Пространство S'

Функции медленного роста:

$$|f(x)| \leqslant C\left(1+|x|^{2p}\right).$$

$$\left\langle \mathcal{P}\frac{1}{x-x_0}, \varphi(x)\right\rangle = \text{v. p.} \int\limits_{-\infty}^{\infty} \frac{\varphi(x)}{x-x_0} dx.$$

$$\mathcal{P}\frac{1}{(x-x_0)^n} = \frac{(-1)^{n-1}}{(n-1)!}\frac{d^{n-1}}{dx^{n-1}}\mathcal{P}\frac{1}{x-x_0}.$$

$$x^n \mathcal{P}\frac{1}{x^n} = 1.$$

$$\left\langle \mathcal{P}\frac{1}{|x|}, \varphi(x)\right\rangle = \\ = \int\limits_{|x|<1} \frac{\varphi(x)-\varphi(0)}{|x|}dx + \int\limits_{|x|\geqslant 1} \frac{\varphi(x)}{|x|}dx.$$

$$x\mathcal{P}\frac{1}{|x|} = \operatorname{sign} x.$$

$$\langle \partial^\alpha f, \varphi\rangle = (-1)^{|\alpha|}\langle f, \partial^\alpha \varphi\rangle.$$

$$\frac{d}{dx}\ln|x| = \mathcal{P}\frac{1}{x}.$$

$$S'\left(\mathbb{R}^3\right): \langle \delta_{S_a}(x), \varphi(x)\rangle = \int\limits_{|x|=a} \varphi(x) dS_x.$$

Преобразование Фурье

$$F[\varphi(x)](y) = \int\limits_{\mathbb{R}^m} e^{i(x,\,y)}\varphi(x)dx.$$

$$\langle F[f(x)](y), \varphi(y)\rangle = \langle f(x), F[\varphi(y)](x)\rangle.$$

$$F^{-1}[\varphi(x)](y) = \frac{1}{(2\pi)^m}\int\limits_{\mathbb{R}^m} e^{-(x,\,y)}\varphi(x)dx.$$

$$\left\langle F^{-1}[f(x)](y), \varphi(y)\right\rangle = \left\langle f(x), F^{-1}[\varphi(y)](x)\right\rangle.$$

$$F^{-1}[F[f](y)](x) = F\left[F^{-1}[f](y)\right](x) = f(x).$$

$$F^{-1}[f(x)](y) = \frac{1}{(2\pi)^m}F[f(x)](-y) = \\ = \frac{1}{(2\pi)^m}F[f(-x)](y).$$

$$\partial^\alpha F[f(x)](y) = F\left[(ix)^\alpha f(x)\right](y).$$

$$F\left[\partial^\alpha f(x)\right](y) = (-iy)^\alpha F[f(x)](y).$$

$$F\left[f(x-x_0)\right](y) = e^{i(y,\,x_0)}F[f(x)](y).$$

$$F^{-1}\left[f(x-x_0)\right](y) = e^{-i(y,\,x_0)}F^{-1}\left[f(x)\right](y).$$

$$F\left[\delta(x-a)\right](y) = e^{iay}.$$

$$F[\delta(x)](y) = 1.$$

$$F^{-1}[1](y) = \delta(y).$$

$$F^{-1}\left[e^{iax}\right](y) = \delta(y-a).$$

$$F\left[e^{iax}\right](y) = 2\pi F^{-1}\left[e^{-iax}\right](y) = \\ = 2\pi\delta(y+a).$$

$$F[1](y) = 2\pi\delta(y).$$

$$F^{-1}\left[\delta(x-a)\right](y) = \frac{1}{2\pi}e^{-iay}.$$

$$F[\sin x](y) = \pi i\left(\delta(y+1)-\delta(y-1)\right).$$

$$F[\operatorname{sign}(x)](y) = 2i\mathcal{P}\frac{1}{y}.$$

$$F\left[\mathcal{P}\frac{1}{x}\right](y) = \pi i \operatorname{sign} y.$$

$$F[\theta(x)](y) = i\mathcal{P}\frac{1}{y} + \pi\delta(y).$$

$$F\left[|x|\right](y) = -2\mathcal{P}\frac{1}{y^2}.$$

$$F\left[\frac{1}{x+b+ia}\right](y) = \\ = -2\pi i \operatorname{sign}(a)\theta(-ay)e^{ay-iby}.$$

$$S'\left(\mathbb{R}^3\right): F\left[\frac{1}{|x|}\right](y) = \frac{4\pi}{|y|^2}.$$

$$F\left[\delta_{S_a}(x)\right](y) = \frac{4\pi a}{|y|}\sin\left(a|y|\right).$$

$$S'\left(\mathbb{R}^3\right): F\left[|x|\right](y) = -4\pi\Delta\left(\frac{1}{|y|^2}\right).$$

$$S'\left(\mathbb{R}^3\right): F[1](y) = (2\pi)^m\delta(y).$$

$$S'\left(\mathbb{R}^m\right) \to S'\left(\mathbb{R}^n\right): \\ \langle f(Ax+b), \varphi(x)\rangle = \\ = \left\langle F^{-1}[f](y), e^{i(b,\,y)}F[\varphi(x)]\left(A^T y\right)\right\rangle.$$

$$S'\left(\mathbb{R}^n\right): \langle f(Ax+b), \varphi(x)\rangle = \\ = \left\langle f(y), \frac{\varphi\left(A^{-1}(y-b)\right)}{|\det A|}\right\rangle.$$

Функции Грина

ЛЧ	$\mathcal{E}_a(t,x)$
$\frac{\partial^2 u}{\partial t^2} - a^2\frac{\partial^2 u}{\partial x^2}$	$\frac{\theta(at-\vert x\vert)}{2a}$
$\frac{\partial^2 u}{\partial t^2} - a^2\Delta u$	$\frac{\delta(at-\vert x\vert)}{4^2 t}$
$\Delta u - a^2 u$	$-\frac{e^{-a\vert x\vert}}{4\pi\vert x\vert}$
Δu	$-\frac{1}{4\pi\vert x\vert}$
$\frac{\partial u}{\partial t} - a^2\Delta u$	$\frac{\theta(t)}{(2a\sqrt{\pi}t)^n}e^{-\frac{\vert x\vert^2}{4a^2t}}$

.

Задачи Коши

$$\langle (f(t,x)) * (\mathcal{E}_a(t,x)), \varphi(t,x)\rangle = \\ = \lim_{R\to+\infty}\left\langle f(t,x), \eta_1\left(\frac{t}{R}, \frac{x}{R}\right)\times \\ \times \langle \mathcal{E}_a(\tau,y), \varphi(t+\tau, x+y)\rangle\right\rangle.$$

Формула Даламбера для одномерного волнового уравнения с правой частью специального вида

$$\delta'(t)u_0(x) + \delta(t)u_1(x),$$

где $u_0(x)$, $u_1(x)$ — медленного роста:

$$u(t,x) = \frac{\theta(t)}{2}\left(u_0(x+at)+u_0(x-at)\right) + \\ + \frac{\theta(t)}{2a}\int\limits_{x-at}^{x+at} u_1(\xi)\,d\xi.$$

Формула Кирхгоффа для трёхмерного ВУ -//-:

$$u(t,x) = \frac{\partial}{\partial t}\left(\frac{\theta(t)}{4\pi a^2 t}\int\limits_{|z-x|=at} u_0(z)dS_z\right) + \\ + \frac{\theta(t)}{4\pi a^2 t}\int\limits_{|z-x|=at} u_1(z)dS_z.$$

$$\langle \mathcal{E}(t,x), \varphi(t,x)\rangle = \frac{1}{4\pi a^2}\int\limits_{\mathbb{R}^3}\frac{\varphi\left(\frac{|x|}{a}, x\right)}{|x|}dx.$$

$$f(x) * \left(-\frac{1}{4\pi|x|}\right) = -\frac{1}{4\pi}\int\limits_{\mathbb{R}^3}\frac{f(y)}{|x-y|}dy.$$