Top-20 Training Program
(Recursion & Divide-Conquer)

Apply Recursive/Divide-conquer thought process to solve the following problems.

Problem1: Anchor Index

Let A be sorted array of n distinct integers, some of which may be negative. An index is called an anchor if a[i] = i. Find an efficient algorithm for finding an anchor in A if one exists.

Problem2: Phone Pad Letter Combiner

Phone pads have letters associated with digits: 3 letters for each digit except #0. Assume the following letter mappings for each button digit:

    1 -> a, b, c
    2 -> d, e, f
    3 -> g, h, i
    4 -> j, k, l
    5 -> m, n, o
    6 -> p, q, r
    7 -> s, t, u
    8 -> v, w, x
    9 -> y, z

Write a function that prints all possible combinations of your home phone number in letters. The order in which the result is printed doesn't matter and you can pass the input mappings as 2-d array.

Example

Input phone number: 283

The output should be:

"dvg", "evg", "fvg", "dwg", "ewg", "fwg", "dxg", "exg", "fxg",
"dvh", "evh", "fvh", "dwh", "ewh", "fwh", "dxh", "exh", "fxh",
"dvi", "evi", "fvi", "dwi", "ewi", "fwi", "dxi", "exi", "fxi".

Problem3: Simple Number Multiplication

A number is called simple if it can be written by repeating just one digit. For example, 4444, 6666 are simple numbers. Write an efficient function to multiply two simple numbers of n digits each in O(n) time, where we count of one digit addition or multiplication as basic operation. Assume that n is always a power of 2.

Problem4: All Combinations of String Letters

Given a string S, write a function that displays all the combinations of the characters from the string. Assume that input string will not contain any repetition of characters.
Example
Input String: "ABC",
Output combinations: "A", "B", "C", "AB", "AC", "BC" and "ABC"

Problem5: Bi-repetitive Matrix Addition and Multiplication

For n a power of 2, an n×n matrix M is Bi-Repetitive if either n=1 or when n>1, M has the form

    M = B B
          C C

where, B and C are in turn $(n/2) \times (n/2)$ Bi-Repetitive matrices.

a) Write an efficient function to add two Bi-Repetitive matrices. What is the time complexity of your solution in terms of number of arithmetic operations?

b) Write an efficient function to multiply two Bi-Repetitive matrices. What is the time complexity of your solution in terms of number of arithmetic operations?