Homework 2: Value-at-Risk Computed Using Various Methods
Due Monday, 6 February 2017 at 5:00 p.m.
Total 10 points

Instructions. This is a group assignment. Groups may include up to five people. Please prepare your solutions in the form of a Word or pdf file and submit your solutions via the Compass site prior to 5:00 p.m. on Monday, 6 February 2017. Please also submit the R, Matlab, or Python script and data file(s) that you use.[1]

When you submit your homework via the Compass site, please do the following:

(1) Name your file HW2_GroupName.xyz, where .xyz is the appropriate file type. For example, if your group name is GoodGroup, then the file name should be HW2_GoodGroup.xyz.
(2) Please include the names of all group members at the top of the first page.
(3) When you include the names of your group members, please capitalize the family name and write it before the given name. That is, write the names as FAMILYNAME Givenname. For example, PEARSON Neil.
(4) Please write the names as they appear in the University's records, that is do not use nicknames. (Or include both given names and nicknames.)
(5) Please include the netid of each group member.

1. Historical simulation method. (3 points) Collect data on the historical daily values of the S&P 500, FT-SE 100, and DAX stock indexes, and the GBP/USD and EUR/USD exchange rates. You will need to collect the daily data for each trading day during 2016, and also for at least 2,000 days prior to the beginning of 2016.

Assume that you are a USD investor, and that you held a position of $1 million invested in each of the three indexes (for total of $3 million) during each trading day in 2016. Specifically, you held this position from the close of trading on 31 December 2015 until the end of the first day of 2016, then adjusted the position so that you had a position of $1 million invested in each of the three indexes and held this position to the next day, and then adjusted the position again and held it to the next day, etc. The last day you should consider is 30 December 2016; this is the gain or loss on the position that you established at the close of trading on 29 December 2016 and held until the close of trading on 30 December 2016.

Use the historical simulation method with a one-day horizon and a confidence level of 99% (equivalently, a probability of 1%) to compute the value-at-risk of your portfolio during each trading day of 2016. In doing this, for each day use the returns on the market factors during the previous 2,000 days. (Remember that the indexes are expressed in their local currencies, but that you are exposed to the USD-denominated return.)

Create a graph to show how your estimates of value-at-risk varied throughout 2016. That is, the horizontal axis should show the dates in 2016, and the graph should show the estimates of the

[1] You must use R, Matlab, or Python; you may not use Excel.

value-at-risk for each date. (Your graph should show all four time series of VaR's for Questions 1-4 on the same graph.) Include the graph in the .doc or .pdf file that you submit via the Compass site.

Remark. What should you do about days when not all index values or exchange rates are available? For this question, you should drop days in which not all relevant data are available. For example, if the value of one of the indexes is missing on day t then drop day t and treat the return from day $t - 1$ to day $t + 1$ as a one-day return. (Note that this is not the approach that usually would be used in the "real world.")

2. Delta-Normal method. (2 points) Use the delta-Normal method with a one-day horizon and a confidence level of 99% (equivalently, a probability of 1%) to compute the value-at-risk of your portfolio during each trading day of 2016. In doing this, for each day use the ordinary returns on the market factors during the most recent 2,000 days to estimate the covariance matrix. In estimating the covariance matrix use the equally weighted estimator on p. 68 of Christoffersen (2012), and assume that the expected returns are zero.

Remark. Use simple (ordinary) returns, not continuously compounded returns.

3. Delta-Normal method using the exponentially weighted covariance estimator. (2 points) Read about the RiskMetrics exponentially weighted variance estimator on pp. 69-70 of Christoffersen (2012). Although Christoffersen does not discuss it, the generalization to the exponentially weighted covariance estimator should be obvious. Compute the delta-Normal value-at-risk for each trading day in 2016, where for each day the covariance matrix is estimated using the exponentially weighted estimator.

4. Historical simulation using rescaled returns. (3 points) For each day t and index i, you have a database of past returns; that is, at the end of day t for index i you have the returns $R^i_{t+1-\tau}$, for $\tau = 1, 2, \ldots, 2{,}000$ (that is, for the 2,000 trading days up to and including date t). From your solution to Question 3, you have, for each day, a volatility estimate for each index return. Using σ^i_t to denote the volatility estimate for date t and index i, for each date t and index i you have a database of past volatilities $\sigma^i_{t+1-\tau}$, for $\tau = 1, 2, \ldots, 2{,}000$. You also have a volatility estimate for the next day, σ^i_{t+1}.

For each date t, construct a database of normalized returns $u^i_{t+1-\tau} = (\sigma^i_{t+1} / \sigma^i_{t+1-\tau}) \times R^i_{t+1-\tau}$, for $\tau = 1, 2, \ldots, 2{,}000$. The idea is that the standard deviation of these normalized returns is equal (assuming the model used to compute σ^i_t is correct) to the standard deviation σ^i_{t+1} of the next day's return R_{t+1}, but that the normalized returns still reflect whatever non-Normality is in the original returns R_t. Use the normalized returns to compute the historical simulation value-at-risk for each day during 2016.

Please submit your R, Matlab, or Python computer program via the Compass site.