probability of 5% ("confidence level" of 95%). Assume that the means of the continuously compounded returns on the indexes are zero. What is the Value-at-Risk of your portfolio?

Hint: Use the (log) changes in the VIX and VXD as proxies for the (log) change in the implied volatilities of the SPX and DJX options, respectively. That is, simulate the log changes in the VIX and the VXD, and assume that the log changes in the option implied volatilities are equal to these simulated log changes in the VIX and VXD.

4. **Expected Shortfall.** (3 points) Use the portfolio, data, and assumptions from Questions 1-3 to estimate the expected shortfall

$$E[\text{loss} \mid \text{loss} \geq \text{VaR}],$$

where VaR is the Value-at-Risk you estimated in Question 3.

5. **Stress scenarios** (total 1 point) Assume that you work in the risk management group of a bank from the home country of one member of your group. (If your group includes members from different countries then please select one of the countries.)

(a) (1/2 point) Describe the two most important stress scenarios that your bank should consider.

(b) (1/2 point) Please briefly explain why you think the stress scenarios from part (a) are relevant.