浙江大学 2007－2008 学年秋冬季学期

《数值分析》课程期末考试试卷

开课学院：计算机学院，考试形式：开卷，允许带_教材、笔记、计算器等任何材料_入场

考试时间：_2008_年_1_月_24_日， 所需时间：_120_分钟，任课教师：_陈越_

考生姓名：________________学号：________________________专业：____________

题序	一	二	三	四	五	六	七	总 分
得分								
评卷人								

I. Please fill in the blanks with your answers. (22 points)

1. Suppose that $A\vec{x}=\vec{b}$ is a linear system of equations where $A\in R^{n\times n}$ and $\vec{x},\vec{b}\in R^n$. Please answer "T" for true and "F" for false for the following statements: (2 points each)

(1) If A is positive definite, then Gaussian elimination can be done without any row or column exchanges. __________

(2) If A is non-singular, then AA^t is positive definite. __________

(3) For a 3-dimentional vector $\vec{x}=(x_1,x_2,x_3)^t$, $2|x_1|-|x_2|+5|x_3|$ defines a norm of the vector.__________

(4) If $\|A\|_\infty<1$, then its spectral radius $\rho(A)<1$. __________

(5) If the eigenvalues of A satisfies $|\lambda_1|=|\lambda_2|>|\lambda_3|\ge\cdots\ge|\lambda_n|\ge 0$, then the original power method must fail. __________

2. Given that the function $S(x)=\begin{cases} x^3+x^2 & 0\le x\le 1 \\ ax^3+bx^2+cx-1 & 1\le x\le 2 \end{cases}$ is a cubic spline.

Then a = _______; b = _______ ; c = _______. (2 points each)

3. Given $\{\varphi_k(x)\}_{k=0}^{\infty}$ as a family of orthogonal polynomials defined on $[0,1]$ with weight function $\rho(x)=x$. If all the leading coefficients are 1, then $\varphi_0(x)=1$, $\varphi_1(x)=$ ___________ (2 points), and $\varphi_2(x)=$ ________________________ (4 points).

II. Given the coordinates of 20 points on this cute baby mouse. Are the following methods appropriate for fitting this mouse's outline curve? Please answer "Yes" or "No" (1 point each), and briefly explain your answers (3 points each).

(1) Lagrange interpolating polynomial of degree 19: ________

Explain:

(2) Piecewise Hermite interpolating polynomial of degree 3, provided that all the points are sorted according to their x coordinates: ___________

Explain:

(3) Piecewise linear interpolation for the paired neighboring points: __________

Explain:

(4) Two least-square approximating polynomials of degree 4 for the upper and lower curves: ___________

Explain:

(5) Piecewise quadratic interpolation for every three neighboring points: ______

Explain:

Note: For each of the following problems, a simple answer without explanation will NOT be graded.

III. Given $x_{n+1} = 1 + \frac{1}{x_n^2}$ as a fixed-point iteration scheme for finding the root of $x^3 - x^2 - 1 = 0$ in the interval $I = [1.3, 1.6]$.

(1) Does this scheme converge for all $x_0 \in I$? (2 points)

(2) If your answer is “YES”, please give the order of convergency and prove your conclusion. If your answer is “NO”, please find an $x_0 \in I$ from which the scheme diverges, and prove your conclusion. (6 points)

IV. Please draw the unit discs with respect to the norms $\| \|_1$, $\| \|_2$ and $\| \|_\infty$ in space R^2. (Note: a unit disc is a disc of radius 1 and is centered at the origin.) (9 points)

V. For what values of ω that the iterative method

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \omega \left(\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix} \vec{x}^{(k)} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$$

will converge? When will the method have the fastest convergence? (14 points)

VI. Determine values for A, B, and C that make the formula

$$\int_0^2 x f(x) dx \approx A f(0) + B f(1) + C f(2)$$

exact for all polynomials of degree as high as possible. Is the formula a Gaussian quadrature? Why or why not? (12 points)

VII. For solving IVP of ODE $\begin{cases} y' = f(t,y) \\ y(t_0) = y_0 \end{cases}$, please determine α and β in the multistep scheme $w_{i+1} = w_{i-2} + h(\alpha f_{i-1} + \beta f_{i-2})$ where $f_i = f(t_i, y_i)$, so that the scheme has the order of accuracy as high as possible. (15 pionts)