STA3105-01 Bayesian Statistics
Homework 1
DUE Friday, September 23

Copying homework solutions from others lead to 0 score. No late submission is allowed. Your solution should contain both code and corresponding explanation for the answer. Submit your HW (a single pdf file) through LearnUs.

1. We observe 52 number of car accidents at Seoul during September 2007. Let's consider our hierarchical model as

$$\text{Likelihood}: \ X|\theta \sim \text{Poisson}(\theta)$$

$$\text{Prior}: \ \theta \sim \text{Gamma}(a, b),$$

where, Gamma distribution is defined as $p(\theta) = \frac{1}{\Gamma(a)b^a}\theta^{a-1}\exp\{-\frac{\theta}{b}\}$ for $\theta \in (0, \infty)$.

   (a) (10 points) Based on the previous study information, we know that the mean number of accidents is 30, and the variance is 180. Based on this information, set the hyperparameter a, b in the prior distribution.

   (b) (20 points) Obtain the posterior distribution in this problem. You should specify the kernel of the likelihood, prior and posterior.

   (c) (10 points) Draw the likelihood, prior and posterior distributions as a function of θ.

   (d) (20 points) Report the posterior mode as well. You can calculate the posterior mode numerically by using `optim` function in `R`.

   (e) (20 points) Report the posterior mean, 95% symmetrical density interval, and 95% highest posterior density interval. Are there any differences between the symmetrical density interval and the highest posterior density interval?

   (f) (20 points) Based on the most recent study, we know that the mean number of accidents is 40, and the variance is 200. Based on this updated prior information, repeat the procedures in (a)-(d). Explain the differences between posterior distributions.