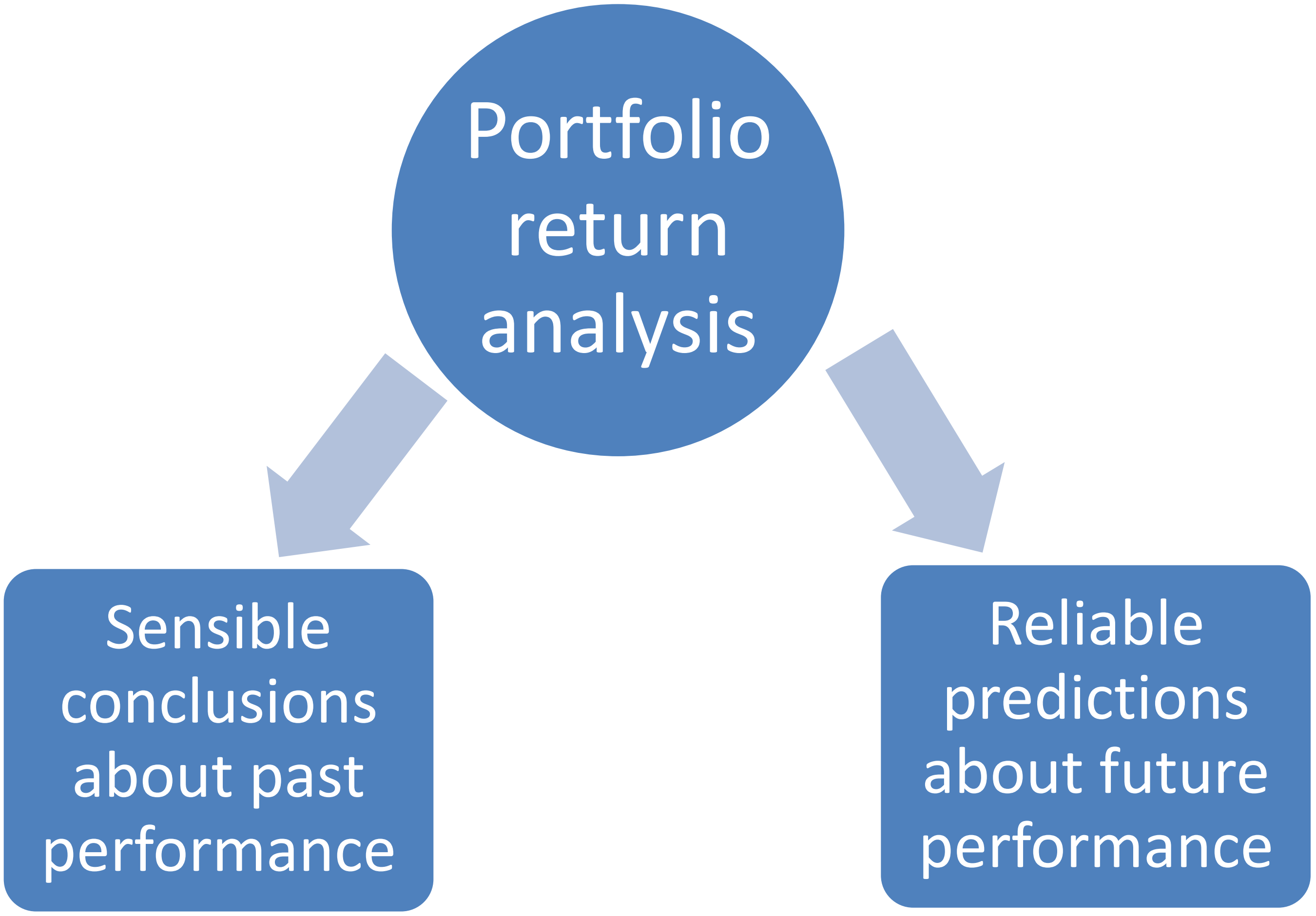
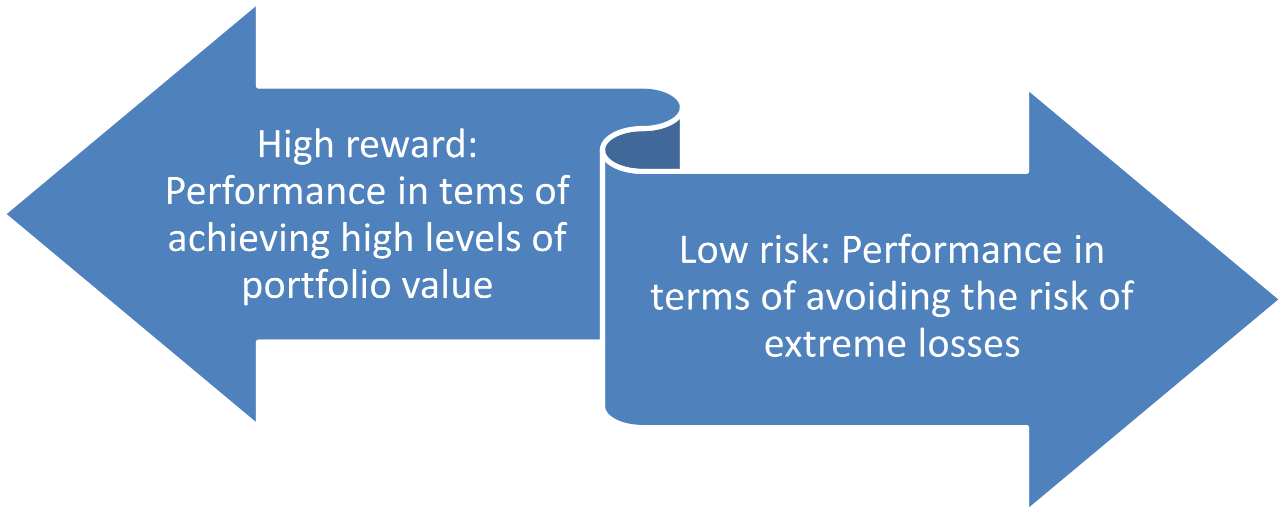
Video 1: The different dimensions of portfolio performance

Interpretation of portfolio returns



Risk versus reward



Need for performance measures

Portfolio returns

Performance and risk measures computed from those returns

Interpretation

Arithmetic mean return

- Focus on mean return and volatility
- Assume a sample of T portfolio return observations:

$$R_1, R_2, \dots, R_T$$

- [Measure for reward] Arithmetic mean return is given by:

$$\hat{\mu} = \frac{R_1 + R_2 + \cdots + R_T}{T}$$

- It shows how large the portfolio return is on average.

Portfolio volatility

- [Measure for risk] Variance of the portfolio return:

$$\hat{\sigma}^2 = \frac{(R_1 - \hat{\mu})^2 + (R_2 - \hat{\mu})^2 + \cdots + (R_T - \hat{\mu})^2}{T - 1}$$

- Portfolio return volatility:

$$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$$

No linear compensation in returns

- Mismatch between average return and effective return
- Illustration: + 50% return and – 50% return, then:

$final\ value = initial\ value * (1+0.5)*(1-0.5)$

$= 0.75*initial\ value$

Average return = (0.5 – 0.5)/2 = 0

- Solution: Use of geometric return.

Geometric mean return

- General formula geometric mean for a sample of T portfolio return observations $R_1, R_2, \ldots,$ R_T:

$$geometric\ mean = [(1+R_1)*(1+R_2)*\cdots(1{+}R_T)]^{\frac{1}{T}}\text{ -1}$$

- Example of a +50% and -50% return:

$$geometric\ mean = [(1+0.50)*(1-0.50)]^{\frac{1}{2}}\text{ -1=}[0.75]^{\frac{1}{2}}\text{ -1=0.866-1=-13.4\%}$$

Application to the S&P 500 portfolio

http://dextergonzaga.info/indexed-account-options/

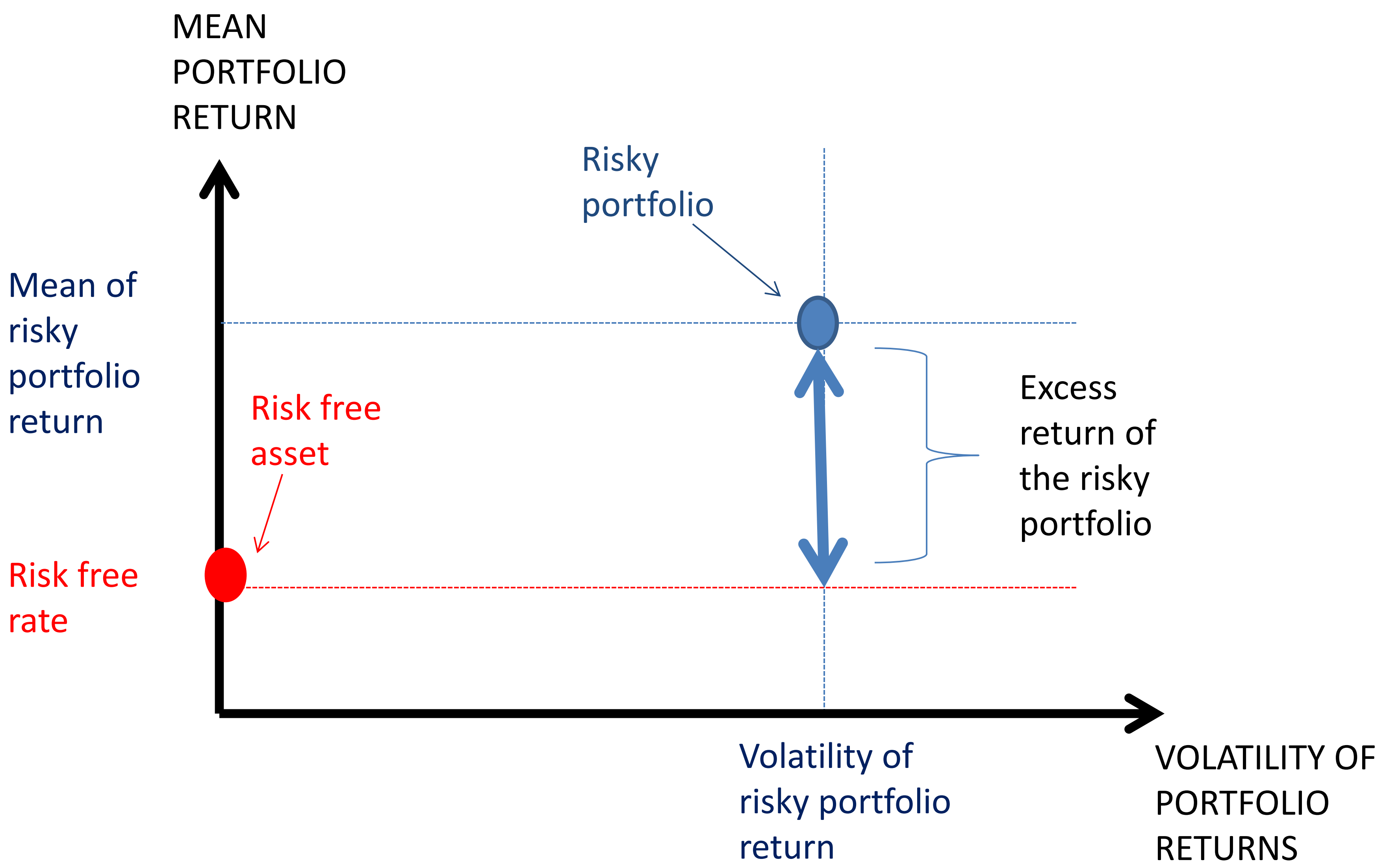
Video 2: The (annualized) Sharpe ratio

Benchmarking performance with the risk free asset

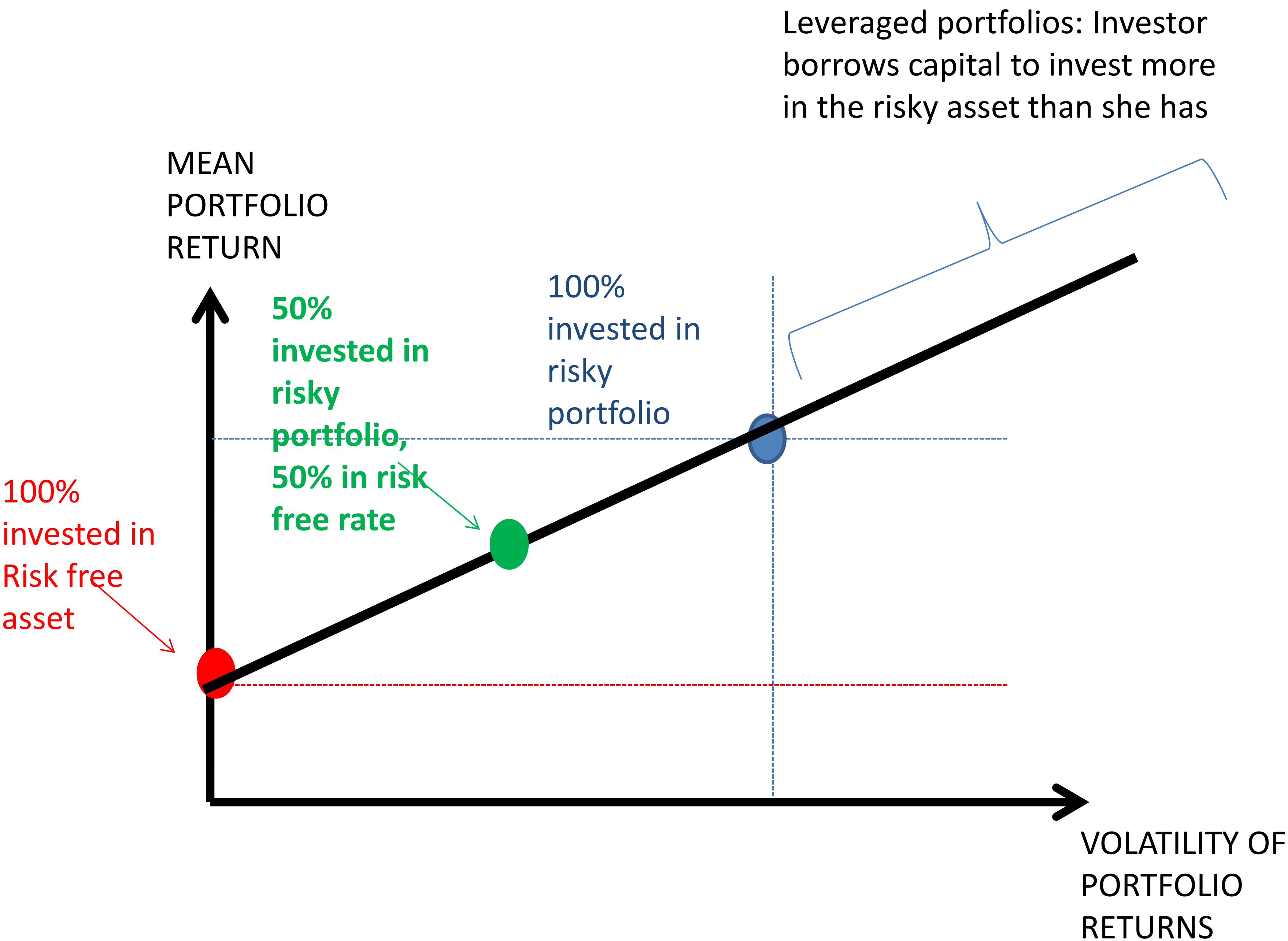
Risky portfolio **E.g. portfolio invested in stocks, bonds, real estate, commodities**
Reward measured by the mean portfolio return
Risk measured by the volatility of the portfolio returns

Risk free asset **E.g. US Treasury Bill**
Reward measured by risk free rate
No risk: the return is always exactly equal to the risk free rate, and volatility is thus 0.

Title: The risk-return trade-off visualized

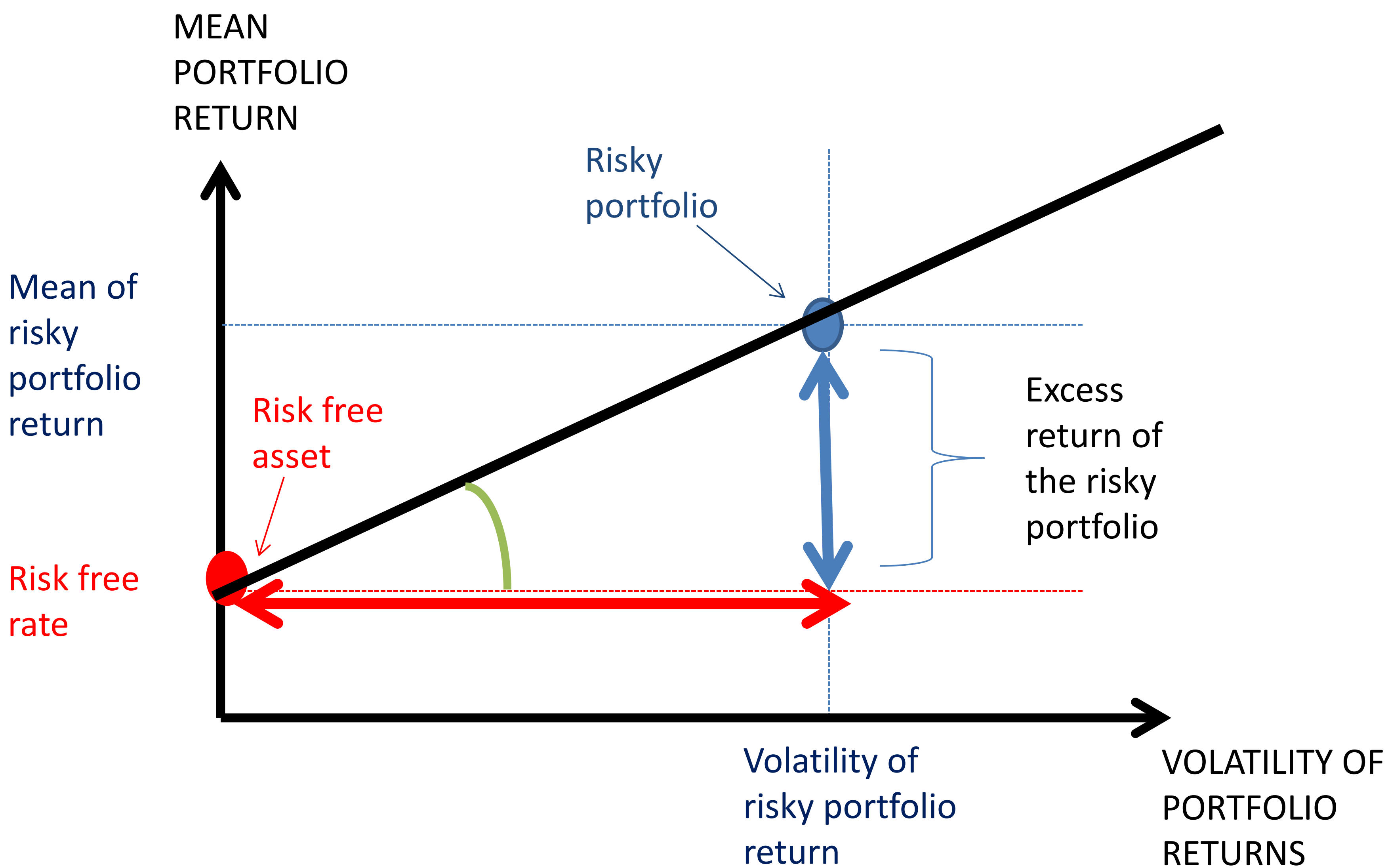


Title: Capital Allocation Line

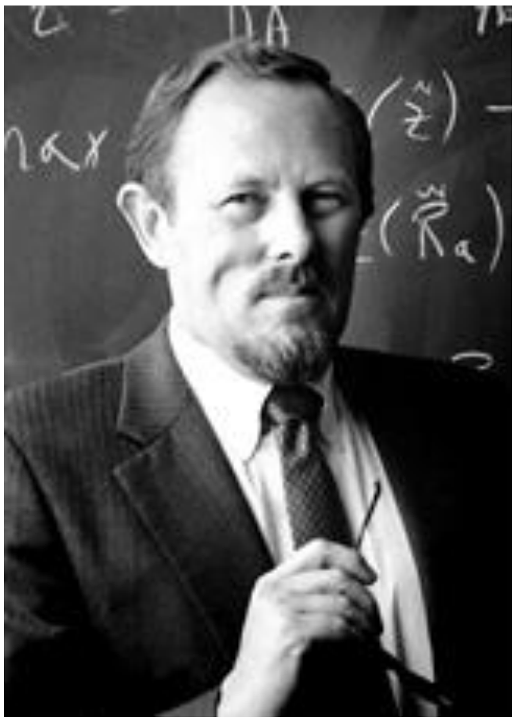


Title: Sharpe ratio

$$\textbf{Slope} = \frac{Excess\ mean\ return}{Volatility\ of\ the\ risky\ portfolio\ returns} = \text{Sharpe ratio}$$



- William Sharpe

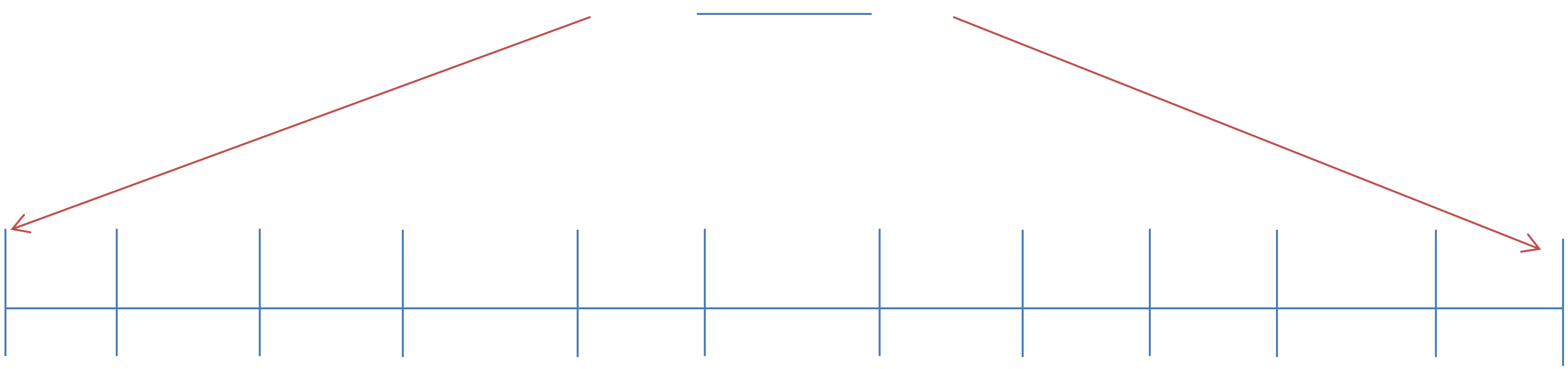


http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1990/sharpe-bio.html

Performance statistics in action

```
> library(PerformanceAnalytics)
> sample_returns <- c(-0.02 , 0.00 , 0.00 , 0.06 ,
0.02 , 0.03 , -0.01 , 0.04)
> mean(sample_returns) # arithmetic mean
[1] 0.015
> mean.geometric(sample_returns) #geometric mean
[1] 0.01468148
> StdDev(sample_returns) #volatility
[1] 0.02725541
> Rf <- 0.004 # risk free
> #Sharpe ratio with arithmetic mean
> (mean(sample_returns)-Rf)/StdDev(sample_returns)
[1] 0.4035897
```

Annualize monthly performance



- Arithmetic mean: monthly mean*12
- Geometric mean, when R_i are monthly returns: $[(1+R_1)*(1+R_2)*\cdots(1+R_T)]^{\frac{12}{T}}$ -1
- Vol: monthly vol*sqrt(12)   Square root of time

```
> mean(sample_returns) # arithmetic mean
[1] 0.015
> mean.geometric(sample_returns) #geometric mean
[1] 0.01468148

> StdDev(sample_returns) #volatility
[1] 0.02725541
>
 Rf <- 0.004 # risk free
> #Sharpe ratio with arithmetic mean
> (mean(sample_returns)-Rf)/StdDev(sample_returns)
[1] 0.4035897
```

* 12

* sqrt(12)

* sqrt(12)

```
Return.annualized(sample_returns,scale=12,geometric=FALSE)
[1] 0.18 > Return.annualized(sample_returns,scale=12,geometric=TRUE)
[1] 0.1911235
> StdDev.annualized(sample_returns,scale=12)
[1] 0.0944155

Return.annualized(sample_returns-Rf,scale=12)/StdDev.annualized(sample_returns,scale=12)
[1] 1.398076
```