

STA3105-01 Bayesian Statistics
Homework 3
DUE Friday, October 21

Copying homework solutions from others lead to a 0 score. No late submission is allowed. Your solution should contain both code and a corresponding explanation for the answer. Submit your HW through LearnUs. **You should submit (1) a report file (pdf) and (2) a relevant code file.** In this assignment, we will implement MCMC algorithms for a Bayesian GLM as follows:

$$\mathbf{Y} \sim \text{Poisson}(\mu(\mathbf{X}))$$

$$\log(\mu(\mathbf{X})) = \mathbf{X}\beta$$

$$\beta_j \sim N(0, 10) \text{ for } j = 1, \dots, 4$$

1. (5 points) Simulate the dataset as follows.
 - (a) Let $X_i \in \mathbb{R}^4$ be the predictors for i th observation. For $i = 1, \dots, 1,000$, simulate $X_i \sim N(0, \mathbf{I})$ independently. Here \mathbf{I} is an identity matrix.
 - (b) For $i = 1, \dots, 1,000$, simulate $Y_i \sim \text{Poisson}(\exp(X_i' \beta))$ independently. Set the true regression coefficient value as $\beta = (0.5, -0.5, 0, 1)$.
2. (30 points) Implement the MCMC algorithm using the simulated dataset in Problem 1. Here, you should write down the code without using any packages (e.g., `nimble`, `adaptMCMC`). Report the trace plots, density plots, 95% HPD intervals, posterior mean, acceptance probability, and effective sample size for all parameters.
3. (30 points) Implement the MCMC algorithm using the simulated dataset in Problem 1. Here, you should use `nimble` package to implement the algorithm. Report the trace plots, density plots, 95% HPD intervals, posterior mean, acceptance probability, and effective sample size for all parameters.
4. (30 points) Implement the MCMC algorithm using the simulated dataset in Problem 1. Here, you should use `adaptMCMC` package to implement the algorithm. Report the trace plots, density plots, 95% HPD intervals, posterior mean, acceptance probability, and effective sample size for all parameters.
5. (5 points) Compare the posterior densities obtained from Problems 2-4. Especially, you should overlap the density of each β_j for all different methods. Draw the true β_j as a vertical line in each posterior density. Discuss whether all MCMC algorithms can recover the true β_j well.