STA3105-01 Bayesian Statistics
Homework 2
DUE Friday, October 7

Copying homework solutions from others lead to 0 score. No late submission is allowed. Your solution should contain both code and corresponding explanation for the answer. Submit your HW through LearnUs. You should submit (1) a report file (pdf) and (2) a relevant code file.

1. Consider the Cauchy distribution with

$$f(x) = \frac{1}{\theta\pi\Big[1 + (\frac{x-\eta}{\theta})^2\Big]}, \quad \infty < x < \infty, \theta > 0$$

   (a) (30 points) Implement the Metropolis-Hastings algorithm to generate 10,000 samples from the Cauchy distribution with $\theta = 2, \eta = 0$. Here, θ is a scale parameter and η is a location parameter. You should tune the proposal distribution that have acceptance probability around 0.2-0.5. Report the trace plot, histogram, acceptance probability of your MCMC samples.

   (b) (20 points) Compute the sample percentiles and compare with the Cauchy distribution ($\theta = 2, \eta = 0$) percentiles. Especially, compare the 25th, 50th, 75th percentiles. You can use `qcauchy` function to obtain the Cauchy distribution percentiles. Does the sample we generated follows the Cauchy distribution?

2. Now we have $x_1, \cdots, x_{1,000}$ samples generated from the Cauchy distribution. These samples are stored as `x` in (`hw02.RData`). We will use the prior of $\eta \sim N(0, 100)$ and $\theta \sim U(0, 10)$ for the Bayesian inference. Our goal is to generate (θ, η) from the posterior distribution.

   (a) (40 points) Implement the Metropolis-Hastings algorithm to generate 10,000 samples of (θ, η) from the posterior distribution. You should tune the proposal distribution that have acceptance probability around 0.2-0.5 for both θ and η. Report the trace plots, histograms, acceptance probabilities of your MCMC samples.

   (b) (10 points) Report the posterior means, 95% HPD intervals of θ and η.