# FINANCIAL DERIVATIVES (FIN 512)

### CITIGROUP EXCHANGE OFFER CASE STUDY

JOONHA YOON SUNJIE HOU VEERAJ GADDA

#### **SOLUTION 1:**

Definition of put-call-parity for European options on non-dividend paying stocks:

$$S_t = c(S, K, t, T) - p(S, K, t, T) + K \cdot B_{t,T}$$

 $B_{t,T}$  is the current price of a \$1 face-value bond that matures at T

While when it comes to American options, and whose stock has a non-zero borrowing cost, let C=c, P≥p, we can get a pair of inequalities.

$$P \ge C + B \cdot K - S - borrowing cost$$

And using no-arbitrage,

$$c + K \ge P + S + borrowing cost$$

Then rewriting the above inequalities, and using C=c, we get

$$K \ge S + borrowing cost - C + P \ge B \cdot K$$

The American put-call-parity is violated at the March 18 closing prices. Because according to the stock, put and call prices, and the 3-month T-bill rate 0.25%, we can get

$$K = 5$$
,  $S = 3.08$ ,  $C = 0.20$ ,  $P = 3.00$ ,  $B = e^{-r_t t} = 0.9994$ 

and from the Exhibit 2 in the Citigroup's Exchange Offer(B), we can get the Annualized Citigroup Stock Borrow Costs on March 18<sup>th</sup>, 2009, which was 77.00%, thus the 3-month borrowing would cost 19.25%,

$$5 \ge 3.08 + 3.08 \times 19.25\% - 0.20 + 3.00 = 6.4729 \ge 4.9968$$

which is not right according to March 18th prices.

So the relationship is violated.

### **SOLUTION 2:**

Our assumption is that there will be no borrow cost to sell 7.308 shares of stock when dealing with this problem. Because, in this question, the term 'selling' is used rather than term 'short'. Also, as per answers below, we would get better by synthesizing the equivalent short position out of options. Thus, even if we don't hold the stock, we still need not short

the stock. Also, it would be ambiguous to determine how long we should borrow the stock, which is related to its borrowing cost.

And in fact, if we have the stock and can sell them, it is better to offer to somebody who want to borrow, and earn the borrowing cost. We will use the option to build the stock.

According to the case, only the Series F and Series AA preferred share can be exchanged, which had paid a rate of interest of 8.50%, and a face value of \$25. Thus it can exchange for 7.308 shares of Citigroup common stock. On March 18th, the common stock price is \$3.08, while the Exchange ratio is calculated as follows:

Exchange ratio = 
$$\frac{\$25 \times 0.95}{\$3.25}$$
 = 7.308

The exchange conversion price of common shares is \$3.25. And since the common stock price on March 18<sup>th</sup> is \$3.08, exchange ratio other should be

Exchange ratio other = 
$$\frac{$25 \times 0.95}{$3.08}$$
 = 7.711

thus I would be better off to sell 7.308 shares of stock for one share of the Series F exchangeeligible preferred.

And using the synthesized stock out of options,

$$K + C - P \ge S \ge B \cdot K + C - P$$

the synthesized stock price should be in the scope below

$$2.20 \ge S \ge 2.1968$$

where the exchange ratio should be

$$10.811 = \frac{\$25 \times 0.95}{\$2.1968} \ge Exchange\ ratio\ sythesized \ge \frac{\$25 \times 0.95}{\$2.20} = 10.795$$

This is better than just shorting shares of stock. So the synthesizing the equivalent short position out of options would make us better, i.e., using the options to synthesized the stock and short it, and exchange it with the one share of the exchange-eligible preferred at the exchange ratio of 7.308.

# **SOLUTION 3:**

The case has given us the number of common shares, approximately 5.5 billion, and according to Citi management announcement, the commitments from private investors in Singapore is to convert at least \$12 billion. The management had also allotted a maximum of \$14.9 billion in converted preferred securities from the public investors. The U.S. Treasury had pledged a dollar-for-dollar match of the combined private and public conversion amount, up to a maximum of \$25 billion, at the same conversion price. If the private investors converted their maximum allotment (which is 12.5 billion), and public investors did not participate at all, the U.S. government would convert \$12.5 billion.

Thus, the least convert is \$24 billion made by private investors in Singapore and the U.S. government, and the maximum of the convert would be made by private, public and U.S. government, \$12.5+\$14.9+\$25=\$52.4 billion.

And from the Exhibit 2c, Citigroup Historic Capital Rations, we can get to know the Book Value of Preferred Equity is 70.664 billion on December 31st, 2008.

Since we get the exchange capital to be  $52.4\ billion \ge exchange\ capital \ge 24\ billion$ , the fraction of shares tendered to Citigroup will be about  $74.15\% = \frac{52.4\ billion}{70.664\ billion} \ge exchange\ fraction \ge \frac{24\ billion}{70.664\ billion} = 33.96\%$ .

Furthermore, we can calculate the expected stock price when  $52.4\ billion \ge exchange\ capital \ge 24\ billion$ .

It had to be noticed that the reason why Citigroup want to exchange the preferred with the stock. By transferring capital from the security to the stock, the stock price will go up although the number of stock shares will go up.

The case has given us the number of common shares, approximately 5.5 billion, and thus, suppose the accepted fraction of shares tendered to Citigroup is x, thus the real value of stock price should be stock capital/stock amount,

$$real \ price = \frac{original \ capital + exchange \ capital}{original \ stock \ amount + exchange \ amount}$$
$$= \frac{5.5 \ billion \times \$3.08 + x \times \$70.664 \ billion}{5.5 \ billion + x \times \frac{\$70.664 \ billion}{\$25} \times 7.31}$$

thus,  $$3.3302 \ge real \ price \ge $3.2706$ .

## Question 4

Using the following assumptions, we can calculate the arbitrage strategy:

- 1. There are no transaction fees or other fees except the borrowing cost
- 2. The borrowing cost is calculated by the days
- 3. The prices of stock prices, convert ratio and the preferred price wouldn't change so that we can always short and buy at the same prices

Since the Annualized Citigroup Stock Borrow Costs on March 18<sup>th</sup>, 2009, was 77.00%. And the Publicly Traded Preferred Shares' and stock price details are as follows,

		Series F				Series AA			
Date	stock	Series F	Conv.	Parity	Spread	Series	Conv.	Parity	Spread
		Price	Ratio	Price		AA	Ratio	Price	
						Price			
18-	3.08	15.00	7.31	22.51	7.51	15.00	7.31	22.51	7.51
Mar									

While the common stock price on March 18<sup>th</sup> was \$3.08, thus the short position for x days would cost  $(3.08\times0.77)\times\frac{x}{365}$ , thus with the conversion Ratio to be 7.31, suppose we short 7.31 shares of common stock and buy one preferred share, we will get

$$return = 3.08 \times 7.31 - (3.08 \times 0.77) \times \frac{x}{365} \times 7.31 - 15.00 = 7.5148 - 0.04749698x$$
  
So when  $return \ge 0$ , we get  $x \le 158.21$ .

So if we want to make money by buying the the preferred and short the common on March 18th, we should hold the shorting position no more than 159 days, which would give us a profit greater than zero.