Семинар №6

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Задача 9.

Решение.

$$A_x = A_z = 0, \quad A_y = \mathcal{H}x.$$

$$\mathbf{H} = \operatorname{rot}\mathbf{A} = \mathcal{H}\mathbf{k}.$$

$$\mathbf{A} = \begin{pmatrix} 0 & \mathcal{H}x & 0 \end{pmatrix}, \quad \varphi = 0.$$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi = 0.$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}}.$$

$$H = \frac{1}{2m}\left(\mathbf{P} - \frac{e}{c}\mathbf{A}\right)^2 + e\varphi.$$

$$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}} + \frac{e}{c}\left(\mathbf{A}, \mathbf{v}\right) - e\varphi.$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{e}{c}\mathbf{A} = \mathbf{p} + \frac{e}{c}\mathbf{A}.$$

Значит

$$\mathbf{p} = \mathbf{P} - \frac{e}{c}\mathbf{A}.$$

$$\widehat{\mathrm{H}} = \frac{1}{2m}\left(\widehat{\mathbf{P}} - \frac{e}{c}\mathbf{A}\cdot\widehat{1}\right) - \underbrace{(\widehat{\boldsymbol{\mu}}, \mathbf{H})}_{=0,\ \text{т. к } s=0}.$$

$$\widehat{\mathrm{H}} = \frac{1}{2m}\left(\widehat{\mathrm{p_x}}^2 + \left(\widehat{\mathrm{p_y}} - \frac{e}{c}\mathcal{H}\widehat{\mathrm{x}}\right)^2 + \widehat{\mathrm{p_z}}^2\right).$$

$$\widehat{\mathrm{P_x}} \equiv \widehat{\mathrm{p_x}}, \quad \widehat{\mathrm{P_z}} = \widehat{\mathrm{p_z}}.$$

$$\widehat{\mathrm{H}}\psi = E\psi.$$

$$\left[\widehat{\mathrm{P_y}}, \widehat{\mathrm{H}}\right] = \left[\widehat{\mathrm{P_z}}, \widehat{\mathrm{H}}\right] = 0, \quad \left[\widehat{\mathrm{P_x}}, \widehat{\mathrm{H}}\right] \neq 0.$$

$$[\widehat{\mathrm{x}}, \widehat{\mathrm{p_x}}]\, i\hbar\cdot\widehat{1} \neq 0.$$

$$\widehat{\mathbf{P}} = -i\hbar\nabla = -i\hbar\begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix}.$$

$$[\widehat{\mathrm{y}}, \widehat{\mathrm{p_z}}] = 0; \quad [\widehat{\mathrm{y}}, \widehat{\mathrm{p_y}}] = i\hbar\cdot\widehat{1}.$$

P_y и $P_z = p_z$ — интегралы движения.

$$\psi = C \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x).$$

$$P_y, P_z \in \left(-\infty; \infty\right).$$

Т. к. $A_z = 0$, то $P_z \equiv p_z = m v_z$.

$$\frac{1}{2m}\left(\widehat{\mathrm{P_x}}^2 + \left(\widehat{\mathrm{P_y}} - \frac{e}{c}\mathcal{H}\widehat{\mathrm{x}}\right)^2 + \widehat{\mathrm{P_z}}^2\right) \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x) = E \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x).$$

1. $\widehat{\mathrm{P_x}}^2 \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x) = -\hbar^2 \exp\left(P_y y + P_z z\right) f(x).$

2. $\widehat{\mathrm{P_z}}^2 \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x) = P_z^2 \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right) f(x)\right)$

3. $\left(\widehat{\mathrm{P_y}} - \frac{e}{c}\mathcal{H}\widehat{\mathrm{x}}\right)^2 \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x) = \left(P_y - \frac{e}{c}\mathcal{H}x\right)^2 \exp\left(\frac{i}{\hbar}\left(P_y y + P_z z\right)\right) f(x)$

$$-\frac{\hbar^2}{2m} f''(x) + \frac{P_z^2}{2m} f(x) + \frac{1}{2m}\left(P_y - \frac{e}{c}\mathcal{H}x\right)^2 f(x) - E f(x) = 0.$$

$$f''(x) + \frac{2m}{\hbar^2}\left(E - \frac{P_z^2}{2m} - \frac{\left(P_y - \frac{e}{c}\mathcal{H}x\right)^2}{2m}\right) f(x) = 0.$$

$$f''(x) + \frac{2m}{\hbar^2}\left(E - \frac{P_z^2}{2m} - \frac{1}{2m} - \frac{1}{2m}\frac{e^2\mathcal{H}^2}{c^2}\left(x - \frac{cP_y}{e\mathcal{H}}\right)^2\right) f(x) = 0.$$

$$x_0 = \frac{cP_y}{e\mathcal{H}}, \quad \omega_H = \frac{e\mathcal{H}}{mc}.$$

$$f''(x) + \frac{2m}{\hbar^2}\left[\left(E - \frac{P_z^2}{2m}\right) - \frac{m\omega_H^2}{2}(x - x_0)^2\right] f(x) = 0.$$

$$\widehat{\mathrm{H}} = \frac{\widehat{\mathrm{p}}^2}{2m} + \frac{m\omega^2\widehat{\mathrm{x}}^2}{2}.$$

$$\psi(x) + \frac{2m}{\hbar^2}\left(E - \frac{m\omega^2 x^2}{2}\right)\psi(x) = 0.$$

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$$E_n - \frac{p_z^2}{2m} = \hbar\omega\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

$$E_n = \frac{p_z^2}{2m} + \hbar\omega\left(n + \frac{1}{2}\right).$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n \cdot n! a_H \sqrt{\pi}}} H_n\left(\frac{x - x_0}{a_H}\right) e^{-\frac{1}{2}\left(\frac{x - x_0}{a_H}\right)^2}, \quad a_H = \sqrt{\frac{\hbar}{m\omega}}.$$

$$H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n}\left(e^{-\xi^2}\right).$$

$$\psi_{n p_y p_z} = C \exp\left(\frac{i}{\hbar}\left(P_y y + P_z\right)\right) f_n(x).$$

$$S = L_x L_y = L^2, \quad \psi(y+L) = \psi(y).$$

$$\psi(z+L) = \psi(z) \implies p_y = \frac{2\pi\hbar}{L} n_y, \quad p_z = \frac{2\pi\hbar n_y}{L}.$$

$$0 < x_0 < L, \quad 0 < \frac{c p_y}{e\mathcal{H}} < L \implies \Delta p_y = \frac{e\mathcal{H}L}{c}.$$

$$\Delta p_y \Delta y = \frac{e\mathcal{H}L}{c} L = \frac{e\mathcal{H}L^2}{c}.$$

$$\Delta y = L, \quad \Delta z \Delta P_z = L \Delta P_z.$$

$$N = \frac{\Delta P_y \Delta y}{2\pi\hbar} = \frac{\Delta P_z \Delta z}{2\pi\hbar} = \frac{e\mathcal{H}V\Delta P_z}{c(2\pi\hbar)^2}.$$