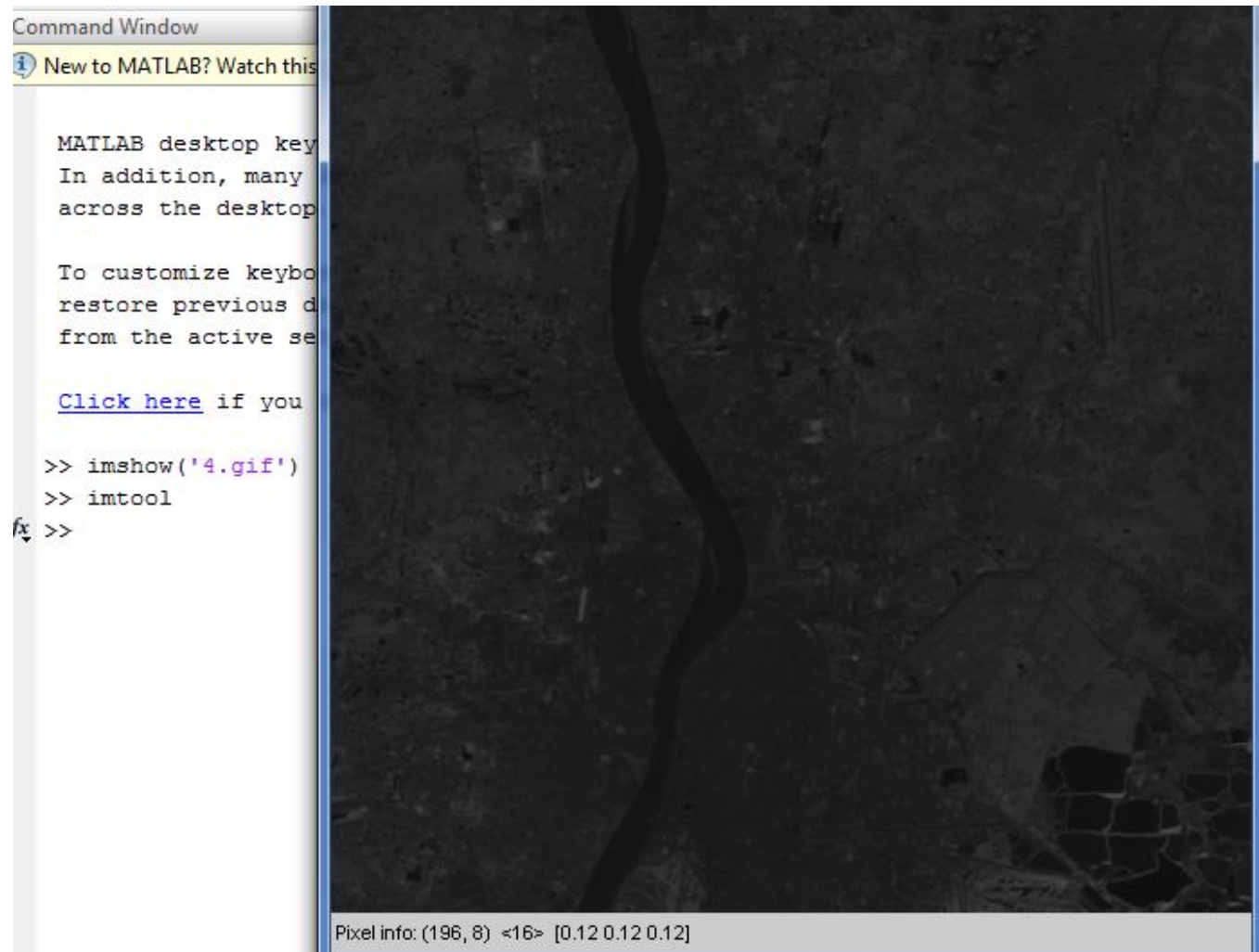


# **Practical Implementation on Baye's Decision Rule**

# ***How to Implement***

- *Four satellite Images of Kolkata (**Rband, Gband, Bband and Iband**) are given to you with equal image size (512 \* 512).*
- *The feature vector dimension is 4*
- *Each pixel location we have four values.*
- *Two Classes are given (River and NonRiver)*
- *Take 50 sample points (Pixel location's corresponding pixel values) from river class for training for each band*
- *Take 100 sample points (Pixel location's corresponding pixel values) from non river class for training for each band.*
- *Take (512 \* 512) sample points (Pixel location's corresponding pixel values) for testing for each band.*
- *Apply baye's decision rule to classify all the test sample either in river or nonriver class denoting 0 and 255 at corresponding pixel locations.*
- *Show the result in image form with black and white image (either 0 and 255)*

# How to choose sample points



```
Xcord_Riv = [154 164 160 168 176 174 177 168 177 165 162 161 161 162 178 174 189 194 195 190 199 206 202 201 203 216 212 215 217 211 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 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2148 2149 2150 2151 2152 2153 2154 2155 2156 2157 2158 2159 2160 2161 2162 2163 2164 2165 2166 2167 2168 2169 2170 2171 2172 2173 2174 2175 2176 2177 2178 2179 2180 2181 2182 2183 2184 2185 2186 2187 2188 2189 2190 2191 2192 2193 2194 2195 2196 2197 2198 2199 2200 2201 2202 2203 2204 2205 2206 2207 2208 2209 2210 2211 2212 2213 2214 2215 2216 2217 2218 2219 2220 2221 2222 2223 2224 2225 2226 2227 2228 2229 2230 2231 2232 2233 2234 2235 2236 2237 2238 2239 2240 2241 2242 2243 2244 2245 2246 2247 2248 2249 2250 2251 2252 2253 2254 2255 2256 2257 2258 2259 2260 2261 2262 2263 2264 2265 2266 2267 2268 2269 2270 2271 2272 2273 2274 2275 2276 2277 2278 2279 2280 2281 2282 2283 2284 2285 2286 2287 2288 2289 2290 2291 2292 2293 2294 2295 2296 2297 2298 2299 2300 2301 2302 2303 2304 2305 2306 2307 2308 2309 2310 2311 2312 2313 2314 2315 2316 2317 2318 2319 2320 2321 2322 2323 2324 2325 2326 2327 2328 2329 2330 2331 2332 2333 2334 2335 2336 2337 2338 2339 2340 2341 2342 2343 2344 2345 2346 2347 2348 2349 2350 2351 2352 2353 2354 2355 2356 2357 2358 2359 2360 2361 2362 2363 2364 2365 2366 2367 2368 2369 2370 2371 2372 2373 2374 2375 2376 2377 2378 2379 2380 2381 2382 2383 2384 2385 2386 2387 2388 2389 2390 2391 2392 2393 2394 2395 2396 2397 2398 2399 2400 2401 2402 2403 2404 2405 2406 2407 2408 2409 2410 2411 2412 2413 2414 2415 2416 2417 2418 2419 2420 2421 2422 2423 2424 2425 2426 2427 2428 2429 2430 2431 2432 2433 2434 2435 2436 2437 2438 2439 2440 2441 2442 2443 2444 2445 2446 2447 2448 2449 2450 2451 2452 2453 2454 2455 2456 2457 2458 2459 2460 2461 2462 2463 2464 2465 2466 2467 2468 2469 2470 2471 2472 2473 2474 2475 2476 2477 2478 2479 2480 2481 2482 2483 2484 2485 2486 2487 2488 2489 2490 2491 2492 2493 2494 2495 2496 2497 2498 2499 2500 2501 2502 2503 2504 2505 2506 2507 2508 2509 2510 2511 2512 2513 2514 2515 2516 2517 2518 2519 2520 2521 2522 2523 2524 2525 2526 2527 2528 2529 2530 2531 2532 2533 2534 2535 2536 2537 2538 2539 2540 2541 2542 2543 2544 2545 2546 2547 2548 2549 2550 2551 2552 2553 2554 2555 2556 2557 2558 2559 2560 2561 2562 2563 2564 2565 2566 2567 2568 2569 2570 2571 2572 2573 2574 2575 2576 2577 2578 2579 2580 2581 2582 2583 2584 2585 2586 2587 2588 2589 2590 2591 2592 2593 2594 2595 2596 2597 2598 2599 2600 2601 2602 2603 2604 2605 2606 2607 2608 2609 2610 2611 2612 2613 2614 2615 2616 2617 2618 2619 2620 2621 2622 2623 2624 2625 2626 2627 2628 2629 2630 2631 2632 2633 2634 2635 2636 2637 2638 2639 2640 2641 2642 2643 2644 2645 2646 2647 2648 2649 2650 2651 2652 2653 2654 2655 2656 2657 2658 2659 2660 2661 2662 2663 2664 2665 2666 2667 2668 2669 2670 2671 2672 2673 2674 2675 2676 2677 2678 2679 2680 2681 2682 2683 2684 2685 2686 2687 2688 2689 2690 2691 2692 2693 2694 2695 2696 2697 2698 2699 2700 2701 2702 2703 2704 2705 2706 2707 2708 2709 2710 2711 2712 2713 2714 2715 2716 2717 2718 2719 2720 2721 2722 2723 2724 2725 2726 2727 2728 2729 2730 2731 2732 2733 2734 2735 2736 2737 2738 2739 2740 2741 2742 2743 2744 2745 2746 2747 2748 2749 2750 2751 2752 2753 2754 2755 2756 2757 2758 2759 2760 2761 2762 2763 2764 2765 2766 2767 2768 2769 
```

# Bayes decision rule

- $M$  classes
- class conditional density functions
$$p_1(\underline{x}), p_2(\underline{x}), \dots, p_M(\underline{x})$$
$$\underline{x} \in \mathfrak{R}^N$$
- prior probabilities  $P_1, P_2, \dots, P_M$ 
$$0 < P_i < 1 \quad \forall i = 1, 2, \dots, M$$
$$\sum_{i=1}^M P_i = 1$$
- Put  $\underline{x}$  in class  $i$  if
$$P_i p_i(\underline{x}) \geq P_j p_j(\underline{x}); \forall j \neq i$$
- Best decision rule  
Minimizes the prob. of misclassification

# Examples on Satellite images of Kolkata: Given Input Images



R Band  
Image  
(512 \* 512)



**G Band  
Image  
(512 \* 512)**





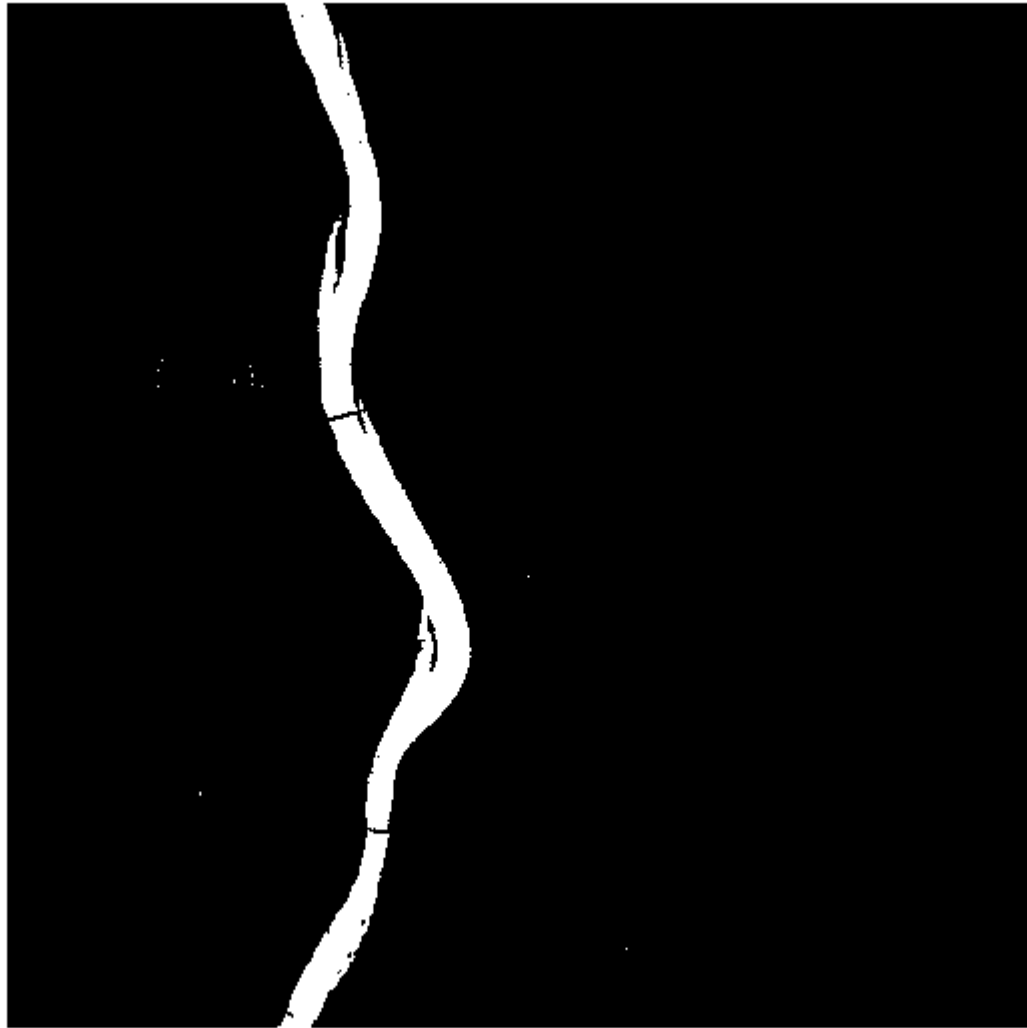
**B Band  
Image  
(512 \* 512)**



I Band Image  
(512 \* 512)



# Output Image (River Class and Non River Class)



**Bayes Rule:  $P1 = 0.3$  and  $P2 = 0.7$**

**Bayes Rule:  $P1 = 0.7$  and  $P2 = 0.3$**



**Bayes Rule:  $P1 = 0.5$  and  $P2 = 0.5$**



# Implementation process of Density Function

- **Step 1: Calculate Mean of River Class :  $T1 = [Mean1; Mean2; Mean3; Mean4]$ ;**  
Mean1 = mean of Rband image for 50 sample points  
Mean2 = mean of Gband Image for 50 sample points  
Mean3 = mean of Bband image for 50 sample points  
Mean4 = mean of Iband image for 50 sample points
- **Step 2: Calculate Mean of NonRiver Class :  $T2 = [Mean1; Mean2; Mean3; Mean4]$ ;**  
Mean1 = mean of Rband image for 100 sample points  
Mean2 = mean of Gband Image for 100 sample points  
Mean3 = mean of Bband image for 100 sample points  
Mean4 = mean of Iband image for 100 sample points
- **Step 3: Calculate the Covariance Matrix for River Class** for 50 samples which is  $4 * 4$  dimensions. Basically  $(X - T1)$  deviation and  $(Y - T1)$  deviation and multiply it and summing up where X and Y represents all the sample points considered for training ( R, G, B and I band image) we will get  $2^4 = 16$  values in the covariance matrix for possible combinations of 4 band images. We are doing the deviation of sample points from the mean vector.  
(Apply covariance matrix calculation formula)

- **Step 4:** Calculate the **Covariance Matrix for Non River Class** for 100 samples which is **4 \* 4 dimensions** also by applying same process explained in step 3.
- **Step 5:** Take **whole image** for test data where : **test\_data= [Rband\_img(i,j) Gband\_img(i,j) Bband\_img(i,j) lband\_img(i,j)]**; i = 1 to 512; and j = 1 to 512;
- **step 6:** The dimension of test data is **(4 \* (512 \* 512))**;
- **Step 7:** For each pixel location of test image Run the loop from **i = 1 to (512\*512)** Do
- **Step 8:** For **river class** calculate **(test\_data - T1) deviation and (test\_data - T1)<sup>T</sup>** Then **Multiply it :**  

$$\text{River\_class} = (\text{Test\_data} - T1)^T * \text{Inverse}(\text{Covariance\_matrix\_Riverclass}) * (\text{Test\_data} - T1)$$
- **Step 9:** For **Non\_river class** calculate **(test\_data - T2) deviation and (test\_data - T2)<sup>T</sup>** Then **Multiply it :**  

$$\text{Nonriver class} = (\text{Test\_data} - T2)^T * \text{Inverse}(\text{Covariance\_matrix\_NonRiverclass}) * (\text{Test\_data} - T2)$$

- **Step 10:** Calculate density function **p1** for **river class** where **P1 = 0.3** given  

$$p1 = (-0.5) * 1/\sqrt{\text{Determinant of Covariance\_matrix\_Riverclass}} * \exp(\text{River\_class});$$
(Here we apply multivariate Normal Distribution)
- **Step 11:** Calculate density function **p2** for **nonriver class** where **P2 = 0.7** given  

$$p2 = (-0.5) * 1/\sqrt{\text{Determinant of Covariance\_matrix\_nonRiverclass}} * \exp(\text{NonRiver\_class});$$
- **Step 12:** For **each pixel location** of test image apply baye's rule **(P1 \* p1) >= (P2 \* p2)** then  
Out\_image(i) = 255 (River class)  
Else  
Out\_image(i) = 0; (Nonriver class)
- **Step 13:** Goto **step 7**;
- **Step 14:** Show the three output image Image using imshow function for three cases:  
Case 1 : River class (Prior Prob: ) = 0.3 , Nonriver class(Prior Prob) = 0.7  
Case 2 : River class (Prior Prob: ) = 0.7 , Nonriver class(Prior Prob) = 0.3  
Case 3 : River class (Prior Prob: ) = 0.5 , Nonriver class(Prior Prob) = 0.5



# Multivariate Normal Distribution

Multivariate Distribution:

$$X \sim N(\underline{\mu}, \Sigma)$$

$$\text{If } f(x) = \frac{1}{(\sqrt{2\pi})^M |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2}(\underline{x} - \underline{\mu})' \Sigma^{-1} (\underline{x} - \underline{\mu})\right\}$$

$$\Sigma \rightarrow \text{Nonsingular} = \text{Det of } \Sigma = \text{nonzero}$$

$$\Sigma \rightarrow \text{Strictly positive definite ; Eigen values } \geq 0$$

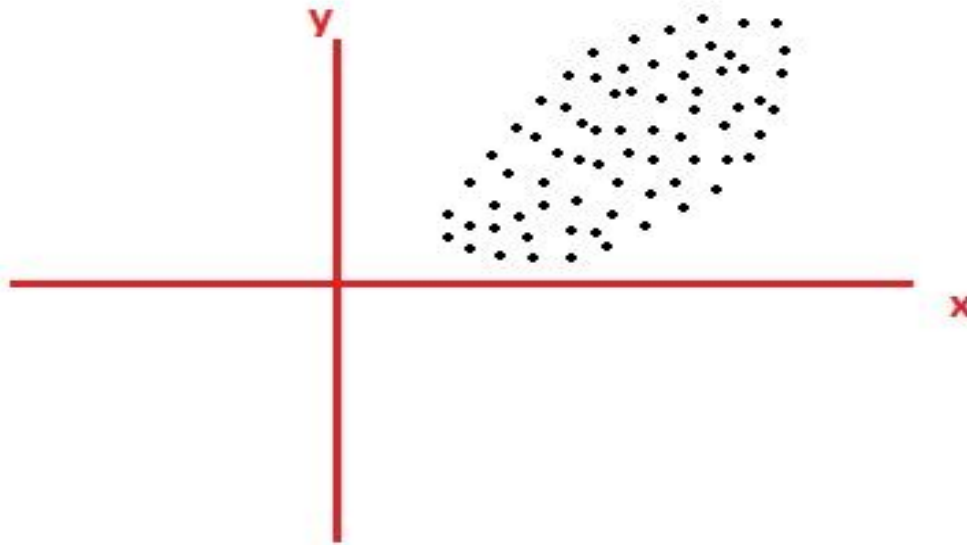
$X \rightarrow$  takes uncountable values – continuous distribution

# Concept of Covariance

Estimate the mean of the population on the basis of mean of sample;

Sample point:

*Let  $(x_i, y_i); i = 1, 2, \dots, n$  be given*

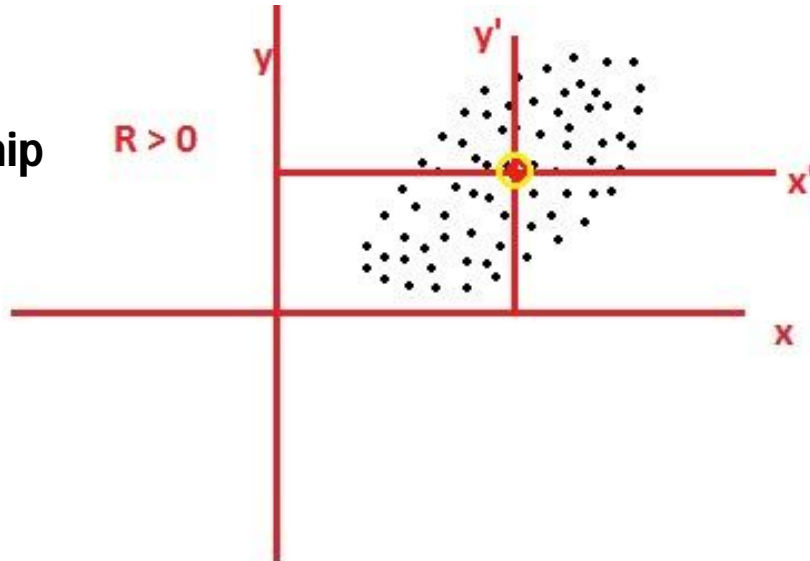


Relationship of x and y on the basis of sample points

Case 1

Positive  
Relationship

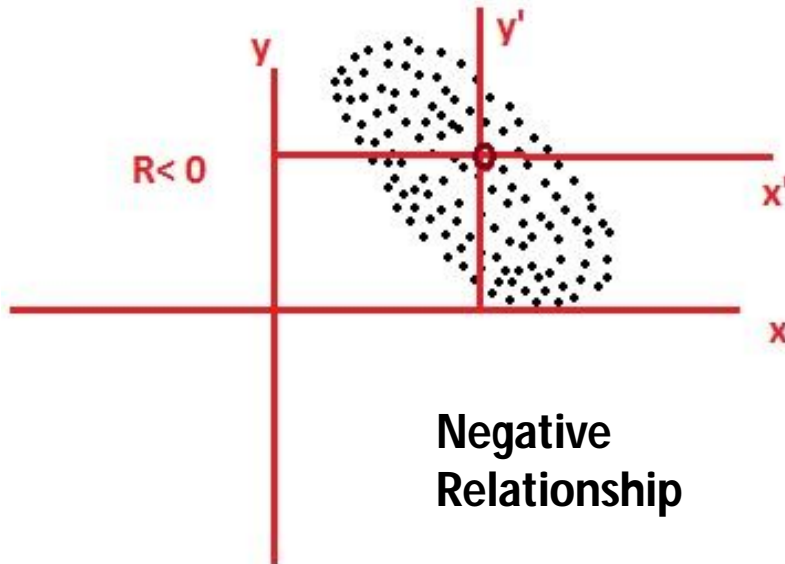
$$R > 0$$



Here  $R$  denotes the relationship between two variables on the basis of sample points:

$$R < 0$$

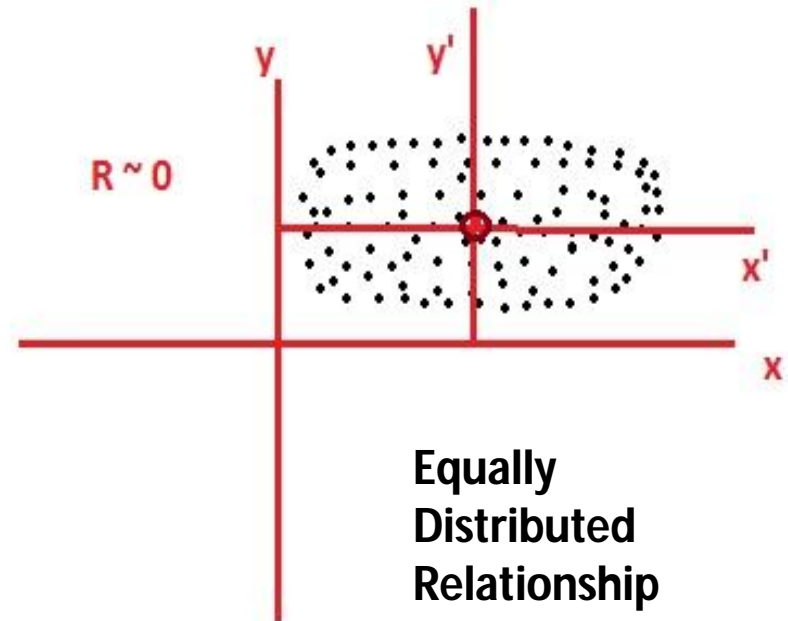
Negative  
Relationship



Case 2

$$R \sim 0$$

Equally  
Distributed  
Relationship



Case 3

Formula:

$$\sum_{i=1}^n (x_i - \bar{x}) \times (y_i - \bar{y}) > 0$$

$\Downarrow$                        $\Downarrow$   
New x Coordinate      New y Coordinate

Now Covariance (x,y) is defined as:-

$$\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x}) \times (y_i - \bar{y})$$

Correlation Coefficient is identified by  $r_{xy}$  is defined as

$$r_{xy} = \frac{cov(x, y)}{\sqrt{var(x) \times var(y)}} ; \text{ where } -1 \leq r_{xy} \leq 1 \text{ always;}$$

# Probability Density Function

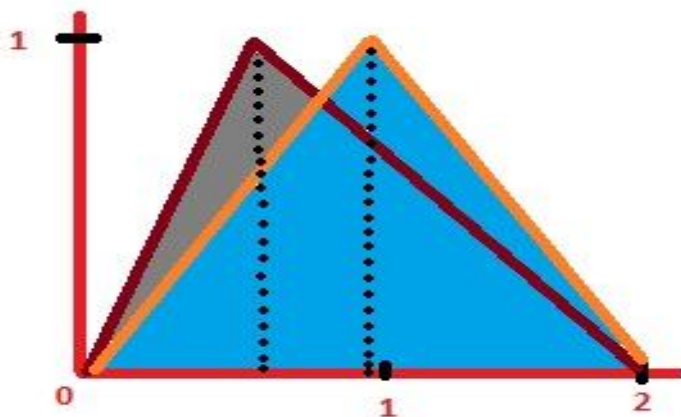
Let  $f(x); x \in \mathbb{R}$  is said to be probability density function if

$$f(x) \geq 0 \quad \forall x \in \mathbb{R}; \text{ and } \int_{-\infty}^{\infty} f(x) dx = 1$$

Triangular Distribution:

$$f(x) \geq 0$$

The area under the curve =  $\frac{1}{2} \times \text{base} \times \text{height}$



$A = \mathbb{R}^N \rightarrow N \text{ dimensional Real Space}$

$$= \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \dots \dots \dots \times \mathbb{R}$$

$$\mathbb{R} = (-\infty, \infty) \text{ (set)}$$

$$\mathbb{R}^2 = (-\infty, \infty) \times (-\infty, \infty)$$

# Normal Distribution

Normal Distribution:

$X \sim N(\mu, \sigma^2)$ ;  $X$  follows mean  $\mu$  and variance  $\sigma^2$

Its probability density function  $f$  is given as

$$f(x) = \frac{1}{\sqrt{2\pi}} \times \frac{1}{\sigma} \exp \left\{ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2 \right\}$$

Where  $-\infty < x < \infty$ ;  $-\infty < \mu < \infty$ ;  $\sigma > 0$ ;

$x \rightarrow$  variable;  $\mu$  and  $\sigma$  have fixed values;

$\mu = 0$ ;  $\sigma = 1$ ; or  $\mu = -10$ ;  $\sigma = 1$ ;  $\text{Max } e^{-x} = e^0 = 1 \text{ } x \geq 0$

Stat Book: **C. R. Rao**: Linear Statistical Inference and its Applications



$$\text{Let } \frac{x-\mu}{\sigma} = y$$

$$dy = \frac{dx}{\sigma}$$

$$x = \sigma y + \mu$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left[ \int_{-\infty}^{\infty} (\sigma y + \mu) e^{-\frac{1}{2}y^2} dy \right]$$

$$= \underbrace{\mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}y^2\right\} dy}_{\text{Proof is there}} + \underbrace{\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y \exp\left\{-\frac{1}{2}y^2\right\} dy}_{\text{Odd Function}}$$

Proof is there

Odd Function

$$g(y) = -g(-y) \text{ [odd function]}; \text{Integration} = 0;$$

$$E(x) = \mu;$$

$$\text{variance} = E(X - E(X))^2$$

$$= \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \sigma^2$$