1

In[∘]:=

```
ClearAll[f, Subscript]
a = 1.;
τ = .01;
h = .01;
L = 1.;
T = 1.;
```

$f_0[x\_] := (x-1)^2$

$f_1 = 1.;$

```
(*Первый порядок:*)
```

$$f1_{i\_,n\_} := f1_{i,n} = \begin{cases} f_1 & i == 0 \\ f_0[i\ h] & n == 0\ \&\&\ 0 < i \leq \frac{L}{h} \\ f1_{i,n-1} - a\ \tau\ \frac{f1_{i,n-1} - f1_{i-1,n-1}}{h} & \text{True} \end{cases}$$

```
(*Второй порядок:*)
```

$$f2_+[i\_,\ n\_] := 0.5\ \left(f2_{i+1,n} + f2_{i,n}\right) - 0.5\ a\ \tau\ \frac{f2_{i+1,n} - f2_{i,n}}{h}$$

$$f2_-[i\_,\ n\_] := 0.5\ \left(f2_{i,n} + f2_{i-1,n}\right) - 0.5\ a\ \tau\ \frac{f2_{i,n} - f2_{i-1,n}}{h}$$

$$f2_{i\_,n\_} := f2_{i,n} = \begin{cases} f_1 & i == 0 \\ f_0[i\ h] & n == 0\ \&\&\ 0 < i \leq \frac{L}{h} \\ 0 & n == 0\ \&\&\ i > \frac{L}{h} \\ f2_{i,n-1} - a\ \tau\ \frac{f2_+[i,n-1] - f2_-[i,n-1]}{h} & \text{True} \end{cases}$$

```
(*Точное решение:*)
sol = FullSimplify[DSolve[
    {∂t f[t, x] + a ∂x f[t, x] == 0, f[0, x] == f0[x], f[t, 0] == f1}, f[t, x], {t, x}]];
fexact[η_, ξ_] := f[t, x] /. sol[[1]] /. {t → η, x → ξ};
(*Построение*)
frames = Table[Show[
    ListLinePlot[{Table[{i h, f1i,n}, {i, 0, L/h}], Table[{i h, f1i,n}, {i, 0, L/h}]},
     PlotRange → {{0, L}, {0, 1}}, PlotLabel → StringTemplate["t=``"][n τ]],
    Plot[fexact[n τ, x], {x, 0, L}, PlotStyle → ColorData[97, "ColorList"][[3]]]], {n,
    0, T/τ}];
rasterizedFrames = Map[Image, frames];
Export["~/Desktop/gif.gif", frames];
SystemOpen["~/Desktop/gif.gif"]
"exact sol:"
fexact[t, x]
```

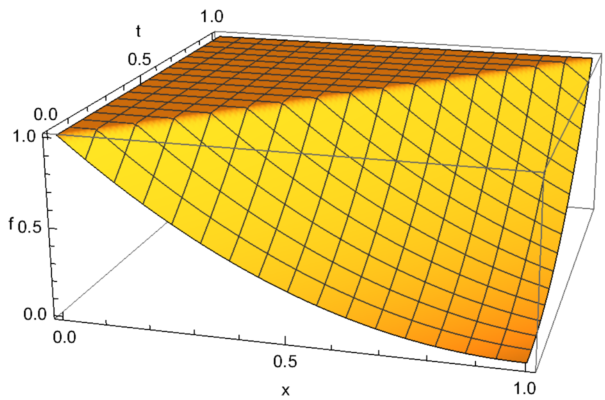
```
"3D num sol plot 1:"
ListPlot3D[Flatten[Table[{i h, n τ, f1_{i,n}}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
"3D num sol plot 2:"
ListPlot3D[Flatten[Table[{i h, n τ, f2_{i,n}}, {i, 0, L/h}, {n, 0, T/τ}], 1],
 PlotRange → All, AxesLabel → {"x", "t", "f"}]
(*ListPlot3D[Flatten[Table[{i h,n τ,RealAbs[f_{i,n}-f_exact[n τ,i h]]},
   {i,0,L/h},{n,0,T/τ}],1],AxesLabel→{"x","t","Δf"},PlotRange→All]*)
"max error 1: "
Max[Table[{RealAbs[f1_{i,n} - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
"max error 2: "
Max[Table[{RealAbs[f2_{i,n} - f_exact[n τ, i h]]}, {i, 0, L/h}, {n, 0, T/τ}]] /.
 HeavisideTheta[0.`] → 1
```

Out[•]= exact sol:

Out[•]= $1. + \left(1.\,t^2 + t\,(2. - 2.\,x) + x\,(-2. + 1.\,x)\right) \text{HeavisideTheta}[-1.\,t + x]$

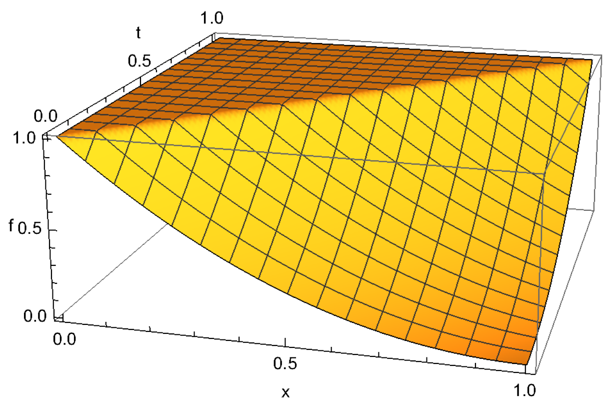
Out[•]= 3D num sol plot 1:

Out[•]=



Out[•]= 3D num sol plot 2:

Out[•]=



Out[•]= max error 1:

Out[•]= 4.44089×10^{-16}

Out[•]= max error 2:

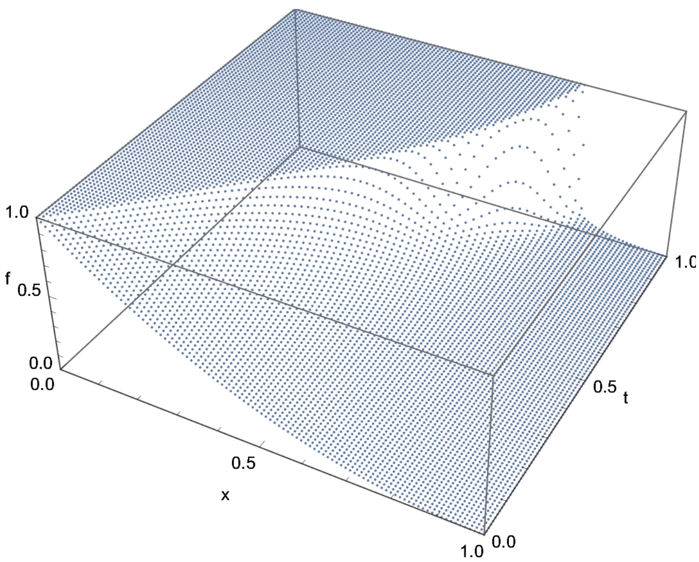
Out[•]= 8.88178×10^{-16}

2

```
In[264]:= ClearAll[f, Subscript, c]
(*$RecursionLimit=10000*)
L = 1.;
T = 1.;
f0[x_] := (x - 1)^2
f1 = 1.;
h = .01;
τ = {.01, .01};
c[x_] := τ/h x
f2Plus[i_, n_] :=
 0.5 (f[i + 1, n]⟦2⟧ + f[i, n]⟦2⟧) - 0.5 c[f[i, n]⟦2⟧]⟦2⟧ ( f[i + 1, n]⟦2⟧ - f[i, n]⟦2⟧)
f2Minus[i_, n_] := 0.5 (f[i, n]⟦2⟧ + f[i - 1, n]⟦2⟧) -
  0.5 c[f[i, n]⟦2⟧]⟦2⟧ ( f[i, n]⟦2⟧ - f[i - 1, n]⟦2⟧)
                                   f1                                  i == 0 && c[f1]⟦1⟧ ≤ :
                                   f0[i h]                             n == 0 && 0 < i ≤ L/h &&
                                   f[i, n - 1]⟦1⟧ - c[f[i, n - 1]⟦1⟧]⟦1⟧    n ≥ 0 && i ≥ 0 && c[f|
f[i_, n_] := f[i, n] = {                ( f[i, n - 1]⟦1⟧ - f[i - 1, n - 1]⟦1⟧)
                                   Indeterminate                       n ≥ 0 && i ≥ 0 &&
                                                                        (f[i, n - 1]⟦1⟧ ===
                                                                          c[f[i, n - 1]⟦1⟧
                                   0                                   True

ıмальных шагов τ*)
ₔs[f];

-
—, i = 1;
k⟧

·, If[f[i, n]⟦k⟧ === Indeterminate || c[f[i, n]⟦k⟧]⟦k⟧ > 1, τ⟦k⟧ = τ⟦k⟧/2;

es[f] = dw;
```

Out[281]= 3D num sol plot 1:

Out[282]=



Out[283]= 3D num sol plot 2:

Out[284]=