Домашняя работа по вычислительной математике

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0.1 Задача 6.6

Решение. Пусть
$$f'(x_0) = \frac{a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2)}{h}.$$
$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f^{IV}(x_0+\theta h).$$
$$f(x_0+2h) = f(x_0) + 2hf'(x_0) + \frac{4h^2}{2}f''(x_0) + \frac{8h^3}{6}f'''(x_0) + \frac{16h^4}{24}f^{IV}(x_0+2\theta h).$$

$$f'(x_0) = \frac{1}{h}\left(a_0 f(x_0) + a_1\left(f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0)\right) + \right.$$
$$\left. + a_2\left(f(x_0) + 2hf'(x_0) + \frac{4h^2}{2}f''(x_0)\right)\right) + O(h^2).$$

$O(h^2)$:
$$\begin{cases} f: a_0 + a_1 + a_2 = 0 \\ f'h: a_1 + 2a_2 = 1 \\ \dfrac{f''h^2}{2}: a_1 + 4a_2 = 0 \end{cases}.$$
$$(a_0, a_1, a_2) = \left(-\frac{3}{2}, 2, -\frac{1}{2}\right).$$

$$\varepsilon_{\text{мет}} = \left|\frac{a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2)}{h} - f'(x_0)\right| =$$
$$= \left|\frac{1}{h}\left(a_0 f(x_0) + a_1\left(f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0+\theta h)\right) + a_2\left(f(x_0) + 2hf'(x_0) + \frac{4h^2}{2}f''(x_0+2\theta h)\right)\right)\right|.$$

Пусть
$$f'(x_0) = \frac{a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2) + a_3 f(x_3)}{h}.$$
$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(x_0) + \frac{h^4}{24}f^{IV}(x_0) + O(h^5).$$

$$f(x_0+2h) = f(x_0)+2hf'(x_0)+\frac{4h^2}{2}f''(x_0)+\frac{8h^3}{6}f'''(x_0)+\frac{16h^4}{24}f^{IV}(x_0)+O(h^5).$$

$$f(x_0+3h) = f(x_0)+3hf'(x_0)+\frac{9h^2}{2}f''(x_0)+\frac{27h^3}{6}f'''(x_0)+\frac{81h^4}{24}f^{IV}(x_0)+O(h^5).$$

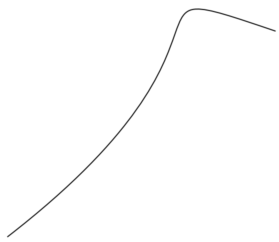
$$f'(x_0) = \frac{1}{h}\left(a_0 f(x_0) + a_1\left(f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0)\right) + \right.$$
$$\left. +a_2\left(f(x_0) + 2hf'(x_0) + \frac{4h^2}{2}\right) + a_3\left(f(x_0) + 3hf'(x_0) + \frac{9h^2}{2}\right)\right) + O(h^2).$$

$O(h^2)$:

$$\begin{cases} f : a_0 + a_1 + a_2 + a_3 = 0 \\ f'h : a_1 + 2a_2 + 3a_3 = 1 \\ \dfrac{f''h^2}{2} : a_1 + 4a_2 = 0 \end{cases}.$$

$$(a_0, a_1, a_2) = \left(-\frac{3}{2}, 2, -\frac{1}{2}\right).$$

$$\varepsilon_{\text{мет}} = \left|\frac{a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2)}{h} - f'(x_0)\right|.$$



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