Date
Note

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1. (a) $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

(b) $P(-1 \le z \le 1) = 1 - 2 \cdot (1 - P(z \le 1))$

$= 1 - 2 \cdot (1 - 0.8413)$

$= 1 - 0.3174$

$= 0.6826$

(c) $2(1-Y) = 0.05 \Rightarrow Y = 0.975$

$P(z \le x) = 0.975$

$x = 1.96$ (查表)

(d) $f_Q(q) = \frac{1}{\sqrt{2}\,\Gamma(\frac{1}{2})} \cdot q^{-\frac{1}{2}} \cdot e^{-\frac{q}{2}}$

(e) $E(Q) = V = 1$

(f) $std(Q) = \sqrt{\sigma^2} = \sqrt{2V} = \sqrt{2}$

(g) $P(Q \le 1) = 1 - P(Q \ge 1) = 1 - 0.242 = 0.758$

2. (a) $f_T(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases}$

(b) $E[T] = \beta = 1$

(c) $\text{Std}[T] = \sqrt{\sigma^2} = \sqrt{1^2} = 1$

(d) $P(T>1) = e^{-1} = 0.3679$

(e) $f_{T_3}(t) = \dfrac{t^2 e^{-t} \cdot 1}{P(3)} = \dfrac{t^2 e^{-t}}{2}, \quad t>0$

(f) $E[T_3] = 3 \cdot 1 = 3$

(g) $\text{Std}[T_3] = \sqrt{\sigma^2} = \sqrt{3 \cdot 1^2} = \sqrt{3}$

(h) $P(T_3>3) = 1 - \sum_{t=1}^{3} P(T_3 = t)$

$= 1 - 0.6781$

$= 0.3213$

(i) 使用七年以上 $\Rightarrow P(T_3>7)$

$P(T_3>7) = 1 - \sum_{t=1}^{7} P(T_3 = t)$

$= 1 - 0.9764$

$= 0.0236$