



INTRODUCTION TO PORTFOLIO ANALYSIS

Modern portfolio theory of Harry Markowitz



Portfolio weights are optimal...

 When they optimize an objective function while satisfying the constraints

Possible objective functions	Possible constraints
Maximize expected return	Only positive weights
Minimize the variance	The weights sum to 1 (all capital needs to be invested)
Maximize the Sharpe ratio	Portfolio expected return equals a target value

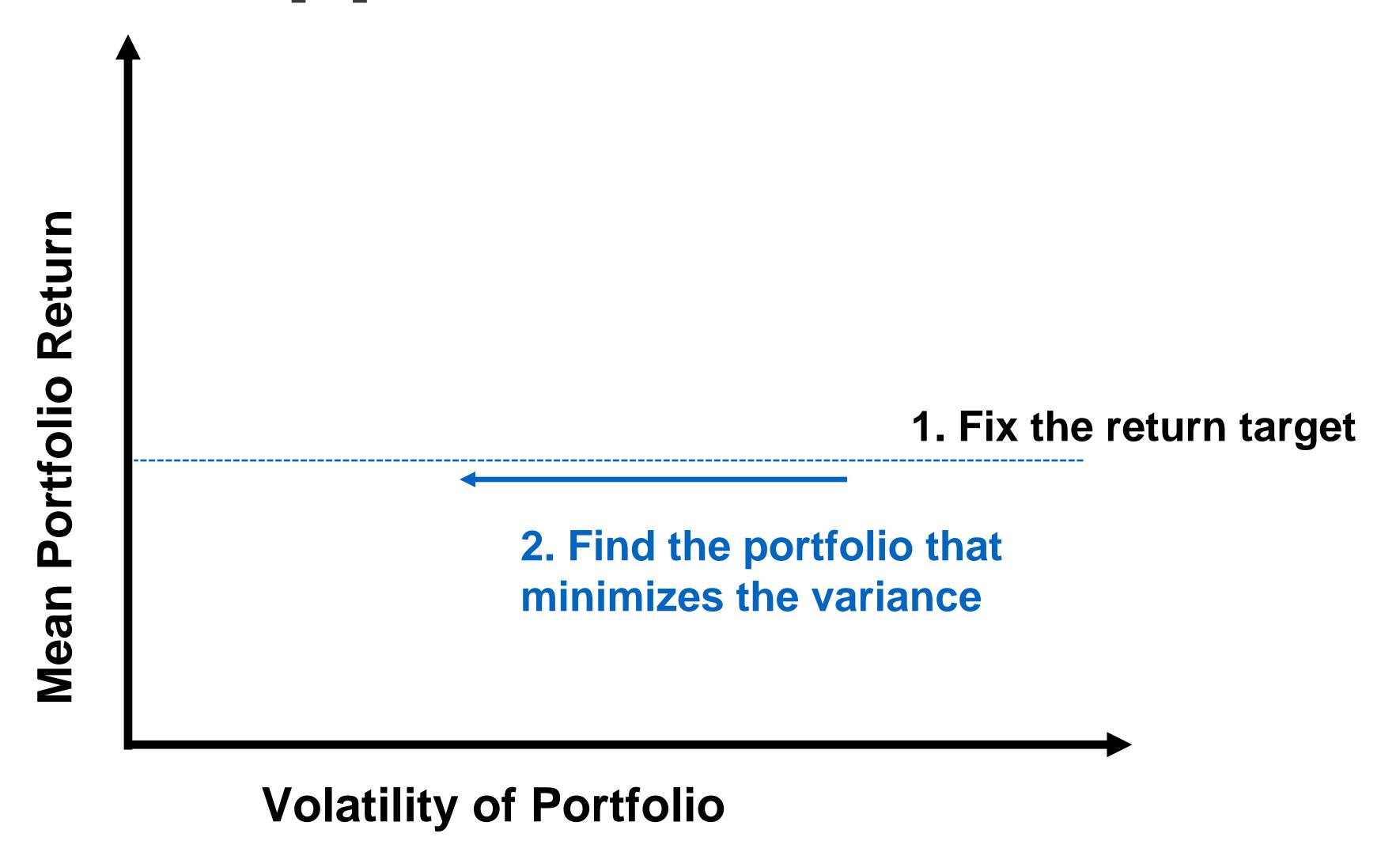




Harry Markowitz

 Minimize the portfolio variance under the constraint that the expected return should be equal to a prespecified return target

The approach of H. Markowitz







Mean

The solution is mean-variance

efficient

Portfolio with lower volatility and higher expected return do not exist

1. Fix the return target

2. Find the portfolio that minimizes the variance



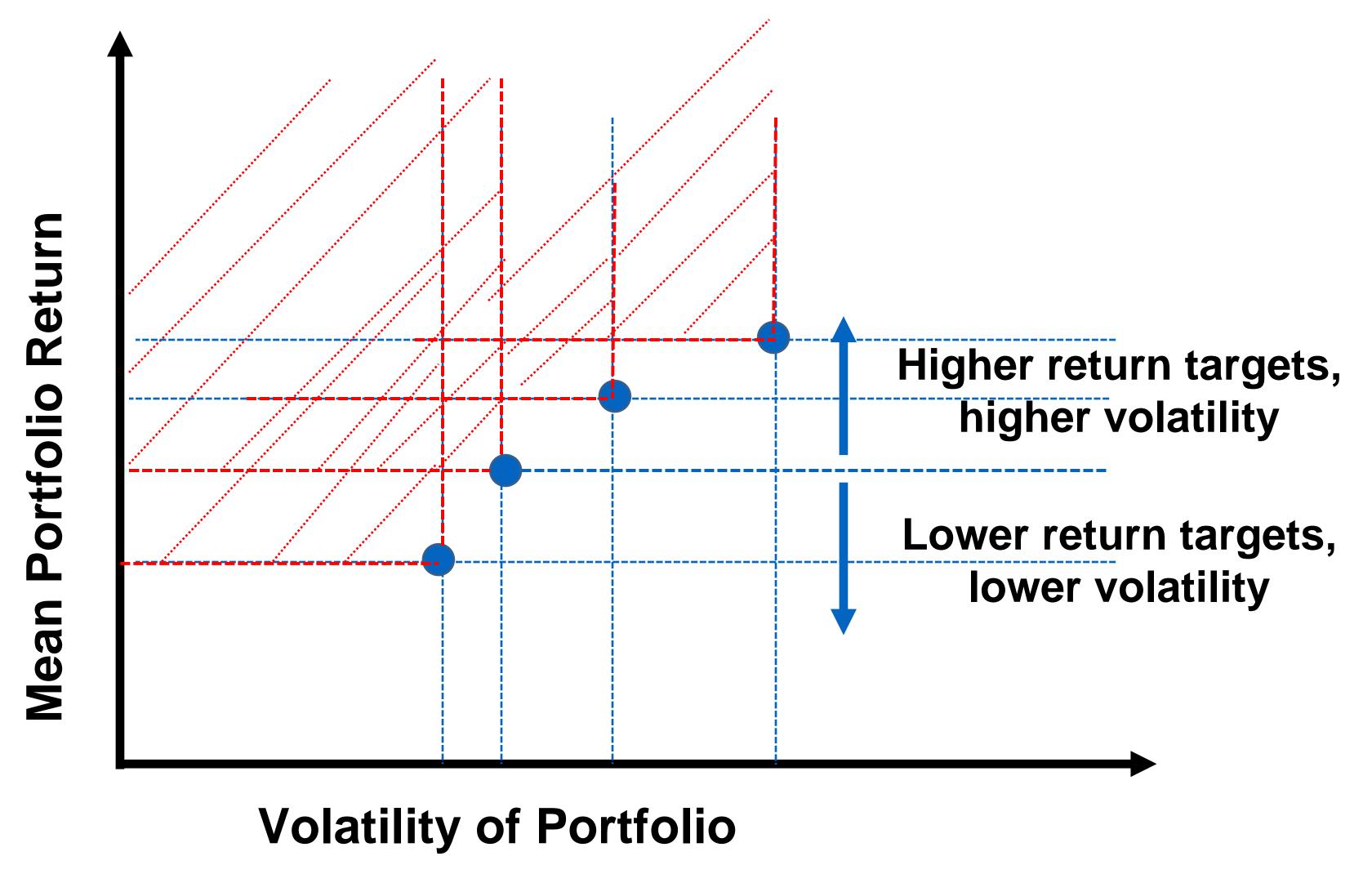




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The efficient frontier

Changing the return target





Return

Portfolio

Mean

The efficient frontier

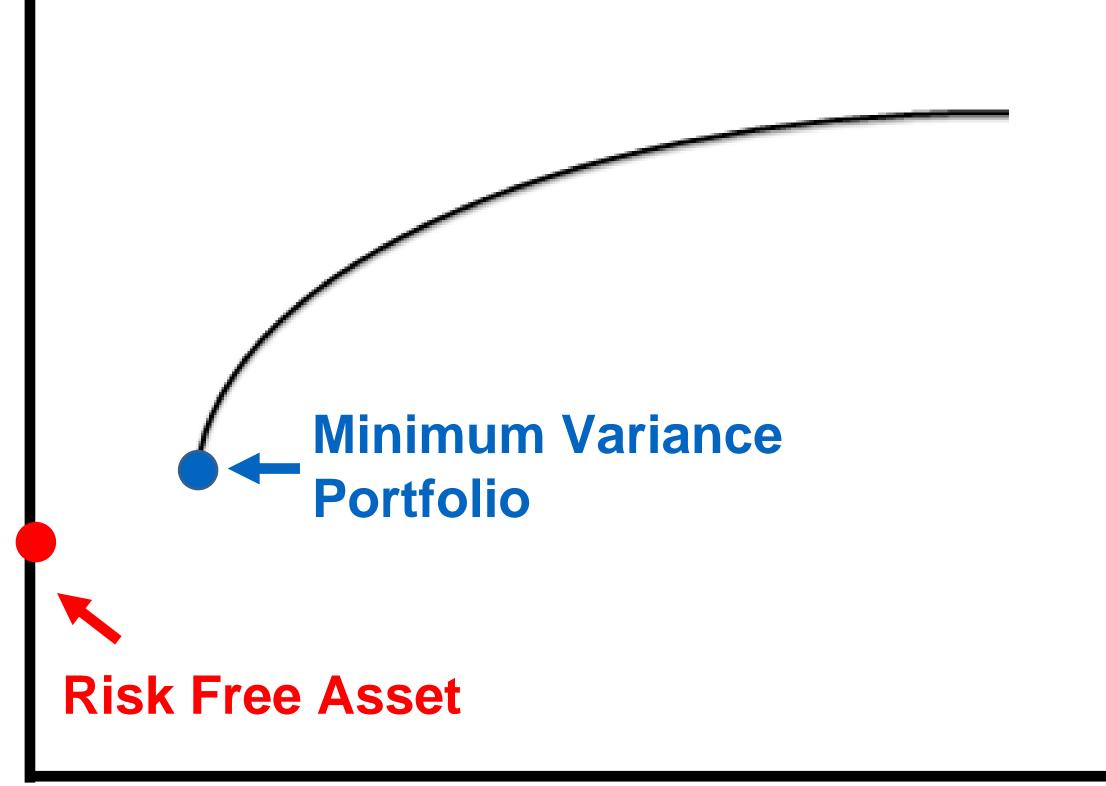
Each point on the efficient frontier is an optimal combination of assets: it minimizes the variance for a given return target These dots represent portfolios that are below the frontier and are thus dominated: there exist other portfolio with higher return at a lower volatility





The minimum variance portfolio

Mean Portfolio Return







The maximum Sharpe ratio

portfolio **Tangency portfoliois the** maximum Sharpe ratio portfolio Return Mean Portfolio **Risk Free Asset**



Time for practice

→ Construct the efficient frontier through a for-loop over a sequence of return targets

Maximum target return

Minimum target return







INTRODUCTION TO PORTFOLIO ANALYSIS

In-sample versus out-of-sample evaluation





Bad news: Estimation error

Limitation to data-driven portfolio allocation:

What we are using in practice



What we should be using in theory

Estimated mean $\hat{\mu}$

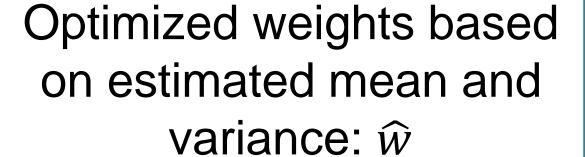
True (unkown) mean µ

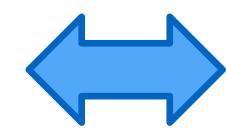
Estimated variance $\hat{\sigma}^2$

True (unkown) variance σ^2









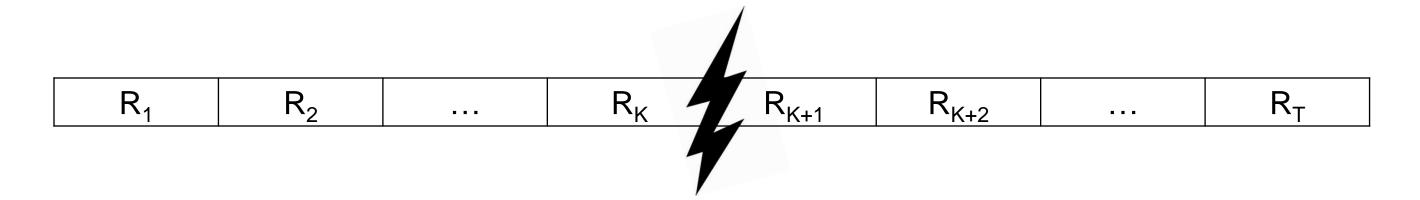
True optimal portfolio: w





Good news: Opportunities for data analysts

- Do not ignore estimation error;
- Use split-sample analysis to do a realistic evaluation of portfolio performance



Estimation sample used to find the optimal weights

Out-of-sample evaluation sample to give a realistic view on portfolio performance





No look-ahead bias in the optimized weights

Split-sample design matches with investor who

Uses at time K the returns $R_1,...,R_K$ to compute optimal weights

Invests between time K and time T using those optimized weights

Time

Function `window' to do split-sample analysis in R.