

**Homework 7 Intraday Realized Variance**

Due Sunday, April 2, 2017

Total 10 points

*Instructions.* This is a group assignment. Groups may include up to 5 people. Please submit a Word or .pdf document with your solutions via the Compass site prior to 5:00 p.m. on Friday, March 31. Please also submit the R, Matlab, or Python scripts/computer programs and data files that you used. (You must use R, Matlab, Python or some other similar package; you may not use Excel.)

**1. Realized variance when the market is open for a highly liquid stock.** (2 points) The file F567C.s2017.HW6.GOOG data.xlsx contains the prices of Google at 1-minute frequency from 6 February 2013 to 20 February 2013, a total of 10 days. These data have been prepared by filling in missing prices with the previous price. For each day, use the 1-minute continuously compounded returns from 8:30 a.m. to 3:00 p.m. to compute the realized variance during the period when the market is open. (This is what the textbook calls  $RV_t^{\text{OPEN}}$ .) What are the 10 estimates of the realized variance? What is the average of the 10 estimates?

**2. Average realized variance estimator** (total 3 points) Compute 2-minute, 5-minute, 10-minute, and 15-minute continuously compounded returns on GOOG. Use these returns and the average realized variance estimator to estimate the realized variance on each of the 10 days based on returns at 2-minute, 5-minute, 10-minute, and 15-minute frequency. (With 10 days and 4 return frequencies you should compute a total of 40 estimates.)

(a) (1 point) Put these 40 realized variance estimates in a table along with the 10 estimates from Question 1. Your table should have 5 columns, corresponding to the 5 return frequencies, and 10 rows, corresponding to the 10 days. Add an eleventh row reporting, for each return frequency, the average across the 10 days of the average realized variance estimators.

(b) (1 point) When the return interval is greater than one minute and the market is not open 24 hours, the average realized variance estimator when the market is open,  $RV_t^{\text{OPEN}}$ , might seem not quite right. For example, there are 26 15-minute returns based on the prices at 8:30, 8:45, 9:00, ..., 14:45, 15:00, but only 25 based on the prices at 8:31, 8:46, 9:01, ..., and 14:46. There are also only 25 returns in the other series of 15-minute returns. Should you somehow adjust for this? If so, how?

*Remark:* This is not in the textbook or lecture notes. You have to think about it, and figure out what to do.

(c) (1 point) Do your estimates of realized variance increase or decrease as the return interval becomes longer? What is the ratio of the average (across days) average realized variance based on 15-minute returns to the average (across days) realized variance based on 1-minute returns? What is the ratio of the average (across days) average realized variance based on 15-minute returns to the average (across days) average realized variance based on 10-minute returns? [Use your results from Question 2(b).]

**3. Realized variance when the market is open for a less liquid stock .** (total 3 points) GE is highly liquid, but less liquid than GOOG.<sup>1</sup> Also, it has a lower price, so the bid-ask spread is larger relative to the stock price. The file F567C.s2017.HW7.GE data.xlsx contains the prices of GE at 1-minute frequency from 6 February 2013 to 20 February 2013, a total of 10 days. These data have been prepared by filling in missing prices with the previous price.

(a) (1 point) Repeat the analysis in Questions 1 and 2(b), but using the prices and returns of GE rather than GOOG. Put the 50 realized variance estimates in a table. As in Question 2, your table should have 5 columns, corresponding to the 5 return frequencies, and 10 rows, corresponding to the 10 days. Add an eleventh row reporting, for each return frequency, the average across the 10 days of the average realized variance estimators.

(b) (1 point) Do your estimates of realized variance increase or decrease as the return interval becomes longer? What is the ratio of the average (across days) average realized variance based on 15-minute returns to the average (across days) realized variance based on 1-minute returns? What is the ratio of the average (across days) average realized variance based on 15-minute returns to the average (across days) average realized variance based on 10-minute returns?

(c) (1 point) Are your estimates of realized variance for GE more or less dependent on the return interval than your estimates for GOOG?

**4. 24-hour realized variance estimator.** (2 points) Use the 15-minute average realized variance estimates from Question 2(b) for each of the 10 days and the two different approaches on slides 57 and 58 of the lecture notes RealizedVariance.ppt to compute estimates of the 24-hour variance of GOOG for each of the 10 days. (Because there are two approaches and 10 days, you should compute a total of  $2 \times 10 = 20$  estimates of the realized variance.)

In using the approach on slide 57, first use the 10 close-to-close returns starting with the close-to-close return from 5 to 6 February and finishing with the close-to-close return from 19 to 20 February to compute the sum of squared close-to-close returns. (The 5 February price is not in the file; you will have to collect it yourself.) Then compute the sum of the 10 realized variance estimators for the 10 days 6 February, 7 February, ..., 20 February. Then compute the ratio of the sum of squared close-to-close returns to the sum of the realized variance estimators. Finally, use the ratio to rescale each of the 10 realized variance estimators.

For the approach on slide 58, simply use the formula on the top of slide 58.

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<sup>1</sup> I am not asking you to analyze the returns for a genuinely illiquid stock because too much data cleaning is required. ☹