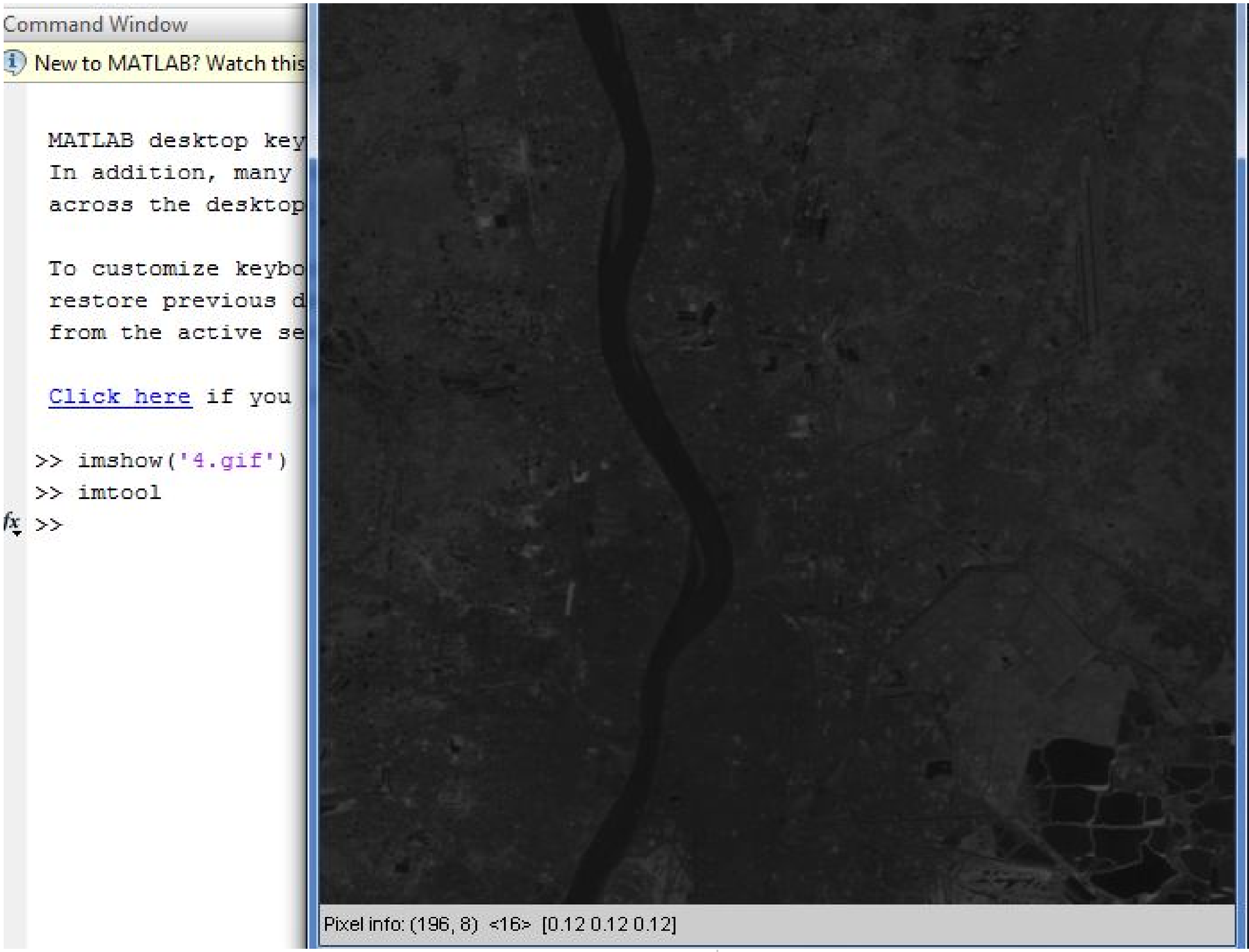
Practical Implementation on Baye’s Decision Rule

How to Implement

- *Four satellite Images of Kolkata (**Rband, Gband, Bband and Iband**) are given to you with equal image size (512 * 512).*
- *The feature vector dimension is 4*
- *Each pixel location we have four values.*
- *Two Classes are given (River and NonRiver)*
- *Take 50 sample points (Pixel location's corresponding pixel values) from river class for training for each band*
- *Take 100 sample points (Pixel location's corresponding pixel values) from non river class for training for each band.*
- *Take (512 * 512) sample points (Pixel location's corresponding pixel values) for testing for each band.*
- *Apply baye's decision rule to classify all the test sample either in river or nonriver class denoting 0 and 255 at corresponding pixel locations.*
- *Show the result in image form with black and white image (either 0 and 255)*

How to choose sample points

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Xcord_Riv = [154 164 160 168 176 174 177 168 177 165 162 161 161 162 178 174 189 194 195 190 199 206 202 201 203 216 212 215 217 211 2
Ycord_Riv = [19 26 37 52 68 88 105  100 136 154 183 198 196 197 230 228 246 260 262 253 265 278 273 274 270 295 289 301 299 327 334 34


Xcord_NonRiv = [25 54 42 134 110 100 111 64 115 140 101 13 156 115 121 90 85 167 115 180 144 17 61 150 81 20 144 105 97 100 72  57 202
Ycord_NonRiv = [29 65 154 97 131 156 212 241 303 221 274 248 251 280 304 67 320 316 335 323 368 414 406 404 423 483 454 487 492 500 48
```

Bayes decision rule

- M classes
- class conditional density functions

  $p_1(\underline{x}), p_2(\underline{x}), \ldots\ldots, p_M(\underline{x})$

  $\underline{x} \in \Re^N$
- prior probabilities $P_1, P_2, \ldots\ldots P_M$

  $0 < P_i < 1 \qquad \forall i = 1,2,\ldots\ldots M$

  $$\sum_1^M P_i = 1$$
- Put $\underline{x}$ in class i if

  $P_i p_i(\underline{x}) \geq P_j p_j(\underline{x}); \forall j \neq i$
- Best decision rule

  Minimizes the prob. of misclassification

Examples on Satellite images of Kolkata: Given Input Images

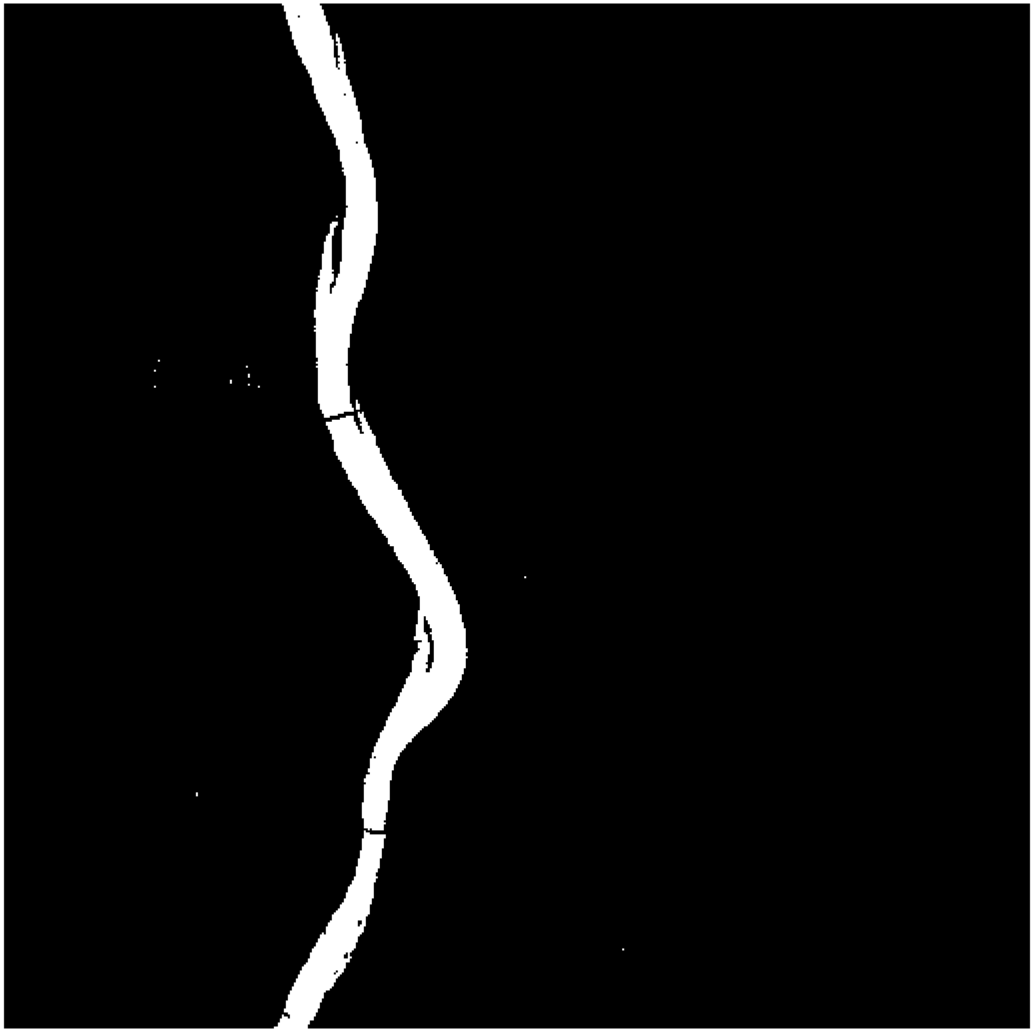
R Band Image (512 * 512)

G Band Image (512 * 512)

B Band
Image
(512 * 512)

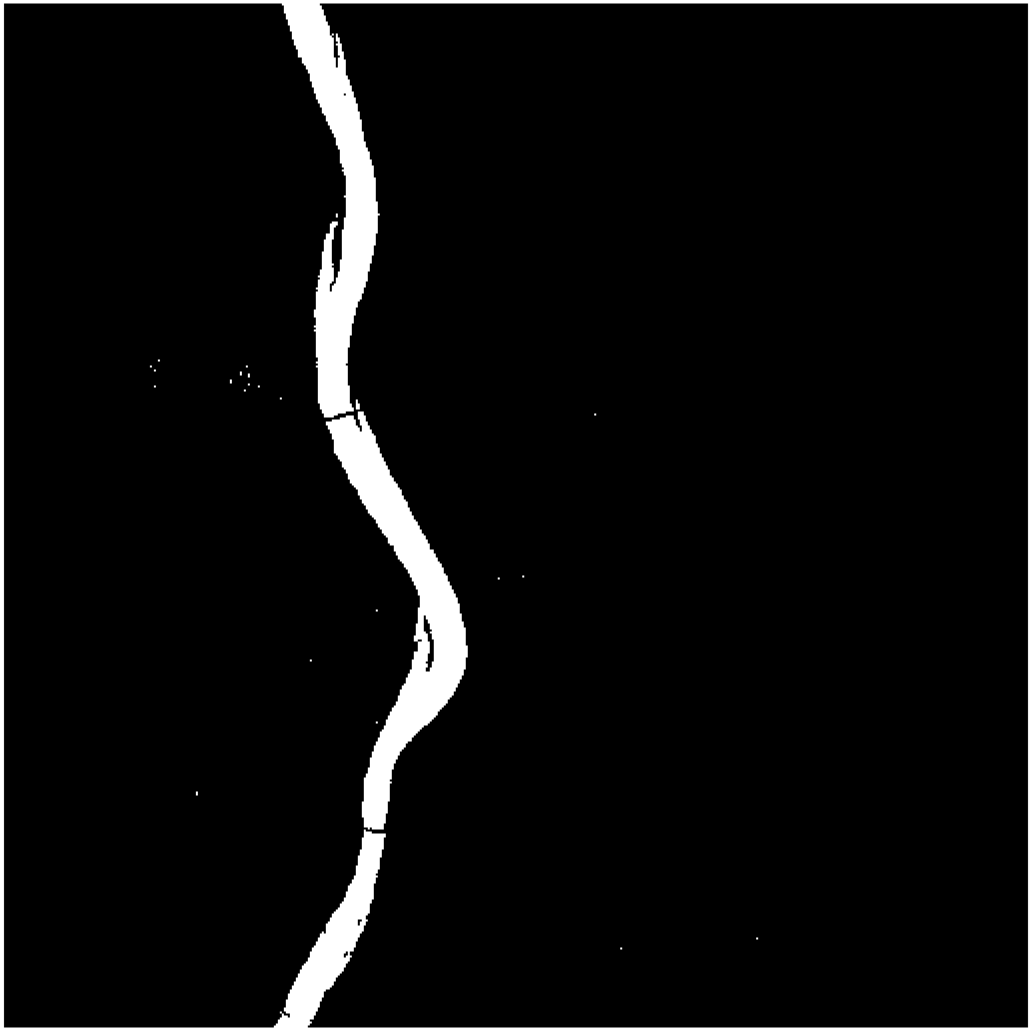
I Band Image
(512 * 512)

Output Image (River Class and Non River Class

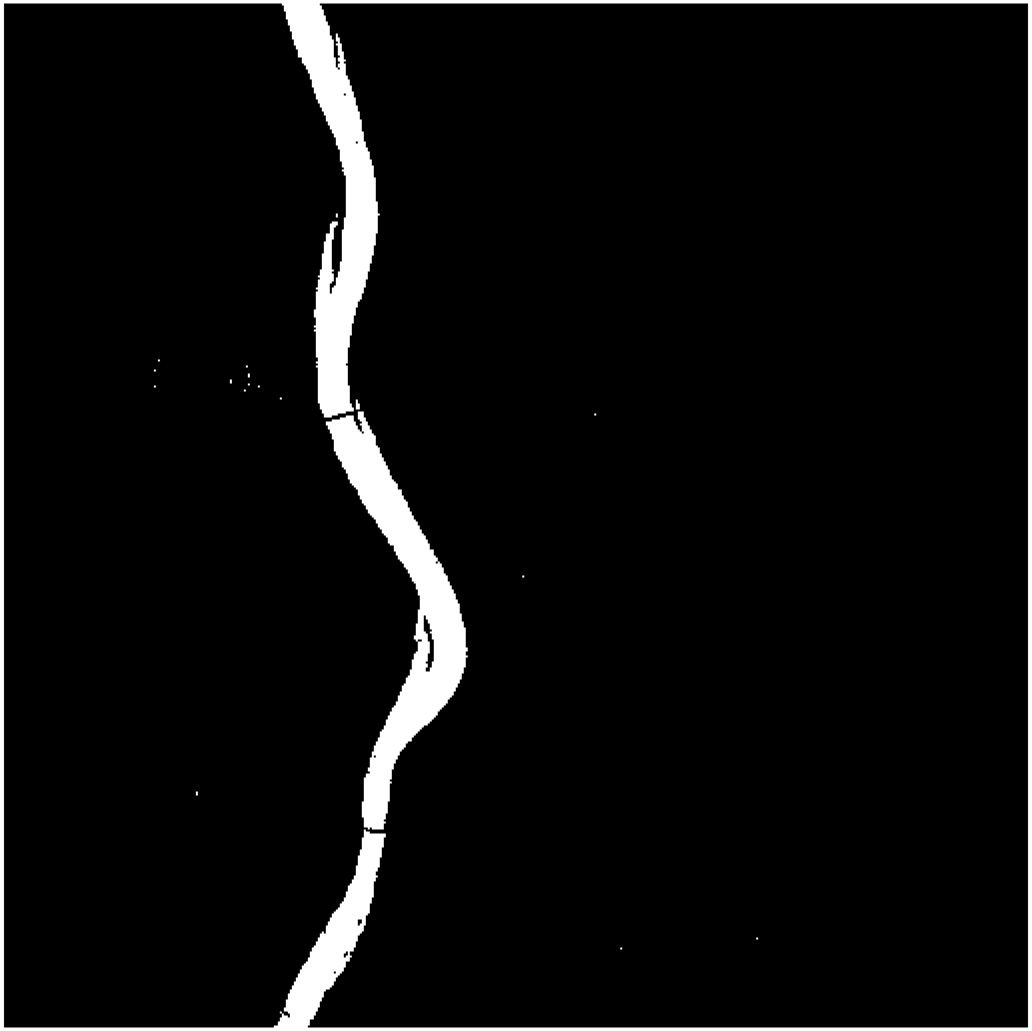


Bayes Rule: P1 = 0.3 and P2 = 0.7

Bayes Rule: P1 = 0.7 and P2 = 0.3



Bayes Rule: P1 = 0.5 and P2 = 0.5



Implementation process of Density Function

- ***<u>Step 1</u>***: **Calculate Mean of River Class : T1 = [Mean1; Mean2; Mean3 ;Mean4];**

  Mean1 = mean of Rband image for 50 sample points

  Mean2 = mean of Gband Image for 50 sample points

  Mean3 = mean of Bband image for 50 sample points

  Mean4 = mean of Iband image for 50 sample points

- ***<u>Step 2</u>***: **Calculate Mean of NonRiver Class : T2 = [Mean1; Mean2; Mean3; Mean4];**

  Mean1 = mean of Rband image for 100 sample points

  Mean2 = mean of Gband Image for 100 sample points

  Mean3 = mean of Bband image for 100 sample points

  Mean4 = mean of Iband image for 100 sample points

- ***<u>Step 3</u>***: **Calculate the Covariance Matrix for River Class** for 50 samples which is **4 * 4** dimensions. Basically **(X – T1)** deviation and **(Y – T1)** deviation and multiply it and summing up where X and Y represents all the sample points considered for training (R, G, B and I band image) we will get 2^4 = 16 values in the covariance matrix for possible combinations of 4 band images. We are doing the deviation of sample points from the mean vector.

  (Apply covariance matrix calculation formula)

- ***<u>Step 4</u>***: Calculate the **Covariance Matrix for Non River Class** for 100 samples which is **4 * 4 dimensions** also by applying same process explained in step 3.

- ***<u>Step 5:</u>*** Take **whole image** for test data where : **test_data= [Rband_img(i,j) Gband_img(i,j) Bband_img(i,j) Iband_img(i,j)]**; i = 1 to 512; and j = 1 to 512;

- ***<u>step 6:</u>*** The dimension of test data is **(4 * (512 * 512));**

- ***<u>Step 7:</u>*** For each pixel location of test image Run the loop from **i = 1 to (512*512)** Do

- ***<u>Step 8:</u>*** For **river class** calculate **(test_data – T1) deviation and (test_data – T1)^T Then Multiply it** :

  River_class = (Test_data – T1)^T * Inverse (Covariance_matrix_Riverclass) *(Test_data – T1)

- ***<u>Step 9:</u>*** For **Non_river class** calculate (test_data – T2) deviation and (test_data – T2)^T Then Multiply it :

  Nonriver class = (Test_data – T2)^T * Inverse (Covariance_matrix_NonRiverclass) *(Test_data – T2)

- ***Step 10:*** Calculate density function **p1** for **river class** where **P1 = 0.3** given
  p1 = (-0.5) * 1/sqrt(Determinant of Covariance_matrix_Riverclass) * exp(River_class);
  (Here we apply multivariate Normal Distrubution)

- ***Step 11:*** Calculate density function **p2** for **nonriver class** where **P2 = 0.7** given
  p2 = (-0.5) * 1/sqrt(Determinant of Covariance_matrix_nonRiverclass) * exp(NonRiver_class);

- ***Step 12:*** For **each pixel location** of test image apply baye's rule **(P1 * p1) >= (P2 * p2)** then
  Out_image(i) = 255 (River class)
  Else
  Out_image(i) = 0; (Nonriver class)

- ***Step 13 :*** Goto **step 7;**

- ***Step 14:*** Show the three output image Image using imshow function for three cases:
  Case 1 : River class (Prior Prob:) = 0.3 , Nonriver class(Prior Prob) = 0.7
  Case 2 : River class (Prior Prob:) = 0.7 , Nonriver class(Prior Prob) = 0.3
  Case 3 : River class (Prior Prob:) = 0.5 , Nonriver class(Prior Prob) = 0.5

Multivariate Normal Distribution

Multivariate Distribution:

$$X \sim N\left(\underline{\mu}, \Sigma\right)$$

If $f(x) = \dfrac{1}{\left(\sqrt{2\pi}\right)^{M} |\Sigma|^{1/2}}\, exp\left\{-\frac{1}{2}\left(\underline{x}-\underline{\mu}\right)' \Sigma^{-1} \left(\underline{x}-\underline{\mu}\right)\right\}$

$$\sum \quad \rightarrow Nonsingular = Det\ of \sum \quad = nonzero$$

$$\sum \quad \rightarrow Strictlypositive\ definite\ ; Eigen\ values\ \geq 0$$

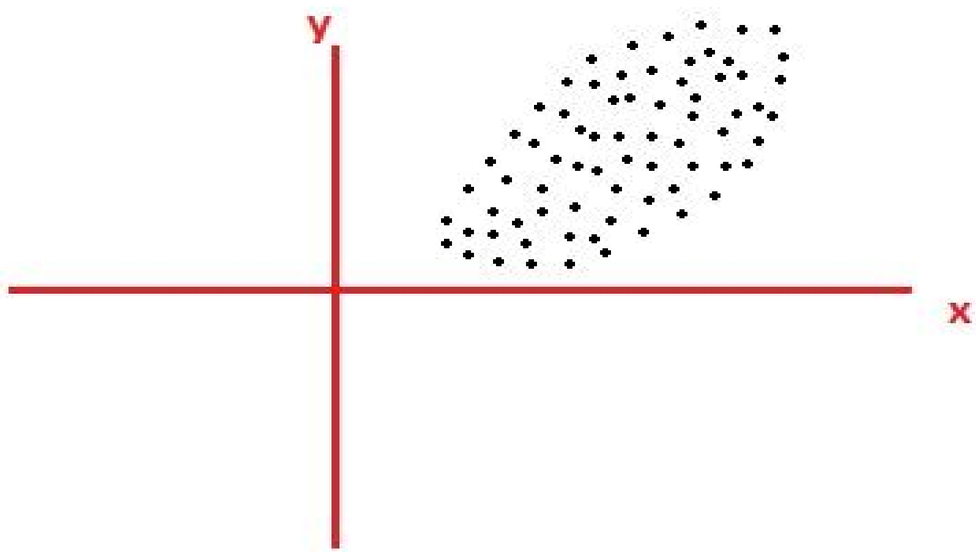
$$X \rightarrow takes\ uncountable\ values - continuous\ distrubution$$

Concept of Covariance

Estimate the mean of the population on the basis of mean of sample;

Sample point:

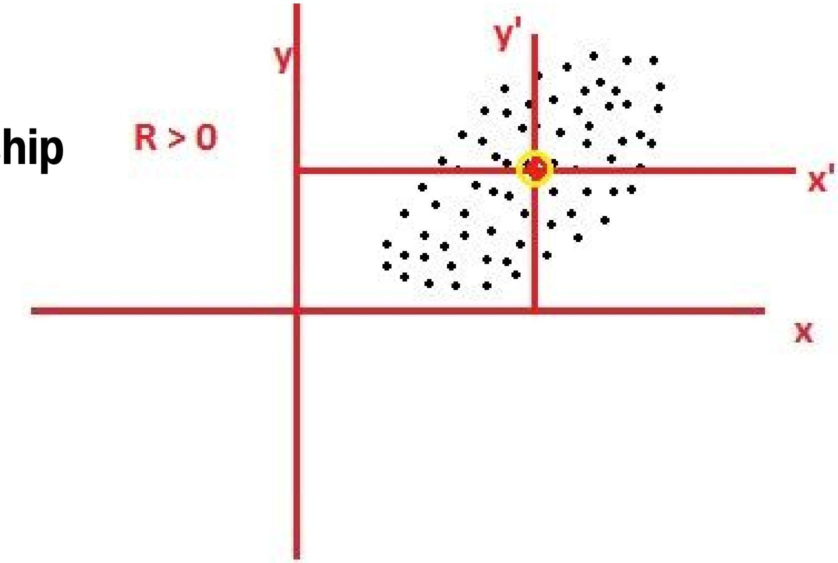
$Let\ (x_i, y_i); i = 1,2, \dots \dots \dots \dots ..n\ be\ given$



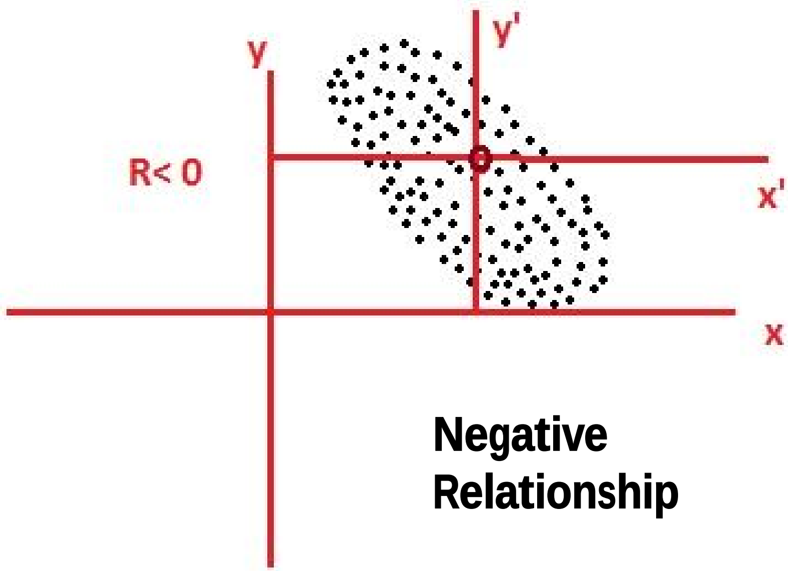
Relationship of x and y on the basis of sample points

Case 1

Positive Relationship

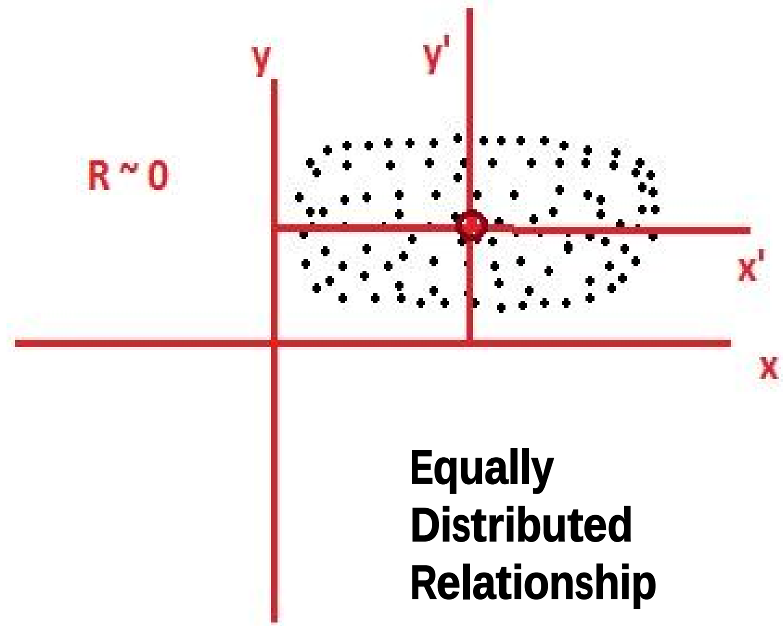


Here R denotes the relationship between two variables on the basis of sample points:



Negative Relationship

Case 2



Equally Distributed Relationship

Case 3

Formula:

$$\underbrace{\sum_{i=1}^{n}(x_i - \overline{x})}_{\text{New x Coordinate}} \times \underbrace{(y_i - \overline{y})}_{\text{New y Coordinate}} > 0$$

Now Covariance (x,y) is defined as:-

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x}) \times (y_i - \overline{y})$$

Correlation Coefficient is identified by r_{xy} is defined as

$$r_{xy} = \frac{cov(x,y)}{\sqrt{var(x) \times var(y)}} \; ; \; where -1 \leq r_{xy} \leq 1 \; always;$$

Probability Density Function

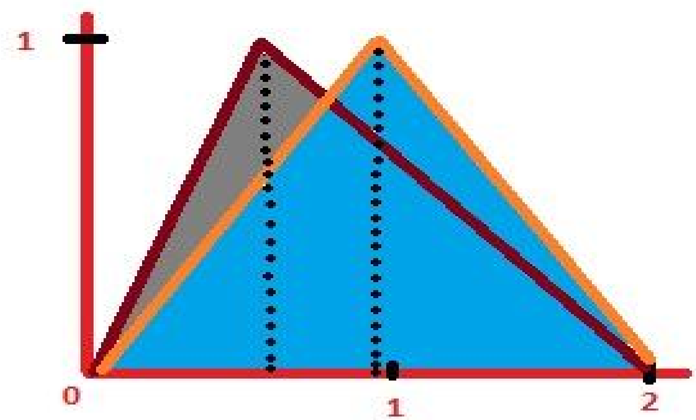
Let $f(x); x \in \mathbb{R}$ *is said to be probability density function if*

$$f(x) \geq 0 \;\; \forall x \;\in \mathbb{R}\,; and \quad \int_{-\infty}^{\infty} f(x)dx = 1$$

Triangular Distribution :

$$f(x) \geq 0$$

The area under the curve = $\frac{1}{2} \times base \times height$



$$A = \;\mathbb{R}^N \;\rightarrow N \; dimensional \; Real \; Space$$

$$= \mathbb{R} \times \;\mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \ldots\ldots\ldots\ldots \times \mathbb{R}$$

$$\mathbb{R} = (-\infty, \infty) \quad (set)$$

$$\mathbb{R}^2 = (-\infty, \infty) \times (-\infty, \infty)$$

Normal Distribution

Normal Distribution:

$X \sim N(\mu, \sigma^2)$; X *follows mean* μ *and variance* σ^2

Its probability density function f is given as

$$f(x) = \frac{1}{\sqrt{2\Pi}} \times \frac{1}{\sigma} \exp\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\}$$

Where $-\infty < x < \infty;\ -\infty < \mu < \infty;\ \sigma > 0;$

$x \rightarrow$ *variable*; μ *and* σ *have fixed values*;

$\mu = 0;\ \sigma = 1$; *or* $\mu = -10;\ \sigma = 1$; *Max* $e^{-x} = e^0 = 1\ x \geq 0$

Stat Book: C. R. Rao: Linear Statistical Inference and its Applications

Let $\frac{x-\mu}{\sigma} = y$

$$dy = \frac{dx}{\sigma}$$

$$x = \sigma y + \mu$$

$$= \frac{1}{\sigma\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} (\sigma y + \mu)\, e^{-\frac{1}{2}y^2}\, dy\right]$$

$$= \underbrace{\mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}y^2\right\} dy}_{\text{Proof is there}} + \underbrace{\frac{\sigma}{\sqrt{2\pi}} \int y\, exp\left\{-\frac{1}{2}y^2\right\} dy}_{\text{Odd Function}}$$

$$g(y) = -g(-y)\ [\,odd\ function]; Integration = 0;$$

$$E(x) = \mu;$$

$$variance = E(X - E(X)^2)$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \sigma^2$$