FINANCIAL DERIVATIVES

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SOLUTION 1:

As per the diffusion process, we know that

$$\frac{dS}{S} = \mu dt + \frac{w}{\sqrt{S}} dW^S$$

After reading the case study, we realize that the Molycorp's stock also follows the diffusion process with w=2

Therefore, we should solve the PDE:

$$\frac{1}{2} w^2 S \frac{\partial^2 F}{\partial S^2} + \frac{\partial F}{\partial t} - rF + rS\frac{\partial F}{\partial S} + \Gamma = 0$$

This PDE should be solved based on the conditions that:

1. The riskless rate is constant at 1 percent.
2. Only if the stock price hits \$16 after September 1st 2015, will the company be able to recover the bonds at par.
3. The bondholders can exercise their conversion option before the redemption date if company calls the bonds. Which means, per \$1000 princpal amount of notes, they get \$83.333 shares of common stock.
4. The only way Molycorp can default on the bond is if S is 0 and there is no payment through dividend till date of bond maturity.

We then value the bonds in the below manner:

1. First of all, a grid of S is generated from 0 to 20 by 0.005.
2. Then because dt= 1/52, T = 5*52 +1 = 261 steps
3. Then we set the boundary conditions (vector $F(S_i, T)$) for the PDE above:

$$F(ST, T) = \max[1000, 83.3333*St] \text{ for all } St > 0$$

$$F(St, t) \geq 83.3333*St, \text{ for all } t \leq T$$

$$F(0, t) = 0 \text{ for all } t \leq T$$

$$Ft(St \geq 16) \leq 1000, \text{ for all } t > 3*52=156$$

4. After that, we build the transition matrix:

$$K = \begin{bmatrix} \dots & & & & & & \\ 0 & k_{i-1}^{+} & k_{i-1}^{0} & k_{i-1}^{-} & 0 & \dots & \\ \dots & 0 & k_{i}^{+} & k_{i}^{0} & k_{i}^{-} & 0 & \dots \\ & \dots & 0 & k_{i+1}^{+} & k_{i+1}^{0} & k_{i+1}^{-} & 0 \\ & & & & & & \dots \end{bmatrix}, \text{ where } \begin{aligned} k_i^{+} &= -w^2 S_i \frac{dt}{2(dS)^2} - rS_i \frac{dt}{2dS} \\ k_i^{-} &= -w^2 S_i \frac{dt}{2(dS)^2} + rS_i \frac{dt}{2dS} \\ k_i^{0} &= 1 + rdt + w^2 S_i \frac{dt}{(dS)^2} \end{aligned} .$$

(we approximate $\frac{\partial^2 F}{\partial S^2} \approx \frac{F(S_{i+1}) - 2F(S_i) + F(S_{i-1})}{(dS)^2}$ and $\frac{\partial F}{\partial t} = \frac{F(t) - F(t-dt)}{dt}$.)

5. Hence, $K^{-1}*F(S_i, 261)=F(S_i, 260)$ can be used to get bond's value on t=260. Thus, by left multiplying the inverse matrix of K, we can calculate the bond's value. But, we need to check the condition:

*F (St, t) ≥ 83.3333*St, for all t ≤ T,*

*Ft(St≥16) ≤ 1000, for all t >3*52=156,*

for every $F(S_i, t)$ in every loop. Therefore, when we get $F(S_i, t=1)$, that is the bond's value for now.

The graph of the bond value versus the stock as of today is shown in Figure 1

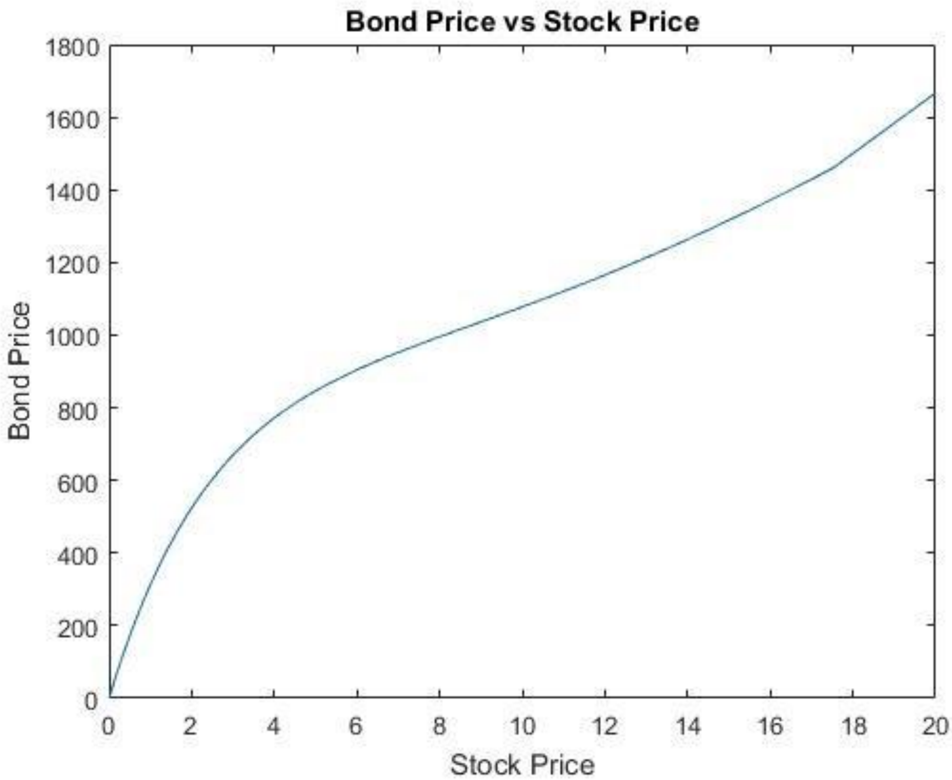
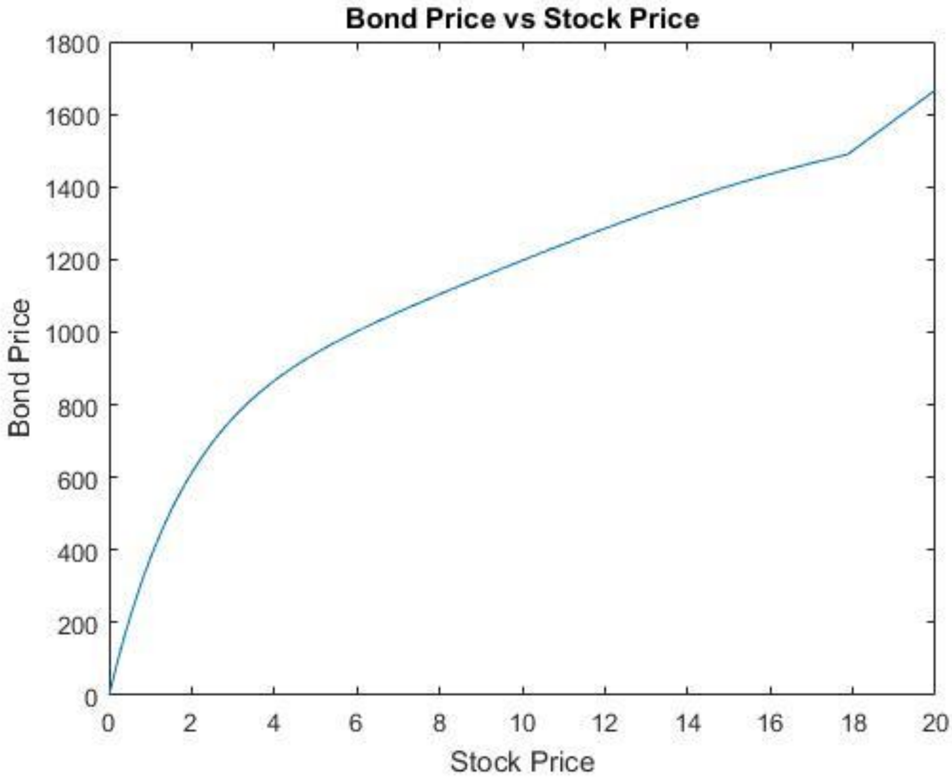


Figure 1: Bond price in dollars vs Stock price in dollars for borrowing rate of 1%

When we set the value of S= $10, we get the bond's value as $1077.8. So, for the stock trading at $10 today, it would be profitable to buy bonds for their face value ($1000) (taking into account borrowing costs given), since $1000 < $1077.8

SOLUTION 2:



As per the graph, at S=$10, we get the bond value as $1197.4.

To calculate the borrowing expense, we first need to calculate Delta. Also, the borrowing expenses rise as compared to previous question due to increase in borrowing costs by 20%. The slope of the graph gives us the delta. At S=$10, we get the delta as 45.3387. Therefor our borrowing expense will be given as,

Borrowing Expense= S * Delta * Rate

Borrowing Expense = 10 * 45.39 * 0.2 = $90.78 per year

As the bond pays $60 every year as its coupon rate is 6%. We realize here that our coupon payment is less than the borrowing cost. As a result, it would be a loss for us if we buy the bond. So, buying the bond is loss-trade and hence, in this current conditions, the company will not be able to sell the bonds.

Also, selling the bonds is not an option due to higher interest rates and Molycorp would not be able to generate funds. Hence, there was a big risk of higher borrowing rates associated with the financial package which poses the threat that Molycorp will not be able to generate funds together, hence its stock price might have declined.

SOLUTION 3:

We initially purchase bonds worth $10million and short the stocks at borrowing costs 1%. Daily, the positioning in stocks is adjusted to delta hedge it with respect to bond position. The missing data is interpolated for bond prices. Thus, Calculating the PnL strategy using information, we get a loss of $2806033. We get a standard deviation of the PL of 274092.2

We also see that the model did a good job for prices near 0. The drawback associated with this model is that the risk-free rate is constant. Also, regarding delta hedging the number of shares to perfectly hedge the bond position might be in decimal if we take precise units of delta. This is not possible; however it will have a very small impact on the overall portfolio. We have also assumed 0 transaction costs.