Video 2: The general case using matrix notation

The variables at stake for N assets:

- w: The Nx1 column-matrix of portfolio weights

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

- R: The Nx1 column-matrix of asset returns

$$\mathrm{R} = \begin{bmatrix} \mathrm{R}_1 \\ \mathrm{R}_2 \\ \vdots \\ \mathrm{R}_\mathrm{N} \end{bmatrix}$$

- μ: The Nx1 column-matrix of expected returns

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_\mathrm{N} \end{bmatrix}$$

The variables at stake for N assets:

- Σ: The NxN covariance matrix of the N asset returns

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

Covariances outside the diagonal

Variances on the diagonal

Generalizing from 2 to N assets

Portfolio return

$w_1 * R_1 + w_2 * R_2$	$w_1 * R_1 + \ldots + w_N * R_N$

Portfolio expected return

$w_1 * \mu_1 + w_2 * \mu_2$	$w_1 * \mu_1 + \ldots + w_N * \mu_N$

Portfolio variance

$w_1^2 * var(R_1) + w_2^2 * var(R_2) + 2 * \mathrm{w}_1 * \mathrm{w}_2 * cov(R_1, R_2)]$	$w_1^2 * var(R_1) + \cdots + w_N^2 * var(R_N) + 2 * \mathrm{w}_1 * \mathrm{w}_2 * cov(R_1, R_2)] + \ldots + 2 * \mathrm{w}_1 * \mathrm{w}_N * cov(R_1, R_N)] + 2 * \mathrm{w}_2 * \mathrm{w}_3 * cov(R_2, R_3)] + \ldots + 2 * \mathrm{w}_2 * \mathrm{w}_N * cov(R_2, R_N)] + \ldots + 2 * \mathrm{w}_{N-1} * \mathrm{w}_N * cov(R_{N\_1}, R_N)]$

Matrices simplify the notation

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

$$\mathrm{w}' = \begin{bmatrix} \mathrm{w}_1 & \mathrm{w}_2 & \cdots & \mathrm{w}_N \end{bmatrix}$$

Matrices simplify the notation

Portfolio return

$w_1 * R_1 + \ldots + w_N * R_N$	$w'R$

Portfolio expected return

$w_1 * E[R_1] + \ldots + w_N * E[R_N]$	$w'\mu$

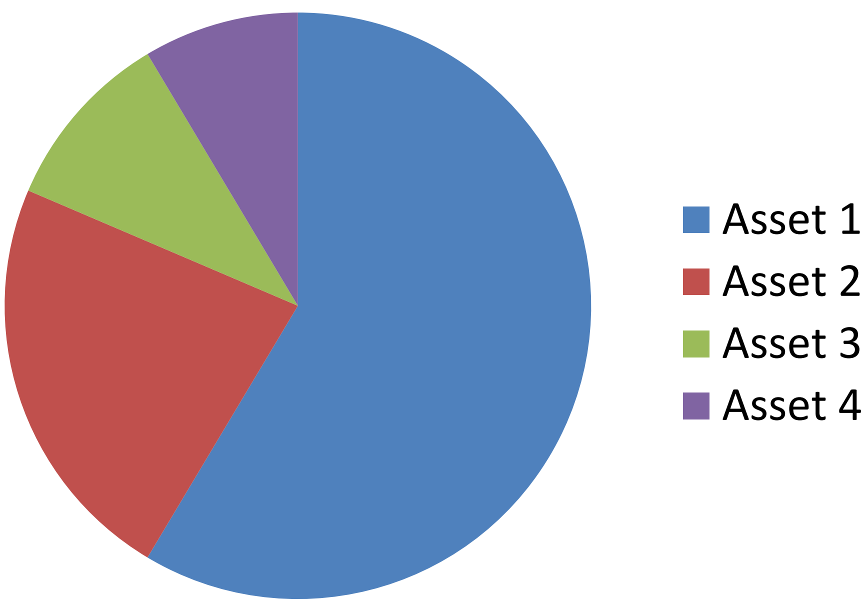
Portfolio variance

$w_1^2 * var(R_1) + \cdots + w_N^2 * var(R_N) + 2 * w_1 * w_2 * cov(R_1, R_2)] + \ldots + 2 * w_1 * w_N * cov(R_1, R_N)] + 2 * w_2 * w_3 * cov(R_2, R_3)] + \ldots + 2 * w_2 * w_N * cov(R_2, R_N)] + \ldots + 2 * w_{N-1} * w_N * cov(R_{N\_1}, R_N)]$	$w'\Sigma w$

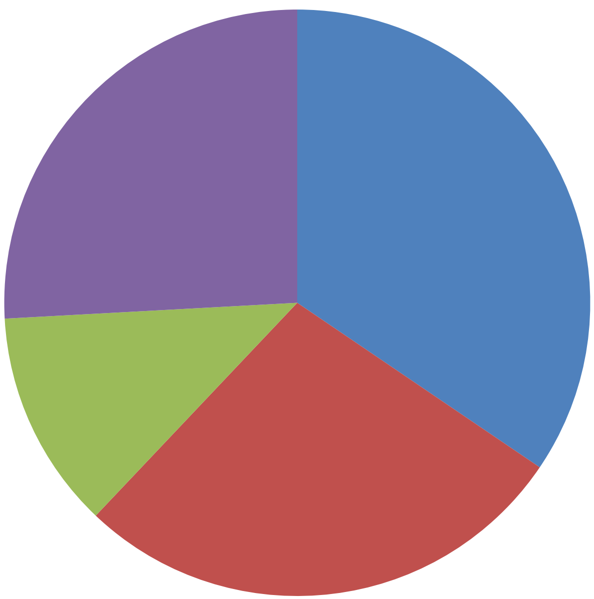
Video 3: The portfolio risk budget

Who did it?

The capital allocation budget showing the percentage of total capital invested in each asset



The risk budget showing the percentage of portfolio volatility risk caused by each asset



Decomposing portfolio volatility in risk contributions

$$Portfolio\ volatility = \sum_{i=1}^{N} RC_i$$

where: $RC_i = \frac{(\Sigma w)_i}{\sqrt{w'\Sigma w}}$

! Note that the risk contribution of asset i depends not only on its own weights, but on the complete matrix of weights w and on the full covariance matrix Σ

The percentage risk contribution

$$\%RC_i = \frac{RC_i}{portfolio\ volatility} \quad \text{with: } \sum_{i=1}^{N} \%RC_i = 1$$

Relatively more risky assets:

$\%RC_i > w_i$

Relatively less risky assets:

$\%RC_i < w_i$