A Comparative Study for Single Image Blind Deblurring

CVPR 2016

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Bradley–Terry model

- Given pairwise comparison votes, we want to fit a global score/ranking for each evaluated method.

- Input: a winning matrix $C \in R^{N \times N}$ where

$$C_{ij} = \begin{cases} \#\text{times that } i \text{ beats } j & , if\ i \neq j \\ 0 & , if\ i = j \end{cases}$$

  – N is the number of evaluated methods

- Output: N scores $s_1, s_2, \cdots, s_N$

Bradley–Terry model

- Assume the probability that users chooses i over j is:

$$P_{ij} = \frac{e^{s_i}}{e^{s_i} + e^{s_j}}$$

- We want to maximize the likelihood:

$$L(s) = \prod_{i=1}^{N} \prod_{j=1, j \neq i}^{N} \left(P_{ij}\right)^{C_{ij}}$$

Bradley–Terry model

- Minimize the negative log likelihood:

$$E(s) = -\sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C_{ij} \log P_{ij}$$

$$= \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C_{ij} \log(e^{s_i} + e^{s_j}) - \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} C_{ij} \log(e^{s_i})$$

$$= E_1(s) - E_2(s)$$

Bradley–Terry model

$$E_1(s) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} C_{ij} \log(e^{s_i} + e^{s_j})$$

$$\frac{\partial E_1(s)}{\partial s_k} = \boxed{\sum_{j=1, j\neq k}^{N} \frac{C_{kj} \cdot e^{s_k}}{e^{s_k} + e^{s_j}}} + \boxed{\sum_{i=1, i\neq k}^{N} \frac{C_{ik} \cdot e^{s_k}}{e^{s_i} + e^{s_k}}}$$

when $i = k$ when $i \neq k$

$$= \sum_{v=1, v\neq k}^{N} \frac{C_{kv} \cdot e^{s_k}}{e^{s_k} + e^{s_v}} + \sum_{v=1, v\neq k}^{N} \frac{C_{vk} \cdot e^{s_k}}{e^{s_v} + e^{s_k}} = \sum_{v=1, v\neq k}^{N} \frac{N_{kv} \cdot e^{s_k}}{e^{s_k} + e^{s_v}}$$

replace j with v replace i with v

where $N_{kv} = C_{kv} + C_{vk}$

Bradley–Terry model

$$E_2(s) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} C_{ij} \log(e^{s_i})$$

$$= \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} C_{ij} s_i$$

$$\frac{\partial E_2(s)}{\partial s_k} = \sum_{j=1, j\neq k}^{N} C_{kj}$$

Bradley–Terry model

- Minimize the negative log likelihood:

$$\frac{\partial E(s)}{\partial s_k} = \frac{\partial E_1(s)}{\partial s_k} - \frac{\partial E_2(s)}{\partial s_k}$$

$$= \sum_{v=1, v\neq k}^{N} \frac{N_{kv} \cdot e^{s_k}}{e^{s_k} + e^{s_v}} - \sum_{j=1, j\neq k}^{N} C_{kj}$$

- Since $C_{ii} = 0, N_{ii} = 0$:

$$\frac{\partial E(s)}{\partial s_k} = \sum_{v=1}^{N} \frac{N_{kv} \cdot e^{s_k}}{e^{s_k} + e^{s_v}} - \sum_{j=1}^{N} C_{kj}$$

Bradley–Terry model

- Let $\frac{\partial E(s)}{\partial s_k} = 0$, we can derive the following EM-like iterative form:

$$e^{s_k^{t+1}} = \frac{\sum_{j=1}^{N} C_{kj}}{\sum_{v=1}^{N} \frac{N_{kv}}{e^{s_k^t} + e^{s_v^t}}}$$

$$s_k^{t+1} = \log\left(\frac{\sum_{j=1}^{N} C_{kj}}{\sum_{v=1}^{N} \frac{N_{kv}}{e^{s_k^t} + e^{s_v^t}}}\right)$$