FULL LEGAL NAME	LOCATION (COUNTRY)	EMAIL ADDRESS	MARK X FOR ANY NON-CONTRIBUTING MEMBER
Regulavalasa Krishna Vamsi	India	krishnavamsi8262@gmail.com	
Aditya Raj	INDIA	aadityaa.1301@gmail.com	
Sithembiso Mnqobi Ndwandwe	South Africa	ndwaby@gmail.com	

Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
Team member 1	Regulavalasa Krishna Vamsi
Team member 2	Aditya Raj
Team member 3	Sithembiso Mnqobi Ndwandwe

Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. Note: You may be required to provide proof of your outreach to non-contributing members upon request.

STEP 1:

Q5)

Pricing of the ATM European call and put option with the Heston model and Monte Carlo simulation can be performed through the first step of initializing the model and simulation parameters, the second step of generating random numbers for the asset and volatility process, next the third step of constructing a covariance matrix to account for the correlation as specified by ρ = -0.30, the fourth step of simulating the volatility paths, and the final step simulating the asset price paths by using the volatility paths. Next, for call and put options at maturity, calculate the option payoffs; discount this payoff using the risk-free rate to present value; and average across all simulations. The average should represent the estimated prices of ATM European call and put options using the Heston model by Monte Carlo simulation by Schumacher 2020.

On the basis of the calculation, In general, European call and put options are closely related to each other. The put options are a little bit cheaper because it is not wise to say only opportunities for the put, as it is better to keep more cash than the security itself.

European Call Price under Heston with S0 = 80 and rho = -0.3: 2.8155

European Put Price under Heston with S0 = 80 and rho = -0.3: 2.8738

Put-Call Parity Check (difference): -1.052125

Q6)

There is a big change in the price of the ATM European call and put option, when the correlation value of the Heston model changes from -0.30 to -0.70, that is, a $2.06 value of the call and a $3.5 value of the put. This would actually be because of the impact of correlation on the dynamics in option pricing. This is because, when the asset price goes up, the asset is no more volatile because of the greater negative correlation of -0.70 between the asset and the volatility, entails, which causes the downswing in the cost of the call option. On the other hand, the put option can, nevertheless, utilize the maximum negative correlation to boost its value by defending the increased fall of the index, which then tends to make the put option more expensive. It also hints at the workflow of correlation dynamics in option pricing sensitivity, market sentiment, and model calibration.

European Call Price under Heston with S0 = 80 and rho = -0.7: 2.0664

European Put Price under Heston with S0 = 80 and rho = -0.7: 3.5043

Put-Call Parity Check (difference): -2.431661

Q7)

For the pricing of options with the Heston Model and Monte Carlo simulation, the underlying quantities which are the two most important Greek letters that show the serious risks that options can have are "delta" and "gamma".

Delta is the price sensitivity of an option to the underlying asset. This is often termed as the derivative of the option. For instance, the delta in the Heston Model and Monte Carlo simulation can be calculated by simulating a slight change in the asset price and increasing it for each simulation. Thus, the option price is then reviewed at option price at each simulation and change is averaged. It can assume a value between -1 and 1 in the case of calls and between -1 and 0 in the case of puts. Specifically, the Heston Model and Monte Carlo simulation with a rho of -0.30, the delta that corresponds to the call option is 0.73, while for a put option, the delta obtained is 5.12.

Conversely, the Gamma is the bias which option delta fluctuates at as the price goes up or down on the underlying.

Call Delta for rho = -0.3: 0.7300

Call Gamma for rho = -0.3: 5.1260

Put Delta for rho = -0.3: -0.6831

Put Gamma for rho = -0.3: 7.3697

Call Delta for rho = -0.7: 0.9235

Call Gamma for rho = -0.7: 74.4477

Put Delta for rho = -0.7: -0.6294

Put Gamma for rho = -0.7: -131.3735

Result for Q12 (Moneyness, Heston Model)

European Call Price under Heston with moneyness = 0.85 and rho = -0.3: 0.0961

European Call Price under Heston with moneyness = 0.90 and rho = -0.3: 0.3945

European Call Price under Heston with moneyness = 0.95 and rho = -0.3: 1.2180

European Call Price under Heston with moneyness = 1.00 and rho = -0.3: 2.8147

European Call Price under Heston with moneyness = 1.05 and rho = -0.3: 5.1078

European Call Price under Heston with moneyness = 1.10 and rho = -0.3: 7.7942

European Call Price under Heston with moneyness = 1.15 and rho = -0.3: 10.5515

European Call Price under Heston with moneyness = 0.85 and rho = -0.7: 0.0098

European Call Price under Heston with moneyness = 0.90 and rho = -0.7: 0.1189

European Call Price under Heston with moneyness = 0.95 and rho = -0.7: 0.6793

European Call Price under Heston with moneyness = 1.00 and rho = -0.7: 2.0674

European Call Price under Heston with moneyness = 1.05 and rho = -0.7: 4.1912

European Call Price under Heston with moneyness = 1.10 and rho = -0.7: 6.7109

European Call Price under Heston with moneyness = 1.15 and rho = -0.7: 9.3586

Jumper Modeler:

For the Merton model, we use the following parameters:

μ = -0.5

δ = 0.22

Q8)

Merton's model is a jump-diffusion option pricing model. The process of the stock price is lognormal with the possibility of jumps in the stock price, governed by a jump intensity parameter, λ.

European Call Option Price with jump intensity of 0.75: 8.28

European Put Option Price jump intensity of 0.75: 7.20

Q9)

European Call Option Price with jump intensity of 0.25: 6.81

European Put Option Price jump intensity of 0.25: 5.79

Q10)

Measures of sensitivity are used in options pricing. Delta and gamma are measures of sensitivity of an option price to a change in the price of the underlying asset.

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Gamma of european call with intensity of 0.75:: 72667.1700
Delta of european call with intensity of 0.75: 21.8533
Gamma of european call with intensity of 0.25:: -1043.0658
Delta of european call with intensity of 0.25: -32.3934
Delta of european put with intensity of 0.75: 11.5227
Gamma of european put with intensity of 0.75:: 11233.2869
Delta of european put with intensity of 0.25: 24.2567
Gamma of european put with intensity of 0.25:: -63593.7835
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Q11)

The put-call parity is $C_0 = -Ke^{-rT} + S_0 + P_0$,

For : Q5) $C_0=2.86$, $-Ke^{-rT}=-78.91$, $S_0=80$, $P_0 = 2.83$.

Q6) $C_0=2.09$, $-Ke^{-rT}=-78.91$, $S_0=80$, $P_0= 3.45$.

Q8) $C_0=8.28$, $-Ke^{-rT}=-78.91$, $S_0=80$, $P_0= 7.20$.

Q9) $C_0 =6.81$, $-Ke^{-rT}=-78.91$, $S_0=80$, $P_0= 5.79$.

So that $C_0 \neq -Ke^{-rT}+S_0+P_0$, Meaning Put-Call parity is not satisfied under the Heston Model & Merton Model for (Q5, Q6) & (Q8, Q9) respectively.

Q12)

Summary of Results

Summary of Results

	Q #s	Type	Exer	GWP 3 Method	StrikesPrice
12	Call	European	Heston Model in combination with a correlation value of -0.30	68	12.15
12	Call	European	Heston Model in combination with a correlation value of -0.30	72	8.55
12	Call	European	Heston Model in combination with a correlation value of -0.30	76	5.38
12	Call	European	Heston Model in combination with a correlation value of -0.30	80	2.92
12	Call	European	Heston Model in combination with a correlation value of -0.30	84	1.33
12	Call	European	Heston Model in combination with a correlation value of -0.30	88	0.52
12	Call	European	Heston Model in combination with a correlation value of -0.30	92	0.18
12	Put	European	Heston Model in combination with a correlation value of -0.30	68	0.19
12	Put	European	Heston Model in combination with a correlation value of -0.30	72	0.54
12	Put	European	Heston Model in combination with a correlation value of -0.30	76	1.31
12	Put	European	Heston Model in combination with a correlation value of -0.30	80	2.8
12	Put	European	Heston Model in combination with a correlation value of -0.30	84	5.16
12	Put	European	Heston Model in combination with a correlation value of -0.30	88	8.3
12	Put	European	Heston Model in combination with a correlation value of -0.30	92	11.9

Summary of Results

	Q #s	Type	Exer	GWP 3 Method	StrikesPrice
12	Call	European	Merton Model in combination with jump intensity of 0.75	68	16.29
12	Call	European	Merton Model in combination with jump intensity of 0.75	72	13.4
12	Call	European	Merton Model in combination with jump intensity of 0.75	76	10.63
12	Call	European	Merton Model in combination with jump intensity of 0.75	80	8.24
12	Call	European	Merton Model in combination with jump intensity of 0.75	84	6.33
12	Call	European	Merton Model in combination with jump intensity of 0.75	88	4.69
12	Call	European	Merton Model in combination with jump intensity of 0.75	92	3.38
12	Put	European	Merton Model in combination with jump intensity of 0.75	68	3.4
12	Put	European	Merton Model in combination with jump intensity of 0.75	72	4.36
12	Put	European	Merton Model in combination with jump intensity of 0.75	76	5.56
12	Put	European	Merton Model in combination with jump intensity of 0.75	80	7.22
12	Put	European	Merton Model in combination with jump intensity of 0.75	84	9.14
12	Put	European	Merton Model in combination with jump intensity of 0.75	88	11.55
12	Put	European	Merton Model in combination with jump intensity of 0.75	92	14.17

Q13)

Summary of Results

Q #s	Type	Exer	GWP 3 Method	Price
5	Call	European	Heston Model in combination with Monte-Carlo simulation with a correlation value of -0.30	2.83
13	Call	American	Heston Model in combination with Monte-Carlo simulation with a correlation value of -0.30	3.62

Comment on any differences you observe from original Questions 5

Because of this early exercise feature that allows the holder to exercise the option any time before expiration, the usual pricing of an American call option is higher than that of a European call option. American options, preceded by stochastic volatility and optimal timing of exercise, have greater values. The ability to take advantage of favorable price movements before maturity enhances the value of American options over European options, which can be exercised only at expiration.

Moreover, the concept of optimal exercise time, while pricing an American option with the use of simulations under the Heston model, pinpoints the significance of timing at each simulation level. This consists of the evaluation of the calculated option price in contrast to the intrinsic value of the option, which is considered the difference between the current price of the asset and the strike price. In instances when this intrinsic value is higher than the option price, the American call option becomes exercisable. Concerning European options, taking away such flexibility might make the option prices expensive.

Last but not least, the Heston model introduces stochastic volatility, further making the dynamics of option pricing volatile, besides considering path dependence. It is this path-dependent nature that makes American options beneficiaries, where volatility conditions fall in their favor to hike up the values of options. In contrast, European options are dependent solely upon the asset price at expiration and thus path-independent. Another reason for the higher pricing of the American call option in the Heston model is the combination of stochastic volatility and path-dependence.

Comment on differences observed from original question 8

Within the Merton model, American options with early exercise are more sensitive to jump parameters such as jump intensity, mean, and volatility of jumps than European options. This is explained by the fact that the value of the option depends more on sudden changes in price, allowing the holder to advantageously exercise the option owing to such jumps.

Jump Impact on Early Exercise: The presence of a possibility of big upward jumps-u--positive skewness-reduces the tendency to exercise American calls prematurely. If a worthwhile price jump is likely at some point in the future,

Computational Complexity: From the more practical point of view, American option pricing in Merton's model is much more computationally demanding due to the presence of optimal stopping times at every step in time. Such is a coupling between early exercise and jump dynamics, an interaction absent in the setup for European options.

Summary of Results

Q #s	Type	Exer	GWP 3 Method	Price
5	Call	European	Heston Model in combination with Monte-Carlo simulation with a correlation value of -0.30	2.83
13	Call	American	Heston Model in combination with Monte-Carlo simulation with a correlation value of -0.30	3.62

Q14)

Summary of Results

Q #s	Type	Exer	GWP 3 Method	Price
5	Call	European	ATM European Call for a simple Heston Model	2.01
5	Call	American	Up-and-in Call option (UAI) with a barrier level/strike price $95	0.01

Now compare this price with that of the simple European call:

The Down-and-In put option is cheaper than the standard ATM European put because it will be only activated when the underlying stock price cuts through a certain barrier level, usually pre-set lower than at which the original stock price was. This prerequisite reduces the chance of the option being in profit since it could quite easily expire worthless should the barrier not be breached. By contrast, the payoff for the ATM European put does not have that prerequisite-it pays off when the stock price is below the strike at expiration and is thus more likely to be in-the-money. It therefore usually sells at a higher price. The price difference reflects the risk-reward tradeoff associated with the additional conditions of the DAI option.

Q15)

Given this, the price for the European Down-and-In Put Option stands at $9.96, a number significantly higher than that of a European Put Option with jump intensity of 0.75 at strike $65, which is $3.72. The resultant implied that the barrier option is more valuable under this situation, and this is probably due to the higher possibility of breaching the barrier and, hence, triggering the option.

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