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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

Step 1

Q1.

5a) Certainly! Here it is broken down into points

1. A put option gives the holder the right, but not the obligation, to sell the underlying asset at a predetermined strike price.

2. A call option provides the holder the right to buy the underlying asset at a specified strike price, without any obligation.

3. To calculate option prices using the Black-Scholes closed form:

- Time-based adjustments are made.

- Intermediate variables, d^+ and d^-, are computed.

- The cumulative distribution function is used for these calculations.

Output for Q5, Q6 & Q7 European Call and Put (Closed Form Solution)

Q#	Type	Exer	GWP1 Method	GWP2 Method	GWP1 Price	GWP2 Price	%Diff
5	ATM Call	EUR	Binomial	Closed Form	$4.61	$4.61	-0.0011
5	ATM Put	EUR	Binomial	Closed Form	$3.36	$3.37	-0.0038
6	Call Delta	EUR	Binomial	Closed Form	0.5695	0.5624	0.0125
6	Put Delta	EUR	Binomial	Closed Form	-0.4305	-0.4265	0.0092
7	Call Vega	EUR	Binomial	Closed Form	19.60	19.65	-0.0026
7	Put Vega	EUR	Binomial	Closed Form	19.60	19.65	-0.0026

5b)

Stock Price Movement: We assumed that the stock price moves according to "geometric Brownian motion," which means it changes smoothly and randomly with respect to time.

Call Option Pricing: The case of a call option involves computing the likelihood of the stock going above the strike price at option expiration. We will use a special math function, called the cumulative distribution function, to do so.

Discounting: We worked out what the call option might pay out in the future, and then we reduce the value—discount—to find out what that means in today's terms, using the risk-free interest rate.

Put Option Pricing: For the put option, we have done the opposite: calculate the probability that the stock price will fall below the exercise price and then find its current equivalent by discounting.

Black-Scholes equation: This formula helped us figure out the fair price for European call and put options by looking at things like the stock price, strike price, interest rate, time to expiration, and volatility.

Here's a revised version of your points:

6a)

• Delta for a European Call Option: It is 0.56, implying that a small rise in the price of the underlying will lead to an increase in the price of the Call Option by about 0.56.

• On the other side, Greek delta for European Put Option corresponds to a value of – 0.43. This simply implies that given a small rise in the price of the underlying, the price of the put option falls by roughly its 0.43 times change in price of underlying.

6b)

This makes sense, for a positive Delta for a European call option increases in value along with a rise in the underlying price of the stock. This happens just because the holder of the option gets to buy the underlying asset from the writer of the option at a fixed price, so the more the price is in the market, the more the buyer of the option benefits by buying from the writer.

7a)

When volatility increases by 5 percent, a European Call option value increases from 4.61 to 4.81.

- A European Put Option valued at 3.37 also moved up to 3.57 for a similar 5percent increase in volatility.

7b) Call Option: The price moved up from $4.61 to $4.81. Intuitively, the more volatile it becomes, the better for the holder of a call option—who is in the business of making money from rising prices. This increase of $0.20 indicates that call options are more valuable when there is a possibility of larger price jumps.

Put Option: The price moved up from $3.37 to $3.57. Though it may sound a bit counterintuitive, the fact of the matter is that volatility also favors put options because it makes extreme drops more likely; that is helpful for holders of put options. To that end, the increase of $0.20 shows put options increase in value with higher volatility due to their favorable involvement with falling prices.

Q2.

5) This program approximates the price, Delta, and Vega of a European Call and Put option by simulating many paths that future stock prices may take using Monte Carlo methods. The code varies the stock price and volatility to show how this would affect the value of the option and checks against benchmark values to estimate accuracy. Computed prices and sensitivities are summarized in Table, with differences with respect to the benchmark values well evident.

6. Delta is expressed as how much the option price will change with a change in the underlying price of the stock. For calls, Delta is positive, meaning that an increase in the underlying price raises the value of the option. Options with a negative payoff are those options whose value falls when that of the underlying stock goes up. All these make sense: call options increase in value as the price of the underlying stock goes up, and put options increase in value as the price of the stock goes down. Delta provides the estimate of the sensitivity of the price of the option to a change in the price of the underlying stock.

7. When volatility changes from 20% to 25%, the price of both the call and the put option rises. This is because there will be more volatility, in which there are chances of big swings in price, which would be liked by options. We compute Vega, telling how much with a change in volatility, the price of an option will change. In the case of call options, if volatility increases by 5%, then the price is higher. For puts, we have a higher price. Basically, Vega tells us how sensitive the price of an option is with respect to changes in volatility. Higher Vegas indicate a greater sensitivity of the option's price to such changes.

Output for Q5, Q6 & Q7 European Call and Put

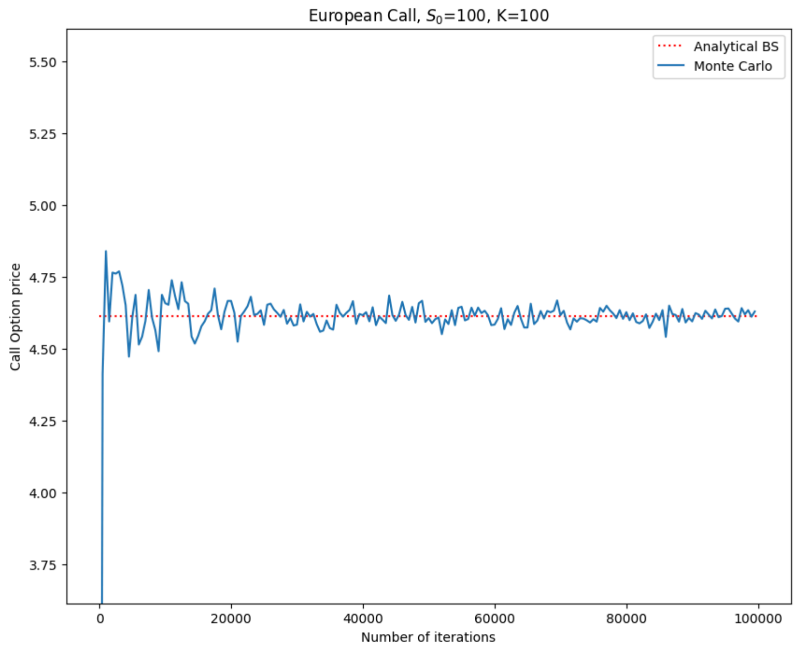
Q#	Type	Exer	GWP1 Method	GWP2 Method	GWP1 Price	GWP2 Price	%Diff
5	ATM Call	EUR	Binomial	MC	$4.61	$4.63	-0.0033
5	ATM Put	EUR	Binomial	MC	$3.36	$3.36	-0.0004
6	Call Delta	EUR	Binomial	MC	0.5695	0.5764	-0.0121
6	Put Delta	EUR	Binomial	MC	-0.4305	-0.4363	-0.0135
7	Call Vega	EUR	Binomial	MC	19.60	19.97	-0.019
7	Put Vega	EUR	Binomial	MC	19.60	19.87	-0.0136

Q3.

a) Put-Call Parity is satisfied.

b) As runs increase, the simulated prices of options in the Monte Carlo approach converge to the theoretical prices derived in the Black-Scholes model.

The reason why this occurs is that a large sample of possible outcomes generates option prices through the Monte Carlo method. The law of big numbers will drive this sample to estimate the expected value of the option payout when the number of iterations goes to infinity. In our case, with 100,000 iterations, this Monte Carlo simulation provides an extremely accurate estimate, very close to the analytical conclusions of the Black-Scholes model. The variance of the Monte Carlo estimate decreases with the sample size, producing a more accurate estimate that will, in theory, converge to the Black-Scholes value where there are enough simulations. We can see how the European Call price converges; then, according to the formal parity, also the European Put price converges.



As we can see the European Call price converge,so that the European Put price also converge according to the formal parity.

Q4.

5a) The American Call option price of $17.99 is the simulation-based estimate of the option's value with consideration of early exercise options in numerous paths for stock prices. Thus, it is an average payoff from all the simulations. Using the Monte Carlo technique, the value of the option is arrived at because of the random price movements considered in it, along with an optimum exercise strategy for each of these paths.

The Delta 0.01 means that should a stock price increase by 1%, the price of the option would go up 0.01. Considering the volatility went up by 1%, Vega, being 4.99, would translate to a rise that can be able to push the price of an option up to about 4.99. The value derived from such measures is the sense of how shifts in stock price and volatility would rattle around in the value of an option.

5b) The Greek Delta for the American call option at time 0 is 0.01.

i. The delta of the American call is 0.01; hence, price is minimally sensitive to movements in the stock price, compared to that for an American Put of –0.49, reflecting significant intrinsic value and potential for early exercise.

(ii) For a Call option, its positive delta of 0.01 interprets that on every 1% increase in the price of the stock, the Call option's price is viewed to move up by 0.01, thus showing the responsive movement to an upward stock price.

iii) It would mean the sensitivity of an option price to a change in the underlying stock price, which guides the traders on how much change may affect their option positions.

Long call option— The American call requires a positive delta. Yes, this does make sense because it rightly states that the value of the option will increase with an underlying stock price increase. After all, it benefits from the fact that buying at a lower strike price and selling at a higher market price creates an opportunity.

6a)

The American Call option value increases from 17.99 to 22.08 in case of a 5percent increase in volatility. This thus, proves that increased volatility will cause an increase in the value of the option since it opens up the possibility of extreme swings either upwards or downwards in price of the underlying asset; this further raises the likelihood that this option may expire in-the-money, thereby increasing the market price of the option

6b)

That the American Call option price itself increased from 17.99 to 22.08, when volatility went up from 20 percent to 25 percent, does clearly reveal that the American call option price does go up with rising market uncertainty. Increased volatility will enlarge both its potential gains and risks, therefore reiterating that, especially in such volatile market conditions, very careful risk-reward analysis has to be supported with efficient risk management strategies.

Q5.

Monte carlo American Put Option Price:3.1499

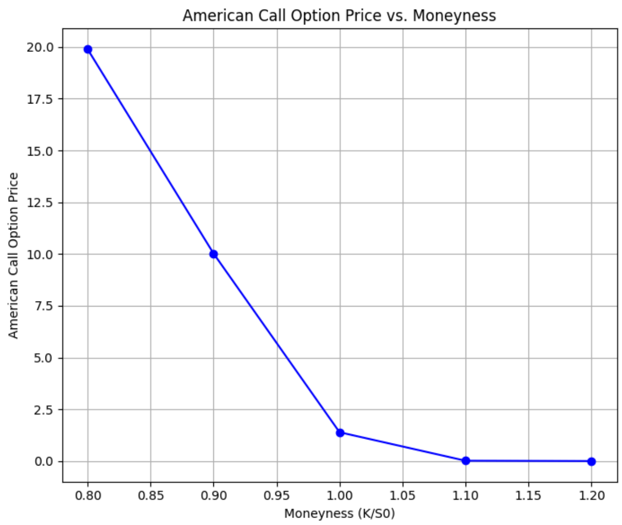
Monte carlo Delta for American Put Option:-0.4790.

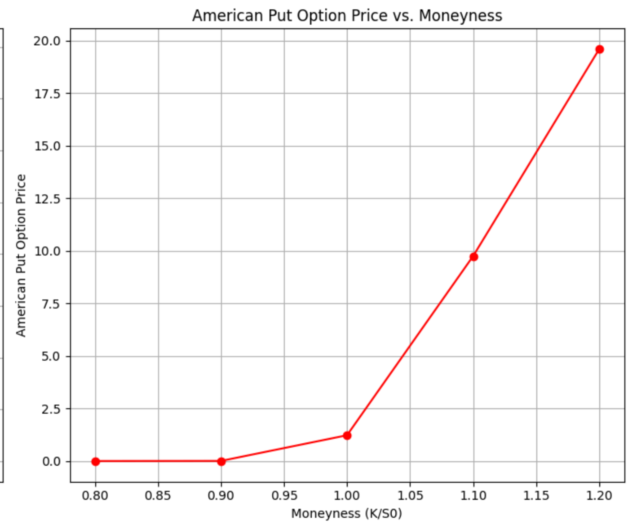
i) A negative Delta for a put option implies that its price falls if that of the stock goes higher, it essentially drops in value when the price of the stock goes up and rises in value when the price of the stock drops. A positive Delta on puts is quite rare and weird since it would mean that the higher the price of a stock, the higher the price of an option, quite contrary to the usual action of puts. A negative Delta is expected for American puts since an increase in stock prices brings down intrinsic value and profit potential for the holder of a put option.

ii) Therefore, with the early exercise feature in high-volatility environments, the American put option has to be worth more than the European put. Given high volatility, the probability that the option will be deep in-the-money before expiration is higher, thereby giving the holder more opportunities for advantageous early exercises. These improved potentials for downward movements and the more optimal opportunities for exercise are then reflected in the price: an increasing price of the American put in response to rising volatility.

Q6.

6a, 6b) Deep OTM: Because the call option is far from the strike price and hence less likely to be lucrative, it will have the lowest price. OTM: Because it is somewhat out-of-the-money, the price will be higher than Deep OTM but still quite affordable. ATM: Because the option is at the money and has the highest intrinsic value, the price will be greater than for OTM options. ITM: Since the option is now in-the-money and has intrinsic value, the price will rise even more. Deep ITM: Due to its substantial intrinsic value and deep in-the-money position, the call option will have the highest price. As moneyness rises, the graph usually displays a rising trend in the call option price.





Q7.

a)

Pricing a European call option at 110% moneyness and European put option with 95% moneyness with the Black-Scholes model yields a call price equal to 1.19, while the Put is at 1.53. It thus shows that the call option was relatively cheap as opposed to the put option.

The pricing difference stems from intrinsic characteristics of the options. The Call option becomes more valuable when the price of an underlying asset goes up, and a Put option would gain value when an asset's price falls.

This is also reflected in the computed deltas, wherein the Call option has a positive delta of 0.22, as it is an instrument that will appreciate in value as the underlying stock price goes up, while the Put option has a negative delta of -0.25, as it is an instrument whose value drops as the stock price rises.

b(i)

Portfolio Delta = Call Delta + Put Delta = 0.22 + (-0.25) = -0.03

This means that there will be a slight reduction in portfolio value due to a small change in the underlying price.

b. (ii) To delta-hedge a portfolio with a delta of −0.03, one needs to take a long position in the underlying to offset that negative delta. This will involve the purchase of approximately 0.03 units of the underlying per unit of the portfolio. You're doing this in order to make your portfolio less sensitive to small price changes in the underlying security, or more balanced in risk exposure.

C(i) The portfolio's delta when one is long a European Call option and short a European Put option, would be derived by subtracting a Put option's delta from that of a Call option:

Portfolio Delta = Call Delta – Put Delta = 0.22 – (-0.25) = 0.47

This positive delta of 0.47 indicates that the value of the portfolio will increase if there is a rise in price for the underlying asset.

c. ii. Since this portfolio will have a positive delta of 0.47, you will want to delta-hedge by going short the underlying asset. Sell about 0.47 units of the underlying asset to offset the positive delta of this instrument. This will leave you with a balanced portfolio regarding price changes, seeking a more stable risk exposure.

Q8, Q9.

Monte-Carlo methods with daily time steps to price an Up-and-Out (UAO) barrier option. The option is currently ATM with a barrier level of 141 and:

$S_0 = 120$; $r = 6\%$; $\sigma = 30\%$; $T = 8$ months

Output for Q8 & Q9

Option Type	Price
Up-and-Out Call Option Price	$0.70
Up-and-In Call Option Price	$13.22
Vanilla Call Option Price	$13.92

- Vanilla options are the most expensive due to their flexibility and lack of barrier conditions.

- UAI options are priced intermediate, reflecting their potential barrier activation.

- UAO options are the least expensive because their barrier condition reduces their value.

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