1

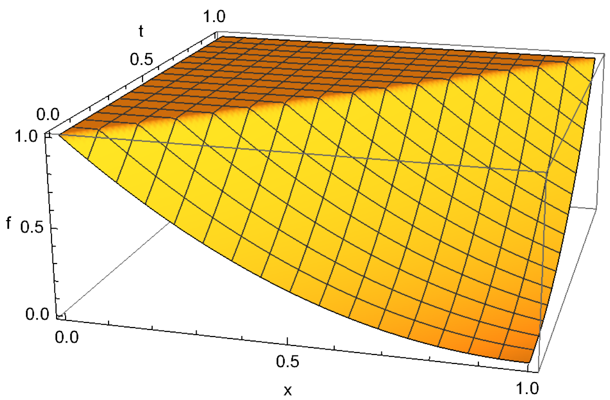
```
ClearAll[f, Subscript]
a = 1.;
τ = .01;
h = .01;
L = 1.;
T = 1.;
f0[x_] := (x - 1)^2
f1 = 1.;
(*Первый порядок:*)
f1[i_,n_] := f1[i,n] = Piecewise[{
   {f1, i == 0},
   {f0[i h], n == 0 && 0 < i ≤ L/h},
   {f1[i,n-1] - a τ (f1[i,n-1] - f1[i-1,n-1])/h, True}}]
(*Второй порядок:*)
f2+[i_, n_] := 0.5 (f2[i+1,n] + f2[i,n]) - 0.5 a τ (f2[i+1,n] - f2[i,n])/h
f2-[i_, n_] := 0.5 (f2[i,n] + f2[i-1,n]) - 0.5 a τ (f2[i,n] - f2[i-1,n])/h
f2[i_,n_] := f2[i,n] = Piecewise[{
   {f1, i == 0},
   {f0[i h], n == 0 && 0 < i ≤ L/h},
   {0, n == 0 && i > L/h},
   {f2[i,n-1] - a τ (f2+[i,n-1] - f2-[i,n-1])/h, True}}]
(*Точное решение:*)
sol = FullSimplify[DSolve[
    {∂t f[t, x] + a ∂x f[t, x] == 0, f[0, x] == f0[x], f[t, 0] == f1}, f[t, x], {t, x}]];
fexact[η_, ξ_] := f[t, x] /. sol[[1]] /. {t → η, x → ξ};
(*Построение*)
frames = Table[Show[
    ListLinePlot[{Table[{i h, f1[i,n]}, {i, 0, L/h}], Table[{i h, f1[i,n]}, {i, 0, L/h}]},
     PlotRange → {{0, L}, {0, 1}}, PlotLabel → StringTemplate["t=``"][n τ]],
    Plot[fexact[n τ, x], {x, 0, L}, PlotStyle → ColorData[97, "ColorList"][[3]]], {n,
    0, T/τ}];
rasterizedFrames = Map[Image, frames];
Export["~/Desktop/gif.gif", frames];
SystemOpen["~/Desktop/gif.gif"]
"exact sol:"
fexact[t, x]
```

```
"3D num sol plot 1:"
ListPlot3D[Flatten[Table[{i h, n τ, f1[i,n]}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
"3D num sol plot 2:"
ListPlot3D[Flatten[Table[{i h, n τ, f2[i,n]}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
(*ListPlot3D[Flatten[Table[{i h,n τ,RealAbs[f[i,n]-f_exact[n τ,i h]]},
   {i,0,L/h},{n,0,T/τ}],1],AxesLabel→{"x","t","Δf"},PlotRange→All]*)
"max error 1: "
Max[Table[{RealAbs[f1[i,n] - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
"max error 2: "
Max[Table[{RealAbs[f2[i,n] - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
```

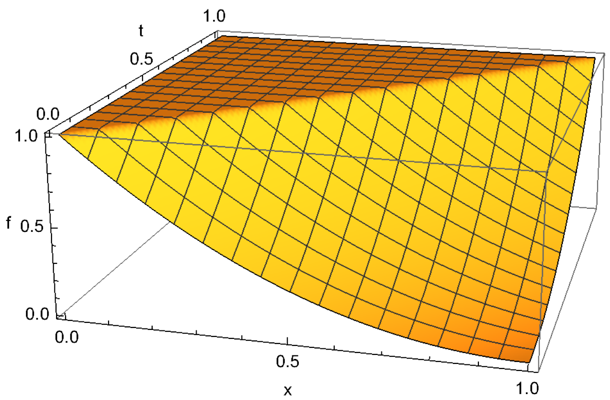
Out[381]= exact sol:

Out[382]= $1. + \left(1.\ t^2 + t\ (2. - 2.\ x) + x\ (-2. + 1.\ x)\right)$ HeavisideTheta$[-1.\ t + x]$

Out[383]= 3D num sol plot 1:

Out[384]=


Out[385]= 3D num sol plot 2:

Out[386]=


Out[387]= max error 1:

Out[388]= 4.44089×10^{-16}

Out[389]= max error 2:

Out[390]= 8.88178×10^{-16}

2

In[391]:=
```
ClearAll[f, Subscript]
τ = .01;
h = .01;
L = 1.;
T = 1.;
f₀[x_] := (x - 1)^2
f₁ = 1.;
F[x_] := x^2/2
Finv[x_] := Sqrt[2 x]
(*Первый порядок*)
F1[i_,n_] := F1[i,n] = Piecewise[{
    {F[f₁], i == 0},
    {F[f₀[i h]], n == 0 && 0 < i ≤ L/h},
    {F1[i,n-1] - τ (F1[i,n-1] - F1[i-1,n-1])/h, True}}]
f1[i_,n_] := f1[i,n] = Finv[F1[i,n]]
(*Второй порядок*)
F2₊[i_, n_] := 0.5 (F2[i+1,n] + F2[i,n]) - 0.5 τ (F2[i+1,n] - F2[i,n])/h
F2₋[i_, n_] := 0.5 (F2[i,n] + F2[i-1,n]) - 0.5 τ (F2[i,n] - F2[i-1,n])/h
F2[i_,n_] := F2[i,n] = Piecewise[{
    {F[f₁], i == 0},
    {F[f₀[i h]], n == 0 && 0 < i ≤ L/h},
    {0, n == 0 && i > L/h},
    {F2[i,n-1] - τ (F2₊[i,n-1] - F2₋[i,n-1])/h, True}}]
f2[i_,n_] := f2[i,n] = Finv[F2[i,n]]
(*Точное решение*)
sol = DSolve[{∂t F[t, x] + ∂x F[t, x] == 0, F[0, x] == F[f₀[x]], F[t, 0] == F[f₁]},
    F[t, x], {t, x}] // FullSimplify;
f_exact[η_, ξ_] := Finv[F[t, x]] /. sol[[1]] /. {t → η, x → ξ};
(*Построение*)
frames = Table[Show[
    ListLinePlot[{Table[{i h, f1[i,n]}, {i, 0, L/h}], Table[{i h, f1[i,n]}, {i, 0, L/h}]},
```

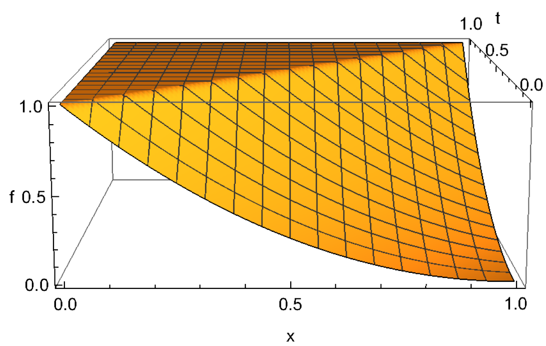
```
    PlotRange → {{0, L}, {0, 1}}, PlotLabel → StringTemplate["t=``"][n τ]],
   Plot[f_exact[n τ, x], {x, 0, L}, PlotStyle → ColorData[97, "ColorList"][[3]]], {n,
   0, T/τ}];
rasterizedFrames = Map[Image, frames];
Export["~/Desktop/gif.gif", frames];
SystemOpen["~/Desktop/gif.gif"]
"exact sol:"
f_exact[t, x]
"3D num sol plot 1:"
ListPlot3D[Flatten[Table[{i h, n τ, f1_{i,n}}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
"3D num sol plot 2:"
ListPlot3D[Flatten[Table[{i h, n τ, f2_{i,n}}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
(*ListPlot3D[Flatten[Table[{i h,n τ,RealAbs[f_{i,n}-f_exact[n τ,i h]]},
   {i,0,L/h},{n,0,T/τ}],1],AxesLabel→{"x","t","Δf"},PlotRange→All]*)
"max error 1: "
Max[Table[{RealAbs[f1_{i,n} - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
"max error 2: "
Max[Table[{RealAbs[f2_{i,n} - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
```

Out[412]= exact sol:

Out[413]= $\sqrt{2}\sqrt{\left(0.5+\left(0.5\,t^4+t^3\,(2.-2.\,x)+3.\,t^2\,(1.-1.\,x)^2-2.\,t\,(-1.+1.\,x)^3+x\,(-2.+x\,(3.+(-2.+0.5\,x)\,x))\right)\text{HeavisideTheta}[-1.\,t+x]\right)}$

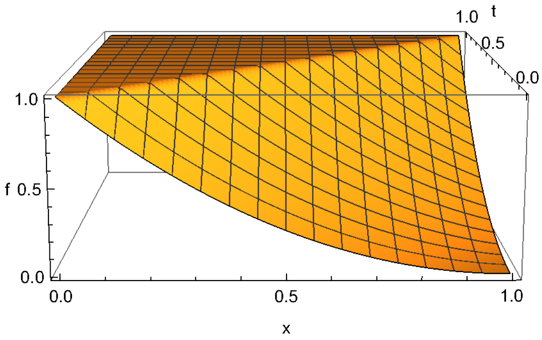
Out[414]= 3D num sol plot 1:

Out[415]=



Out[416]= 3D num sol plot 2:

Out[417]=



Out[418]= max error 1:

Out[419]= 1.36146×10^{-12}

Out[420]= max error 2:

Out[421]= 1.36146×10^{-12}