Using Matrix Notation

Variables at Stake for N Assets

- w: the N x 1 column-matrix of portfolio weights

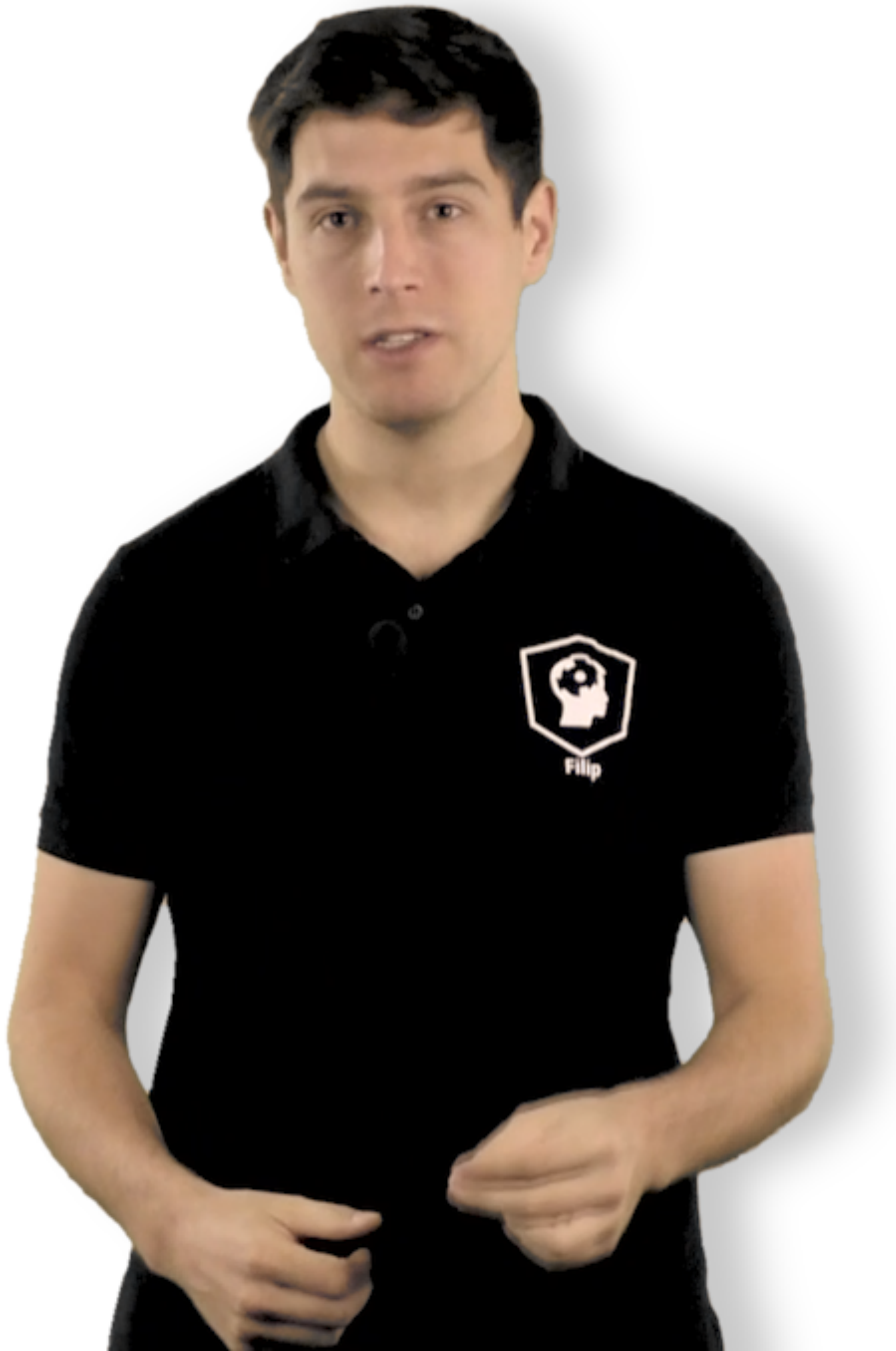
$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

- R: the N x 1 column-matrix of asset returns

$$R = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

- μ: the N x 1 column-matrix of expected returns

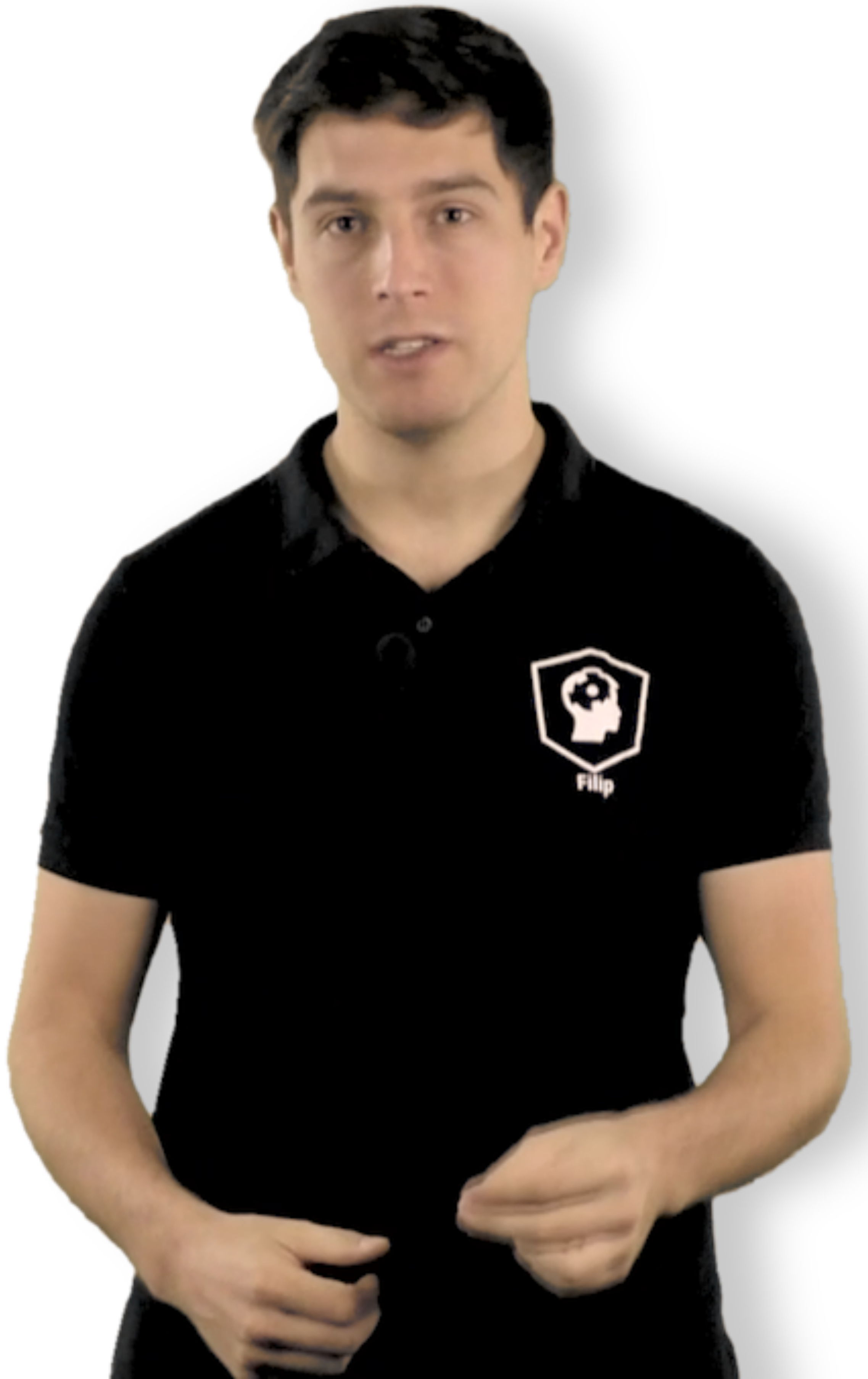
$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{bmatrix}$$



Variables at Stake for N Assets

- Σ: The N x N covariance matrix of the N asset returns:

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{bmatrix}$$



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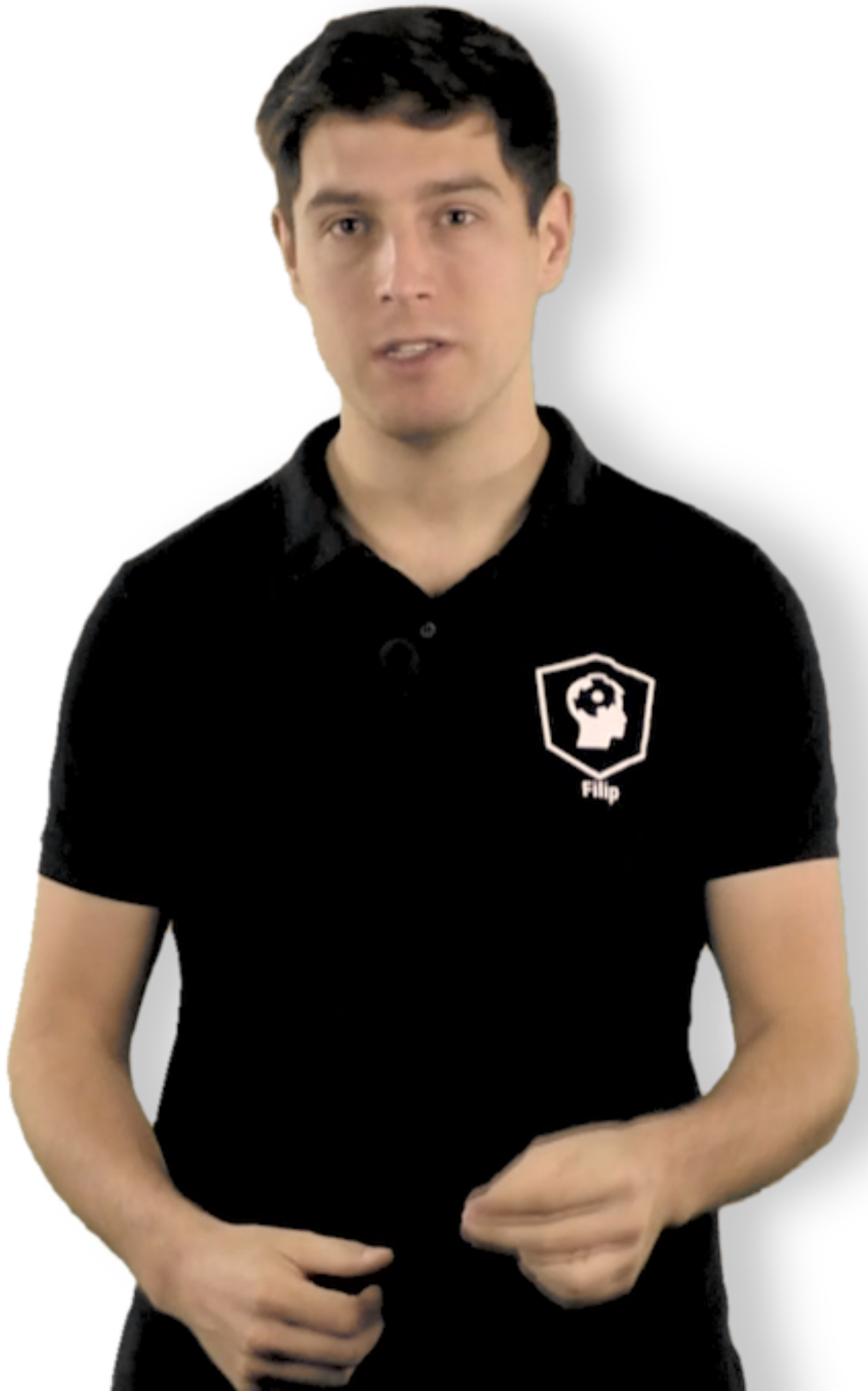

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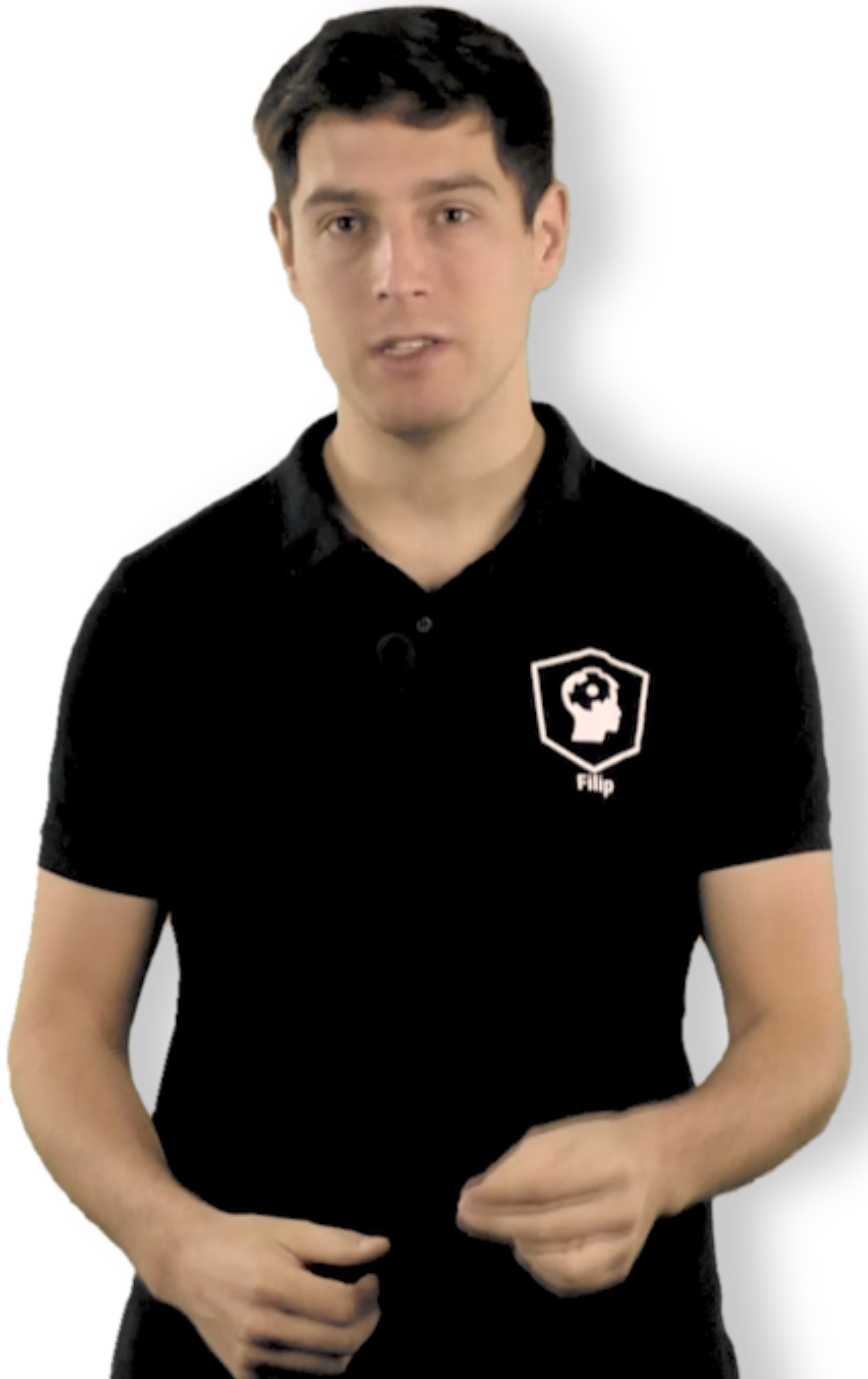
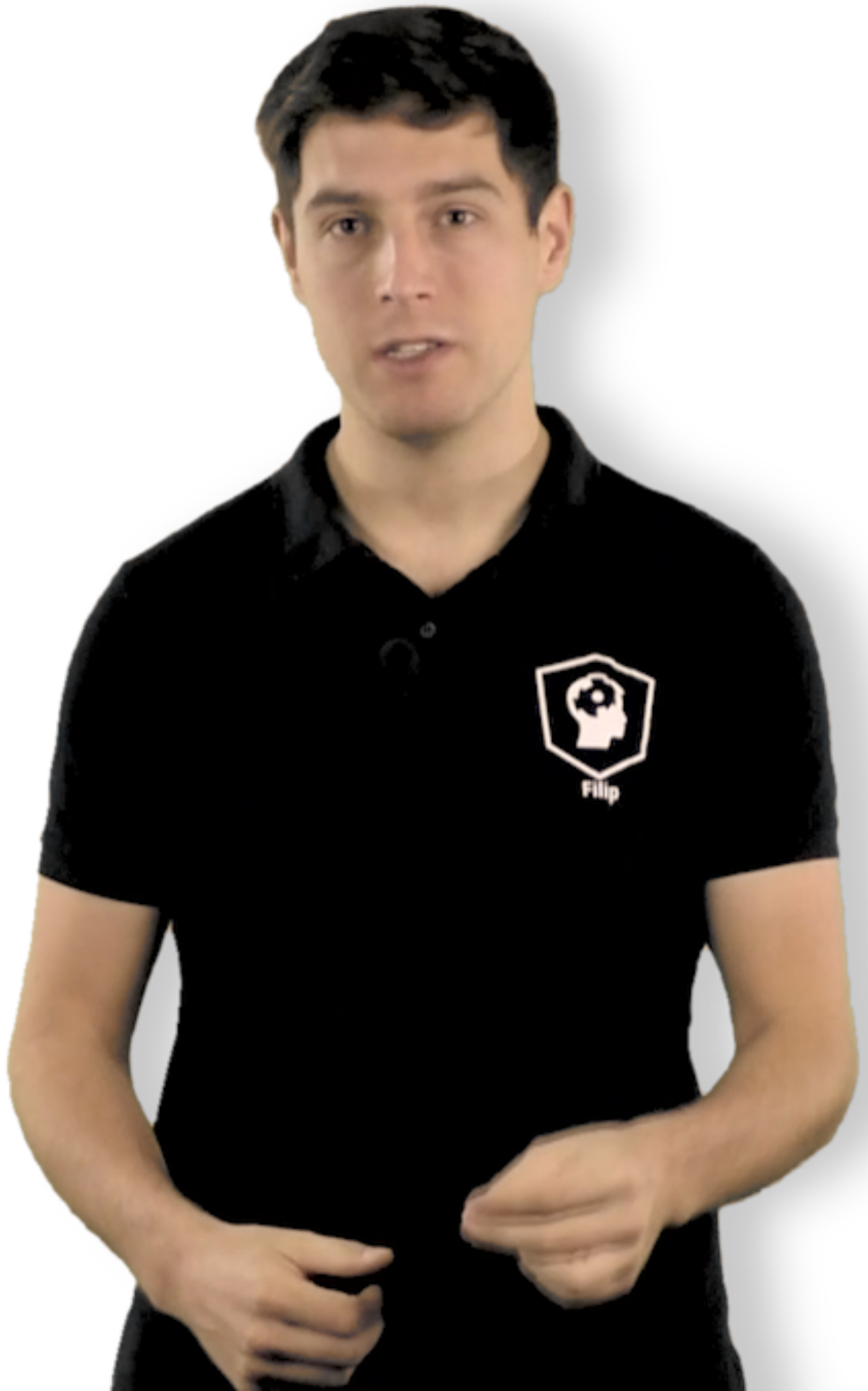
Covariance: Outside Diagonal
Variance: On Diagonal



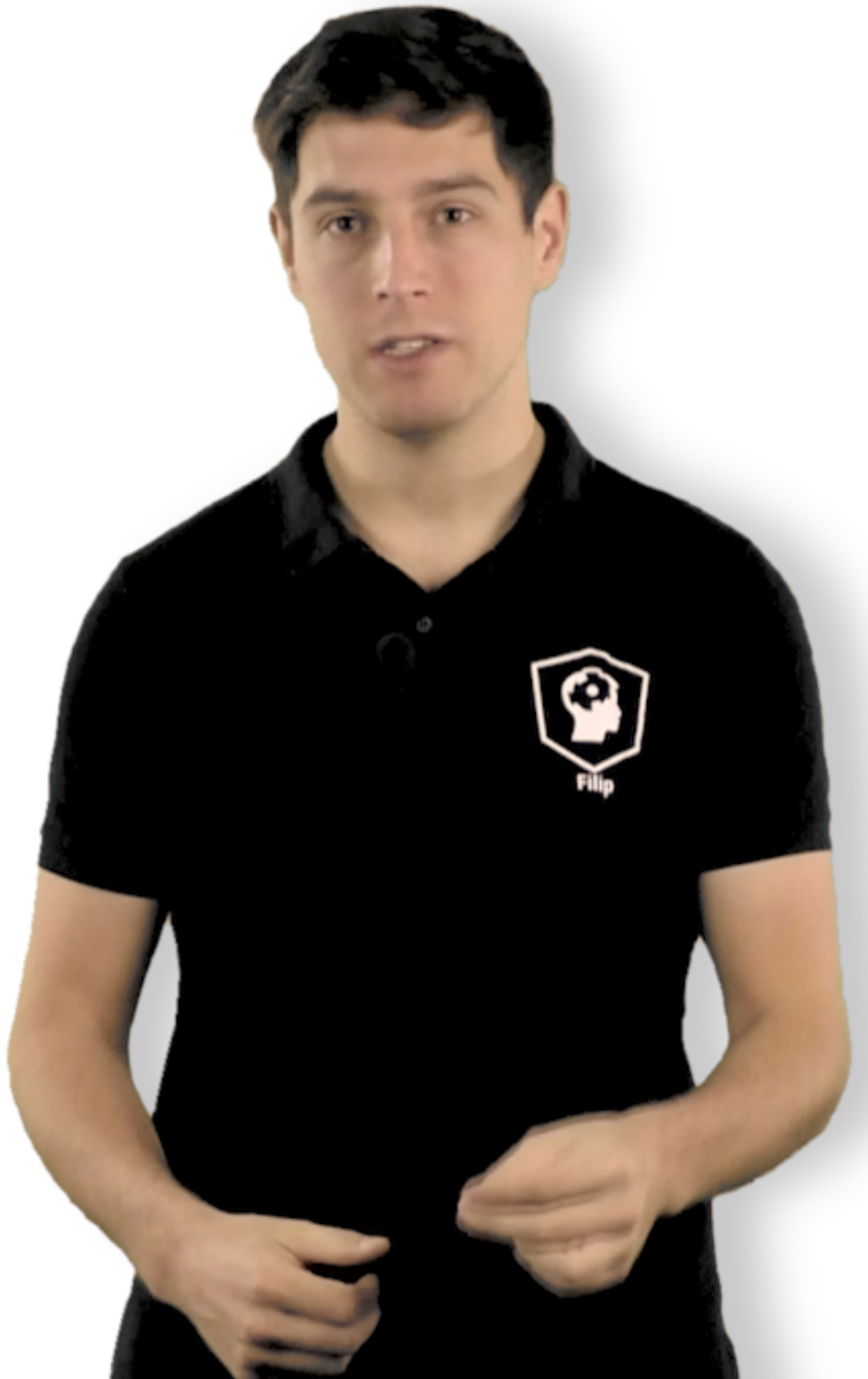
Generalizing from 2 to N Assets



Generalizing from 2 to N Assets

Portfolio Return

$$\boxed{w_1 * R_1 + w_1 * R_2}$$



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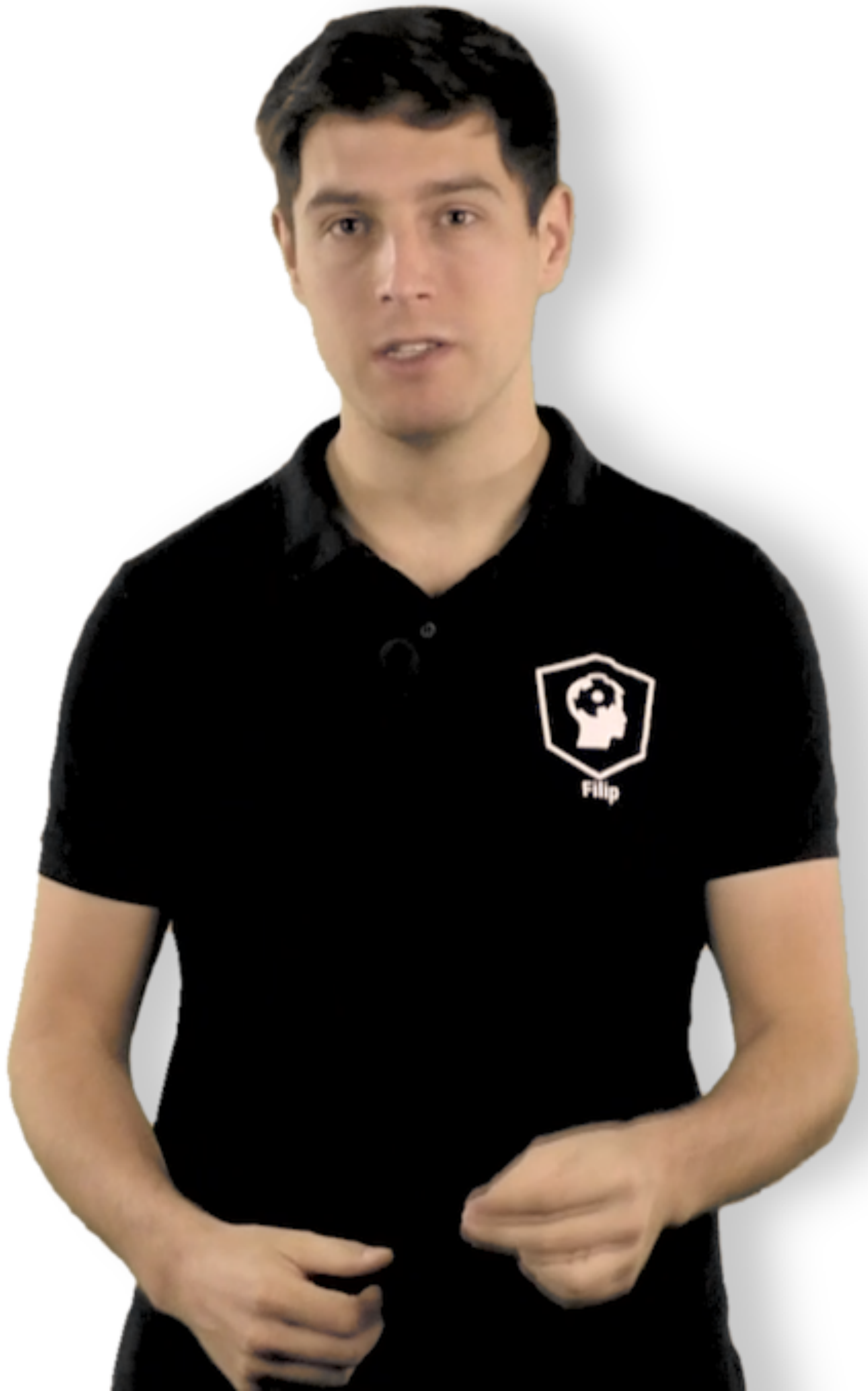
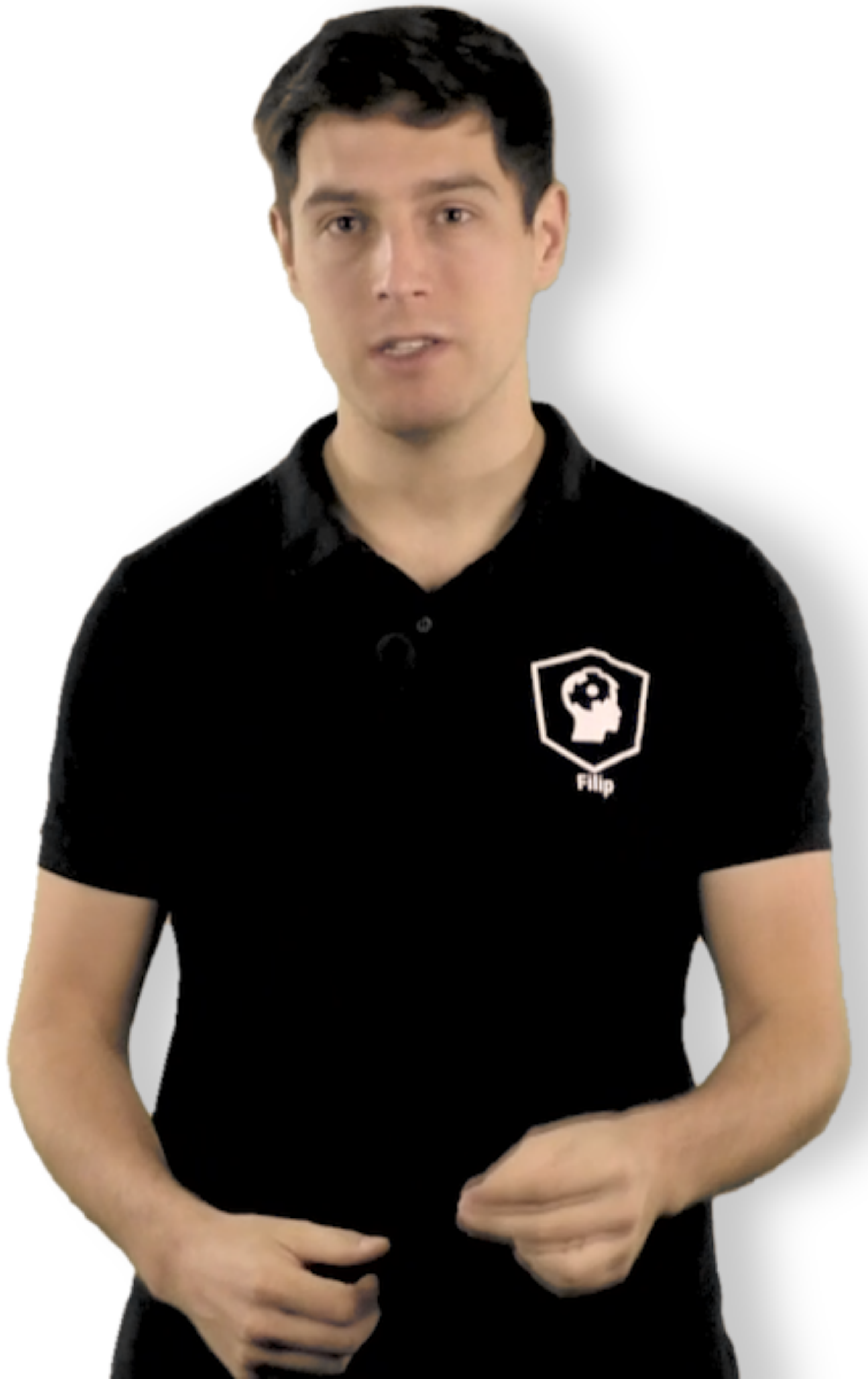


Generalizing from 2 to N Assets

Portfolio Return

$$w_1 * R_1 + w_1 * R_2$$

$$w_1 * R_1 + \ldots + w_N * R_N$$



Generalizing from 2 to N Assets

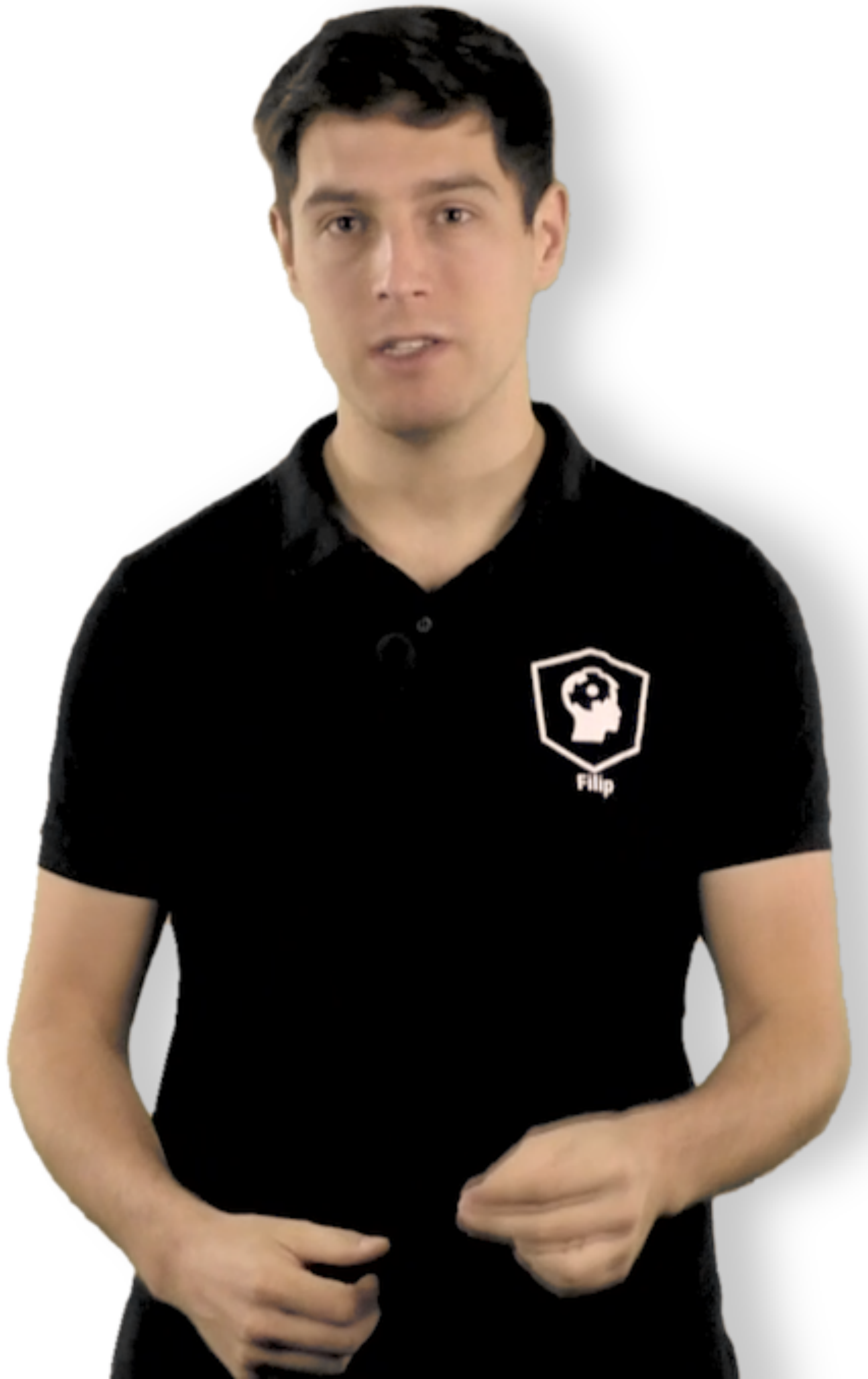
Portfolio Return

$$w_1 * R_1 + w_1 * R_2$$

$$w_1 * R_1 + \ldots + w_N * R_N$$

Portfolio Expected Return

$$w_1 * \mu_1 + w_2 * \mu_2$$



Generalizing from 2 to N Assets

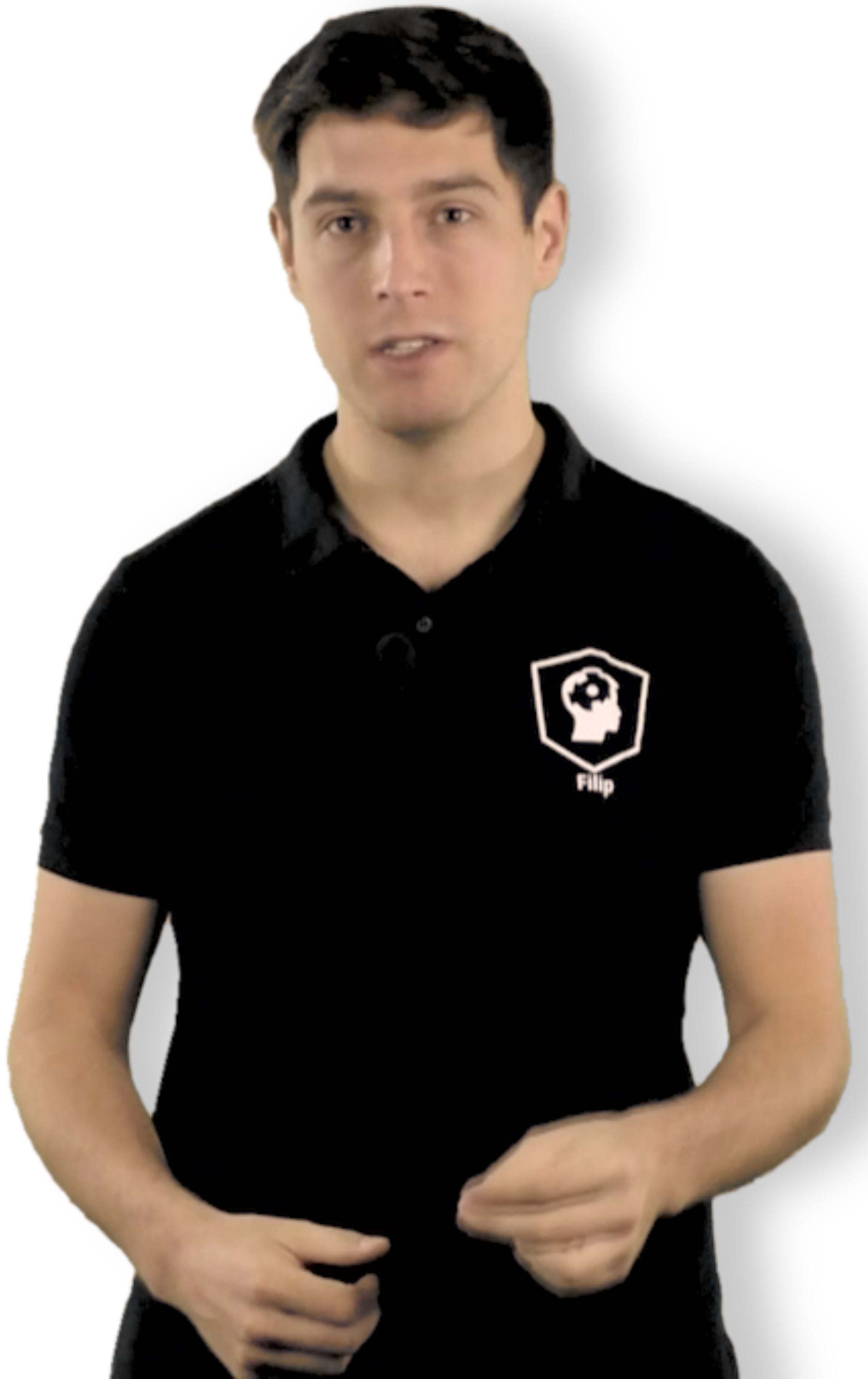
Portfolio Return

$$\boxed{w_1 * R_1 + w_1 * R_2}$$

$$\boxed{w_1 * R_1 + \ldots + w_N * R_N}$$

Portfolio Expected Return

$$\boxed{w_1 * \mu_1 + w_2 * \mu_2}$$



Generalizing from 2 to N Assets

Portfolio Return

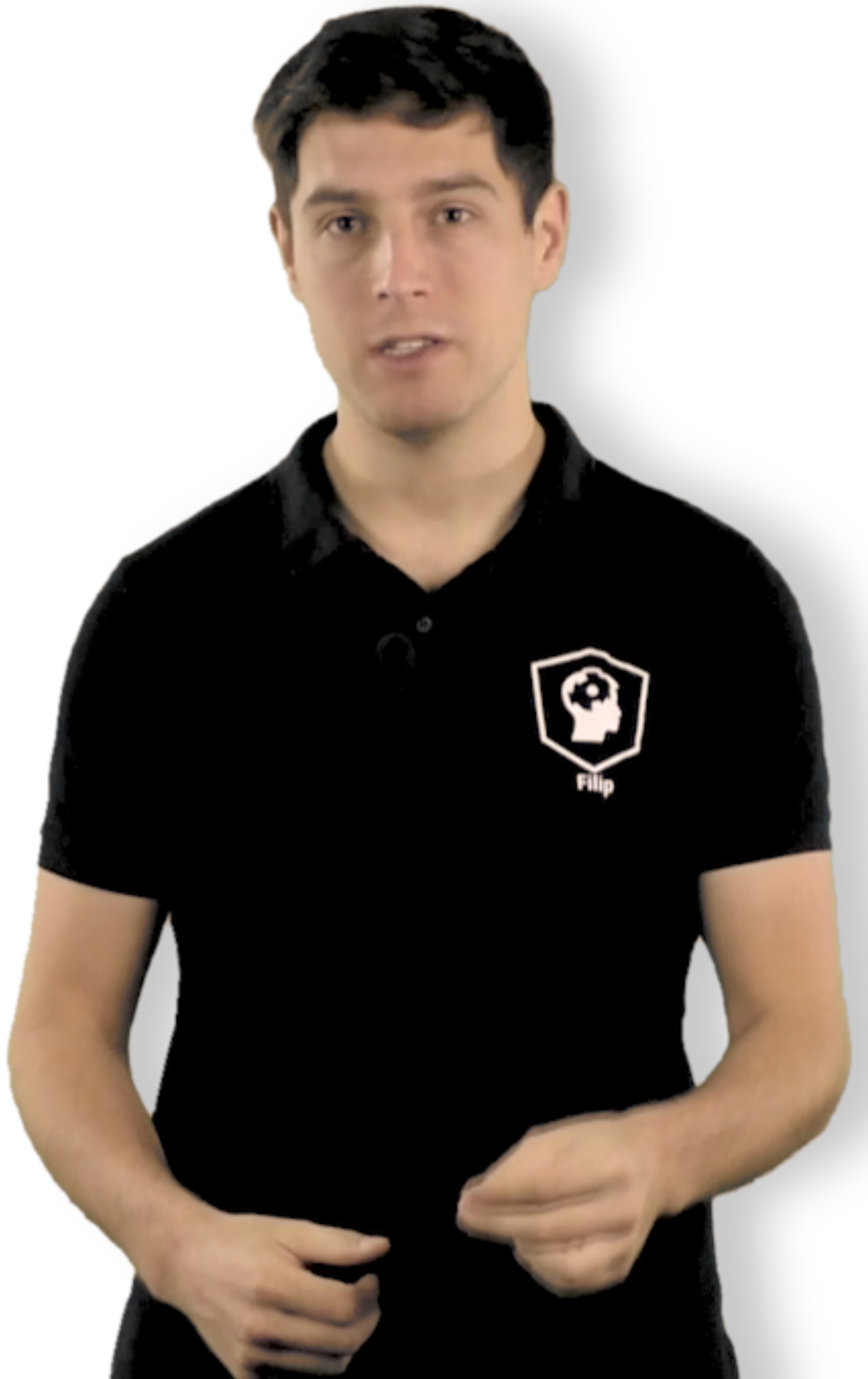
$$w_1 * R_1 + w_1 * R_2$$

$$w_1 * R_1 + \ldots + w_N * R_N$$

Portfolio Expected Return

$$w_1 * \mu_1 + w_2 * \mu_2$$

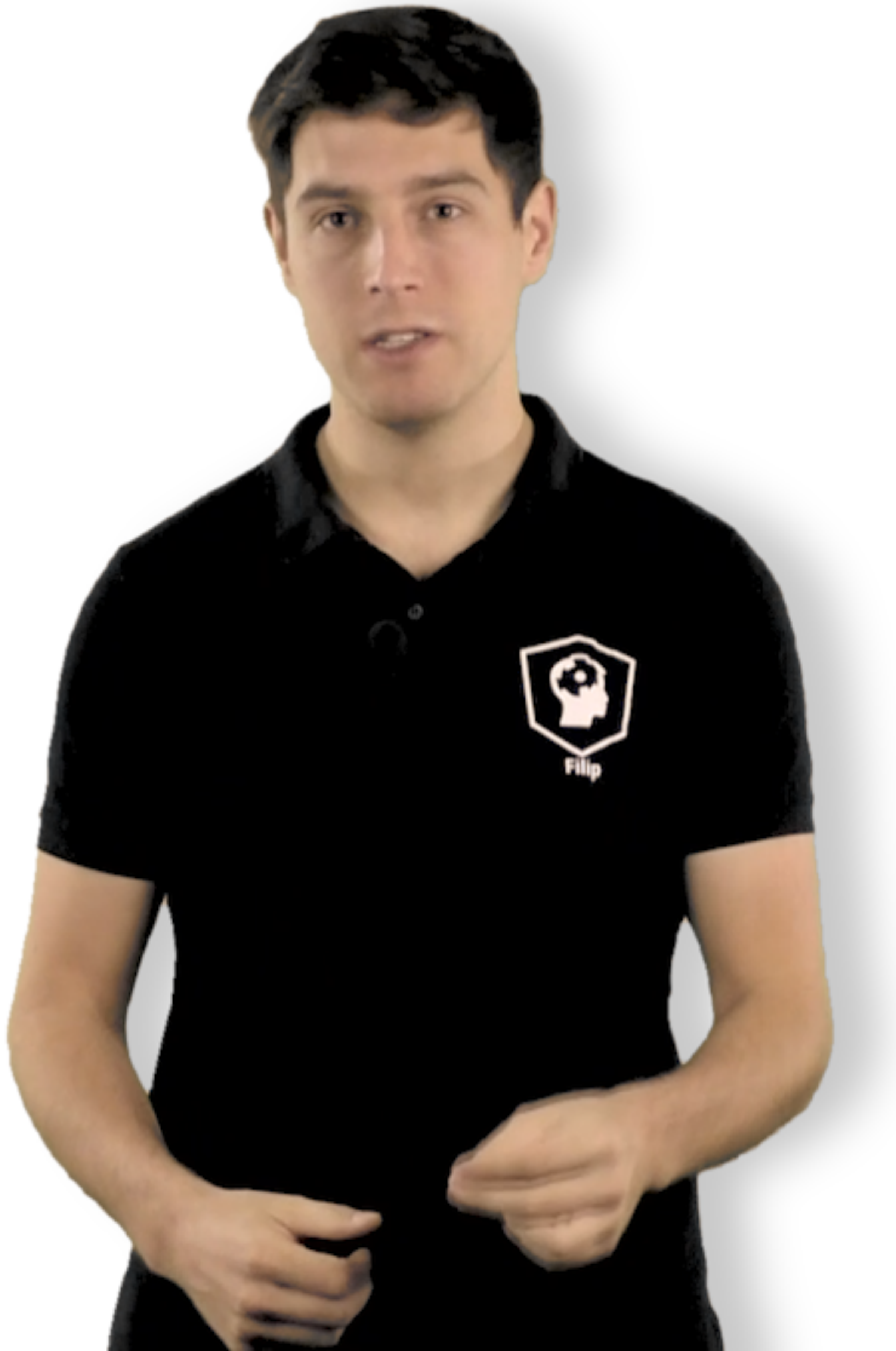
$$w_1 * \mu_1 + \ldots + w_N * \mu_N$$



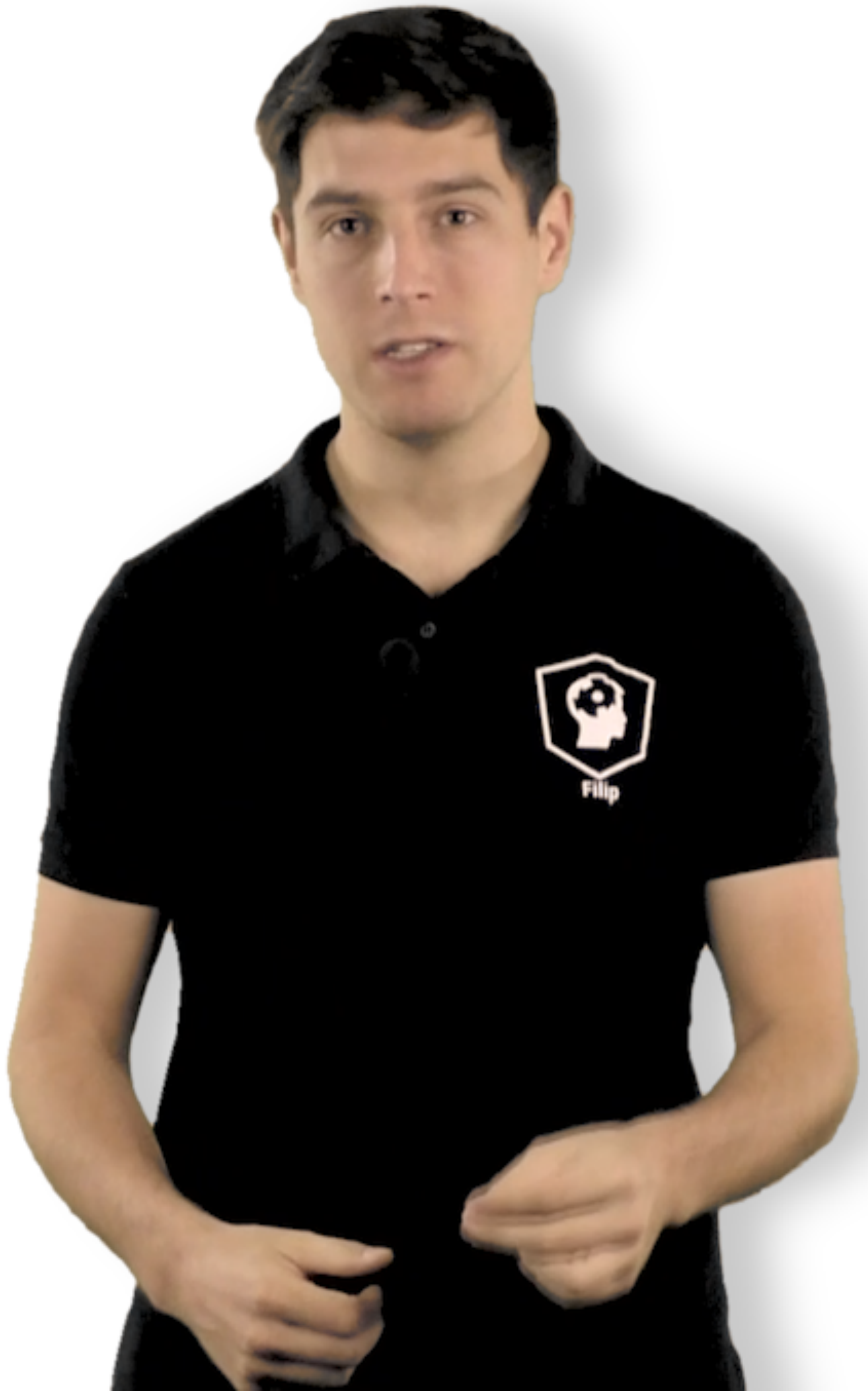
Matrices Simplify the Notation

- Avoid large number of terms by using matrix notation
- We have 4 matrices:
  - weights (w), returns (R), expected returns (μ), and covariance matrix (Σ)

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} \qquad w' = \begin{bmatrix} w_1 & w_2 & \cdots & w_N \end{bmatrix}$$



Simplifying the Notation



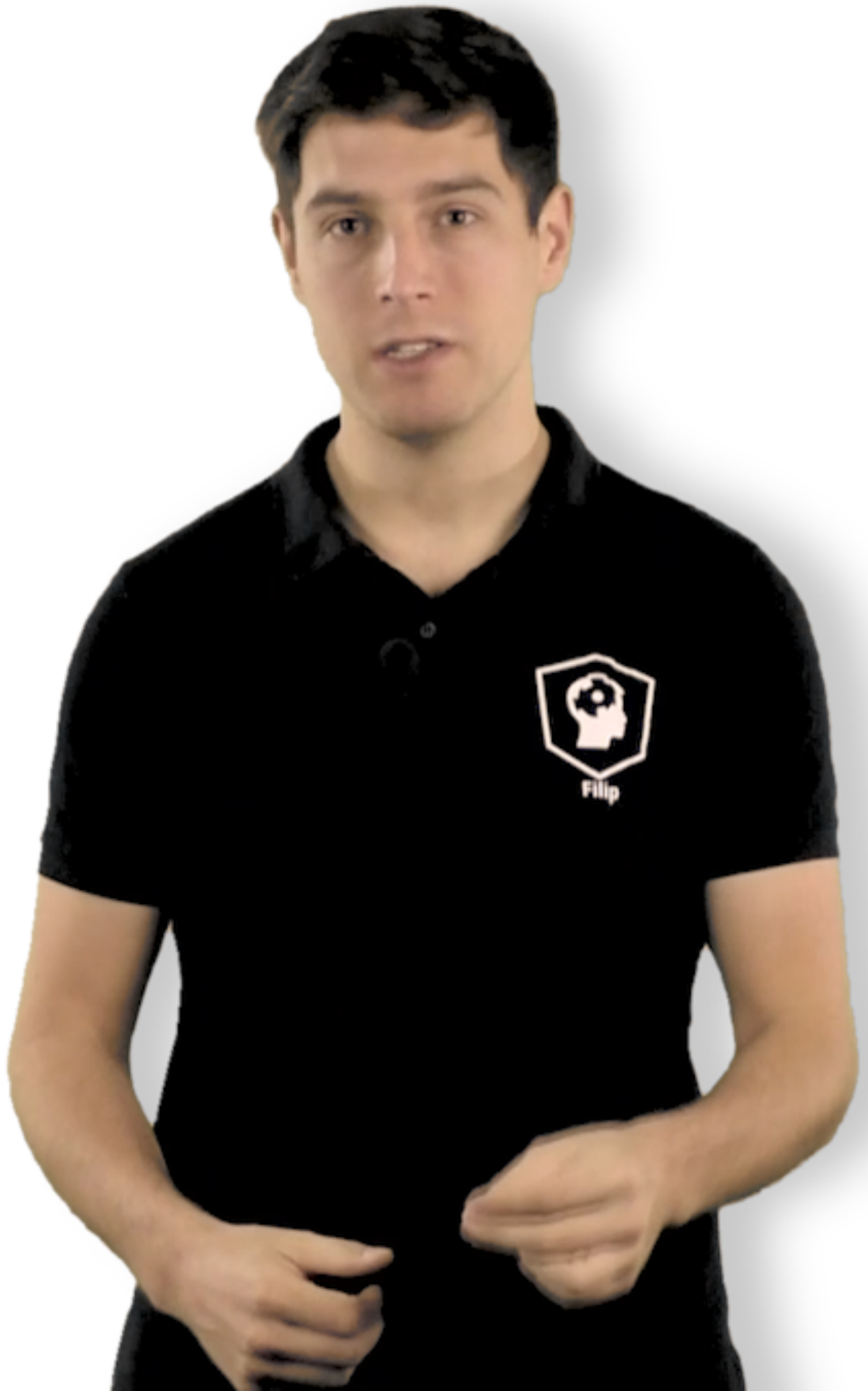
Simplifying the Notation

Portfolio Return

$$\boxed{w_1 * \mu_1 + w_2 * \mu_2}$$

$$\Downarrow$$

$$\boxed{w'R}$$



Simplifying the Notation

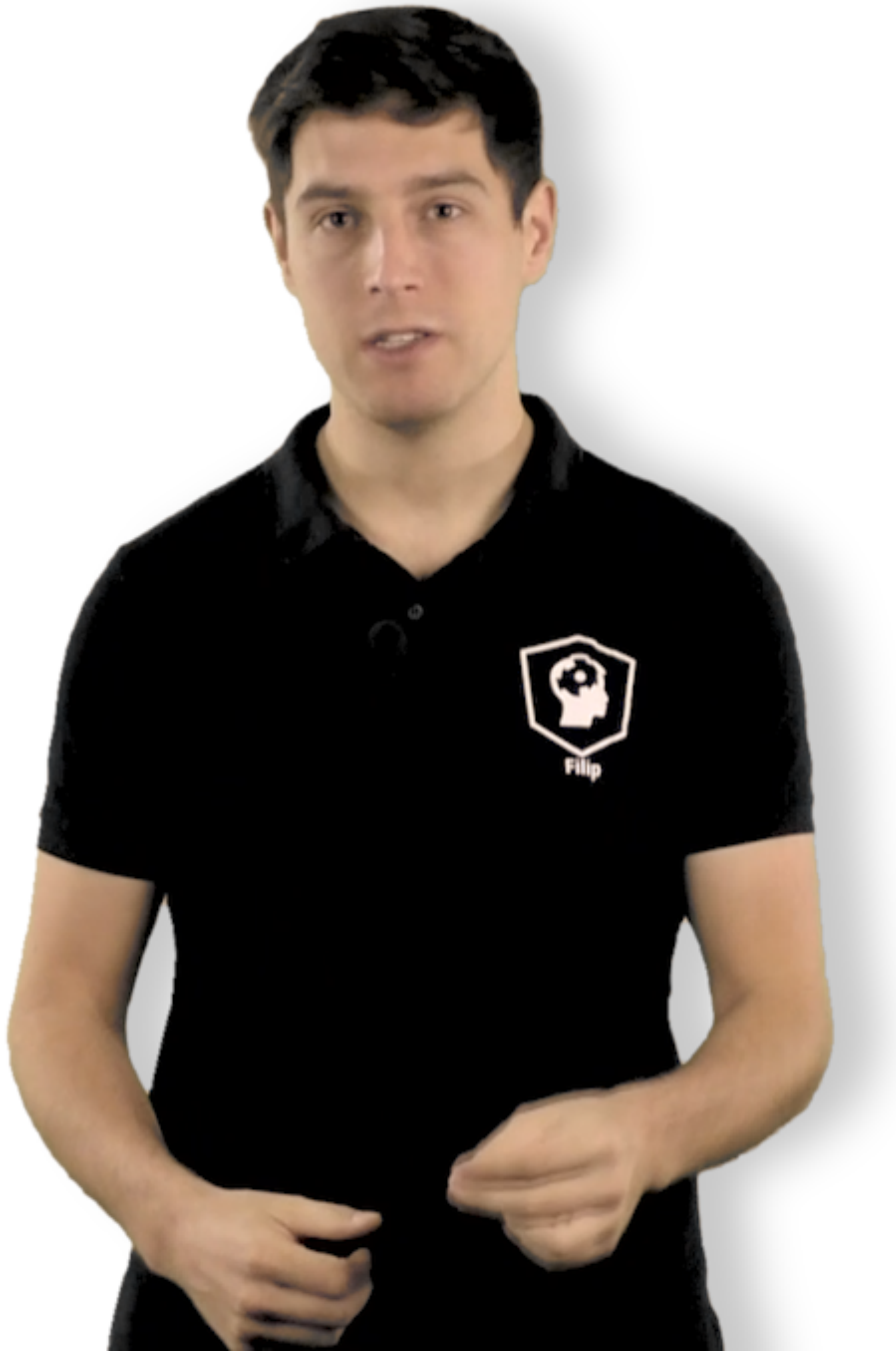
Portfolio Return

$$w_1 * \mu_1 + w_2 * \mu_2$$

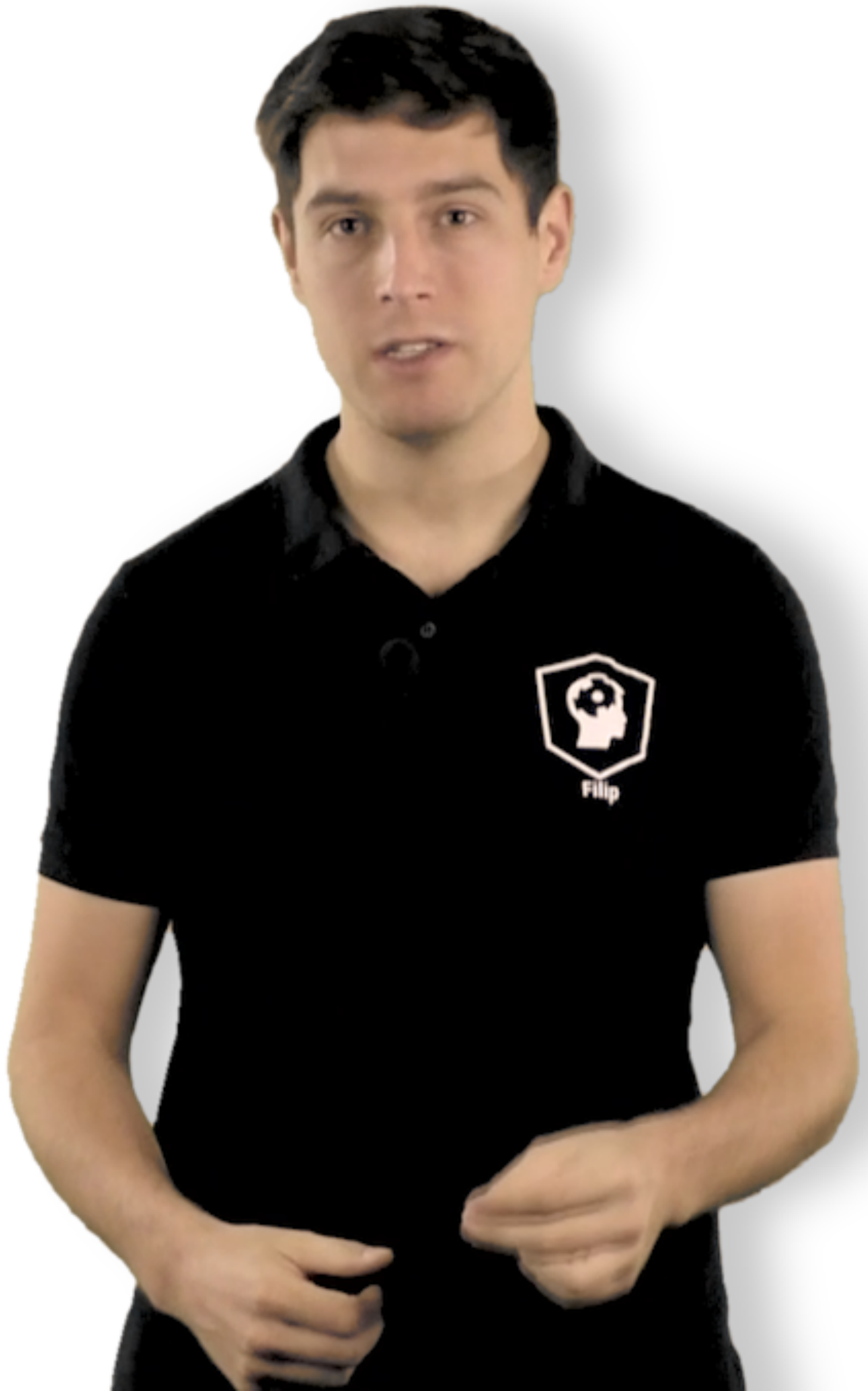
$$w'R$$

Portfolio Expected Return

$$w_1 * E[R_1] + \ldots + w_N * E[R_N] \quad \Rightarrow \quad w'\mu$$



Simplifying the Notation



Simplifying the Notation

Portfolio Variance

$$\boxed{\begin{aligned} & w_1^2 * var(R_1) + \ldots + w_N^2 * var(R_N) \\ & + 2 * w_1 * w_2 * cov(R_1, R_2) + \ldots \\ & + 2 * w_{N-1} * w_N * cov(R_{N-1}, R_N) \end{aligned}}$$

$$\boxed{w' \Sigma w}$$