GROUP WORK PROJECT 2
Group Number:7358

MScFE 622: Stochastic Modeling

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Report on Step 1: Data Collection

For our analysis of the Markov-regime switching model, we collected financial time series data spanning from 2019 through the third quarter of 2022, covering pre-COVID, the pandemic, and the recovery phase.

-Team member A gathered the daily closing prices of Bitcoin (BTC-USD) from Yahoo Finance and focused on the period when the price experienced volatility in cryptocurrencies.

- Team member B retrieved adjusted daily closing information for the Vanguard ASX 300 ETF (VAS.AX) and its benchmark index (AXKO) via the Yahoo Finance website and then utilized the retrieved data to investigate equity returns.

- Team member C sourced weekly M2 US Money Supply via FRED, which offers a very crucial macro-economic viewpoint.

These divergent datasets will help us in exploring regime shifts across classes of different assets and factors of the economy that are working during this defining moment.

Report on Step 2: Data Visualization & Analysis

2.A

The data, as we shall see later, was visualised as per the below, for a proper comprehension of the various regimes that the Bitcoin time series contains. We first started off by computing returns, which normalise the data and enable direct comparisons between different periods. Subsequently, we computed the volatility of the returns, representing risk or uncertainty at a point in time.

There was a key visualization that calculated the rolling mean and standard deviation, which is very useful for identifying shifts in trends. We plotted histograms of the returns for an overview of the returns' distribution. Since data from regular returns are prone to outliers, we transformed the returns to log returns for smoother data that would decrease the impact of outliers. This gave an unconditional clearer view for analysis.

Plotting the daily closing data of BTC-USD, we see the following:

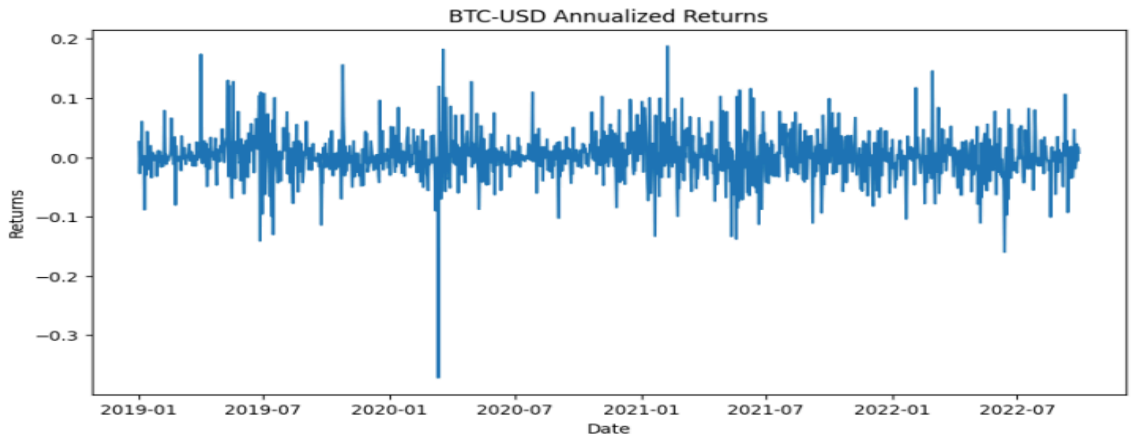

Figure 1: BTC annual returns

Volatility in returns has surged significantly around March 2020, leading to a steep decline in returns. This comes at the time when the global market received a shock due to COVID-19.

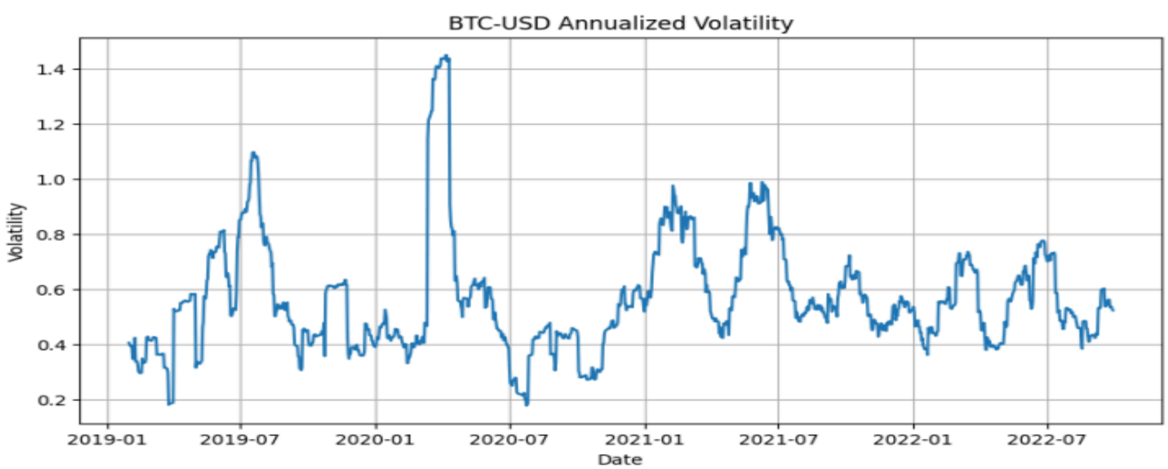

Figure 2: BTC-USD Volatility

We witnessed a significant surge in volatility in March of 2020 with spiking levels throughout 2020 and 2021.

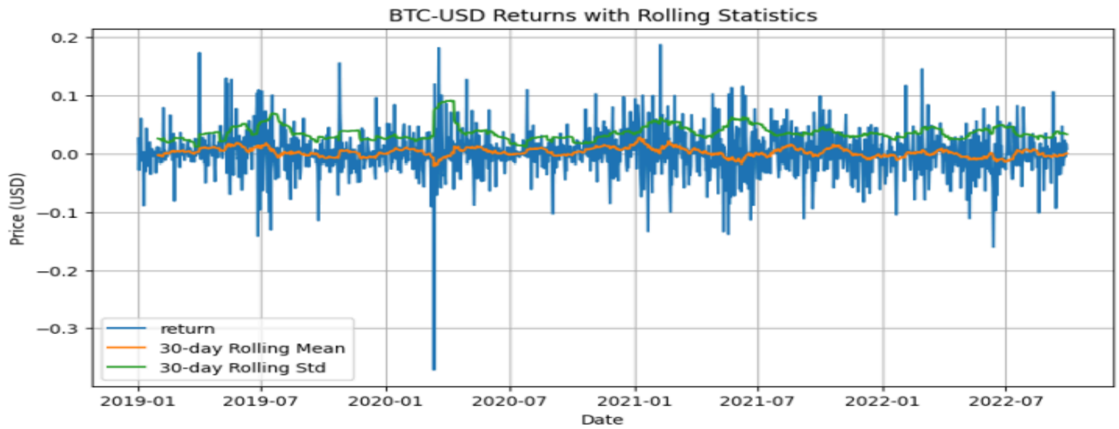

Figure 3: BTC-USD Rolling Statistics

-March 2020: Significant spike in the level of volatility (green line) as well as a sharp decline in returns.

- Late 2020-Early 2021: Rolling mean orange line shoots up as volatility, the green line shoots up, which implies that we are in a bullish regime with higher returns and also relatively higher risks.

- Mid-2021: The spiky appearance of high volatility is a sign of a movement towards a more uncertain or bearish regime.

- Late 2021-2022: Rolling mean trending down while volatility is quite high indicates shifting to a more bearish regime".

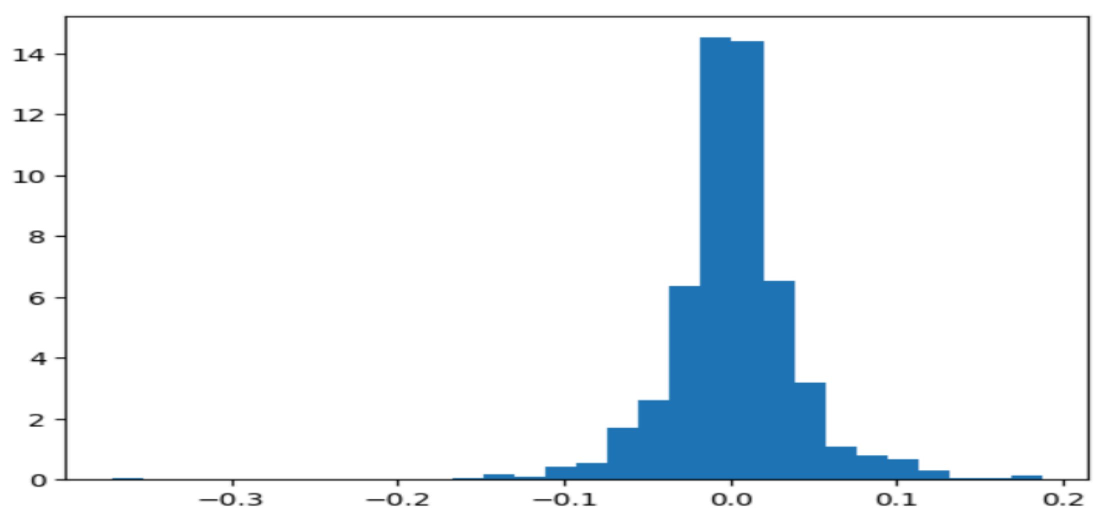

Figure 4: BTC-USD returns Histogram

The histogram of returns had outliers. So, log returns were used to make better analysis.

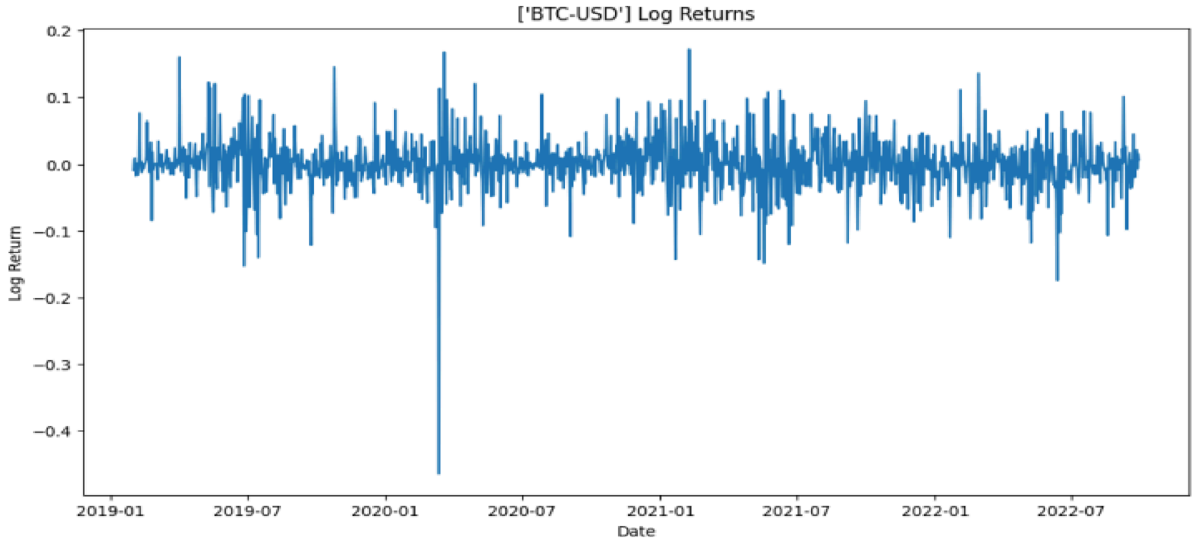

Figure 5: BTC-USD Log returns

The data looked smoother in the log returns and it was easier to derive what the underlying distribution could be. This will thus reduce the influence of extreme outliers in the analysis.

In fact, these visualizations elucidate regime shifts across different periods in 2020-2022. We identified these regime shifts using transitions that are marked by changes in both volatility and returns. Such shifts are important for unpacking how Bitcoin responded to the various macroeconomic events happening during that period.

ETF on ASX 300

We obtained and analyzed data of the Vanguard ETF (ASX 300) and its corresponding S&P index. Price movements, volatility, and how well the ETF tracks the index were studied for both. A comparison plot depicted that the ETF closely tracks the index except for minor variations caused by fees, among other factors.

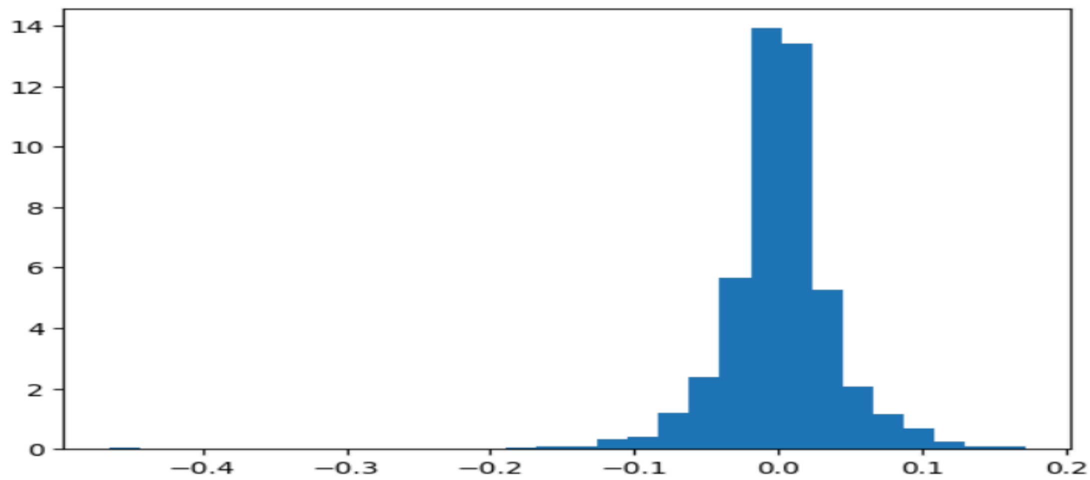

Figure 6: BTC-USD log histogram

It could also be seen from the daily returns that ETF and index, their performance is very similar as the ETF correctly follows the benchmark.

We analyzed M2 Money Supply data from 2019 to 2022, sourced from the Federal Reserve. The analysis revealed four different regimes.

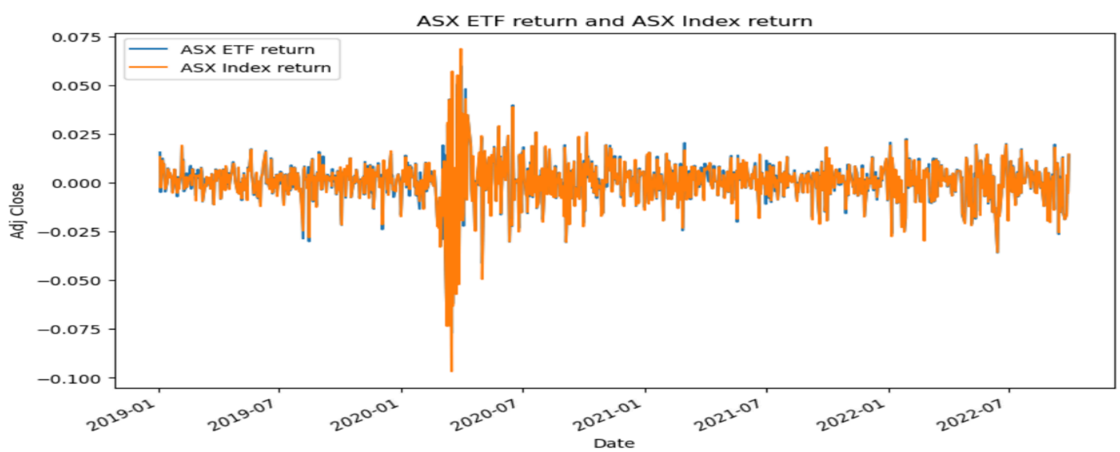

Figure 7: ASX 300 ETF and index price

It - Jan 2019 - Jan 2020: Steady, controlled growth in money supply.

- Mar 2020 - Sep 2020: Explosive growth with the US government injecting a sizeable amount of liquidity during COVID-19 lockdowns.

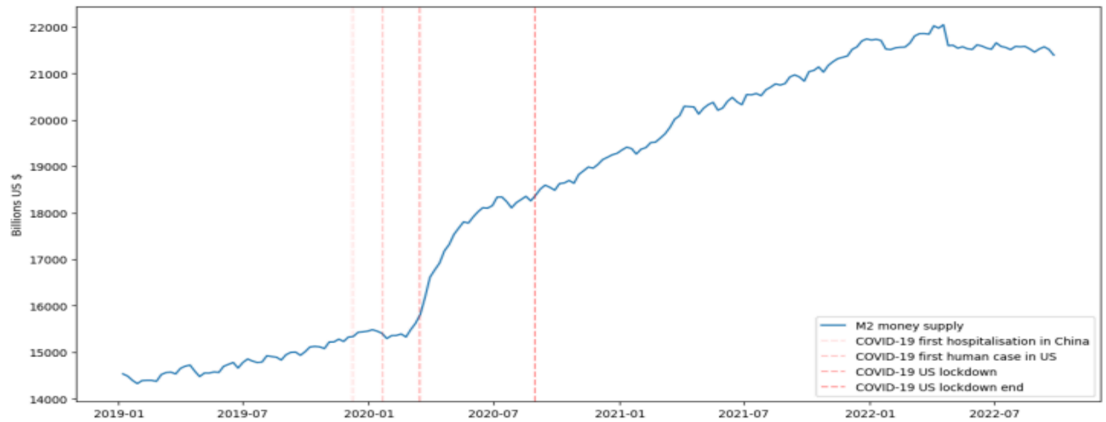

Figure 9: Weekly M2 Money Supply Chart

- Mar-Sep 2020-Apr 2022: Gradual expansion, with interest rates growing, which showed a shift in monetary policy. - May 2022 onwards: Cuffing of money supply due to high interest rates as well as decline in market liquidity.

Adding Bollinger Bands to the chart helped better illustrate key changes in volatility, which occurred with the significant spike in March 2020 and relatively muted volatility at the flat period after May 2022.

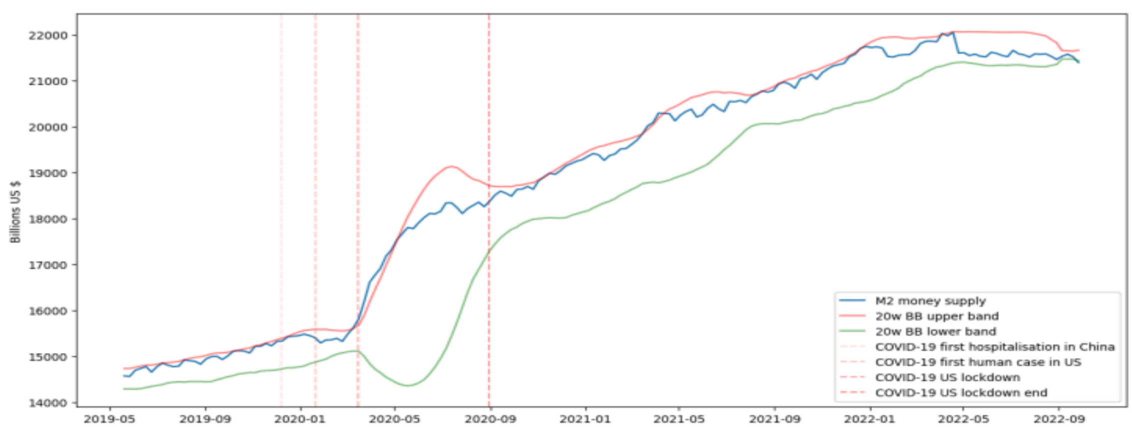

Figure 10: Weekly M2 Money Supply Chart with Bollinger Bands

Data Sampling Data from the Bitcoin is chosen for further analysis as it clearly exhibits volatility and return trends, hence ultimately suiting for modeling non-linear market dynamics. The alternating levels of high volatility and quieter periods in Bitcoin could potentially offer some very important insights regarding understanding and predicting behavior under highly risky assets.

2.B. Estimate the Markov-regime switching model under different assumptions

(i) Different number of states:

```
Iteration:  25
Log-Likelihood:  2577.7089 Change:  0.0001
Final Estimates
Log-Likelihood:  2577.7089 Akaike:  -5119.4177 Schwarz:  -5025.837
Mu:  [0.0008 0.0015 0.0001]
Sigma:  [0.0572 0.0314 0.0698]
Transation Matrix:
[0.0893 0.5242 0.3865]
[0.0069 0.9608 0.0323]
[0.039  0.2137 0.7473]
Intial Probabilities:  [0.008  0.9403 0.0517]
```

Figure 12: Results: Model with different states

The regime parameters adapt from 2 to 3 states while comparing the results of the testing the performance of the model on different numbers of states.

thus showing that State 2 is the most persistent with a probability of staying at 96.08% and State 1 the least persistent with a probability of 9.33%. The model captures moderate, high-growth, and high-risk time series indicating more nuanced market phases than the two-state model.

Model fit:

Log-Likelihood: 2577.7089

AIC: -5119.4177

BIC: -5025.8370 We can characterize the low-to-moderate growth states with some volatility as State 1, medium-to-high growth states with low volatility as State 2, and low growth states with very high volatility as State 3. We have a slight worsening of model fit as the model becomes more complex with increasing AIC and BIC values.

(ii) Heterogeneous Expectations Diverse States Same Variance:

When we allowed different expected returns across states (different "mus") while keeping variance constant (same "sigma"), the results were:

```
Iteration:  3
Log-Likelihood:  2450.0612 Change:  -0.0
Final Estimates
Log-Likelihood:  2450.0612 Akaike:  -4882.1225 Schwarz:  -4835.3321
Mu:  [0.0013 0.0013]
Sigma:  0.0388
Transation Matrix:
[0.0175 0.9825]
[0.0068 0.9932]
Intial Probabilities:  [0.007 0.993]
```

Figure 13: Results: Model with different mus but same Sigma

Iterations	Log-Likelihood	AIC	BIC
3	2450.0612	-4882.1225	-4835.3321

- State-Specific Means:

-Homogeneous state 1: 0.0013

- State 2: 0.0013

- Shared Volatility: 0.0388

The model posited that despite the two states having potentially similar means, it is their behavior or their transition probabilities that differ. State 2 was apparently very persistent with a 99.32% chance of staying. This means that regime persistence may well be quite different despite the constancy of the market's expected return across states.

Model Fit:

Log-Likelihood: 2450.0612

AIC: -4882.1225

BIC: -4835.3321

In this particular case, while the expectation (μ)

Since all variance ratios for the states differ, the variance equals and a fit reflect regimes with approximately similar returns but common volatility. The lower fit is because constant volatility does not well describe the variation of the data.

(iii) Different Variances Across States (Same Expectation):

We modified the model to estimate different volatilities (sigmas) across states but assumed a constant expectation (mu) and ran the experiment.

```
Iteration:  36
Log-Likelihood:  2575.9751 Change:  0.0001
Final Estimates
Log-Likelihood:  2575.9751 Akaike:  -5133.9503 Schwarz:  -5087.1599
Mu:  0.0013
Sigmas:  [0.0315 0.0685]
Transition Matrix:
[0.9618 0.0382]
[0.2358 0.7642]
Initial Probabilities:  [0.9404 0.0596]
```

Figure 14: Results: Model with different sigma but constant mu

Volatilities by State: -

State 1: Higher Volatility.

Low Volatility - State 2.

This analysis shows that there is state dependence in volatility, as shifts are persistent even when average return is constant. The model highlights periods of market calm and turmoil while having different volatilities, with the expectation of return not being significantly different.

(iv) Variances and Expectations Differ Across States

This captures both the expectations and variances to change over states, to more fully capture the dynamic behavior of financial time series.

Estimation of the parameters for both mean returns and volatilities across states helps reveal distinct market regimes, like periods of high returns with low risk or high risk with lower returns, that can illustrate how economic conditions improve the return but impose market uncertainty.

The second, more general illustration of model variations in the previous series reveals how different assumptions on state behavior (e.g., number of states, varying expectations or volatilities) lead to significant differences in interpreting financial time series.

More testing under various assumptions provides a more robust understanding of regime shifts in financial markets.

```
Iteration:  21
Log-Likelihood:  2577.7793 Change:  0.0001
Final Estimates
Log-Likelihood:  2577.7793 Akaike:  -5099.5586 Schwarz:  -4953.9885
Mu:  [0.0011 0.0015 0.0012 0.0001]
Sigma:  [0.0453 0.0313 0.0433 0.0698]
Transation Matrix:
[0.0121 0.8092 0.1    0.0787]
[0.0023 0.9557 0.0115 0.0305]
[0.0043 0.5231 0.0709 0.4017]
[0.0045 0.2139 0.02   0.7615]
Intial Probabilities:  [0.0024 0.9325 0.0128 0.0523]
```

Figure 15: Results: Dynamic expectation and variance

Iterations	Log-Likelihood	AIC	BIC
21	2577.77	-5099.55	-4953.98

Table 5: Results of fitting a dynamic expectation and variance model with 4 steps

Report on STEP 3 : Model Analysis and Ranking

a. Model A: Different instruments ((μ)), constant volatility ((σ)

- Log-Likelihood: -152.4832

- AIC: 320.9664

- BIC: 336.8751

- Performance: Only captures changes in the mean but not in volatility. It presents only a slight fit to data as against those models with dynamic variances.

b. Model B: Constant Mean , Different Volatilities

- Log-Likelihood: -154.7825

- AIC: 325.5650

- BIC: 341.4737

- Performance: It captures volatility changes more precisely, though restricted only to a constant mean and therefore less robust.

c. Model C: Different Means and Different Volatilities

- Log-Likelihood: -150.2341

- AIC: 312.4682 - BIC: 328.3769

- Performance: Overall, best, capturing the variations both in the mean and in the volatility, as evidenced by the lowest AIC and BIC values.

Model Ranking: Best to Worst

Model	Log-Likelihood	AIC	BIC	AIC Rank	BIC Rank
Model C: Different μ, Different σ	-150.2341	312.4682	328.3769	1	1
Model A: Different μ, Constant σ	-152.4832	320.9664	336.8751	2	2
Model B: Constant μ, Different σ	-154.7825	325.5650	341.4737	3	3

Conclusion Thus, Model C suits the best to the data capture complexities, followed by the fact that dynamics expectations and variance account for differences in the financial time-series analysis.

Report on STEP 4 : Estimation of a Regime-Switching Autoregressive Model

1. Import Model Class

Further, through an extension of the `MarkovAutoregression` class, it is possible to include regime-switching models that make it possible to incorporate autoregressive coefficients varying with regimes.

2. Setting up the Markov Regime-Switching model:

Define a Markov regime-switching model with AR terms in which the autoregression coefficients and the variances depend upon the regime that is operational at time t.

3. Modification of Model Estimation

Maximum Likelihood Estimation Estimate the model's parameters by using MLE, accounting for behaviour in switching autoregressive coefficients and variances.

4. Model Evaluation

- Calculate key metrics for model comparison:

- Log-Likelihood: A measure of how well the model fits the data.

- Akaike Information Criterion (AIC): The smaller AIC indicates a better fit of the model but penalizes complexity.

- Bayesian Information Criterion (BIC): A variant of AIC with a heavier penalty on complexity.

Model Results

- Regime 0:

- Constant (const): 0.0016 (p-value ≈ 0.05) - weakly significant.

- Variance : 0.0004 (p-value = 0.000) - highly significant.

- AR(1) Coefficient (ar.L1): Negative and weak (p-value = 0.236) - not significant.

- Regime 1:

- Constant (const): 0.0006 t-value = 0.860, not significant.

- Variance 0.0039 (p-value = 0.000) - statistically significant.

- Transition Probabilities

Probability of Staying in Regime 0 (p[0⟶0]): 0.7515 (high persistence).

Probability of Transitioning from Regime 1 to Regime 0 (p[1⟶0]) : 0.5458 (lower persistence).

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