Homework 5 VECMs
Due Monday, 27 February 2017 at 5:00 p.m.
Total 10 points

1. VECM with known cointegating relationship. (total 4 points) Currently two classes of Google stock are available for trading, Class A (ticker symbol GOOGL) and Class C (ticker symbol GOOG).[1] The Excel spreadsheet Google Class A&C.xlsx contains the prices of the two classes of Google stock from April 3, 2014 to February 17, 2017.

You think that there is a known cointegrating relationship between the natural logarithms of the prices of Classes A and C. In particular, you think the long run relationship is that $\ln S_{A,t} - \ln S_{C,t} = \alpha$, where $S_{A,t}$ and $S_{C,t}$ are the prices of Class A and Class C shares, respectively, and α is a constant.

(a) (2 points) Use the data from April 3, 2014 to December 30, 2016 to estimate a VECM of the returns on Class A and C assuming a known cointegrating vector (1, −1). Specifically, let $y_{1t} = \ln(S_{C,t})$ and $y_{2t} = \ln(S_{A,t})$, and estimate the VECM

$$\Delta y_t = \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} + \begin{pmatrix} \phi_{11} \\ \phi_{21} \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} y_{t-1} + \begin{pmatrix} \phi_{12} & \phi_{13} \\ \phi_{22} & \phi_{23} \end{pmatrix} \Delta y_{t-1} + \varepsilon_t$$

What are the estimates of the coefficients ϕ_{ij}?

Remark: the constant α that appears in the cointegrating relationship $\ln S_{C,t} - \ln S_{A,t} = \alpha$ allows the average levels of the prices of Class C and A to differ, which might occur because the two share classes have different voting rights or different liquidity.[2] You do not need to explicitly consider α because the VECM includes the vector of intercepts (ϕ_{10}, ϕ_{20}).

(b) (1 point) Assume that the current time is at the close of trading on January 3, 2017, and that you know the estimates from part (a) and the prices and returns from all previous days up to and including January 4, 2015. Use the VECM coefficients and the available data to predict the returns of Classes A and C from January 3 to January 4.

(c) (1 point) Repeat the exercise in part (b) for every remaining day in January and February 2017. The result should be a vector of 32 predicted returns on Class A for Jan. 4 to 5, Jan. 5 to 6, Jan. 6 to 9, …, and Feb. 16 to 17, and a vector of 32 predicted returns on Class B for the same dates. If you also consider your answer to part (b), you have 33 predicted returns on Class A and

[1] Class B is held by insiders and does not trade in the public market. Some more information about the different classes of Google common stock can be found at http://economix.blogs.nytimes.com/2014/04/02/the-many-classes-of-google-stock/ and http://www.bloomberg.com/news/articles/2014-03-26/google-traders-see-opportunity-in-confusion-on-new-shares.

[2] The differences in voting rights might be unimportant because Google is controlled by the Class B shares.

33 predicted returns on Class C. What are the 33 predicted returns on the Class A shares? What are the 33 predicted returns on the Class C shares?

2. Profits of a hypothetical trading strategy. (2 points) Let $\hat{R}_{At}$ and $\hat{R}_{Ct}$, for t = 1, 2, …, 33, be the predicted returns on Class A and Class C shares for each of the 33 days between Jan. 4, 2017 and Feb. 17, 2017. (For example, $\hat{R}_{A1}$ is the predicted return on Class A from Jan. 3 to Jan. 4.) Consider a trading strategy in which you buy Class A and short Class C if $\hat{R}_{At} - \hat{R}_{Ct} \geq 0$ and you short Class A and buy Class C if $\hat{R}_{At} - \hat{R}_{Ct} < 0$. Let H_t = 1 if $\hat{R}_{At} - \hat{R}_{Ct} \geq 0$ and let H_t = –1 if $\hat{R}_{At} - \hat{R}_{Ct} < 0$. Thus, each day your profit or loss is

$$\text{P/L}_t = H_t \times (S_{\text{A}t-1} \times (\exp[R_{\text{A}t}] - 1) - S_{\text{C}t-1} \times (\exp[R_{\text{C}t}] - 1)).$$

What are the 33 profits or losses P/L$_t$, for t = 1, 2, …, 33? What is the average daily profit? What is the standard deviation of the daily profit?

3. VECM with 4 lags. (total 2 points) Now include an additional 3 lags of Δy_t on the right-hand side of the VECM, that is use Δy_{t-1} through Δy_{t-5}, and re-estimate the model.

(a) (1 point) What are the estimates of the coefficients ϕ_{ij}?

(b) (1 point) Repeat the exercises in 1(b) and 1(c). What are the predicted $\hat{R}_{At}$ and $\hat{R}_{Ct}$, for t = 1, 2, …, 33? Are they similar to the predicted returns you computed in 1(c)?

4. VECM with unknown cointegating relationship. (2 points) You still think that there is a cointegrating relationship between the natural logarithm of the prices of Classes A and C, but you are not willing to assume that the cointegrating vector is (1, –1). Instead, you want to estimate the model with an unknown cointegrating vector. Specifically, let $y_{1t} = \ln(S_{\text{A},t})$ and $y_{2t} = \ln(S_{\text{C},t})$, and estimate the VECM

$$\Delta y_t = \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} y_{t-1} + \begin{pmatrix} \phi_{13} & \phi_{14} \\ \phi_{23} & \phi_{24} \end{pmatrix} \Delta y_{t-1} + \varepsilon_t$$

What are the estimates of the coefficients ϕ_{ij}?