Семинар №6

Драчов Ярослав
Факультет общей и прикладной физики МФТИ

10 марта 2021 г.

$$S = d^d x \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi, \quad \Delta = \frac{d}{2} - 1.$$

Кл. симм.: токи

$$\partial_\mu J^\mu = 0, \quad J^\mu = \begin{pmatrix} T^\mu_{\text{c}\ \nu} & j^\mu \end{pmatrix} \ldots.$$

$$S = \int d^d x \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right) = \int dt d^{d-1}x \left(\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right) \overset{*}{=\!=} .$$

$$Z = \int D\varphi(x) e^{iS[\varphi]} = \int D\varphi e^{-S_E[\varphi]}.$$

Виковский поворот

$$t = -i\tau.$$

$$\overset{*}{=\!=} i\underbrace{\int d\tau d^{d-1}x \left(\frac{1}{2}(\partial_\tau\varphi)^2 + \frac{1}{2}(\nabla\varphi)^2 + V(\varphi)\right)}_{S_E} = i\int d^d x_E \left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi) + V(\varphi)\right).$$

$$\eta_{MN} = \delta_{MN} \quad d + 1\text{-мерная теория.}$$

$$z^0, z^1 \to w^0, w^1.$$

$$g'^{\mu\nu}(\omega) = \frac{\partial w^\mu}{\partial z^\rho}\frac{\partial w^\nu}{\partial z^\sigma} g^{\rho\sigma} = \Lambda^{-1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad g^{\mu\nu} = \delta^{\mu\nu}.$$

$$0 = g'^{01} = \frac{\partial w^0}{\partial z^\rho}\frac{\partial w^1}{\partial z^\sigma} g^{\rho\sigma} = \frac{\partial w^0}{\partial z^0}\frac{\partial w^1}{\partial z^0} + \frac{\partial w^0}{\partial z^1}\frac{\partial w^1}{\partial z^1} = 0.$$

$$g'^{00} = g'^{11}, \quad \left(\frac{\partial w^0}{\partial z^0}\right)^2 + \left(\frac{\partial w^0}{\partial z^1}\right)^2 = \left(\frac{\partial w^1}{\partial z^0}\right)^2 + \left(\frac{\partial w^1}{\partial z^1}\right)^2.$$

Имеем систему

$$\begin{cases} ab + cd = 0 \\ a^2 + c^2 = b^2 + d^2 \end{cases}.$$

Её решениями являются

$$\begin{cases} b = c \\ a = -d \end{cases}, \quad \begin{cases} a = -d \\ a = d \end{cases}.$$

То есть

$$\begin{cases} \dfrac{\partial w^1}{\partial z^0} = \dfrac{\partial w^0}{\partial z^1} \\ \dfrac{\partial w^0}{\partial^0} = -\dfrac{\partial w^1}{\partial z^1} \end{cases}, \quad \begin{cases} \dfrac{\partial w^1}{\partial z^0} = -\dfrac{\partial w^0}{\partial z^1} \\ \dfrac{\partial w^0}{\partial z^0} = \dfrac{\partial w^1}{\partial z^1} \end{cases}.$$

Это условия голоморфности и антиголоморфности соответственно.

$$g_{\mu\nu} dz^\mu dz^\nu = (dz^0)^2 + (dz^1)^2 = \frac{1}{4} \begin{pmatrix} dz^2 d\overline{z}^2 + 2dzd\overline{z} \\ dz^2 d\overline{z}^2 + 2dzd\overline{z} \end{pmatrix} = dzd\overline{z}.$$

$$z = z^0 + iz^1, \quad \overline{z} = z^0 - iz^1.$$

$$g_{\mu\nu} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}.$$

$$w = w(z) \text{ либо } w = w(\overline{z}).$$

$$g_{\mu\nu} dz^\mu dz^\nu = dzd\overline{z} = \left(\frac{\partial z}{\partial w} \frac{\partial \overline{z}}{\partial \overline{w}} \right) dwd\overline{w} = \left(\frac{d\overline{z}}{dw(\overline{z})} \frac{\partial z}{\partial \overline{w(\overline{z})}} \right) dwd\overline{w}.$$

Квазипримарное поле с $\Delta = 0$

	лок.	глоб
инф.		
конечн.	$\varnothing$	$w = \frac{az+b}{cz+d}$

Таблица 1

$$\varphi'(w, \overline{w}) = \varphi(z, \overline{z}), \quad w(z) = z + \varepsilon(z), \quad \varepsilon \to 0.$$

$$\varphi'(z, \overline{z}) = \varphi(z, \overline{z}) - \varepsilon \partial_z \varphi - \overline{\varepsilon} \partial_{\overline{z}} \varphi.$$

$$\varepsilon(z) = \sum_{n=-\infty}^{\infty} \varepsilon_n z^{n+1},$$

где ε_n — счётное число параметров преобразования.

$$l_n = -z^{n+1} \partial_z \text{ — голом.}, \quad \overline{l}_n = -\overline{z}^{n+1} \partial_{\overline{z}} \text{ — антиголом.}$$

$$\left(\delta_{\varepsilon_1 \overline{\varepsilon}_1} \delta_{\varepsilon_2 \overline{\varepsilon}_2} - \delta_{\varepsilon_2 \overline{\varepsilon}_2} \delta_{\varepsilon_1 \overline{\varepsilon}_1} \right) \varphi = \left[\delta_{\varepsilon_1 \overline{\varepsilon}_1}, \delta_{\varepsilon_2 \overline{\varepsilon}_2} \right] \varphi.$$

$$[l_n, l_m] = (m+1) z^{n+m+1} \partial_z - (n+1) z^{n+m+1} \partial_z = (m-n) l_{n+m}.$$

Алгебра Витта:

$$\left[\overline{l}_n, \overline{l}_m \right] = (m-n) \overline{l}_{n+m}.$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{C}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \alpha & \beta \\ \gamma & \delta = -\alpha \end{pmatrix}.$$

$$w = \frac{az+b}{cz+d} = \frac{(1+\alpha)z + \beta}{\gamma z + 1 - \alpha} = z + \beta + 2\alpha z - \gamma z^2.$$

$$\beta = \varepsilon_{-1}, \quad \alpha = \frac{\varepsilon_0}{2}, \quad \gamma = -\varepsilon_1.$$

$$l_{-1},\, l_0,\, l_1.$$

$$[l_{-1}, l_0] = l_{-1}, \quad [l_0, l_1] = l_1, \quad [l_{-1}, l_1] = 2l_0.$$

$$\delta\varphi = -\varepsilon\partial_z\varphi - \overline{\varepsilon}\partial_{\overline{z}}\varphi = \sum_{n=-\infty}^{\infty} \left(\varepsilon_n l_n \varphi + \overline{\varepsilon}_n \bar{l}_n \varphi\right).$$

$$w(z) = z + \varepsilon(z).$$

Квазипримарные поля

$$\varphi'\left(w, \overline{w}\right) = \left(\frac{\partial w}{\partial z}\right)^{-h} \left(\frac{\partial \overline{w}}{\partial \overline{z}}\right)^{-\overline{h}} \varphi\left(z, \overline{z}\right).$$

$$h = \frac{1}{2}(\Delta + S), \quad \overline{h} = \frac{1}{2}(\Delta - S).$$

$$\varphi_{z_1, \ldots,\, z_h,\, \overline{z}_1, \ldots,\, \overline{z}_{\overline{h}}} = \left(\frac{\partial z}{\partial w}\right)^{h} \left(\frac{\partial \overline{z}}{\partial \overline{w}}\right)^{\overline{h}} \varphi'_{w_1, \ldots,\, w_h,\, \overline{w}_1, \ldots,\, \overline{w}_{\overline{h}}}.$$