GROUP WORK PROJECT # 1
GROUP NUMBER: 6792

MScFE 620: Derivative Pricing

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

Step 1

Q1.

$$C - P = S - Ke^{-rt}$$

Where:

C is the price of the European call option

P is the price of the European put option

S is the current price of the underlying asset

K is the strike price of the options

r is the risk-free interest rate

T is the time to expiration

An arbitrage opportunity would be present if the put-call parity failed to hold. Profits without risk could be made by traders by utilizing the variations in price between put and call options. However, in a well-functioning market, arbitragers quickly eliminate these prospects for profit, preserving the put-call parity relationship.

Q2.

$$C = S - Ke^{-rt} + P$$

Q3.

$$P = C + Ke^{-rt} - S$$

Q4.

No, American options are not subject to put-call parity since they can be exercised at any point before to expiration, which creates uncertainty regarding the timing of cash flows. Since early exercise and dividends can affect the value of the options differently than they can for European options, which can only be exercised at expiration, this flexibility leads to pricing difficulties.

Parameters Set as , $S_0 = 100$; $r = 5\%$; $\sigma = 20\%$; $T = 3$ months

Binomial Tree

Q5.

a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.
Ans:

Number of steps chosen is 100 because, it strikes balance between accuracy and computational efficiency. With that, we find that the call is priced at 4.6 US dollars, while the put is priced at 3.4 US dollars.

b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.
Ans:
b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.

1. We Set the Parameters:
2. Calculated the upward and downward movements:
The up factor (u) represented the stock price's upward movement, and the down factor (d) represented the downward movement.
3. Then we calculate risk-neutral probability (p):
4. Construction of Binomial Tree:
We create a Binomial tree for stock prices from time 0 to T.
Then, we calculate the stock prices at each subsequent time factors.
5. Option Valuation:
 * Compute option values at each final node (at maturity).
 * Work backwards through the tree to calculate the option values at each preceding node using the risk-neutral valuation.
6. Calculate Call and Put Prices:
 * The price at the root of the tree will be the option price at time 0.

Reason for Choosing 100 Steps
 * Accuracy, Efficiency:

because it is an optimal number of steps that allows for high enough prices while maintaining computational efficiency, 100 steps strike a balance between computational feasibility and the precision of the results.

Q6.

a. How do they compare?
The Greek delta of the European Call Option (approximately 0.56 US dollars) is positive, and nearer to 1. We find that the price of the call option will rise by around 0.56 US dollars for every dollar increase in the price of the underlying asset, while the Delta of the put option is negative,and near to 0. (approximately -0.43 US dollars) This indicates that the call option price increases with an increase in the underlying stock price, whereas the put option price decreases.

b. Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?

The Delta for the European call option is positive, indicating that for each dollar increase in the underlying asset's price, the call option's value is expected to rise by about $0.5695. This positive Delta reflects the call option's tendency to gain value as the asset price rises, behaving similarly to holding the underlying asset.

Conversely, the Delta for the European put option is negative, meaning that for each dollar increase in the underlying asset's price, the put option's value is expected to decrease by approximately $0.4305. This negative Delta suggests that the put option loses value as the asset price rises, and its near-zero value implies it is likely out-of-the-money.

Delta as a Sensitivity Measure:

Delta measures an option's sensitivity to changes in the underlying asset's price:

Positive Delta for Call Options: Indicates that call options increase in value with a rise in the asset's price, aligning with bullish strategies.

Negative Delta for Put Options: Shows that put options gain value when the asset's price falls, aligning with bearish or protective strategies.

Delta reflects how options are used to express market views and manage risk, with positive Delta for calls and negative Delta for puts.

Q7.

a. Compute the sensitivity of previous put and call option prices to a 5% increase in volatility (from 20% to 25%). How do prices change with respect to the change in volatility?

A 5% increase in volatility (from 20% to 25%) causes the price of the European Call option price to increase from 4.61 to 5.59. Similarly, a 5% increase in volatility (from 20% to 25%) causes the price of the European Call option price to increase from 3.36 to 4.34.

b. Comment on the potential differential impact of this change for call and put options.
Call Options:

Call options generally have a positive Vega, meaning their value increases when volatility rises. As volatility grows, the chances of the underlying asset's price moving significantly—either up or down—also increase. For call options, this increased potential for price movement is beneficial because it raises the probability of the asset price exceeding the strike price. Thus, with a 5% rise in volatility, the price of a call option tends to increase more substantially due to its positive Vega.

Put Options:

Put options also have a positive Vega, so their value increases with higher volatility. More volatility means there's a greater chance of the underlying asset's price falling below the strike price, which benefits put holders. However, put options are generally less sensitive to volatility changes compared to call options. This is because their primary value comes from downward price movement, and while increased volatility helps, the price increase of a put option might be smaller relative to its initial value when volatility rises by 5%.

Q8.

(Answer sections a and b of Q5 as well)

part a -- Number of steps chosen is 100 as after this point call price converges to the same point.

part b -- For American style option, same steps are followed as the European option except checking for the early exercise price at each nodes. If it's greater than the european option, then the greater value is preffered over the european option price.

Q9.

part a -- Delta for the call option is positive whereas for the put option it is negative.

part b -- Delta shows the sensitivity of option prices to the underlying stock prices. +ve delta signifies that as the stock price is increased, the option price increases accordingly. It shows the direct relationship. Whereas, the negative one shows the inverse relationship.

Q10.

part a -- It is evident from the output table that as the volatility increases, the price of both the call and put option increases.

part b -- Vega, which is generally positive for both types of options, reflects this relationship, as it measures the sensitivity of the option's price to changes in the volatility of the underlying asset. The more volatile the asset, the greater the chance of significant price shifts, either upwards or downwards. While both calls and puts gain from heightened volatility, the degree of their price increase may vary. This variation is affected by several factors, including how in-the-money the options are, the remaining time until expiration, and prevailing interest rates.

Graphs

Q11.

To show that the European call and put satisfy put-call parity consider the following:
Portfolio A:
• One European call option (price C)
• An amount of cash equal to the present value of the strike price K which is Ke^{-rT}

Portfolio B:
• One European put option (price P)
• One share of the underlying asset (price S)

Payoff at Expiration:
If $S_T > K$ then both Portfolios have a payoff of S_T
If $S_T \leq K$ then both Porfolios have a payoff of K

Because of this, in order to avoid arbitrage opportunities, the initial costs of these portfolios must be equal, resulting in:

$$C - P = S - Ke^{-rT}$$

Where:
C is the price of the European call option
P is the price of the European put option
S is the current price of the underlying asset
K is the strike price of the options
r is the risk-free interest rate
T is the time to expiration

Q12.

American options are not subject to put-call parity since they can be exercised at any point before to expiration, which creates uncertainty regarding the timing of cash flows. Since early exercise and dividends can affect the value of the options differently than

they can for European options, which can only be exercised at expiration, this flexibility leads to pricing difficulties.

Q13.

The American call option holder's ability to exercise the option early is the primary distinction between these two types of options. The American call option has additional value because it can be exercised at any point prior to expiration. This is so that, prior to the expiration date, the holder might profit from favorable price changes of the underlying asset.

Q14.

The American put option holder's ability to exercise the option early is the primary distinction between these two types of options. The American put option has additional value because it can be exercised at any point prior to expiration. This is so that, prior to the expiration date, the holder might profit from favorable price changes of the underlying asset. The holder of an American put option has the opportunity to exercise the option early in order to potentially gain from a cheaper price or prevent future reductions when the price of the underlying asset drops below the strike price. The value of the American put option is increased by its capacity to respond to changes in the market

Step 2

Q15.

a. Using the trinomial tree, price the Call option corresponding to the 5 different strikes selected. (Unless stated otherwise, consider input data given in Step 1).

Ans. Prices are stored in the output of the code below.

b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.
Ans. As the strike price increases, option price decreasesbecause higher strike price means lower chance of option being exercised.

Q16.

part a -- Prices are stored in the output of the code below.

part b -- As the strike price increases, option price increases because higher strike price means lower chance of option being exercised.

Team Member A worked with American options using a trinomial tree:

Q17.

a. Using the trinomial tree, price the Call option corresponding to the 5 different strikes selected. (Unless stated otherwise, consider input data given in Step 1).

-- Prices are stored in the output of the code below.

b. Comment on the trend you observe (e.g., increasing/decreasing in moneyness) in option prices and whether it makes sense.

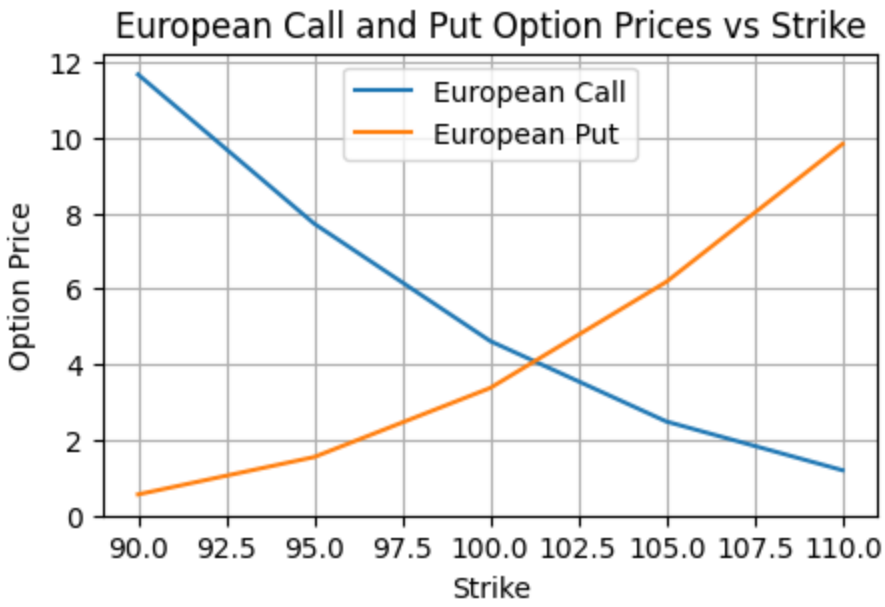
-- As the strike price increases, option price decreasesbecause higher strike price means lower chance of option being exercised.

Q18.

a) Prices are stored in the output of the code Written in NoteBook.

b) As the strike price increases, option price increases because higher strike price means lower chance of option being exercised.

Q19.


European Call and Put Option Prices vs Strike
European Call
European Put
Option Price
Strike
0
2
4
6
8
10
12
90.0
92.5
95.0
97.5
100.0
102.5
105.0
107.5
110.0

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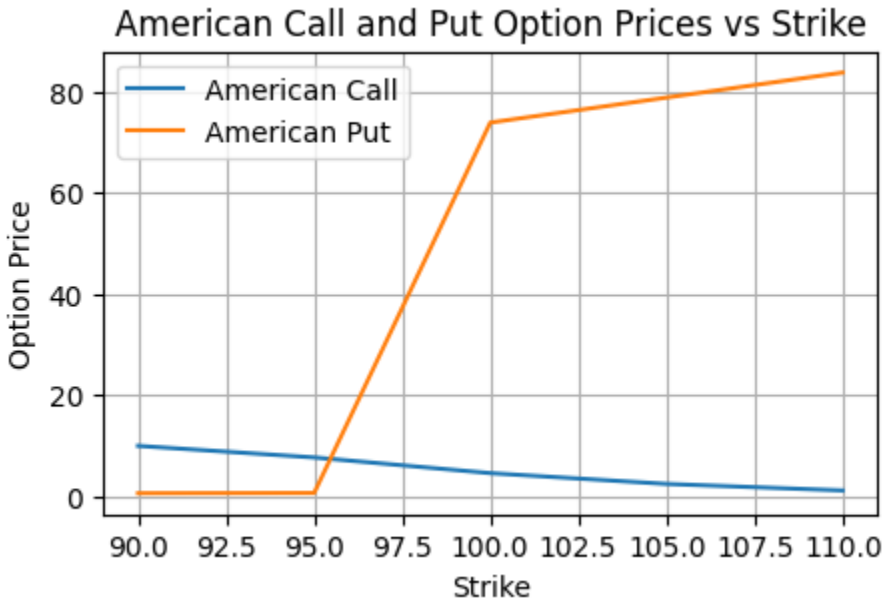
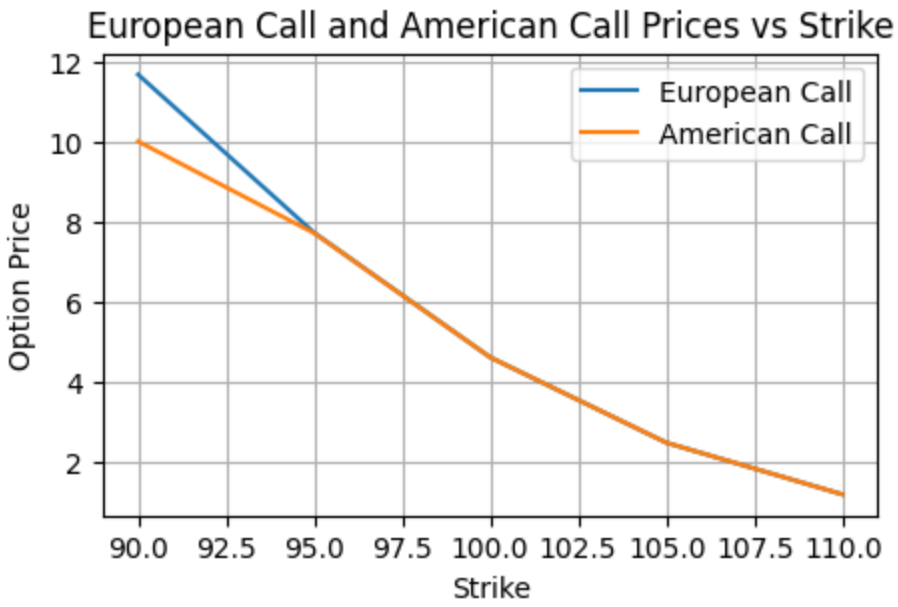
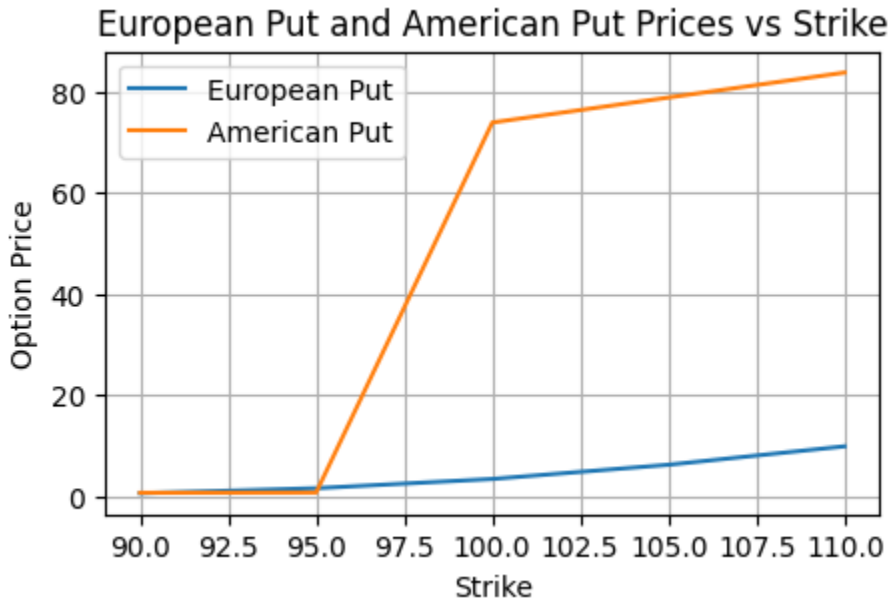
Q20.


American Call and Put Option Prices vs Strike
American Call
American Put
Option Price
Strike
0
20
40
60
80
90.0
92.5
95.0
97.5
100.0
102.5
105.0
107.5
110.0

Q21.


European Call and American Call Prices vs Strike
European Call
American Call
Option Price
Strike
12
10
8
6
4
2
90.0
92.5
95.0
97.5
100.0
102.5
105.0
107.5
110.0

Q22.


European Put and American Put Prices vs Strike
European Put
American Put
Option Price
Strike
80
60
40
20
0
90.0
92.5
95.0
97.5
100.0
102.5
105.0
107.5
110.0

Q23.

As we can see:

```
Equation Results:
--------------------
Set 1:
C_0 = 11.67
Calculated = 11.67
Equation Holds: True
--------------------
Set 2:
C_0 = 7.72
Calculated = 7.72
Equation Holds: True
--------------------
Set 3:
C_0 = 4.61
Calculated = 4.61
Equation Holds: True
--------------------
Set 4:
C_0 = 2.48
Calculated = 2.48
Equation Holds: True
--------------------
Set 5:
C_0 = 1.19
Calculated = 1.20
Equation Holds: False
--------------------
```

So the put-call parity holds (within sensible rounding).

Q24.

```
Equation Results:
--------------------
Set 1:
C_0 = 10.00
Calculated = 11.76
Equation Holds: False
--------------------
Set 2:
C_0 = 7.71
Calculated = 6.87
Equation Holds: False
--------------------
Set 3:
C_0 = 4.61
Calculated = 75.13
Equation Holds: False
--------------------
Set 4:
C_0 = 2.48
Calculated = 75.13
Equation Holds: False
--------------------
Set 5:
C_0 = 1.19
Calculated = 75.14
Equation Holds: False
--------------------
```

So the put-call parity doesn't hold (within sensible rounding).

Step 3

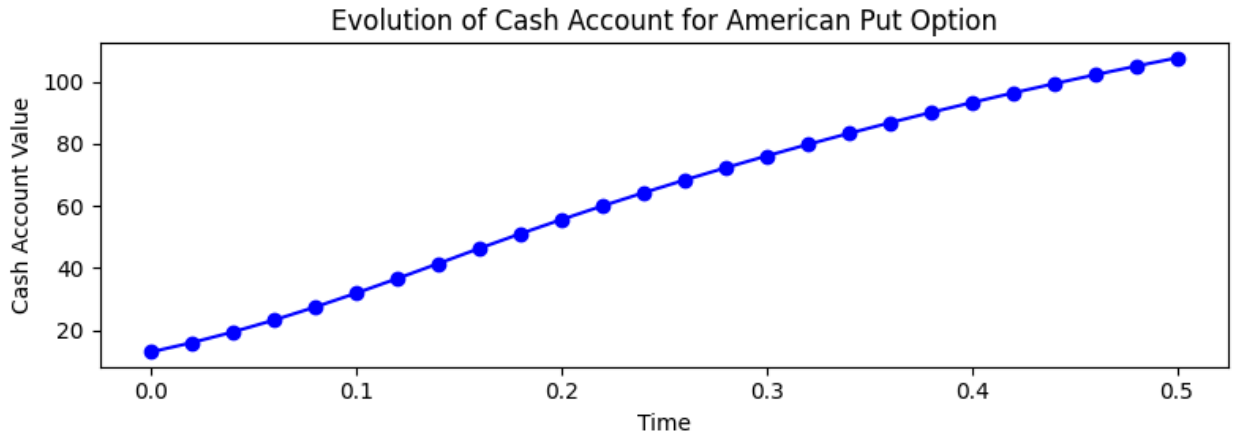
Q25.

a. Price a European Put option with the previous characteristics using a 3-step binomial tree (you do not need code for this).

The price of the European put option is: 0.23

Q26.

The evolution of the cash-account throughout the different steps for one path.



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FinCampus Lecture Hall. Pricing an American Option: 3 Period Binomial Tree Model. YouTube, 26 May 2013, https://www.youtube.com/watch?v=35n7TICJbLc.