Семинар №7

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Из теоремы Нётер

$$\varphi \to \varphi + \varepsilon_i \delta\varphi_i, \quad x^\mu \to x^\mu + \varepsilon_i \delta x^\mu_i.$$

$$J^\mu = \left(\mathcal{L}\delta^\mu_\nu - \frac{\partial \mathcal{L}}{\partial\partial_\mu\varphi}\partial_\nu\varphi\right)\delta x^\nu + \frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi}\delta\varphi.$$

	δx^μ	ε_i	$\varepsilon_i\delta\varphi_i$
P_μ	a^μ	a^μ	0
D	λx^μ	λ	$-\lambda\cdot\Delta\cdot\varphi$
$L_{\mu\nu}$	$\omega^\mu_\nu x^\nu$	$\omega^{\mu\nu}$	$-iS_{\mu\nu}\varphi$
K_μ	$2(b\cdot x)x^\mu - b^\mu x^2$	b^μ	?

Таблица 1

Токи:

1. Трансляция:

$$T_{\text{c}\ \nu}^{\mu} = \mathcal{L}\delta^\mu_\nu - \frac{\partial\mathcal{L}}{\partial(\partial\varphi)}\partial_\nu\varphi, \quad \partial_\mu T_{\text{c}\ \nu}^{\mu} = 0.$$

2. Поворот:

$$M^\mu{}_{\rho\nu} = T_{\text{c}\ \nu}^{\mu} x_\rho - T_{\text{c}\ \rho}^{\mu} x_\nu - i\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\varphi\right)}S_{\nu\rho}\varphi, \quad \partial_\mu M^\mu{}_{\nu\rho} = 0.$$

3. Дилотация

$$J^\mu_{\text{D}} = T_{\text{c}\ \nu}^{\mu} x^\nu - \Delta\cdot\frac{\partial\mathcal{L}}{\partial\partial_\mu\varphi}\varphi.$$

$$\delta S = \int\limits_V d^d x' \mathcal{L}\left(\varphi', \partial'\varphi\right) - S = \int d^d x \partial_\mu \mathcal{K}^\mu(\varphi) = \int\limits_{\partial V} d^d S_\mu \mathcal{K}^\mu(\varphi).$$

Пусть ток

$$\partial_\mu J^\mu = 0$$

сохраняется. Если добавить

$$\partial_\mu\left(J^\mu + \partial_\nu B^{\mu\nu}\right) = 0, \quad B^{\mu\nu} = -B^{\nu\mu}$$

то это выражение также будет сохраняться.

У T есть антисимметричная часть, т. к.

$$0=\partial_\mu M^{\mu\nu\rho}=T^{\rho\nu}-T^{\nu\rho}-i\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\varphi\right)}S_{\nu\rho}\varphi\right).$$

$$T^{\rho\nu}-T^{\nu\rho}=i\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\left(\partial_\mu\varphi\right)}S_{\nu\rho}\varphi\right).$$

$$T^{\mu\nu}\to T_{\text{c}}^{\mu\nu}+\partial_\rho B^{\rho\mu\nu},\quad B^{\rho\mu}=-B^{\mu\rho}.$$

$$T^{\mu\nu}-T^{\nu\mu}=2\partial_\rho B^{\rho\mu\nu}=i\partial_\rho\underbrace{\left(\frac{\partial\mathcal{L}}{\partial\left(\partial_\rho\varphi\right)}S^{\nu\mu}\right)}_{b^{\rho\mu\nu}}.$$

$$B^{\rho\mu\nu}=\frac{i}{4}\left(b^{\rho\mu\nu}+b^{\nu\rho\mu}+b^{\mu\nu\rho}\right).$$

$$M^{\mu\nu\rho}=T^{\mu\nu}x^\rho-T^{\mu\rho}x^\nu.$$

Тождество Уорда:

$$\partial_\mu\left\langle T^{\mu\nu}X\right\rangle=-i\sum_{i=1}^n\delta(x-x_i)\left\langle\varphi_1(x_1)\ldots\underbrace{\widehat{\mathrm{P}}}_{=-i\frac{\partial}{\partial x_\nu}}\varphi_i(x_i)\ldots\varphi_n(x_n)\right\rangle=$$
$$=-\sum_{i=1}^n\delta(x-x_i)\frac{\partial}{\partial x_i}\left\langle X\right\rangle. \quad (1)$$

$$L^{\nu\rho}=-i\left(x^\rho\partial^\nu-x^\nu\partial^\rho\right)+S^{\nu\rho}.$$

$$\partial_\mu\left\langle\left(T^{\mu\nu}x^\rho-T^{\mu\rho}x^\nu\right)X\right\rangle=-\sum_{i=1}^n\delta(x-x_i)\left\langle\varphi_i(x_i)\ldots\widehat{\mathrm{L}}^{\nu\rho}\varphi(x_i)\ldots\varphi_n(x_n)\right\rangle=$$
$$=\sum_{i=1}^n\delta(x-x_i)\left(\left(x_i^\nu\partial_i^\rho-x_i^\rho\partial_i^\nu\right)-iS_i^{\nu\rho}\right)\left\langle X\right\rangle.$$

Используя 1, получаем

$$x^\rho\partial_\mu\left\langle T^{\mu\nu}X\right\rangle-x^\nu\partial_\mu\left\langle T^{\mu\rho}X\right\rangle+\left\langle\left(T^{\rho\nu}-T^{\nu\rho}\right)X\right\rangle.$$

$$\left\langle\left(T^{\rho\nu}-T^{\nu\rho}\right)X\right\rangle=-i\sum_i\delta(x-x_i)S_i^{\nu\rho}\left\langle X\right\rangle.$$

$$\partial_\mu J_{\text{D}}^\mu=T_{\text{c}}^\mu?^\mu{}_\mu-\Delta\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)}\varphi\right)=0.$$

$$\partial_\mu\left\langle J_{\text{D}}^\mu X\right\rangle=x^\nu\partial_\mu\left\langle T^\mu{}_\nu X\right\rangle+\left\langle T^\mu{}_\mu X\right\rangle=$$
$$=-i\sum\delta(x-x_i)\left\langle\varphi_1(x_1)\ldots\left(-ix_i^\rho\partial_\rho^i-i\Delta\right)\varphi_i(x_i)\ldots\varphi_n(x_n)\right\rangle.$$

$$\partial_\mu\left\langle T^{\mu\nu}X\right\rangle=-\sum_{i=1}^n\delta(x-x_i)\frac{\partial}{\partial x_\nu^i}\left\langle X\right\rangle.$$

$$\langle T^{\mu}{}_{\mu} X\rangle = -\sum_{i=1}^{n}\delta(x-x_i)\Delta_i \langle X\rangle .$$

$$\varphi'(w,\overline{w}) = \left(\frac{dw}{dz}\right)^{-h}\left(\frac{d\overline{w}}{d\overline{z}}\right)^{-\overline{h}}\varphi(z,\overline{z}) = e^{-i\varphi S}\varphi(z,\overline{z}) \approx$$

$$\approx (1-i\varphi s)\varphi(z,\overline{z}) = \left(1-\frac{i}{2}\omega^{\mu\nu}S_{\mu\nu}\right)\varphi(z,\overline{z}).$$

$$h = \frac{1}{2}(\Delta+s), \quad \overline{h} = \frac{1}{2}(\Delta-s).$$

$$w = e^{i}\varphi z, \quad \overline{w} = e^{-i\varphi}\overline{z}.$$

$$\omega_{\mu\nu} = \begin{pmatrix} 0 & \varphi \\ -\varphi & 0\end{pmatrix} = \varphi\cdot\varepsilon_{\mu\nu}, \quad S_{\mu\nu} = \varepsilon_{\mu\nu}s.$$

$$\varepsilon_{\mu\nu}\varepsilon^{\mu\nu} = 2.$$

$$S_{\mu\nu} = \varepsilon_{\mu\nu}s.$$

$$T^{\rho\sigma} - T^{\sigma\rho} = \varepsilon^{\rho\sigma}\varepsilon_{\mu\nu}T^{\mu\nu}.$$

$$\langle \varepsilon^{\mu\nu}T_{\mu\nu}X\rangle = -i\sum_{i=1}^{n}\delta(x-x_i)S_i\langle X\rangle .$$

$$z'^{\mu} = z^{\mu} + \varepsilon^{\mu}(z).$$

$$\partial_\mu\left(\varepsilon\nu T^{\mu\nu}\right) = \partial_\mu\varepsilon_\nu T^{\mu\nu} + \varepsilon_\nu\partial_\mu T^{\mu\nu} = \varepsilon_\nu\partial_\mu T^{\mu\nu} + \frac{1}{2}\left(\partial_\rho\varepsilon^{\rho}\right)T^{\mu}{}_{\mu} + \frac{1}{2}e_{\rho\sigma}\partial^{\rho}\varepsilon^{\sigma}e_{\mu\nu}T^{\mu\nu}.$$

$$\partial_\mu\varepsilon_\nu + \partial_\nu\varepsilon_\mu = \frac{1}{2}\left(\partial_\rho\varepsilon^\rho\right)\underbrace{g_{\mu\nu}}_{\delta_{\mu\nu}}.$$

$$\partial_\mu\varepsilon_\nu - \partial_\nu\varepsilon_\mu = e_\mu\nu\left(\varepsilon^{\rho\sigma}\partial_\rho\varepsilon_\sigma\right).$$

$$\delta_\varepsilon\langle X\rangle = \int d^2x\partial_\mu\langle\varepsilon_\nu T^{\mu\nu}X\rangle .$$

Д/з

Задача 1. Проверить что это действительно дельта-функция, подставив в формулы

$$\delta^{(2)}(x) = \frac{1}{\pi}\partial_{\overline{z}}\frac{1}{z} = \frac{1}{\pi}\partial_z\frac{1}{\overline{z}}.$$

$$\int d^2 f(z)\delta^{(2)}(x) = f(z=0).$$

$$d^2xf(\overline{z})\,\delta^{(2)}(x) = f(\overline{z}=0).$$

Задача 2 . Проверить 3 тождества Уорда для

$$\partial_\mu = \begin{pmatrix}\partial_\mu & \partial_{\overline{z}}\end{pmatrix}, \quad T^{\mu\nu} = \begin{pmatrix} T^{zz} & T^{z\overline{z}} \\ T^{\overline{z}z} & T^{\overline{z}\overline{z}}\end{pmatrix}.$$

Задача 3. Вычислить кореллятор

$$\langle T^{zz}X\rangle .$$