Video 1: The drivers in case of two assets

The random nature of future returns

Successfullly optimizing the portfolio requires to form expectations:

* about what the portfolio return will be on average (mean)

* and how far off it may be (variance)

Why? Because the portfolio return is a random variable.

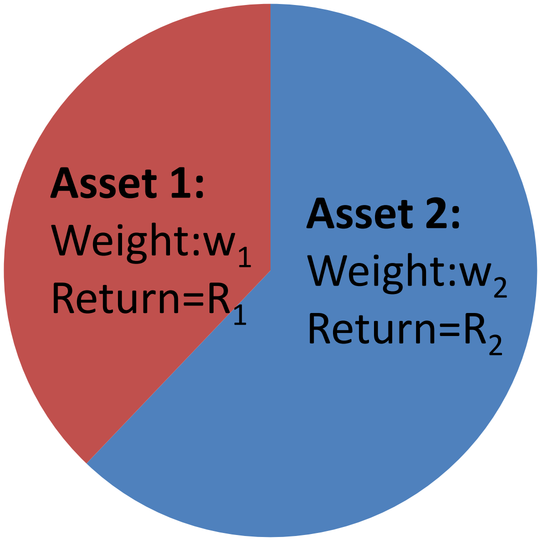
From describing past performance to making expectations about the future

	Mean portfolio return
Computed on a sample of T historical returns	$\hat{\mu} = \frac{R_1 + R_2 + \cdots + R_T}{T}$
When the return is a random variable	$\mu = \mathbf{E}[R]$ (the best possible prediction of the future return)

	Portfolio return variance
Computed on a sample of T historical returns	$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \cdots + (R_T - \hat{\mu})^2}{T - 1}$
When the return is a random variable	$\sigma^2 = \mathbf{E}[(R - \mu)^2]$ (the best possible prediction of the squared deviation of the return from the mean)

What drives the mean and variance?

- Assume two assets:



- Then: Portfolio return = $w_1 * R_1 + w_2 * R_2$
- And thus:

$$\boxed{E[\text{Portfolio return}] = w_1 * E[R_1] + w_2 * E[R_2]}$$

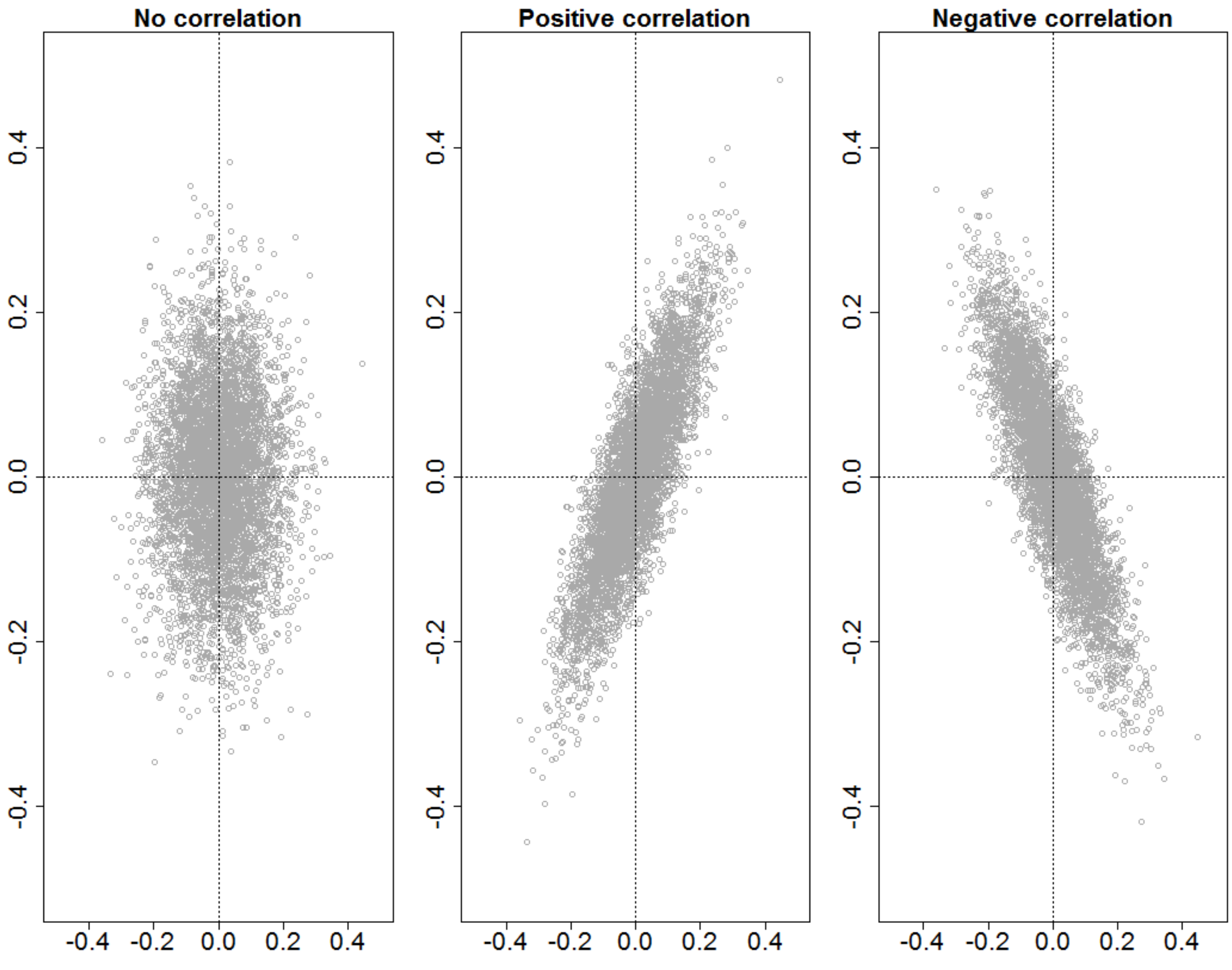
Portfolio return variance

$$var(portfolio\ return)$$
$$= \ w_1^2 * var(R_1) + w_2^2 * var(R_2)$$
$$+2 * \mathrm{w}_1 * \mathrm{w}_2 * \boldsymbol{E[(R_1 - E[R_1]) * (R_2 - E[R_2])]}$$

= the covariance between asset return 2 and 1, written as:

$Cov(R_1, R_2)$ =StdDev(R_1)*StdDev(R_2)*corr(R_1, R_2)

Correlation



Take-away formulas

E[Portfolio return] = w_1*E[R_1]+ w_2*E[R_2]

$$var(portfolio\ return) = w_1^2 * var(R_1) + w_2^2 * var(R_2) + 2 * \mathrm{w}_1 * \mathrm{w}_2 * cov(R_1, R_2)]$$