Семинар №3

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19 февраля 2021 г.

Задача 0-3-1.

$$n = 10^{21}\,\frac{1}{\text{см}}.$$

$$\varepsilon_F \sim 1 \text{ эВ} \simeq 10000 \text{ К}.$$

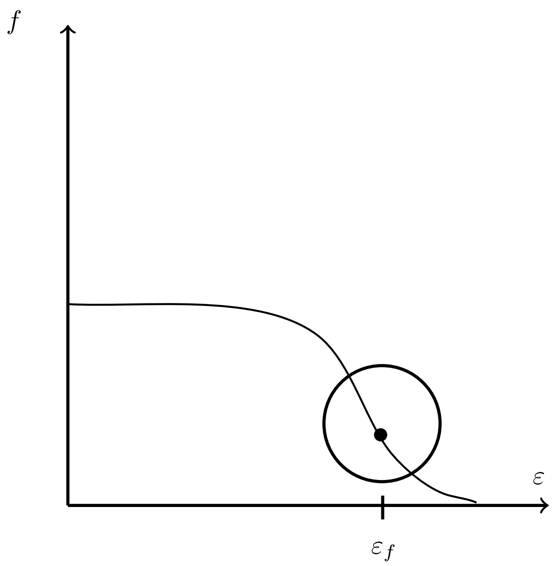

Рис. 1

Задача 0-3-2.

$$\frac{k_F}{k_{\text{Бр}}} = \frac{\sqrt[3]{3\pi^2/a^3}}{\pi/a} = \sqrt[3]{\frac{3}{\pi}} < 1.$$

Задача 3.4.

$$d = 0{,}37 \text{ нм}, \quad \varepsilon = \frac{p^2}{2m^*}, \quad \langle\varepsilon\rangle - ?.$$

Решение.

$$dN = 2\frac{4\pi p^2 dp}{(2\pi\hbar)^3/V}.$$

$$\Delta x \Delta p \sim 2\pi\hbar.$$

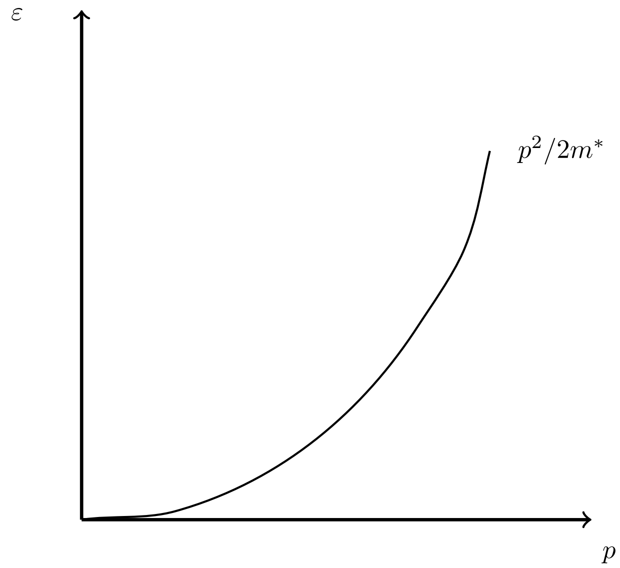

Рис. 2

$$\frac{N}{V} = \int\limits_{o}^{\varepsilon_F} \frac{m^{3/2}\sqrt{2}}{\pi\hbar^3}\sqrt{\varepsilon}d\varepsilon.$$

$$\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}}.$$

$$f = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{kT}}+1}.$$

$$\langle\varepsilon\rangle = \frac{\int\limits_0^\infty \varepsilon f(\varepsilon)g(\varepsilon)d\varepsilon}{\int\limits_0^\infty g(\varepsilon)f(\varepsilon)d\varepsilon} = \frac{\int\limits_o^{\varepsilon_F} \varepsilon^{3/2}d\varepsilon}{\int\limits_0^{\varepsilon_F} \varepsilon^{1/2}d\varepsilon} = \frac{3}{5}\varepsilon_F.$$

$$n = \frac{2}{\left(2d/\sqrt{3}\right)^3}.$$

Задача 3.13.

Решение.

$$\varepsilon_F \sim \left(\frac{n}{g_s}\right)^{2/3}.$$

$$N \sim g_S \varepsilon_F^{3/2}.$$

$$N = A\cdot 2\varepsilon_{F_1}^{3/2} = A\cdot 1\cdot \varepsilon_{F_2}^{3/2}.$$

$$\varepsilon_{F_2} = \varepsilon_{F_1}2^{2/3}.$$

Задача 3.5.

$$\varepsilon_F = 5 \text{ эВ}.$$

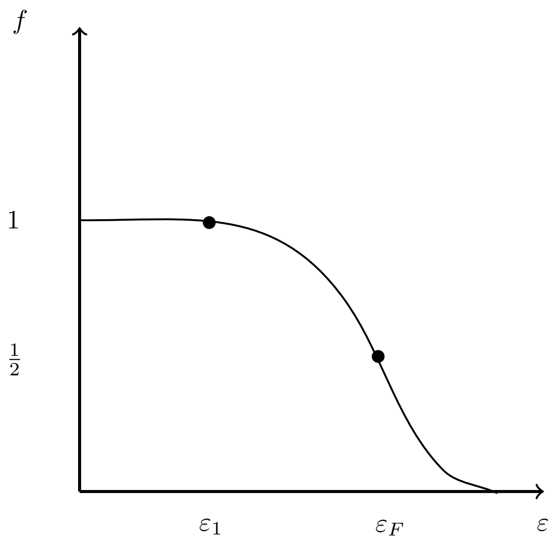

Рис. 3

Решение. 1. C_p доб. эн. $\Delta\varepsilon = k_{\text{Б}}$

2. $\Delta N \sim T \cdot D(\varepsilon_F)$

$$D(\varepsilon_F) = \frac{3}{2}\frac{N}{\varepsilon_F}.$$

$$\frac{\Delta E}{E} = \frac{\Delta N \Delta\varepsilon}{E} = \frac{\Delta N T}{\frac{3}{5}\varepsilon_F} = \frac{3/2N\frac{T^2}{\varepsilon_F}}{3/5\varepsilon_F} = \frac{5}{2}N\left(\frac{T}{\varepsilon_F}\right)^2.$$

Задача 3.44.

Решение.

$$C = C_{\text{эл}} + C_{\text{ф}}.$$

$$T \ll \Theta.$$

$$C(T) = AT + BT^3.$$

$$u = \int\limits_0^\infty f(\varepsilon)g(\varepsilon)\varepsilon d\varepsilon \approx n\frac{k^2T^2}{\varepsilon_F}.$$

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\varepsilon_F}{kT}} + 1}.$$

$$g(\varepsilon) = 2\frac{4\pi p^2 dp}{(2\pi\hbar)^3/V}.$$

$$C_{\text{эл}} = \frac{du}{dT} \approx 2nk^2\frac{T}{\varepsilon_F}.$$

$$C_{\text{ф}} \sim \left(\frac{T}{\Theta}\right)^3.$$

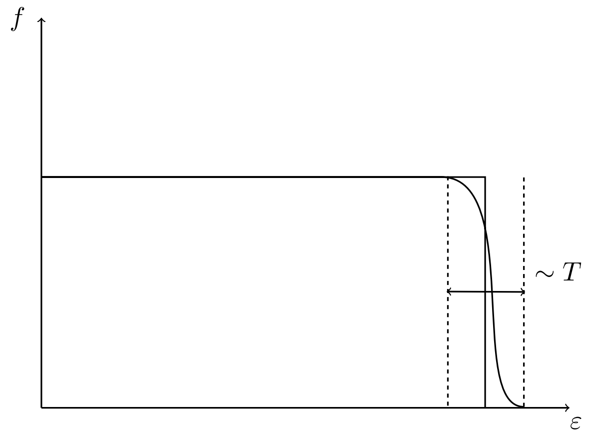

Рис. 4

Задача 3.17.

$$Z = N, \quad A = 238, \quad \varepsilon_\gamma - ?.$$

Решение.

$$\Delta\varepsilon = \frac{1}{D(\varepsilon_F)}.$$

$$\varepsilon_F = An^{2/3}.$$

$$\ln\varepsilon_F = \ln L + \frac{2}{3}\ln n.$$

$$\frac{d\varepsilon_F}{\varepsilon_F} = \frac{2}{3}\frac{dn}{n}.$$

$$\left.\frac{dn}{d\varepsilon}\right|_{\varepsilon_F} = \frac{3}{2}\frac{N}{\varepsilon}.$$

$$dN = 2 \cdot 2\frac{\varepsilon\pi p^2 dp}{(2\pi\hbar)^2/V}.$$

$$\varepsilon = \frac{p^2}{2m}.$$

$$n = \frac{2}{3\pi^2\hbar^3}p_F^3 \text{ — число нукл. на ед. объёма.}$$

Задача 3.22.

$$n = 8{,}5 \cdot 10^{22}\frac{1}{\text{см}^3}.$$

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T.$$

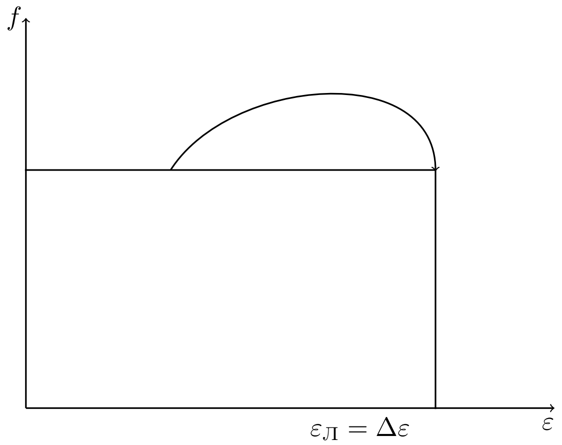

Рис. 5

Решение.

$$p = \frac{2}{3} n \left\langle \varepsilon \right\rangle .$$

$$n = \frac{N}{V}.$$

$$dn = -\frac{N}{V^2} dV.$$

$$\frac{dn}{n} = -\frac{dV}{V}.$$

$$\beta_T = \frac{1}{n} \frac{1}{\left(dp/dn\right)_T}.$$

$$p \sim 270 \text{ атм.}$$

Задача 3.27.

$$\varepsilon_F = 1 \text{ ГэВ}, \quad T = 0 \text{ К}.$$

Решение.

$$\varepsilon = pc.$$

$$dN = 2 \frac{4\pi p^2 dp}{(2n\hbar)^3/V} = 2 \frac{4\pi \varepsilon^2 d\varepsilon}{(2\pi\hbar c)^3} = g(\varepsilon) d\varepsilon = dn.$$

$$\langle \varepsilon \rangle = \frac{\int\limits_0^\infty \varepsilon g(\varepsilon) f(\varepsilon) d\varepsilon}{\int\limits_0^\infty g(\varepsilon) f(\varepsilon) d\varepsilon} = \frac{\int\limits_0^{\varepsilon_F} \varepsilon^3 d\varepsilon}{\int\limits_o^{\varepsilon_F} \varepsilon^2 d\varepsilon} = \frac{3}{4} \varepsilon_F.$$

Задача Т3.1.

$$M, \quad T \leqslant 10^9 \text{ К}, \quad R-?.$$

Решение.

$$E_G = -\frac{3}{5}\frac{GM^2}{R}.$$

$$E_k = N \cdot \frac{3}{5} E_F = \frac{M}{m}\frac{3}{5}\frac{p_F^2}{2m} = \frac{3}{10}\frac{M}{m^2}\left(\frac{9\pi M}{4mR^3}\right)^{2/3}\hbar^2.$$

$$E = E_k + E_G, \quad \frac{\partial E}{\partial R} = 0.$$

$$R = \frac{\hbar}{6M^2}\frac{M^{5/3}}{m^{8/3}}\left(\frac{9\pi}{4}\right)^{2/3}.$$

$$n = \frac{M}{m}\frac{1}{\frac{4\pi}{3}R^3}.$$