

Семинар №6

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Задача 9.

Решение.

$$A_x = A_z = 0, \quad A_y = \mathcal{H}x.$$

$$\mathbf{H} = \text{rot } \mathbf{A} = \mathcal{H}\mathbf{k}.$$

$$\mathbf{A} = (0 \quad \mathcal{H}x \quad 0), \quad \varphi = 0.$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi = 0.$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}}.$$

$$H = \frac{1}{2m} \left(\mathbf{P} - \frac{e}{c} \mathbf{A} \right)^2 + e\varphi.$$

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{e}{c} (\mathbf{A}, \mathbf{v}) - e\varphi.$$

$$\mathbf{P} = \frac{\partial L}{\partial \mathbf{v}} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{e}{c} \mathbf{A} = \mathbf{p} + \frac{e}{c} \mathbf{A}.$$

Значит

$$\mathbf{p} = \mathbf{P} - \frac{e}{c} \mathbf{A}.$$

$$\hat{H} = \frac{1}{2m} \left(\hat{\mathbf{P}} - \frac{e}{c} \mathbf{A} \cdot \hat{\mathbf{1}} \right) - \underbrace{(\hat{\boldsymbol{\mu}}, \mathbf{H})}_{=0, \text{ т. к. } s=0}.$$

$$\hat{H} = \frac{1}{2m} \left(\hat{p}_x^2 + \left(\hat{p}_y - \frac{e}{c} \mathcal{H} \hat{x} \right)^2 + \hat{p}_z^2 \right).$$

$$\hat{P}_x \equiv \hat{p}_x, \quad \hat{P}_z = \hat{p}_z.$$

$$\hat{H}\psi = E\psi.$$

$$[\hat{P}_y, \hat{H}] = [\hat{P}_z, \hat{H}] = 0, \quad [\hat{P}_x, \hat{H}] \neq 0.$$

$$[\hat{x}, \hat{p}_x] i\hbar \cdot \hat{\mathbf{1}} \neq 0.$$

$$\hat{\mathbf{P}} = -i\hbar \nabla = -i\hbar \left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right).$$

$$[\hat{y}, \hat{p}_z] = 0; \quad [\hat{y}, \hat{p}_y] = i\hbar \cdot \hat{\mathbf{1}}.$$

P_y и $P_z = p_z$ — интегралы движения.

$$\psi = C \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x).$$

$$P_y, P_z \in (-\infty; \infty).$$

Т. к. $A_z = 0$, то $P_z \equiv p_z = mv_z$.

$$\frac{1}{2m} \left(\widehat{P}_x^2 + \left(\widehat{P}_y - \frac{e}{c} \mathcal{H} \widehat{x} \right)^2 + \widehat{P}_z^2 \right) \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x) = E \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x).$$

$$1. \widehat{P}_x^2 \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x) = -\hbar^2 \exp (P_y y + P_z z) f(x).$$

$$2. \widehat{P}_z^2 \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x) = P_z^2 \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x)$$

$$3. \left(\widehat{P}_y - \frac{e}{c} \mathcal{H} \widehat{x} \right)^2 \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x) = \left(P_y - \frac{e}{c} \mathcal{H} x \right)^2 \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f(x)$$

$$-\frac{\hbar^2}{2m} f''(x) + \frac{P_z^2}{2m} f(x) + \frac{1}{2m} \left(P_y - \frac{e}{c} \mathcal{H} x \right)^2 f(x) - E f(x) = 0.$$

$$f''(x) + \frac{2m}{\hbar^2} \left(E - \frac{P_z^2}{2m} - \frac{(P_y - \frac{e}{c} \mathcal{H} x)^2}{2m} \right) f(x) = 0.$$

$$f''(x) + \frac{2m}{\hbar^2} \left(E - \frac{P_z^2}{2m} - \frac{1}{2m} - \frac{1}{2m} \frac{e^2 \mathcal{H}^2}{c^2} \left(x - \frac{c P_y}{e \mathcal{H}} \right)^2 \right) f(x) = 0.$$

$$x_0 = \frac{c P_y}{e \mathcal{H}}, \quad \omega_H = \frac{e \mathcal{H}}{mc}.$$

$$f''(x) + \frac{2m}{\hbar^2} \left[\left(E - \frac{P_z^2}{2m} \right) - \frac{m \omega_H^2}{2} (x - x_0)^2 \right] f(x) = 0.$$

$$\widehat{H} = \frac{\widehat{p}^2}{2m} + \frac{m \omega^2 \widehat{x}^2}{2}.$$

$$\psi(x) + \frac{2m}{\hbar^2} \left(E - \frac{m \omega^2 x^2}{2} \right) \psi(x) = 0.$$

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$$E_n - \frac{p_z^2}{2m} = \hbar \omega \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, \dots$$

$$E_n = \frac{p_z^2}{2m} + \hbar \omega \left(n + \frac{1}{2} \right).$$

$$\psi_n(x) = \frac{1}{\sqrt{2^n \cdot n! a_H \sqrt{\pi}}} H_n \left(\frac{x - x_0}{a_H} \right) e^{-\frac{1}{2} \left(\frac{x - x_0}{a_H} \right)^2}, \quad a_H = \sqrt{\frac{\hbar}{m \omega}}.$$

$$H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n} \left(e^{-\xi^2} \right).$$

$$\psi_{n p_y p_z} = C \exp \left(\frac{i}{\hbar} (P_y y + P_z z) \right) f_n(x).$$

$$\begin{aligned}
S &= L_x L_y = L^2, \quad \psi(y+L) = \psi(y). \\
\psi(z+L) = \psi(z) &\implies p_y = \frac{2\pi\hbar}{L} n_y, \quad p_z = \frac{2\pi\hbar n_y}{L}. \\
0 < x_0 < L, \quad 0 < \frac{cp_y}{e\mathcal{H}} < L &\implies \Delta p_y = \frac{e\mathcal{H}L}{c}. \\
\Delta p_y \Delta y &= \frac{e\mathcal{H}L}{c} L = \frac{e\mathcal{H}L^2}{c}. \\
\Delta y &= L, \quad \Delta z \Delta P_z = L \Delta P_z. \\
N &= \frac{\Delta P_y \Delta y}{2\pi\hbar} = \frac{\Delta P_z \Delta z}{2\pi\hbar} = \frac{e\mathcal{H}V \Delta P_z}{c(2\pi\hbar)^2}.
\end{aligned}$$