INTRODUCTION TO PORTFOLIO ANALYSIS

Using Matrix Notation

Variables at Stake for N Assets

- w: the N x 1 column-matrix of portfolio weights

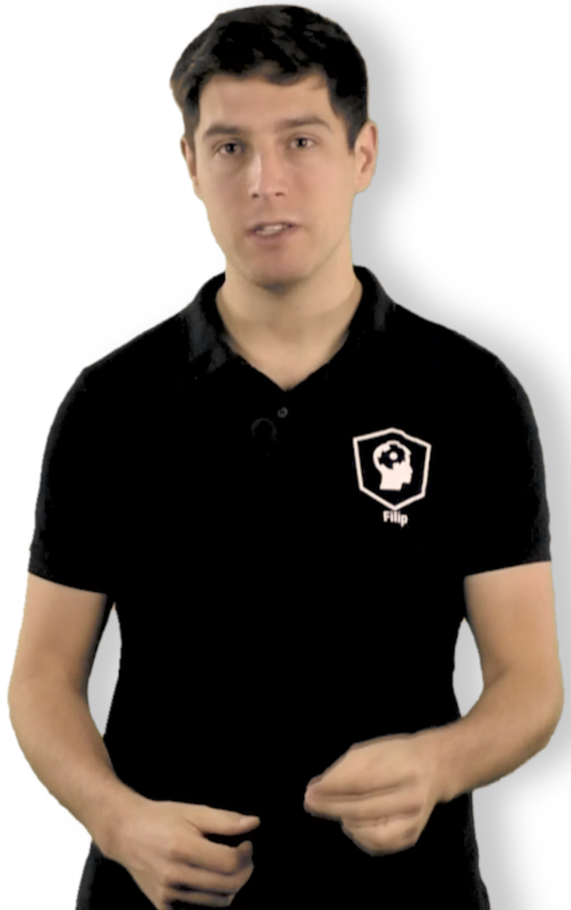
$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

- R: the N x 1 column-matrix of asset returns

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

- μ: the N x 1 column-matrix of expected returns

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix}$$

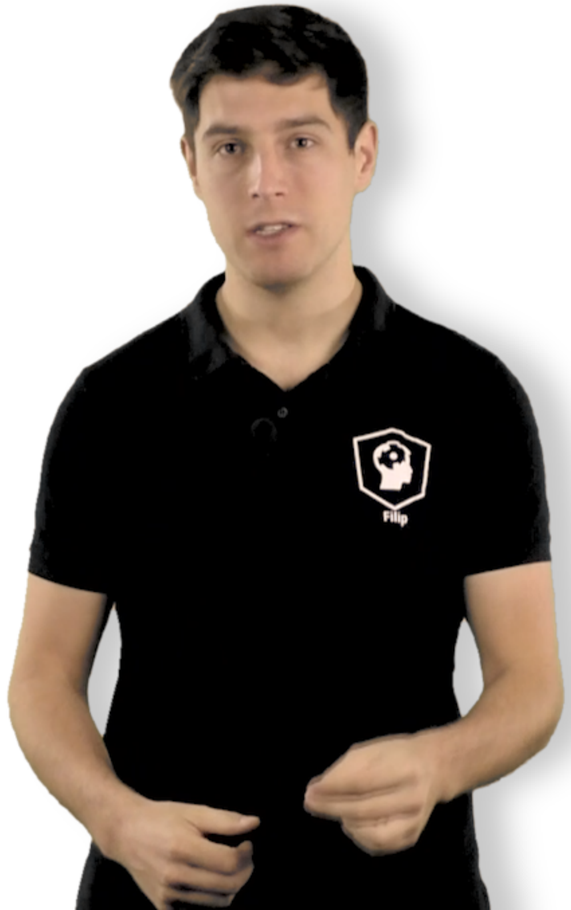


Variables at Stake for N Assets

- Σ: The N x N covariance matrix of the N asset returns:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$

Covariance: Outside Diagonal
Variance: On Diagonal



Generalizing from 2 to N Assets

Portfolio Variance

Portfolio Return

$$w_1^2 * var(R_1) + w_2^2 * var(R_2) + 2 * w_1 * w_2 * cov(R_1, R_2)$$

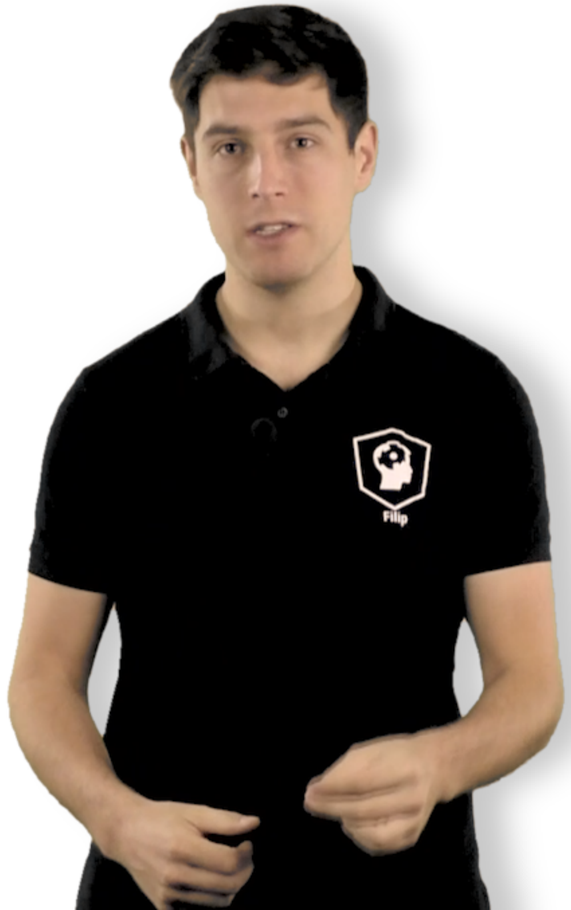
$w_1 * R_1 + w_1 * R_2$

$w_1 * R_1 + \ldots + w_N * R_N$

$$w_1^2 * var(R_1) + \ldots + w_N^2 * var(R_N) + 2 * w_1 * w_2 * cov(R_1, R_2) + \ldots + 2 * w_{N-1} * w_N * cov(R_{N-1}, R_N)$$

$w_1 * \mu_1 + w$

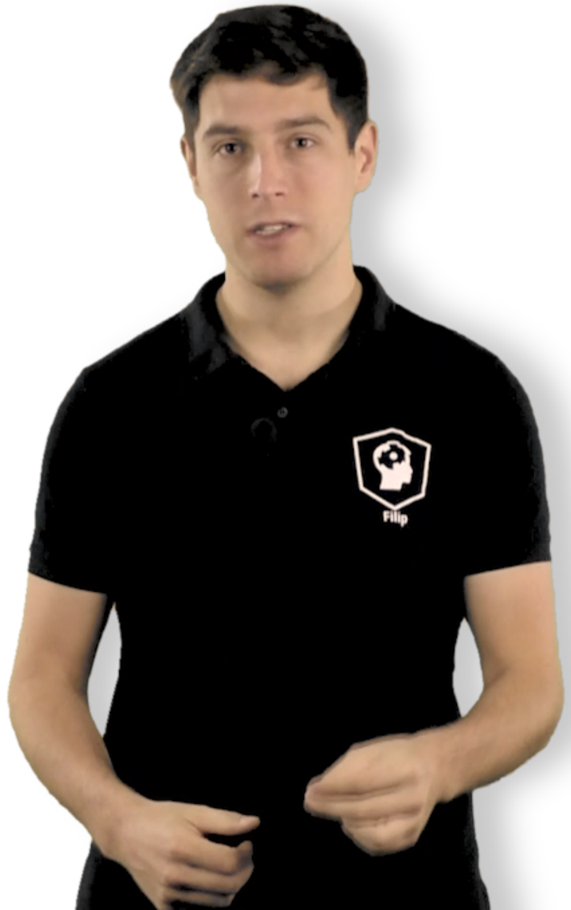
$..+ w_N * \mu_N$



Matrices Simplify the Notation

- Avoid large number of terms by using matrix notation
- We have 4 matrices:
  - weights (w), returns (R), expected returns (μ), and covariance matrix (Σ)

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad \mathbf{w}' = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_N \end{bmatrix}$$



Simplifying the Notation

Portfolio Variance

$w_1 * R_1 + \ldots + w_N * R_N$

$$w_1^2 * var(R_1) + \ldots + w_N^2 * var(R_N) + 2 * w_1 * w_2 * cov(R_1, R_2) + \ldots + 2 * w_{N-1} * w_N * cov(R_{N-1}, R_N)$$

w'R

Portfolio Expected Return

$w_1 * \mu_1 + w_2 * \mu_2$

$w'\Sigma w$

w'μ