Modern portfolio theory of Harry Markowitz

Portfolio weights are optimal...

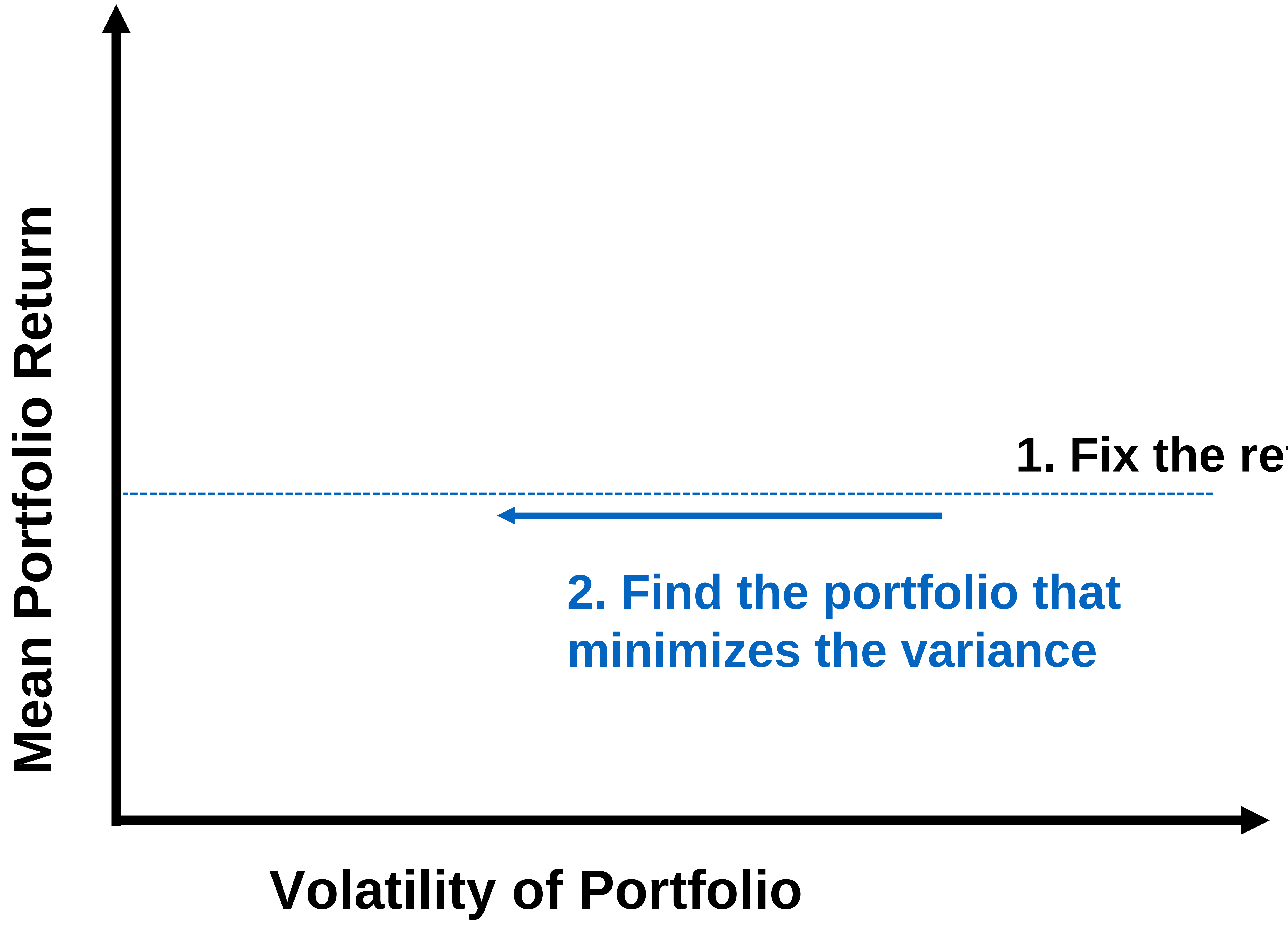
- When they optimize an objective function while satisfying the constraints

Possible objective functions	Possible constraints
Maximize expected return	Only positive weights
Minimize the variance	The weights sum to 1 (all capital needs to be invested)
Maximize the Sharpe ratio	Portfolio expected return equals a target value

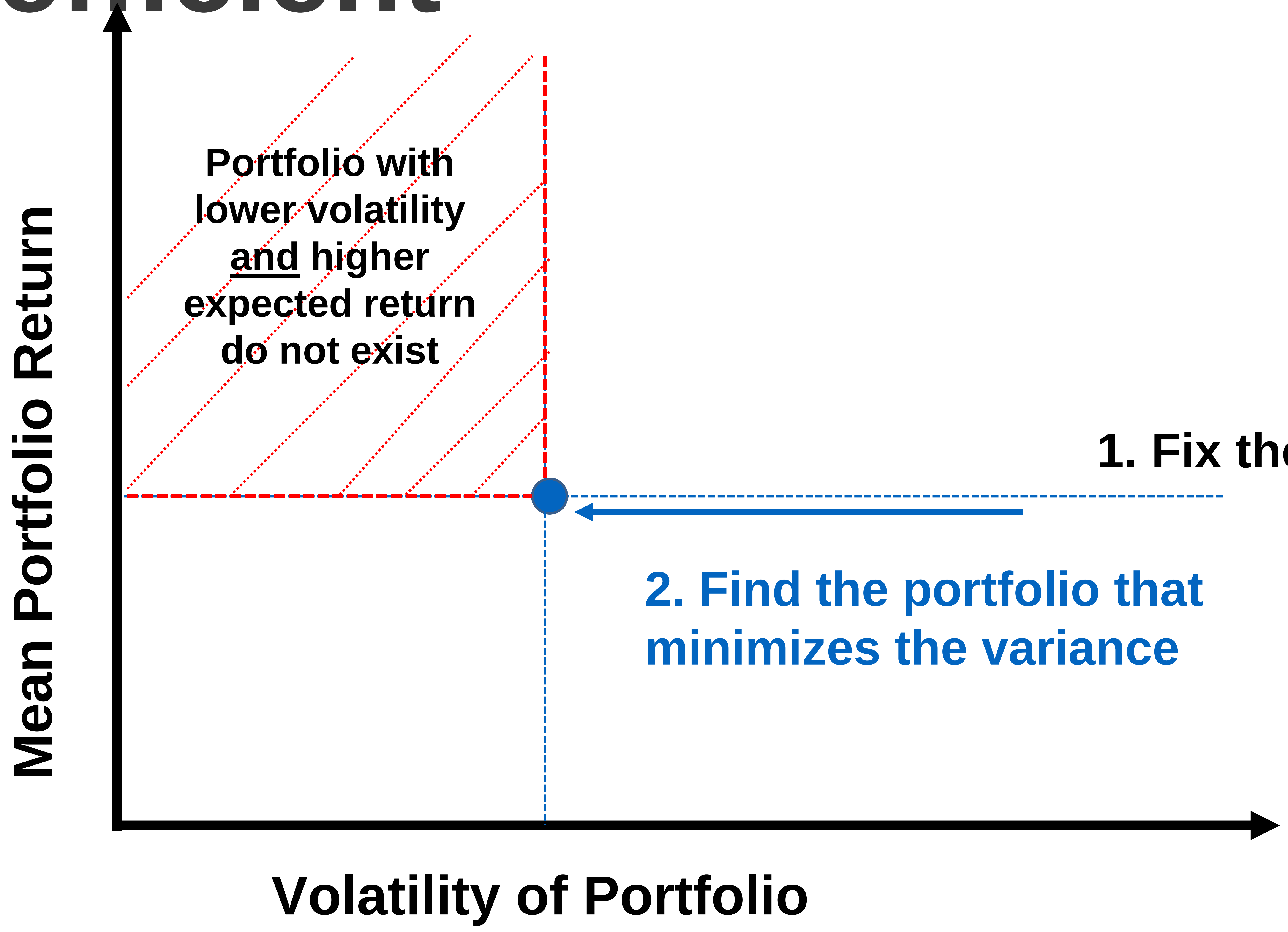
Harry Markowitz

- Minimize the portfolio variance under the constraint that the expected return should be equal to a pre-specified return target

The approach of H. Markowitz

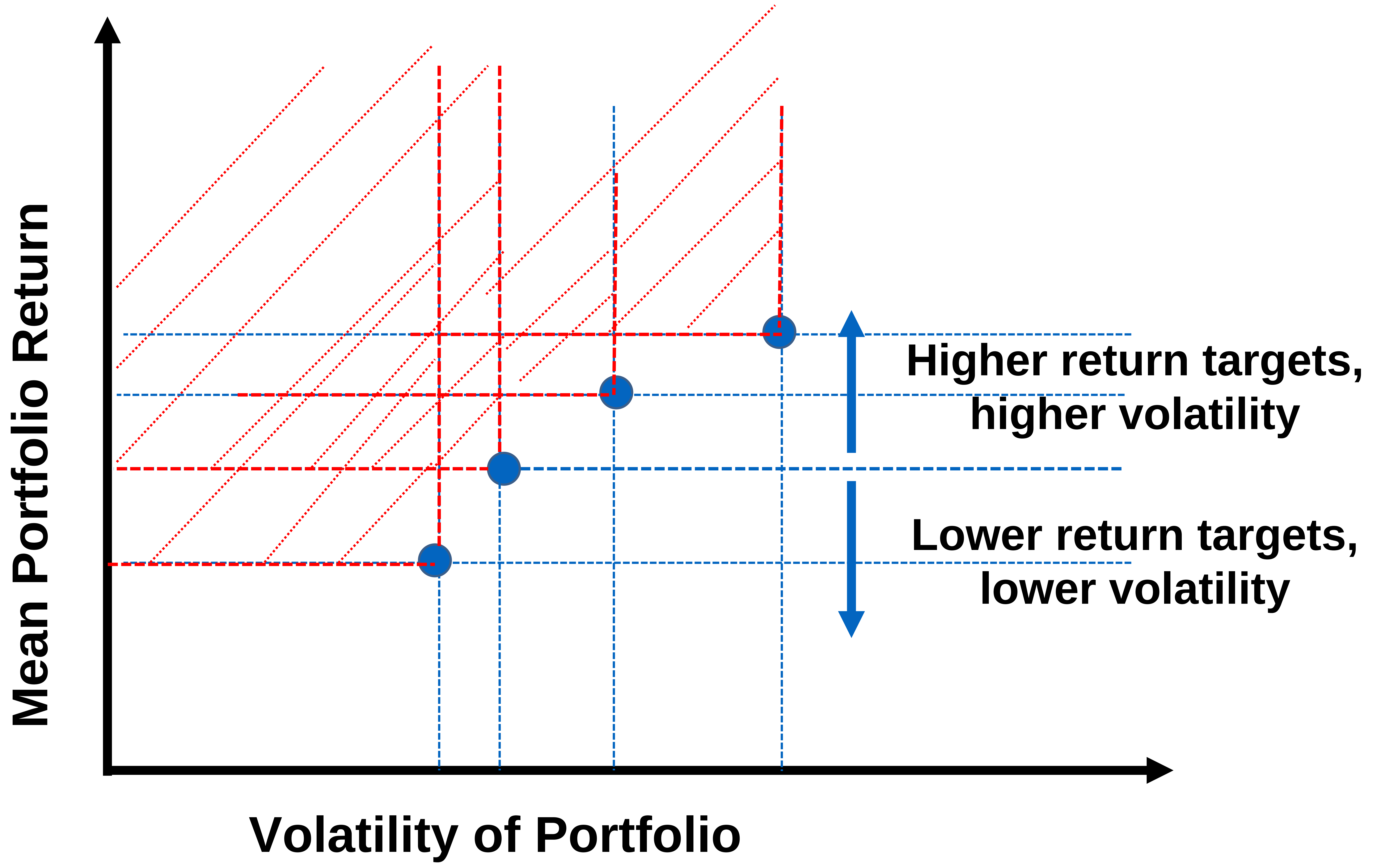


The solution is mean-variance efficient

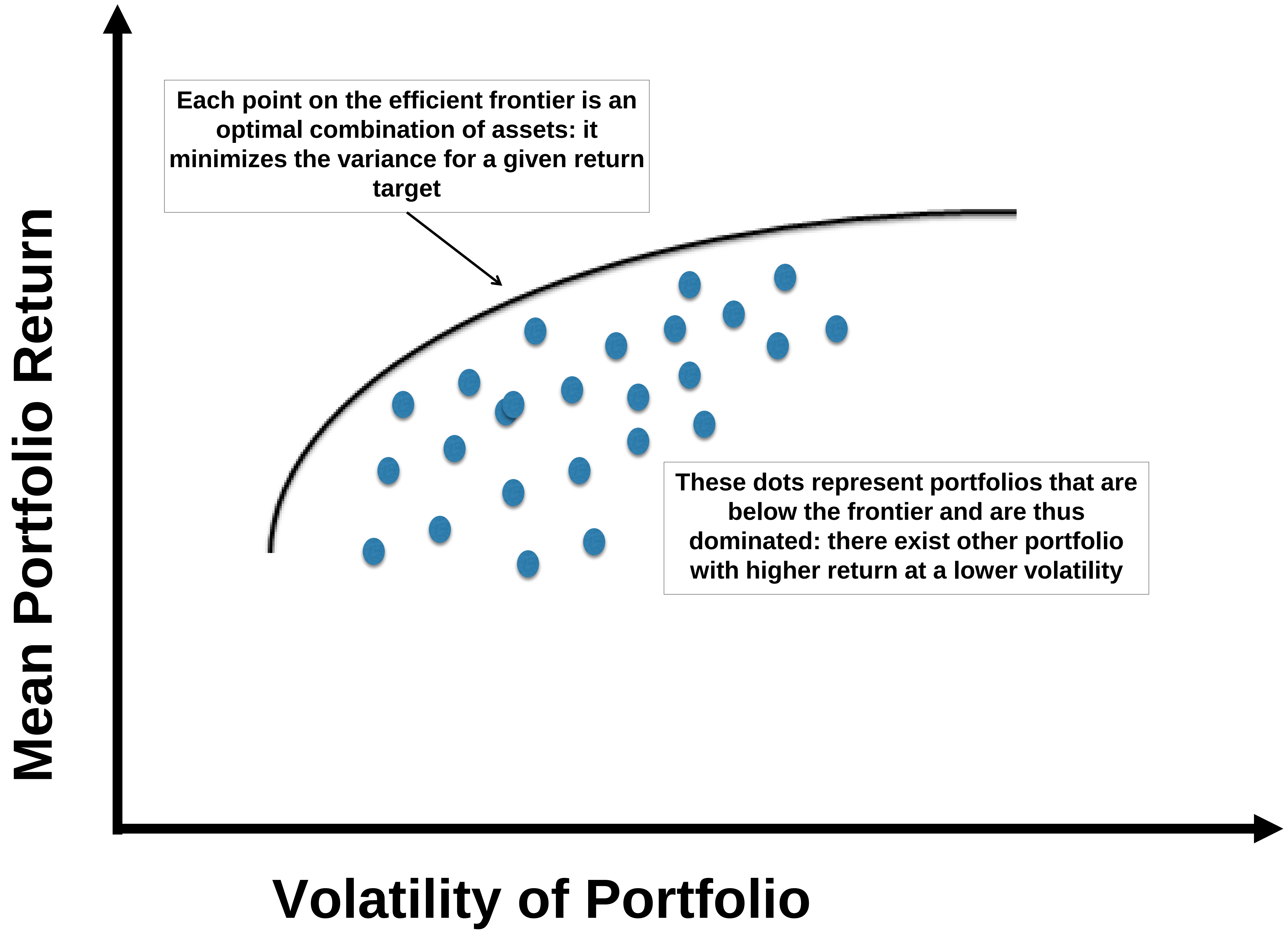


The efficient frontier

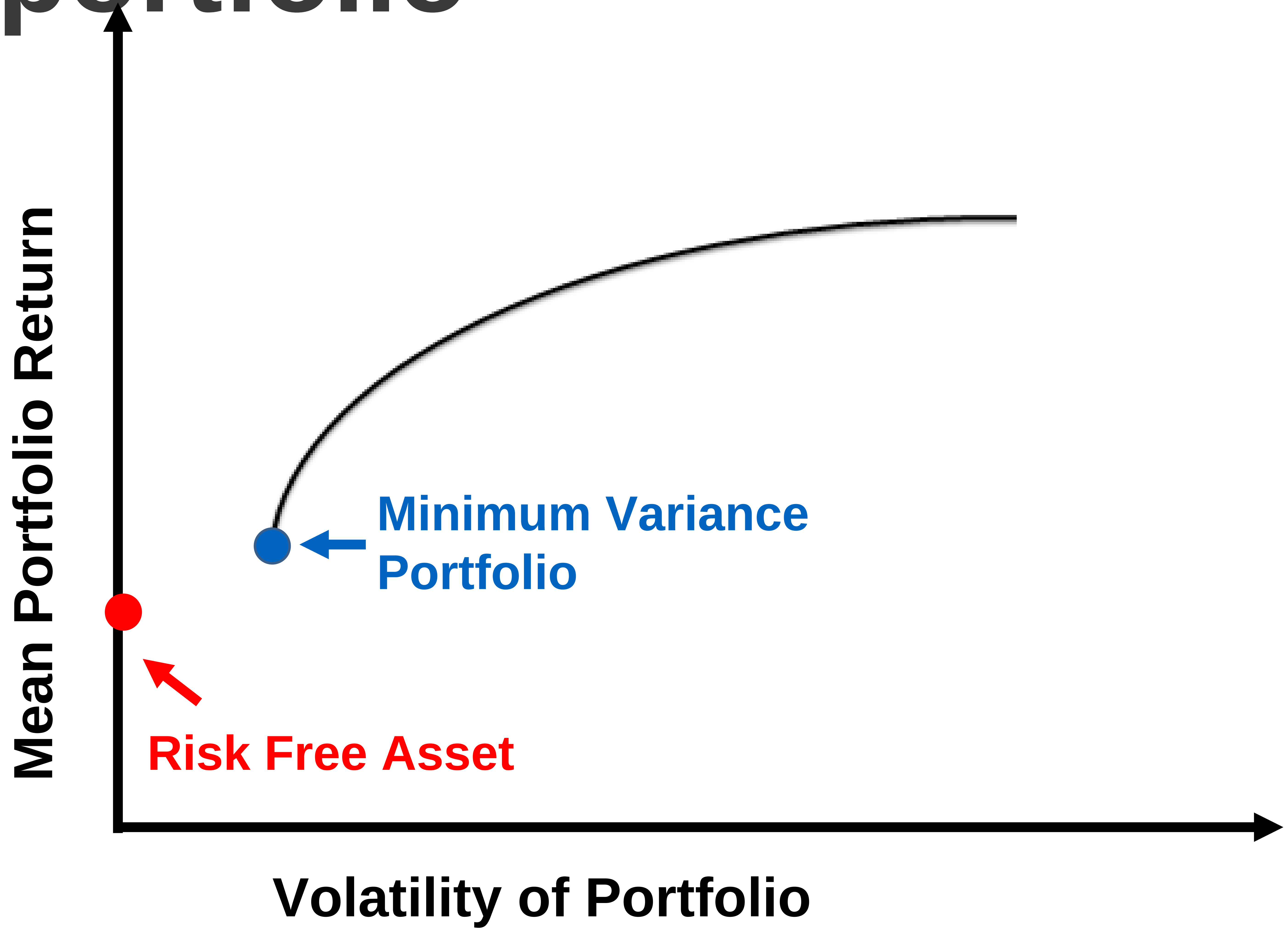
Changing the return target



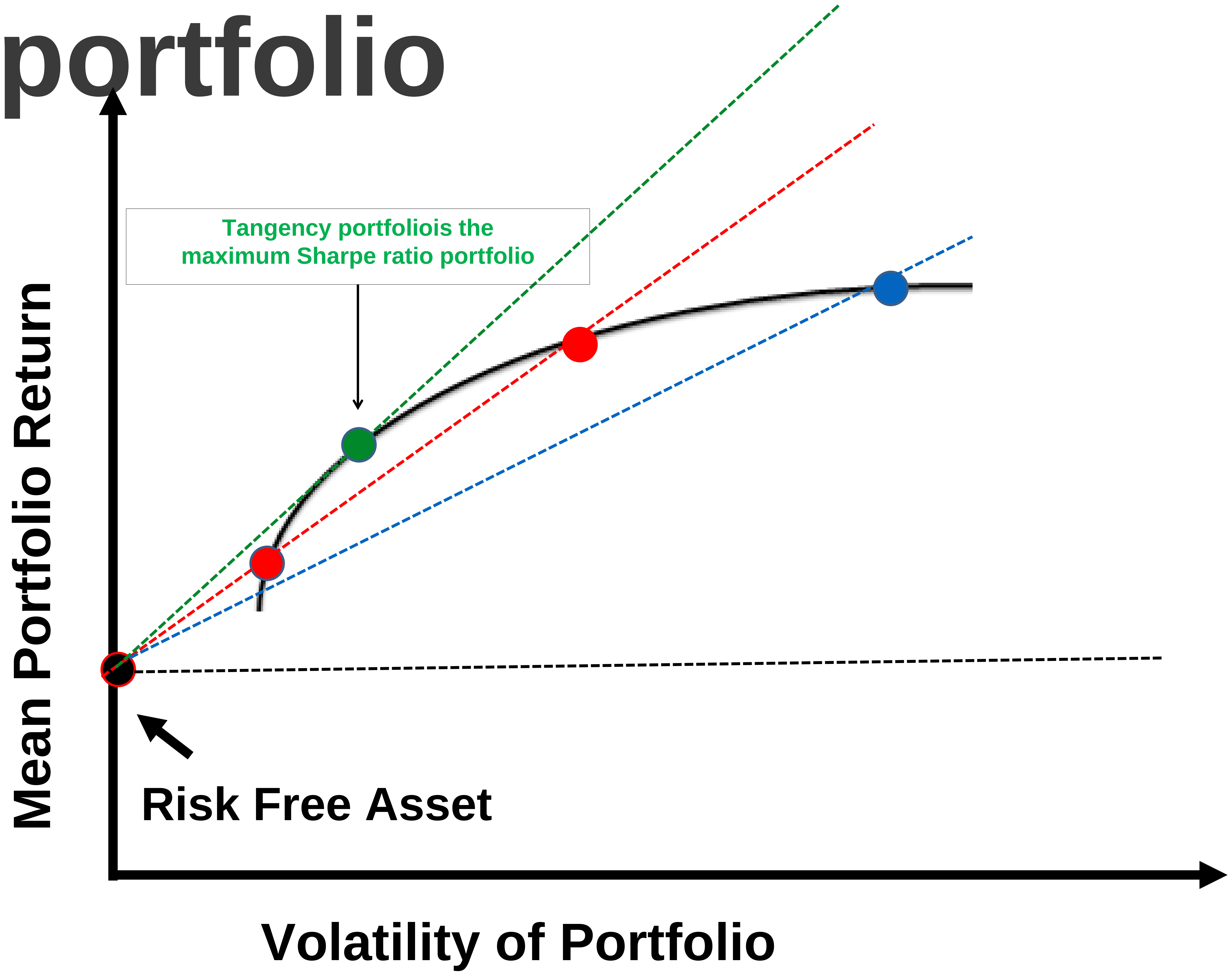
The efficient frontier



The minimum variance portfolio

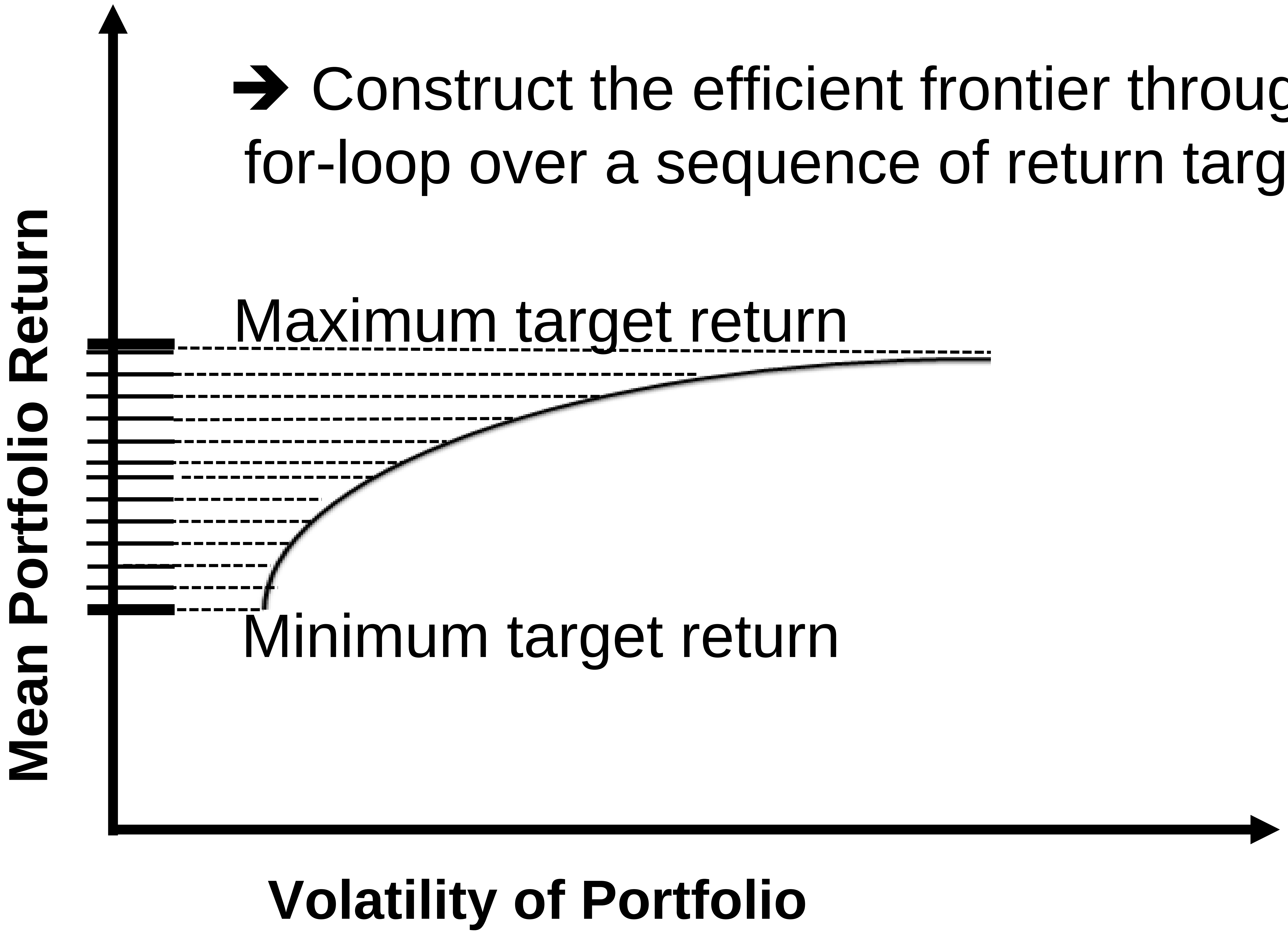


The maximum Sharpe ratio portfolio



Time for practice

➔ Construct the efficient frontier through a for-loop over a sequence of return targets



In-sample versus out-of-sample evaluation

Bad news: Estimation error

- Limitation to data-driven portfolio allocation:

What we are using in practice	What we should be using in theory
Estimated mean $\hat{\mu}$	**True (unkown) mean μ**
Estimated variance $\hat{\sigma}^2$	**True (unkown) variance σ^2**
Optimized weights based on estimated mean and variance: $\hat{w}$	**True optimal portfolio : w**

Good news: Opportunities for data analysts

- Do not ignore estimation error;
- Use split-sample analysis to do a realistic evaluation of portfolio performance

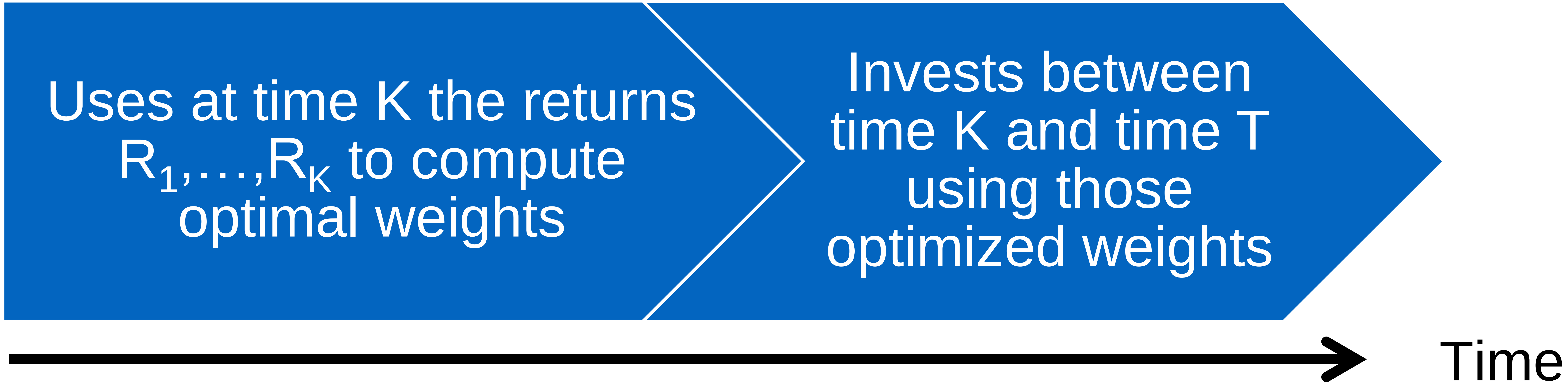
R_1	R_2	…	R_K	R_{K+1}	R_{K+2}	…	R_T

Estimation sample used to find the optimal weights

Out-of-sample evaluation sample to give a realistic view on portfolio performance

No look-ahead bias in the optimized weights

- Split-sample design matches with investor who



- Function `window’ to do split-sample analysis in R.